DC Motor Position Control using State Space Technique

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ABSTRACT— The design and modeling for DC motor position control using state space technique has been presented in this paper. The objective of the work is to improve the steady state performance of the DC motor position control system thereby eliminating the steady state error of the motor position. In this paper, the state space model of the DC motor is considered and state feedback controller design is proposed. The state feedback controller and state feedback controller with integral control are designed using Pole Placement Technique. Ackerman's formula is used to find the value of feedback gain matrix. The simulation results are obtained through MATLAB Software. Graphical User Interface (GUI) is developed for the controllers using MATLAB software. According to the simulation result, state feedback controller with integral control has the better performance in terms of steady state error than the state feedback controller.

Keywords— Ackerman's formula, DC motor, Graphical user interface, Integral control, State feedback controller

I. INTRODUCTION

The DC motor is widely used in many industrial applications, robot manipulators and home appliances where a precise positioning of the motor is required. In this paper, DC motor with armature control and a fixed field is considered as a third order system[1].This paper is focused on studying State-Feedback Controller with Integral control [2] in controlling the DC motor at desired position. DC motor is modeled first to obtain the transfer function between shaft position and armature voltage. The state space model [3] of the DC motor [4] is derived and verified. The State-Feedback controller is designed based on the state space model. After that, State-Feedback controller with integral control [5] is designed to overcome the problem faced in achieving desired response by State-Feedback controller. Pole-Placement design technique is used for the state feedback controller with and without integral control since it has the best performance compared to other controllers in terms of oscillation and settling time[6]. Ackermann's formula [7],[8] is used to find the value of feedback gain matrix. GUI (Graphical User Interface) [9] is developed for the controllers through MATLAB software to represent the information and available action. Each controller performance is analyzed with unit step response. Finally, the MATLAB simulation result of the state feedback Controller and state feedback Controller with integral control is compared [10,11].

II. DC MOTOR MODELING

A DC motor with armature control and a fixed field is assumed and the schematic diagram of the motor is shown in Fig.1.

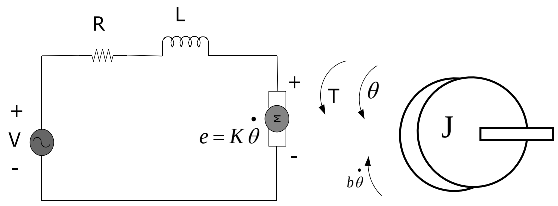

Fig.1: Schematic Diagram of the DC motor

The dynamic behaviour of the DC motor is given by the following equations :

$$V = L\frac{di}{dt} + Ri + e$$

$$T = K_t.i$$

$$T = J\ddot{\theta} + b\dot{\theta}$$

$$e = K_e\dot{\theta}$$

Taking Laplace Transform and after simplification, the transfer function is as below:

$$\theta(S)/V(S) = K/S\left[(LS+R)(JS+b)+K^2\right]$$

where, R= Armature resistance in ohm,
L=Armature inductance in henry,
i= Armature current in ampere,
V=Armature voltage in volts,
e=back emf voltage in volts,
$K=K_e$=electromotive force constant in volt/(rad/sec),
K_t=torque constant in N-m/Ampere,
T=torque developed in N-m,
θ=angular displacements of shaft in radians,
J= moment of inertia of motor and load in Kg-m^2/rad,
b=damping ratio of mechanical system in N-m/(rad/sec).

Numerical values

The following specifications and parameters [1] for the DC motor have been considered:

a)Specifications: 2hp,230volts,8.5amperes,1500rpm

b)Parameters:

R=2.45Ω,

L=0.035H,

K=1.2volt/(rad/sec),

J=0.022Kgm^2/rad,

b=0.0005Nm/(rad/sec).

Using the numerical values of the parameters, the overall transfer function of the system is obtained as:

$$\theta(S)/V(S) = 1.2/\left(0.00077S^3 + 0.0539S^2 + 1.441S\right)$$

The Step response of the system is shown in fig.2

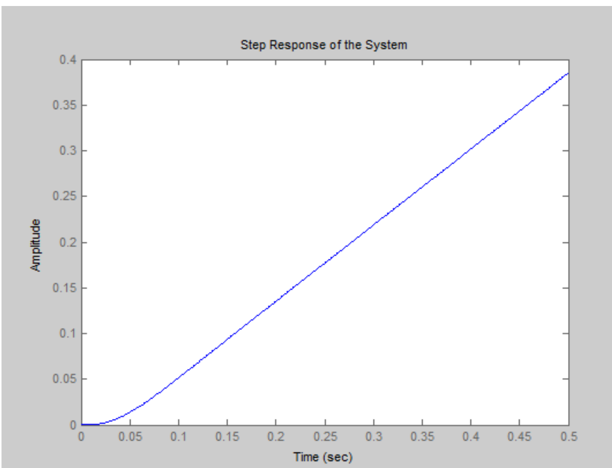

Fig.2: Step response of the system

III.STATE SPACE MODELING

State-space representation is a mathematical model of a physical system as set of input, output and state variables related by first order differential equations. A state model of the DC motor is derived by defining the three states :

The position of the motor shaft, the velocity of the motor shaft and armature current.

The state space representation of the DC motor in matrix form is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K}{J} \\ 0 & -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

IV.STATE-FEEDBACK CONTROLLER

A. *State feedback Controller*

The method of feed-backing all the state variables to the input of the system through a suitable feedback matrix in the control strategy is known as the full-state variable feedback control technique. Using this approach, the poles or eigen values of the closed loop system can be placed at the desired location.. Thus, the aim is to design a feedback controller that will move some or all of the open-loop poles of the measured system to the desired closed-loop pole location as specified. Hence, this approach is also known as the *pole-placement control* design.

In this paper, the state feedback controller is designed using pole placement technique via Ackermann's formula.

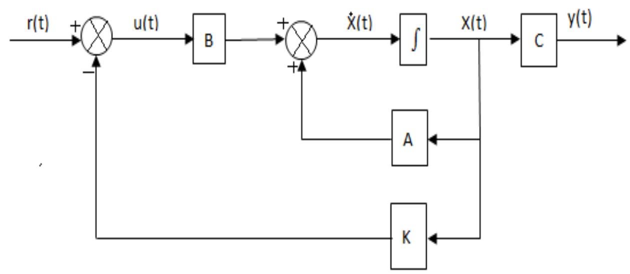

Fig.3: Block diagram of system with State feedback controller

The general state space equation for the block diagram as shown in Fig.3 is

$$\dot{x}(t) = (A-BK)x(t) + Br(t)$$

The Ackermann's formula to determine state feedback gain matrix (K) is,

$$K = [\,0 \;\; 0 \ldots 1\,] \;\; Qc^{-1} \;\; \Delta_d(A)$$

where, K = feedback gain matrix

Qc = Controllability Matrix

$=[\,B \;\; AB \;\; A^2B \ldots A^{n-2}B \;\; A^{n-1}B\,]$

$\Delta_d(A)$ = the matrix polynomial formed with the

coefficient of the desired characteristic equation $\Delta_d(S)$

The step response of full state feedback controller is shown in fig.4

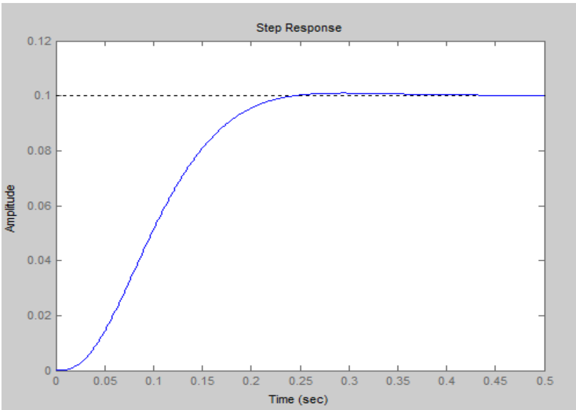

Fig.4: step response of the system with State feedback controller

B. State feedback Controller with Integral control

One of the major disadvantage of state feedback controller design by using only the pole-placement is the introduction of a large steady-state error. In order to compensate this problem, an integral control is added where it will eliminate the steady-state error in responding to a step input.

Figure.5 shows the block diagram of the system with the integral control added into it.

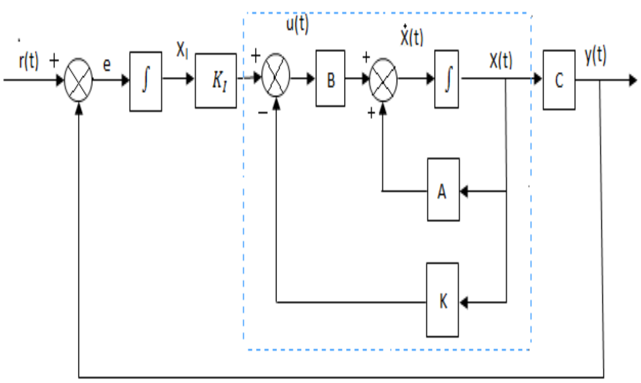

Fig.5: Block diagram of system with State feedback and Integral Controller

To introduce integral control, the state ' x ' as well as the integral of error is feedback by augmenting the state ' x ' with the extra 'integral state' x_i defined by the equation,

$$x_i(t) = \int_0^t \left[r(t) - y(t) \right] dt$$

$$\dot{x}_i(t) = \left[r(t) - y(t) \right] dt = r(t) - Cx(t)$$

$\dot{x}_i(t)$ is easily included by augmenting the original system,

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y = C\,x(t)$$

as follows:

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} -C & 0 \\ A & 0 \end{bmatrix} \begin{bmatrix} x_i(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$$

Defining $U(t) = [K_I \quad -K] \begin{bmatrix} x_i(t) \\ x(t) \end{bmatrix}$ and substituting it into the above equation , the system matrix with integral control is obtained as

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} -C & 0 \\ A - BK & BK_I \end{bmatrix} \begin{bmatrix} x_i(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$$

and $$y(t) = [0 \quad C] \begin{bmatrix} x_i(t) \\ x(t) \end{bmatrix}$$

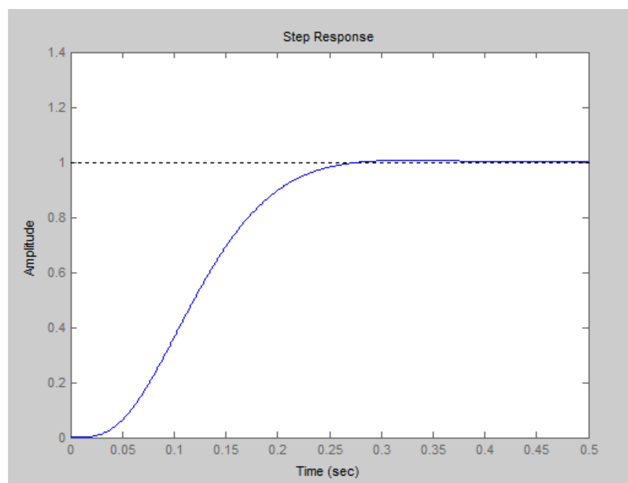

Fig.6: Step response of the system with State feedback and Integral controller

The step response of full state feedback controller with Integral Control is shown in fig.6

V. SIMULINK REPRESENTATION

The DC motor model, state feedback controller and state feedback controller with integral control built in MATLAB Simulink are shown in Fig.7, Fig.8 & Fig.9

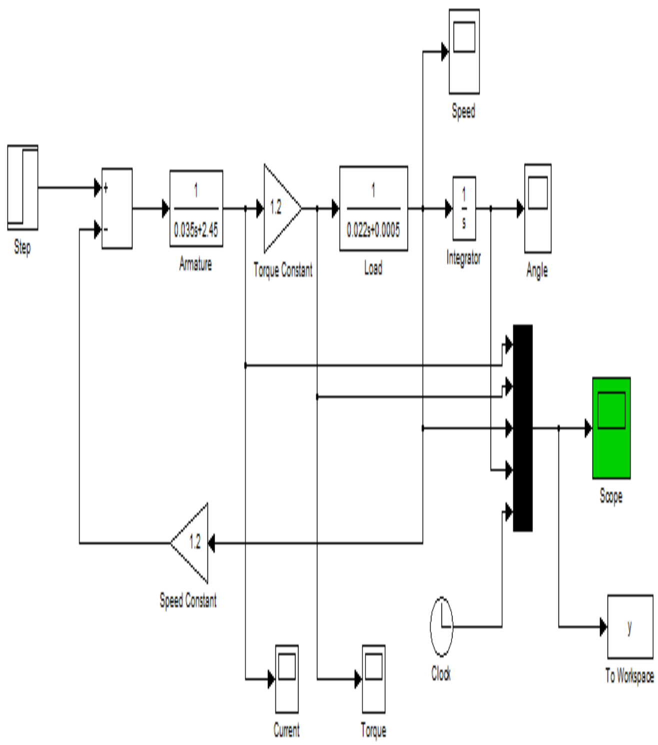

Fig.7: Simulink model of DC motor

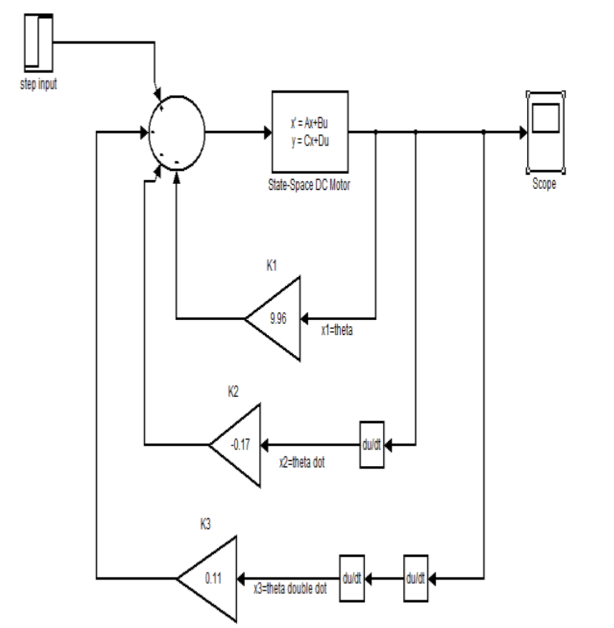

Fig.8: Simulink model of full state-feedback controller

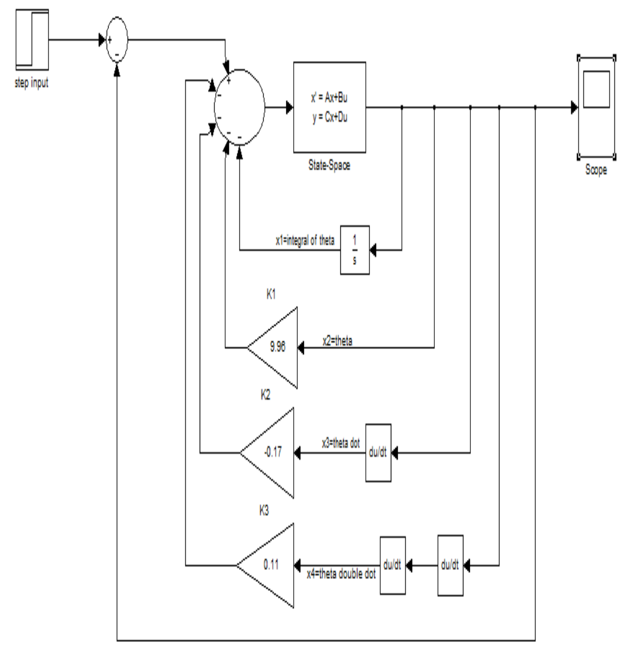

Fig.9: Simulink model of full state-feedback controller with Integral Control

VI. GRAPHICAL USER INTERFACE (GUI)

A graphical user interface is a particular case of user interface for interacting with a computer which employs graphical images and widgets in addition to text to represent the information and actions available to the user.

The Graphical user Interface (GUI) is developed in MATLAB for the adjustment of parameters and viewing, manipulating or running programs.The GUI for the DC motor ,State feedback controller and State feedback controller with integral control are shown in fig.-10, fig.-11, and fig.-12 respectively.

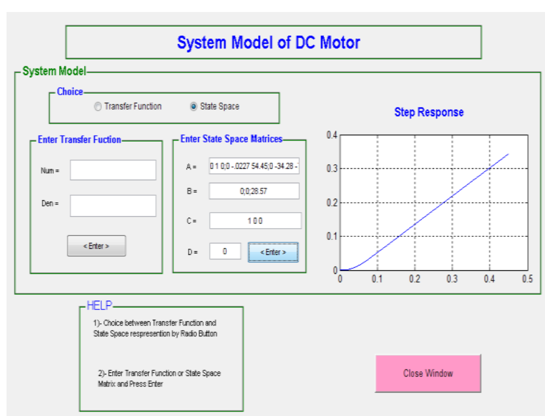

Fig.10: *Graphical User Interface of DC motor response*

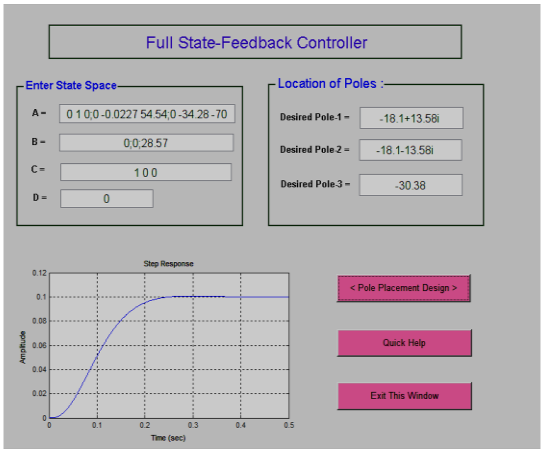

Fig.11: *Graphical User Interface of State feedback controller*

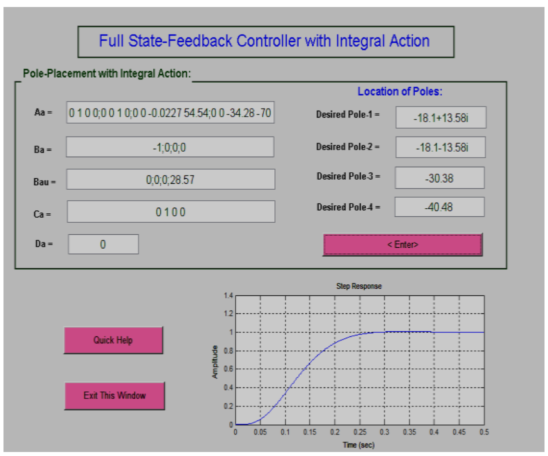

Fig.12: *Graphical User Interface of State feedback controller with Integral control*

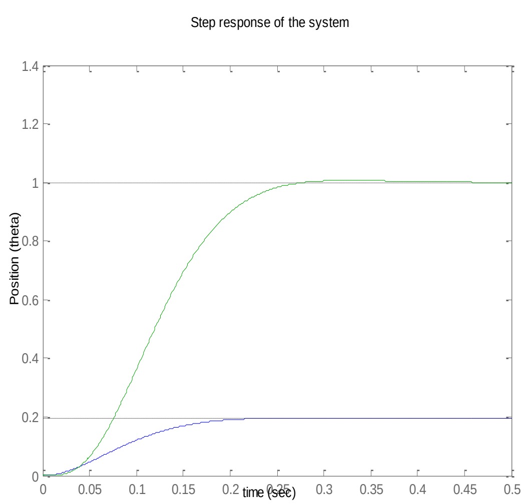

Fig.13: *comparison of output response for State feedback controller and State feedback controller with Integral control*

VII. COMPARISON OF OUTPUT RESPONSE

The comparison of MATLAB simulation results for state feedback controller and state feedback controller with integral control is shown in fig.13. It is seen that a large steady state error is introduced by the state feedback controller. The designed state feedback with integral control gives better performance in terms of percentage of steady state error.

VIII. CONCLUSIONS

In this paper, the steady state performance of the DC motor position control system has been improved. It has been shown that the state feedback controller with integral control eliminates the large steady state error introduced by the state feedback controller. The controller performance is analyzed and compared using the step response through MATLAB. The Graphical user Interface (GUI) is developed in MATLAB. Based on the simulation results, it can be concluded that state feedback controller with integral control is better than the state feedback controller without integral control.

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