Proceedings of the 2001 IEEE
International Conference on Robotics & Automation
Seoul, Korea • May 21-26, 2001

INVERSE KINEMATICS AND DYNAMICS OF THE 3-RRS PARALLEL PLATFORM

Jianfeng Li Jinsong Wang Wusheng Chou* Yuru Zhang* Tianmiao Wang* Qixian Zhang*
Department of precision instruments, Tsinghua University, Beijing, P. R. China, 100084
lijf@post.pim.tsinghua.edu.cn (Jianfeng Li)
*Robotics Institute, Beijing University of Aeronautics and Astronautics

Abstract

This paper presents the formulation of the inverse kinematics and dynamics of the 3-RRS parallel platform with three degrees of freedom. For inverse kinematics, The position analysis is firstly performed. Then the differential motion constraint equations of the movable platform are established, based on which the velocity and acceleration formulae of leg actuators are derived. In inverse dynamic analysis, the parallel platform is decomposed into two parts through parting the three spherical joints, and the forces acting on the parted joints are determined according to the moment (and force) equilibrium's of the legs and the movable platform. Subsequently, the analytic expressions of the driving moments of the leg actuators are derived by means of the moment equilibrium's of the legs. Although only the 3-RRS parallel platform is analyzed in this paper, the presented method and process can also be applied to other less DOF parallel platforms.

1 INTRODUCTION

In recent years, the parallel platforms with less than six degrees of freedom have attracted the researchers and some of them were used in the structure designs of robotic manipulators. For examples, the 3-RPS parallel platform was adopted as a micromanipulator [1-2], the 3-DOF spherical parallel platform was used in the structure design of the "agile eye"[3]. In paper [4], the direct drive DELTA manipulator with three translational degrees of freedom was presented to meet the demand of high efficiency manipulation. Articles [5-6] presented a 4-DOF and a 5-DOF parallel platform and pointed out that they can substitute the general 6-DOF parallel platforms in the cases where less than six degrees of freedom are sufficient for task manipulations. Apart form the direct applications of the less DOF parallel platforms, they can also be serially connected to form hybrid manipulators which can overcome the limited workspaces of parallel platforms and the low stiffness of serial kinematic chains. In paper [7], a hybrid type manipulation system with 10 degrees of freedom was proposed, in which a 7-DOF serial kinematic chain and a 3-DOF parallel platform are serially connected. Paper [8] presented a hybrid robotic manipulator composed of a 2-DOF parallel platform and a 3-DOF parallel platform to obtain higher stiffness and larger workspace for propeller grinding. Hybrid manipulators with similar architecture characteristic were also proposed in other articles [9-12].

To better understand the features of less DOF parallel platforms for their structure designs and practical applications, it is necessary to study their kinematics and dynamics. In papers [1-2], the kinematics and dynamics of the 3-RPS parallel platform were researched by numerical method. Papers [5-6] presented the velocity and singularity analyses of a 4-DOF and a 5-DOF parallel platform. Article [13] studied the instantaneous motion characteristics of the 3-RPS, 3-RRS and 3-UPU parallel platforms using the screw theory. References [14-15] studied the position problems of the 3-RPS parallel platform and the 3-RSR parallel platform respectively, corresponding approaches for the forward and inverse position analysis were provided. In paper [16], the static balancing of the 3-RRS parallel platform using counter-weights and springs was studied.

Although the less DOF parallel platforms have been investigated to some extent, very little has been reported in the literature on the closed-form inverse kinematics and dynamics for them as independent units.

The 3-RRS parallel platform can be used in the form of a unit as a less DOF manipulator or adopted as a sub-structure in hybrid manipulator designs. The purpose of this paper is to formulate its inverse kinematics and dynamics.

2 STRUCTURE AND COORDINATE YSTEMS

The 3-RRS parallel platform investigated in this paper is shown in Fig.1. It is composed of a base platform, a movable platform and three connecting legs. Each leg contains two links, and a succession of revolute, revolute and spherical joints. The two axes of revolute joints on each leg parallel each other and are perpendicular to the leg links.

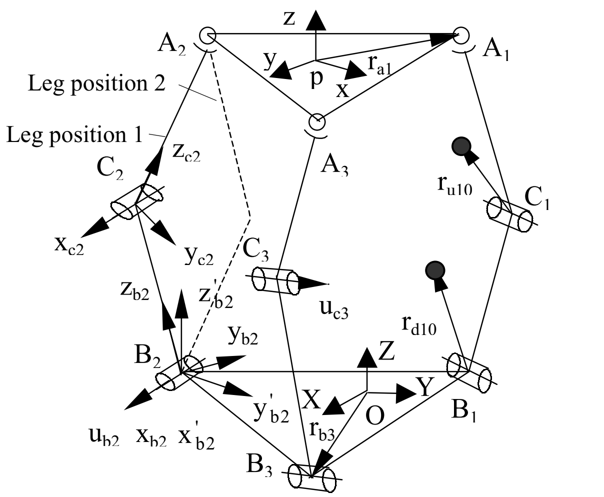


Fig.1 The 3-RRS parallel platform

Frame $\mathbf{P{-}xyz}$ is fixed on the movable platform, P is the mass center of the movable platform and $\mathbf{r}_{ai}$ is the position vector of the center point A_i $(i=1,2,3)$ of the spherical joint expressed in frame $\mathbf{P{-}xyz}$. $\mathbf{r}_{bi}$ is the position vector of the center point B_i $(i=1,2,3)$ of the revolute joint on the base platform expressed in frame $\mathbf{O{-}XYZ}$. Coordinate axis $\mathbf{x}'_{bi}$ of the local firm frame $\mathbf{B}_i{-}\mathbf{x}'_{bi}\mathbf{y}'_{bi}\mathbf{z}'_{bi}$ $(i=1,2,3)$ is identical with the revolute axis $\mathbf{u}_{bi}$, coordinate axis $\mathbf{y}'_{bi}$ is vertical to $\mathbf{x}'_{bi}$, and $\mathbf{z}'_{bi}$ axis is perpendicular to both $\mathbf{x}'_{bi}$ and $\mathbf{y}'_{bi}$. Frame $\mathbf{B}_i{-}\mathbf{x}_{bi}\mathbf{y}_{bi}\mathbf{z}_{bi}$ $(i=1,2,3)$ moves along with the down link, its coordinate axes $\mathbf{x}_{bi}$ and $\mathbf{z}_{bi}$ are identical with the revolute axis $\mathbf{u}_{bi}$ and the link vector $\mathbf{L}_{bci}$ respectively, $\mathbf{y}_{bi}$ axis is perpendicular to both $\mathbf{x}_{bi}$ and $\mathbf{z}_{bi}$. Local frame $\mathbf{C}_i{-}\mathbf{x}_{ci}\mathbf{y}_{ci}\mathbf{z}_{ci}$ $(i=1,2,3)$ moves along with the upper link, its $\mathbf{x}_{ci}$ axis is identical with the revolute axis $\mathbf{u}_{ci}$, $\mathbf{z}_{ci}$ axis is along the link vector $\mathbf{L}_{cai}$, coordinate axis $\mathbf{y}_{ci}$ is perpendicular to both $\mathbf{x}_{ci}$ and $\mathbf{z}_{ci}$. Frame $\mathbf{A}_i{-}\mathbf{x}_{ai}\mathbf{y}_{ai}\mathbf{z}_{ai}$ (not shown in Fig.1) parallels to frame $\mathbf{C}_i{-}\mathbf{x}_{ci}\mathbf{y}_{ci}\mathbf{z}_{ci}$ and its origin is point A_i.

3 INVERSE KINEMATIC ANALYSIS

3.1 Position Analysis

The position and orientation parameters of the movable platform of the 3-RRS parallel platform are six, but only three of them are independent. As in the 3-RPS parallel platform, each center point of the spherical joint of the 3-RRS parallel platform always moves on a plane. For this case, the analytic expression for the relationship of the six motion parameters of the movable platform was presented in paper [14]. For inverse position analysis, if the position and orientation of the movable platform are given according to paper [14], the angles q_{bi} between axes $\mathbf{y}_{bi}$ and $\mathbf{y}'_{bi}$, q_{ci} between axes $\mathbf{y}_{ci}$ and $\mathbf{y}_{bi}$, can be calculated as

For leg position 1

$$q_{bi} = \alpha_i + \beta_i - \pi/2 \qquad q_{ci} = \gamma_i - \pi \quad (i=1,2,3) \tag{1}$$

For leg position 2

$$q_{bi} = \beta_i - \alpha_i - \pi/2 \quad q_{ci} = \pi - \gamma_i \quad (i=1,2,3) \tag{2}$$

In equations (1) and (2), α_i is the angle between vectors $\mathbf{L}_{bai}$ and $\mathbf{L}_{bci}$, β_i is the angle between vector $\mathbf{L}_{bai}$ and coordinate axis $\mathbf{y}'_{bi}$, γ_i is the angle between vectors $\mathbf{L}_{cbi}$ and $\mathbf{L}_{cai}$, and

$$\alpha_i = \arccos\left(\frac{L_{bci}^2 + |\mathbf{L}_{bai}|^2 - L_{cai}^2}{2L_{bci}|\mathbf{L}_{bai}|}\right) \quad \beta_i = \arccos\left(\frac{\mathbf{L}_{bai} \bullet \mathbf{y}'_{bi}}{|\mathbf{L}_{bai}|}\right)$$

$$\gamma_i = \arccos\left(\frac{L_{cai}^2 + L_{bci}^2 - |\mathbf{L}_{bai}|^2}{2L_{cai}L_{bci}}\right)$$

$$\mathbf{L}_{bai} = \mathbf{r}_p + \mathbf{R}_{op}\mathbf{r}_{ai} - \mathbf{r}_{bi}$$

Where $\mathbf{R}_{op}$ is the orientation matrix of frame $\mathbf{P{-}xyz}$, $\mathbf{r}_p$ is the position vector of point P.

According to equations (1) and (2), It is known that the 3-RRS parallel platform has eight inverse position solutions. If the original configuration of the parallel platform is known, one of the eight inverse position solutions corresponds to a given platform pose.

From the leg kinematic chain, link vectors $\mathbf{L}_{bci}$ and $\mathbf{L}_{cai}$ can be obtain as

$$\mathbf{L}_{bci} = \mathbf{R}_{obi}\mathbf{R}_{ubi}\begin{pmatrix}0 & 0 & L_{bci}\end{pmatrix}^T \tag{3}$$

$$\mathbf{L}_{cai} = \mathbf{L}_{bai} - \mathbf{L}_{bci} \tag{4}$$

Where $\mathbf{R}_{obi}$ is the constant matrix describing the orientation of frame $\mathbf{B}_i{-}\mathbf{x}'_{bi}\mathbf{y}'_{bi}\mathbf{z}'_{bi}$ with respect to frame $\mathbf{O{-}XYZ}$, $\mathbf{R}_{ubi}$ is the orientation matrix of frame $\mathbf{B}_i{-}\mathbf{x}_{bi}\mathbf{y}_{bi}\mathbf{z}_{bi}$ with respect to frame $\mathbf{B}_i{-}\mathbf{x}'_{bi}\mathbf{y}'_{bi}\mathbf{z}'_{bi}$, and

$$\mathbf{R}_{ubi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_{bi} & -\sin q_{bi} \\ 0 & \sin q_{bi} & \cos q_{bi} \end{bmatrix}$$

Furthermore, the position vectors of the mass centers of the upper and down links can be obtained as (expressed in the frame whose origin is point B_i and parallels to frame $\mathbf{O-XYZ}$)

$$\mathbf{r}_{ui}=\mathbf{L}_{bci}+\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{R}_{uci}\mathbf{r}_{ui0} \qquad \mathbf{r}_{di}=\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{r}_{di0}$$

In above equations, $\mathbf{r}_{ui0}$ and $\mathbf{r}_{di0}$ are the position vectors of the upper and down links expressed in frames $\mathbf{C}_i-\mathbf{x}_{ci}\mathbf{y}_{ci}\mathbf{z}_{ci}$ and $\mathbf{B}_i-\mathbf{x}_{bi}\mathbf{y}_{bi}\mathbf{z}_{bi}$ respectively, and

$$\mathbf{R}_{uci}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos q_{ci} & -\sin q_{ci}\\ 0 & \sin q_{ci} & \cos q_{ci}\end{bmatrix}$$

3.2 Velocity Analysis

The velocity of point A_i corresponding to the given velocity of the movable platform can be written as

$$\mathbf{v}_{ai}=\mathbf{v}+\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai}) \tag{5}$$

Dot multiply the right side of equation (5) with the unit vector $\mathbf{x}_{bi}$ and according to the kinematic constraint of the leg chain, we have

$$(\mathbf{v}+\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai}))\bullet\mathbf{x}_{bi}=0 \quad (i=1,2,3) \tag{6}$$

Writing equation (6) in matrix form, we obtain

$$\begin{bmatrix}\mathbf{D} & \mathbf{E}\end{bmatrix}\begin{pmatrix}\mathbf{v}\\ \boldsymbol{\omega}\end{pmatrix}=\mathbf{0} \tag{7}$$

Where

$$\mathbf{D}=\begin{bmatrix}\mathbf{x}_{b1}^{T}\\ \mathbf{x}_{b2}^{T}\\ \mathbf{x}_{b3}^{T}\end{bmatrix} \qquad \mathbf{E}=\begin{bmatrix}((\mathbf{R}_{op}\mathbf{r}_{a1})\times\mathbf{x}_{b1})^{T}\\ ((\mathbf{R}_{op}\mathbf{r}_{a2})\times\mathbf{x}_{b2})^{T}\\ ((\mathbf{R}_{op}\mathbf{r}_{a3})\times\mathbf{x}_{b3})^{T}\end{bmatrix}$$

Equation (7) is the velocity constraint equation of the movable platform. Suppose the parallel platform is not in a singular configuration and the linear velocity of the movable platform is given, the angular velocity of the movable platform can be obtained as

$$\boldsymbol{\omega}=-\mathbf{E}^{-1}\mathbf{D}\mathbf{v} \tag{8}$$

From the differential motion of the leg chain, the velocity of point A_i can also be written as

$$\mathbf{v}_{ai}=\boldsymbol{\omega}_{bi}\times(\mathbf{L}_{bci}+\mathbf{L}_{cai})+\boldsymbol{\omega}_{ci}\times\mathbf{L}_{cai} \tag{9}$$

In equation (9), $\boldsymbol{\omega}_{bi}$ and $\boldsymbol{\omega}_{ci}$ are the angular velocities of the two leg revolute joints.

Subjecting equation (5) into (9), and then dot multiply both sides of equation (9) with vectors $\mathbf{L}_{bai}$ and $\mathbf{L}_{cai}$ respectively, we have

$$\boldsymbol{\omega}_{ci}=\frac{(\mathbf{v}+\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai}))\bullet\mathbf{L}_{bai}}{(\mathbf{L}_{cai}\times\mathbf{L}_{bai})\bullet\mathbf{x}_{ci}} \tag{10}$$

$$\boldsymbol{\omega}_{bi}=\frac{(\mathbf{v}+\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai}))\bullet\mathbf{L}_{cai}}{(\mathbf{L}_{bai}\times\mathbf{L}_{cai})\bullet\mathbf{x}_{bi}} \tag{11}$$

Writing equation (11) in matrix form, we obtain

$$\boldsymbol{\omega}_{b}=\begin{bmatrix}\mathbf{G} & \mathbf{H}\end{bmatrix}\begin{pmatrix}\mathbf{v}\\ \boldsymbol{\omega}\end{pmatrix} \tag{12}$$

Where

$$\mathbf{G}=\begin{bmatrix}\dfrac{\mathbf{L}_{ca1}^{T}}{(\mathbf{L}_{ba1}\times\mathbf{L}_{ca1})\bullet\mathbf{x}_{b1}}\\ \dfrac{\mathbf{L}_{ca2}^{T}}{(\mathbf{L}_{ba2}\times\mathbf{L}_{ca2})\bullet\mathbf{x}_{b2}}\\ \dfrac{\mathbf{L}_{ca3}^{T}}{(\mathbf{L}_{ba3}\times\mathbf{L}_{ca3})\bullet\mathbf{x}_{b3}}\end{bmatrix} \qquad \mathbf{H}=\begin{bmatrix}\dfrac{((\mathbf{R}_{op}\mathbf{r}_{a1})\times\mathbf{L}_{ca1})^{T}}{(\mathbf{L}_{ba1}\times\mathbf{L}_{ca1})\bullet\mathbf{x}_{b1}}\\ \dfrac{((\mathbf{R}_{op}\mathbf{r}_{a2})\times\mathbf{L}_{ca2})^{T}}{(\mathbf{L}_{ba2}\times\mathbf{L}_{ca2})\bullet\mathbf{x}_{b2}}\\ \dfrac{((\mathbf{R}_{op}\mathbf{r}_{a3})\times\mathbf{L}_{ca3})^{T}}{(\mathbf{L}_{ba3}\times\mathbf{L}_{ca3})\bullet\mathbf{x}_{b3}}\end{bmatrix}$$

Equations (7) and (12) compose the closed-form velocity formulae of the 3-RRS parallel platform.

3.3 Acceleration Analysis

Corresponding to the given acceleration of the movable platform, the acceleration of point A_i can be written as

$$\dot{\mathbf{v}}_{ai}=\dot{\mathbf{v}}+\dot{\boldsymbol{\omega}}\times(\mathbf{R}_{op}\mathbf{r}_{ai})+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai})) \tag{13}$$

According to the kinematic constraint of the leg chain, we have

$$(\dot{\mathbf{v}}+\dot{\boldsymbol{\omega}}\times(\mathbf{R}_{op}\mathbf{r}_{ai})+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai})))\bullet\mathbf{x}_{bi}=0 \quad (i=1,2,3) \tag{14}$$

Writing equation (14) in matrix form, we obtain

$$\begin{bmatrix}\mathbf{D} & \mathbf{E}\end{bmatrix}\begin{pmatrix}\dot{\mathbf{v}}\\ \dot{\boldsymbol{\omega}}\end{pmatrix}+\mathbf{F}=\mathbf{0} \tag{15}$$

Where

$$\mathbf{F}=\begin{bmatrix}(\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a1})))\bullet\mathbf{x}_{b1}\\ (\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a2})))\bullet\mathbf{x}_{b2}\\ (\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a3})))\bullet\mathbf{x}_{b3}\end{bmatrix}$$

Equation (15) is the acceleration constraint equation of the movable platform. Also suppose the parallel platform is not in a singular configuration and the linear acceleration of the movable platform is given, the angular acceleration of the movable platform can be obtained as

$$\dot{\boldsymbol{\omega}}=-\mathbf{E}^{-1}(\mathbf{D}\dot{\mathbf{v}}+\mathbf{F}) \tag{16}$$

Again from the differential motion of the leg chain, the acceleration of point A_i can be written as follow

$$\begin{aligned}\dot{\mathbf{v}}_{ai}=&\dot{\boldsymbol{\omega}}_{bi}\times(\mathbf{L}_{bci}+\mathbf{L}_{cai})+\dot{\boldsymbol{\omega}}_{ci}\times\mathbf{L}_{cai}+\boldsymbol{\omega}_{bi}\times(\boldsymbol{\omega}_{bi}\times\mathbf{L}_{bci})\\ &+(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times\mathbf{L}_{cai})\end{aligned} \tag{17}$$

Where $\dot{\boldsymbol{\omega}}_{bi}$ and $\dot{\boldsymbol{\omega}}_{ci}$ are the angular accelerations of the two leg revolute joints.

Subjecting equation (13) into (17), and then dot multiply both sides of equation (17) with vectors $\mathbf{L}_{cai}$, we get

$$\begin{aligned}\dot{\boldsymbol{\omega}}_{bi}=&\frac{1}{(\mathbf{L}_{bai}\times\mathbf{L}_{cai})\bullet\mathbf{x}_{bi}}((\dot{\mathbf{v}}+\dot{\boldsymbol{\omega}}\times(\mathbf{R}_{op}\mathbf{r}_{ai}))\bullet\mathbf{L}_{cai}+(\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai})))\bullet\mathbf{L}_{cai}\\ &-((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times\mathbf{L}_{cai}))\bullet\mathbf{L}_{cai}-(\boldsymbol{\omega}_{bi}\times(\boldsymbol{\omega}_{bi}\times\mathbf{L}_{bci}))\bullet\mathbf{L}_{cai})\end{aligned}$$

$$(i=1,2,3) \tag{18}$$

Again subjecting equation (13) into (17), and dot multiply both sides of equation (17) with vectors $\mathbf{L}_{bai}$, we acquire

$$\dot{\boldsymbol{\omega}}_{ci}=\frac{1}{(\mathbf{L}_{cai}\times\mathbf{L}_{bai})\bullet\mathbf{x}_{ci}}((\dot{\mathbf{v}}+\dot{\boldsymbol{\omega}}\times(\mathbf{R}_{op}\mathbf{r}_{ai}))\bullet\mathbf{L}_{bai}+(\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{ai})))\bullet\mathbf{L}_{bai}$$

$$-((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times\mathbf{L}_{cai}))\bullet\mathbf{L}_{bai}-(\boldsymbol{\omega}_{bi}\times(\boldsymbol{\omega}_{bi}\times\mathbf{L}_{bci}))\bullet\mathbf{L}_{bai})$$
(i=1,2,3)

Writing equation (18) in matrix form, we have

$$\dot{\boldsymbol{\omega}}_b=[\mathbf{G}\quad\mathbf{H}]\begin{pmatrix}\dot{\mathbf{v}}\\ \dot{\boldsymbol{\omega}}\end{pmatrix}+\mathbf{K} \tag{19}$$

Where

$$\mathbf{K}=\begin{bmatrix}\frac{1}{(\mathbf{L}_{ba1}\times\mathbf{L}_{ca1})\bullet\mathbf{x}_{b1}}((\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a1})))\bullet\mathbf{L}_{ca1}-((\boldsymbol{\omega}_{b1}+\boldsymbol{\omega}_{c1})\times \\ ((\boldsymbol{\omega}_{b1}+\boldsymbol{\omega}_{c1})\times\mathbf{L}_{ca1}))\bullet\mathbf{L}_{ca1}-\boldsymbol{\omega}_{b1}\times(\boldsymbol{\omega}_{b1}\times\mathbf{L}_{bc1})\bullet\mathbf{L}_{ca1}) \\ \frac{1}{(\mathbf{L}_{ba2}\times\mathbf{L}_{ca2})\bullet\mathbf{x}_{b2}}((\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a2})))\bullet\mathbf{L}_{ca2}-((\boldsymbol{\omega}_{b2}+\boldsymbol{\omega}_{c2})\times \\ ((\boldsymbol{\omega}_{b2}+\boldsymbol{\omega}_{c2})\times\mathbf{L}_{ca2}))\bullet\mathbf{L}_{ca2}-\boldsymbol{\omega}_{b2}\times(\boldsymbol{\omega}_{b2}\times\mathbf{L}_{bc2})\bullet\mathbf{L}_{ca2}) \\ \frac{1}{(\mathbf{L}_{ba3}\times\mathbf{L}_{ca3})\bullet\mathbf{x}_{b3}}((\boldsymbol{\omega}\times(\boldsymbol{\omega}\times(\mathbf{R}_{op}\mathbf{r}_{a3})))\bullet\mathbf{L}_{ca3}-((\boldsymbol{\omega}_{b3}+\boldsymbol{\omega}_{c3})\times \\ ((\boldsymbol{\omega}_{b3}+\boldsymbol{\omega}_{c3})\times\mathbf{L}_{ca3}))\bullet\mathbf{L}_{ca3}-\boldsymbol{\omega}_{b3}\times(\boldsymbol{\omega}_{b3}\times\mathbf{L}_{bc3})\bullet\mathbf{L}_{ca3})\end{bmatrix}$$

Equations (15) and (19) compose the closed-form acceleration formulae of the 3-RRS parallel platform. Moreover, the accelerations of the mass centers of the upper and down links can be obtained as

$$\dot{\mathbf{v}}_{ui}=\dot{\boldsymbol{\omega}}_{bi}\times\mathbf{L}_{bci}+\boldsymbol{\omega}_{bi}\times(\boldsymbol{\omega}_{bi}\times\mathbf{L}_{bci})+(\dot{\boldsymbol{\omega}}_{bi}+\dot{\boldsymbol{\omega}}_{ci})\times(\mathbf{r}_{ui}-\mathbf{L}_{bci})$$
$$+(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times((\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times(\mathbf{r}_{ui}-\mathbf{L}_{bci}))$$
$$\dot{\mathbf{v}}_{di}=\dot{\boldsymbol{\omega}}_{bi}\times\mathbf{r}_{di}+\boldsymbol{\omega}_{bi}\times(\boldsymbol{\omega}_{bi}\times\mathbf{r}_{di})$$

4 INVERSE DYNAMIC ANALYSIS

4.1 Dynamic Analysis of the Upper Link of the Leg

If the 3-RRS parallel platform is parted at the three spherical joints, the constraint force $\mathbf{f}_{ai}=f_{xai}\mathbf{x}_{ai}+f_{yai}\mathbf{y}_{ai}+f_{zai}\mathbf{x}_{ai}$ occurring at the spherical joints becomes the external reaction force. Then, according to the moment equilibrium of the upper link about the revolute axis $\mathbf{u}_{ci}$, we have

$$(m_{ui}(\mathbf{r}_{ui}-\mathbf{L}_{bci})\times(\mathbf{g}-\dot{\mathbf{v}}_{ui})-\mathbf{I}_{ui}(\dot{\boldsymbol{\omega}}_{bi}+\dot{\boldsymbol{\omega}}_{ci})-(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times(\mathbf{I}_{ui}(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci}))$$
$$+f_{ayi}\mathbf{L}_{cai}\times\mathbf{y}_{ai})\bullet\mathbf{x}_{ci}=0 \tag{20}$$

In above equation

$$\mathbf{I}_{ui}=\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{R}_{uci}\mathbf{I}_{ui0}(\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{R}_{uci})^T$$
$$\mathbf{y}_{ai}=\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{R}_{uci}\begin{pmatrix}0 & 1 & 0\end{pmatrix}^T$$

Where $\mathbf{I}_{ui0}$ is the inertia moment of the upper link expressed in frame $\mathbf{C}_i-\mathbf{x}_{ci}\mathbf{y}_{ci}\mathbf{z}_{ci}$.

From equation (20), the constraint force component f_{ayi} can be obtained as

$$f_{ayi}=-\frac{1}{(\mathbf{L}_{cai}\times\mathbf{y}_{ai})\bullet\mathbf{x}_{ci}}((m_{ui}(\mathbf{r}_{ui}-\mathbf{L}_{bci})\times(\mathbf{g}-\dot{\mathbf{v}}_{ui})-\mathbf{I}_{ui}(\dot{\boldsymbol{\omega}}_{bi}+\dot{\boldsymbol{\omega}}_{ci})$$
$$-(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times(\mathbf{I}_{ui}(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})))\bullet\mathbf{x}_{ci})\qquad (i=1,2,3) \tag{21}$$

4.2 Dynamic Analysis of the Movable Platform and Driving Moment of the Leg Actuators

Let the external force and moment acting on the movable platform be $\mathbf{F}_e$ and $\mathbf{M}_e$, the mass and inertia moment of the movable platform be m_p and $\mathbf{I}_p$ (expressed in frame $\mathbf{P}-\mathbf{xyz}$), Considering the force and moment equilibrium of the movable platform, we have

$$\begin{pmatrix}\mathbf{F}_e+m_p\mathbf{g}\\ \mathbf{M}_e\end{pmatrix}-\begin{pmatrix}m_p\dot{\mathbf{v}}\\ (\mathbf{R}_{op}\mathbf{I}_p\mathbf{R}_{op}^T)\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times((\mathbf{R}_{op}\mathbf{I}_p\mathbf{R}_{op}^T)\boldsymbol{\omega})\end{pmatrix}$$
$$-\mathbf{G}_1\mathbf{f}_{ay}-\mathbf{G}_2\mathbf{f}_{axz}=\mathbf{0} \tag{22}$$

Where

$$\mathbf{f}_{ay}=\begin{pmatrix}f_{ay1} & f_{ay2} & f_{ay3}\end{pmatrix}^T$$
$$\mathbf{f}_{axz}=\begin{pmatrix}f_{ax1} & f_{az1} & f_{ax2} & f_{az2} & f_{ax3} & f_{az3}\end{pmatrix}^T$$
$$\mathbf{G}_1=\begin{bmatrix}\mathbf{y}_{a1} & \mathbf{y}_{a2} & \mathbf{y}_{a3}\\ (\mathbf{R}_{op}\mathbf{r}_{a1})\times\mathbf{y}_{a1} & (\mathbf{R}_{op}\mathbf{r}_{a2})\times\mathbf{y}_{a2} & (\mathbf{R}_{op}\mathbf{r}_{a3})\times\mathbf{y}_{a3}\end{bmatrix}$$
$$\mathbf{G}_2=\begin{bmatrix}\mathbf{x}_{a1} & \mathbf{z}_{a1} & \mathbf{x}_{a2} & \mathbf{z}_{a2} & \mathbf{x}_{a3} & \mathbf{z}_{a3}\\ (\mathbf{R}_{op}\mathbf{r}_{a1})\times\mathbf{x}_{a1} & (\mathbf{R}_{op}\mathbf{r}_{a1})\times\mathbf{z}_{a1} & (\mathbf{R}_{op}\mathbf{r}_{a2})\times\mathbf{x}_{a2} & (\mathbf{R}_{op}\mathbf{r}_{a2})\times\mathbf{z}_{a2} & (\mathbf{R}_{op}\mathbf{r}_{a3})\times\mathbf{x}_{a3} & (\mathbf{R}_{op}\mathbf{r}_{a3})\times\mathbf{z}_{a3}\end{bmatrix}$$
$$\mathbf{z}_{ai}=\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{R}_{uci}\begin{pmatrix}0 & 0 & 1\end{pmatrix}^T$$

In terms of equation (22), we obtain

$$\mathbf{f}_{axz}=\mathbf{G}_2^{-1}\begin{pmatrix}\mathbf{F}_e+m_p\mathbf{g}\\ \mathbf{M}_e\end{pmatrix}-\mathbf{G}_2^{-1}\begin{pmatrix}m_p\dot{\mathbf{v}}\\ (\mathbf{R}_{op}\mathbf{I}_p\mathbf{R}_{op}^T)\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times((\mathbf{R}_{op}\mathbf{I}_p\mathbf{R}_{op}^T)\boldsymbol{\omega})\end{pmatrix}$$
$$-\mathbf{G}_2^{-1}\mathbf{G}_1\mathbf{f}_{ay} \tag{23}$$

According to moment equilibrium of the leg about the revolute axis $\mathbf{u}_{bi}$, we obtain the driving moments of the leg actuators on the base platform

$$\tau_{di}=\tau_{dui}+\tau_{ddi}-(\mathbf{L}_{bai}\times\mathbf{f}_{ai})\bullet\mathbf{x}_{bi}\qquad (i=1,2,3) \tag{24}$$

In above equation

$$\tau_{dui}=((\mathbf{I}_{ui}-m_{ui}[\mathbf{L}_{cbi}\times]^2)(\dot{\boldsymbol{\omega}}_{bi}+\dot{\boldsymbol{\omega}}_{ci})+(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci})\times((\mathbf{I}_{ui}-m_{ui}[\mathbf{L}_{cbi}\times]^2)(\boldsymbol{\omega}_{bi}+\boldsymbol{\omega}_{ci}))$$
$$-m_{ui}\mathbf{r}_{ui}\times(\mathbf{g}-\dot{\mathbf{v}}_{ui}))\bullet\mathbf{x}_{bi}$$
$$\tau_{ddi}=(\mathbf{I}_{di}\dot{\boldsymbol{\omega}}_{bi}+\boldsymbol{\omega}_{bi}\times(\mathbf{I}_{di}\boldsymbol{\omega}_{bi})-m_{di}\mathbf{r}_{di}\times(\mathbf{g}-\dot{\mathbf{v}}_{di}))\bullet\mathbf{x}_{bi}$$

and

$$\mathbf{I}_{di}=\mathbf{R}_{obi}\mathbf{R}_{ubi}\mathbf{I}_{di0}(\mathbf{R}_{obi}\mathbf{R}_{ubi})^T$$
$$[\mathbf{L}_{cbi}\times]=\begin{bmatrix}0 & -L_{cbiz} & L_{cbiy}\\ L_{cbiz} & 0 & -L_{cbix}\\ -L_{cbiy} & L_{cbix} & 0\end{bmatrix}$$

Where $\mathbf{I}_{di0}$ is the inertia moment of the upper link expressed in frame $\mathbf{B}_i-\mathbf{x}_{bi}\mathbf{y}_{bi}\mathbf{z}_{bi}$ and L_{cbix}, L_{cbiy}, L_{cbiz} are the three coordinate components of vector $\mathbf{L}_{cbi}$.

5 NUMERICAL EXAMPLE

In this section, a numerical example of the inverse dynamic analysis for the 3-RRS parallel platform is presented. In this example, the movable platform and the base platform are both equilateral triangulars, and the coordinate axes $\mathbf{x}$, $\mathbf{X}$ of frames $\mathbf{P-xyz}$ and $\mathbf{O-XYZ}$ pass through points A_1 and B_1 respectively. The link lengths are $L_{bci}=L_{cai}=1\text{m}\ (i=1,2,3)$, the position vectors of points A_i, B_i, the unit vectors $\mathbf{u}_{bi}\ (i=1,2,3)$ and the orientation matrixes of the local firm frame $\mathbf{B}_i-\mathbf{x}'_{bi}\mathbf{y}'_{bi}\mathbf{z}'_{bi}$ $(i=1,2,3)$ with respect to frame $\mathbf{O-XYZ}$ are as follows

$$\mathbf{r}_{a1}=(0.45\ \ 0\ \ 0)^T(\text{m}) \qquad \mathbf{r}_{a2}=(-0.225\ \ 0.3897\ \ 0)^T(\text{m})$$

$$\mathbf{r}_{a3}=(-0.225\ \ -0.3897\ \ 0)^T(\text{m})$$

$$\mathbf{r}_{b1}=(0.7\ \ 0\ \ 0)^T(\text{m}) \qquad \mathbf{r}_{b2}=(-0.35\ \ 0.6062\ \ 0)^T(\text{m})$$

$$\mathbf{r}_{b3}=(-0.35\ \ -0.6062\ \ 0)^T(\text{m})$$

$$\mathbf{u}_{b1}=(0\ \ 1\ \ 0)^T \quad \mathbf{u}_{b2}=\left(-\sqrt{3}/2\ \ -1/2\ \ 0\right)^T \quad \mathbf{u}_{b3}=\left(\sqrt{3}/2\ \ -1/2\ \ 0\right)^T$$

$$\mathbf{R}_{ob1}=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\ \mathbf{R}_{ob2}=\begin{bmatrix}-\sqrt{3}/2 & 1/2 & 0\\ -1/2 & -\sqrt{3}/2 & 0\\ 0 & 0 & 1\end{bmatrix}\ \mathbf{R}_{ob3}=\begin{bmatrix}\sqrt{3}/2 & 1/2 & 0\\ -1/2 & \sqrt{3}/2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Suppose the position vectors of the mass centers of leg links, the masses of the links and the movable platform and their inertia moments are

$$\mathbf{r}_{ui0}=(0\ \ 0\ \ 0.5)^T(\text{m}) \qquad \mathbf{r}_{di0}=(0\ \ 0\ \ 0.5)^T(\text{m}) \qquad (i=1,2,3)$$

$$m_{ui}=12(\text{kg}) \qquad m_{di}=12(\text{kg})\ (i=1,2,3) \qquad m_p=68(\text{kg})$$

$$\mathbf{I}_{ui0}=\begin{bmatrix}6.2 & 0 & 0\\ 0 & 6.2 & 0\\ 0 & 0 & 0.5\end{bmatrix}(\text{kgm}^2)\ \mathbf{I}_{di0}=\begin{bmatrix}6.2 & 0 & 0\\ 0 & 6.2 & 0\\ 0 & 0 & 0.5\end{bmatrix}(\text{kgm}^2)\ (i=1,2,3)$$

$$\mathbf{I}_p=\begin{bmatrix}28 & 0 & 0\\ 0 & 36 & 0\\ 0 & 0 & 20\end{bmatrix}(\text{kgm}^2)$$

The original configuration the parallel platform is

$$\mathbf{r}_p=\begin{pmatrix}0.1\\ 0\\ 1\end{pmatrix}(\text{m}) \quad \mathbf{R}_{op}=\begin{bmatrix}\cos(-0.25) & 0 & \sin(-0.25)\\ 0 & 1 & 0\\ -\sin(-0.25) & 0 & \cos(-0.25)\end{bmatrix}$$

For this numerical example, the linear motion of the movable platform is given as

$$\dot{\mathbf{v}}=\begin{pmatrix}\dot{v}_x\\ \dot{v}_y\\ \dot{v}_z\end{pmatrix}=\begin{pmatrix}-\dfrac{0.12}{T^2}\left(\dfrac{60t}{T}-\dfrac{180t^2}{T^2}+\dfrac{120t^3}{T^3}\right)\\ \dfrac{0.1}{T^2}\left(\dfrac{60t}{T}-\dfrac{180t^2}{T^2}+\dfrac{120t^3}{T^3}\right)\\ -\dfrac{0.5}{T^2}\left(\dfrac{60t}{T}-\dfrac{180t^2}{T^2}+\dfrac{120t^3}{T^3}\right)\end{pmatrix}(\text{m/s}^2) \quad (T=6)$$

If there is no external force and moment acting on the movable platform and the 3-RRS parallel platform moves 6 seconds, the times-histories of the angular displacement, the velocity, the acceleration of the movable platform, the angular displacements, velocities, accelerations and driving moments of the leg actuators on the base platform are shown in Fig.2.

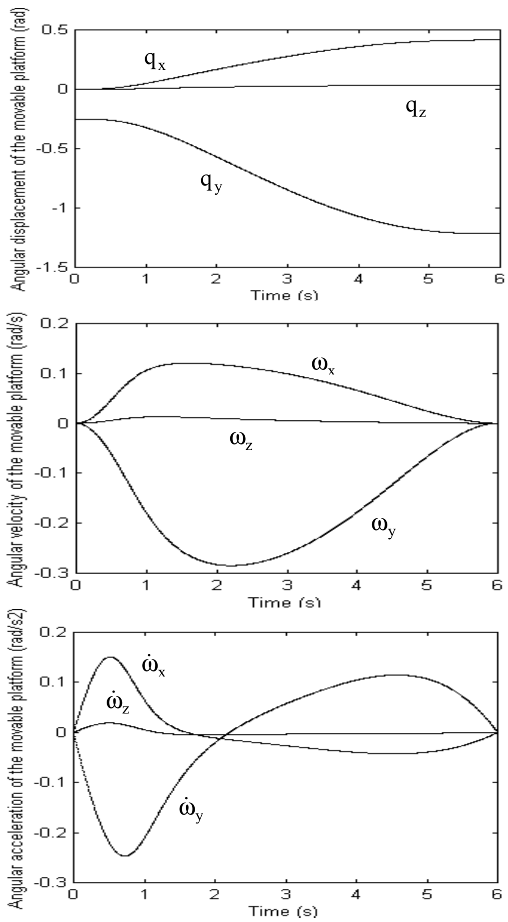
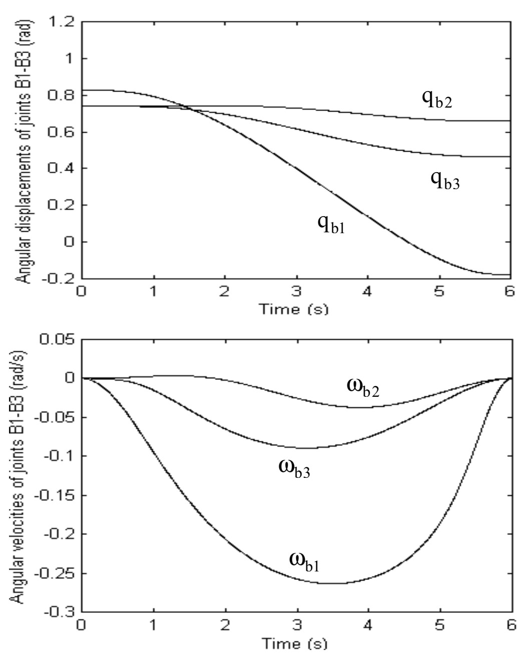


Fig.2.1 Times-histories of angular displacement, velocity and acceleration of the movable platform



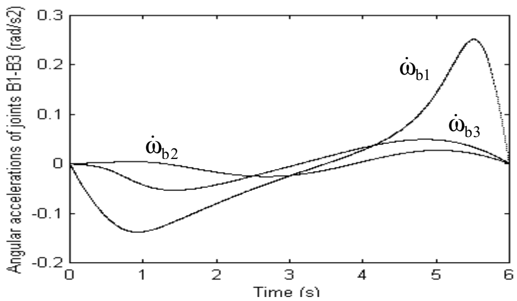


Fig.2.2 Times-histories of angular displacements, velocities and accelerations of B_1-B_3 joints

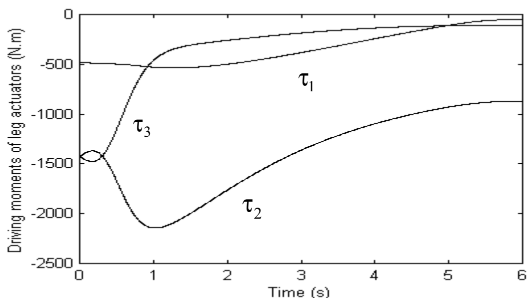


Fig.2.3 Times-histories of the driving moments of leg actuators

6 CONCLUSION

In this paper, the inverse kinematic and dynamic analyses of the 3-RRS parallel platform were presented. The major research results include: (1) The formula to determine the angular positions of leg actuators was obtained. (2) The velocity and acceleration formulae of the 3-RRS parallel platform were acquired in closed-form. This was achieved by introducing the differential motion constraint equations of the movable platform established according to the kinematic constraint of the leg chains. (3) Through directly parting the three spherical joints and using the moment (and force) equilibrium's of the legs and the movable platform, the inverse dynamic equations of the 3-RRS parallel platform were derived.

The present study provides a framework for future researches such as the singularity analysis, the structure design and the control of the 3-RRS parallel platform. Moreover, the method and process presented in this paper is also applicable to other less DOF parallel platforms.

ACKNOWLEDGEMENTS

This paper received the financial support from the National Fund Council of Natural Science of China and China Postdoctoral Science Foundation.

REFERENCES

[1] K. M. Lee, D. K. Shah, Kinematic Analysis of A Three-degree-of-Freedom In-Parallel Actuated Manipulator, *IEEE J. of Robotics and Automation*, 1988, Vol.4, No.3, 354-360

[2] K. M. Lee and D. K. Shah, Dynamic Analysis of A Three-Degree-of-Freedom In-Parallel Actuated Manipulator, *IEEE J. of Robotics and Automation*, 1988, Vol.4, No.3, 361-367

[3] C.M. Gosselin and J. F. Hamel, The Agile Eye: A High-Performance 3-DOF Camera-Orienting Device, *Proceedings of the IEEE Int. Conf. On Robotics and Automation,* 1994, Vol.3, 781-786

[4] R. Clavel, Delat, A Fast Robot with Parallel Geometry, *Proceedings of the 18th Int. Symp. On Industrial Robots*, 1988, Vol.1, 91-100

[5] J. Wang and C. M. Gosselin, Kinematic Analysis and Singularity Loci of Spatial Four-Degree-of-Freedom Parallel Manipulators Using A Vector Formulation, *ASME J. of Mechanical Design*, 1998, Vol.120, 555-558

[6] J. Wang and C. M. Gosselin, Kinematic Analysis and Singularity Representation of Spatial Five-Degree-of-Freedom Parallel Mechanisms, *Int. J. of Robotic Systems*, 1997, Vol.14, 851-869

[7] K. J. Waldron, M. Raghavan and B. Roth, Kinematics of A Hybrid Series-Parallel Manipulation Systems, *ASME J. of Dynamic Systems, Measurement, and Control,* 1989, Vol.111, No.2, 211-221

[8] M. K. Lee and K. W. Park, Kinematic and Dynamic Analysis of A Double Parallel Manipulator for Enlarging Workspace and Avoiding Singularity, *IEEE Trans. on Robotics and Automation*, 1999, Vol.15, No.6, 1024-1034

[9] M. Shahinpoor, Kinematics of A Parallel-Serial (Hybrid) Robot Manipulator, *Int. J. of Robotic Systems*, 1992, Vol.9, No.1, 32-44

[10] H. H. Cheng, J. J. Lee and R. Penkar, Kinematic Analysis of A Hybrid Serial-and-Parallel-Driven Redundant Industrial Manipulator, *Int. J.* of *Robotics and Automation*, 1995, Vol.10, No.4, 159-166

[11] L. Romdhane, Design and Analysis of A Hybrid Serial-Parallel Manipulator, *Mech. Mach. Theory*, 1999, Vol.34, No.7, 1037-1055

[12] T. K. Tanev, Kinematics of A Hybrid (Parallel-Serial) Robot Manipulator, *Mech. Mach. Theory*, 2000, Vol.35, No.6, 1183-1196

[13] Z. Huang, W. S. Tao and Y. F. Fang, Study on the Kinematic Characteristics of 3 DOF In-Parallel Actuated Platform Mechanisms, *Mech. Mach. Theory*, 1996, Vol.31, No.8, 999-1007

[14] K. Buruncuk and Y. Tokad, On the Kinematics of A 3-DOF Stewart Platform, *Int. J. of Robotic Systems*, 1999, Vol.16, No.2, 105-118

[15] R. B. Hertz and P. C. Hughes, Kinematic Analysis of A General Double-Tripod Parallel Manipulator, *Mech. Mach. Theory*, 1998, Vol.33, No.6, 683-696

[16] J. Wang and C. M. Gosselin, Static Balancing of Spatial Three-Degree-of-Freedom Parallel Mechanism, *Mech. Mach. Theory*, 1999, Vol.34, No.3, 437-452