Design and optimization of fuzzy-PID controller for the nuclear reactor power control
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A B S T R A C T
This paper introduces a fuzzy proportional-integral-derivative (fuzzy-PID) control strategy, and applies it
to the nuclear reactor power control system. At the fuzzy-PID control strategy, the fuzzy logic controller
(FLC) is exploited to extend the finite sets of PID gains to the possible combinations of PID gains in stable
region and the genetic algorithm to improve the ‘extending’ precision through quadratic optimization
for the membership function (MF) of the FLC. Thus the FLC tunes the gains of PID controller to adapt
the model changing with the power. The fuzzy-PID has been designed and simulated to control the reactor
power. The simulation results show the favorable performance of the fuzzy-PID controller.

1. Introduction
Nuclear reactors are in nature nonlinear and their parameters vary with time as a function of power level. These characteristics must be considered if large power variations occur in power plant working regimes, such as in load following conditions. Reactor power control has been used in base-load operating conditions traditionally. But with the increasing share of power plants in electricity generation, it seems that the load-follow operation of nuclear reactors will be inevitable in the future. It is hard to get the satisfying performance with the classic control strategy to control nuclear reactor power. Multi-model control is a kind of relatively effective nonlinear time-dependent control strategy (Ciprian et al., 2008; Kolavennu et al., 2001), but it often brings unacceptable error and switch trouble (Luo et al., 2008; Zou et al., 2007). Advanced intelligent control gives a bright future to nonlinear time-dependent control system. The fuzzy logic controller (FLC) is the good representative of them (Ismael and Yu, 2006), but if it is solely used in nuclear reactor power levels control system, it is not easy to handle the ‘precision’ problem compared with the classic controller such as PID. So incorporating fuzzy logic controller and PID to be fuzzy proportional-integral-derivative (fuzzy-PID) has been researched and its excellent properties proved (Rubaai et al., 2007, 2008). Applying the fuzzy-PID to nuclear reactor power control system is the researching objective of this paper. The FLC performance is decided by the shape and type of the membership function (MF) when the rule base has been specified. So if the shape and type of MF are properly selected by some optimizing algorithm, its performance can markedly be improved. The genetic algorithm is a popular and effective optimizing method for the MF of the FLC (Wagner and Hagras, 2007; Narvydas et al., 2007). Basing on genetic algorithm, this paper introduces a quadratic optimizing algorithm. Finally, the optimized fuzzy-PID controller is employed to control the nuclear reactor power. The simulation results show satisfactory performance.

2. Theory
2.1. Fuzzy-PID control
At the fuzzy-PID control strategy, a few sets of PID gains is firstly designed, and then FLC is exploited to extend the finite sets of PID gains to the possible combinations of PID gains in stable region and the genetic algorithm to improve the ‘extending’ precision through quadratic optimization for the membership function (MF) of the FLC. Thus FLC tunes PID gains to adapt the model changing with the power levels. Its schematic diagram is shown as Fig. 1.

In Fig. 1, FLC is the mapping function from the power levels to the PID gains. This kind of mapping relation is constructed as follows: first, several nuclear reactor models at different power levels should be identified (generally, the more models, the fuzzy-PID to be designed gets the better performance). For being accessible and representative, the selected power levels are homogeneous distribution at whole power levels. Secondly, the gains of PID controllers for these models are set according to the actual demands. Finally, the correspondence relation between the power levels and the PID gains is expressed by the FLC.

But this mapping relation is often rough because of the arbitrariness in selecting the MF for the FLC. So some works should be done to assure that this mapping relation is precise enough.

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The objective function is defined as follows:

\[ J = \frac{1}{T} \int_0^T e(t) \, dt \]

where \( e(t) \) is the error function; \( P_f \) is the output of the fuzzy-PID control system at the power levels in the second group; \( P_p \) is the output of the multi-PID control system at the power levels in the second group.

Here, the outputs of multi-PID control system are regarded as desirable outputs.

2.2. Quadratic optimization by genetic algorithm

Genetic algorithm is a global search and optimization algorithm inspired on the biological laws of genetics. It shows the excellent performance in optimization (Marseguerra et al., 2005).

There are many researches in optimizing the shape of MF (Jose et al., 2008; Mohammad and Hadavi, 2008). In fact, when different types of MF are optimized by genetic algorithm for the same FLC, the optimum fitness values of objective function are often different. So this paper presents a kind of optimizing method: first, the typical three kinds of MF, namely triangle-type MF, trapezoid-type MF and gauss-type MF are optimized, respectively, by the genetic algorithm. Then compare three optimum fitness values and select out the maximum which stands for the minimal output error. The MF of this maximum is the optimal MF. Because both the shape and type of the MF are optimized at the same time, this method is entitled as quadratic optimization.

3. Designing

3.1. Fuzzy-PID controller

These nuclear reactor models at power levels 20%, 40%, 60%, 80% and 100% are identified as \( G_1, G_2, G_3, G_4 \) and \( G_5 \). Following the actual control demands, the controllers of PID1, PID2, PID3, PID4 and PID5 are designed independently for each model. The gains of PID1, PID3 and PID5 at the power levels 20%, 60% and 100% are employed to design the fuzzy-PID as follows.

The power levels are taken as the FLC input variable \( P \) and the PID parameters \( K_0, K_i, K_d \) as output variables. The fuzzy sets of \( P \) are defined as PS, PM and PB. Fig. 2 shows the triangle-type MFs of output variables \( K_0, K_i, K_d \). The nine fuzzy sets can be symbolized by \( A_1, A_2, A_3, D_1, D_2, D_3 \). Their corresponding membership grades can be calculated by \( \mu_{A_1}(P) \), \( \mu_{A_2}(K_0) \), \( \mu_{A_3}(K_i) \) and \( \mu_{D_i}(K_d) \). Eqs. (5)–(13) give expressions of \( \mu_{A_1}(P) \), \( \mu_{A_2}(K_0) \), \( \mu_{A_3}(K_i) \) and \( \mu_{D_i}(K_d) \):

\[
\mu_{A_1}(P) = \begin{cases} 
1 & (0 \leq P < 20) \\
-2.5P + 1.5 & (20 \leq P \leq 60) 
\end{cases}
\]

\[
\mu_{A_2}(P) = \begin{cases} 
2.5P - 0.5 & (20 \leq P \leq 60) \\
-2.5P + 2.5 & (60 \leq P \leq 100)
\end{cases}
\]

In the same way, the fuzzy sets of \( K_0, K_i, K_d \) are, respectively, defined as PS, PM and PB. Fig. 3 shows the triangle-type MFs of output variables \( K_0, K_i, K_d \). The nine fuzzy sets can be symbolized by \( B_1, B_2, B_3, C_1, C_2, C_3, D_1, D_2, D_3 \). Their corresponding membership grades can be formalized by \( \mu_{B_i}(K_0), \mu_{C_i}(K_i), \mu_{D_i}(K_d) \). Eqs. (5)–(13) give expressions of \( \mu_{B_i}(K_0), \mu_{C_i}(K_i), \mu_{D_i}(K_d) \):

\[
\mu_{B_1}(K_0) = \begin{cases} 
K_0 - K_{P_1} & (K_0 \leq K_{P_1}) \\
K_{P_1} - K_{P_2} & (K_{P_1} \leq K_0 \leq K_{P_2}) \\
K_{P_2} - K_P & (K_{P_2} \leq K_0)
\end{cases}
\]

\[
\mu_{B_2}(K_0) = \begin{cases} 
K_0 - K_{P_1} & (K_0 \leq K_{P_1}) \\
K_{P_1} - K_{P_2} & (K_{P_1} \leq K_0 \leq K_{P_2}) \\
K_{P_2} - K_P & (K_{P_2} \leq K_0)
\end{cases}
\]

\[
\mu_{B_3}(K_0) = \begin{cases} 
K_0 - K_{P_1} & (K_0 \leq K_{P_1}) \\
K_{P_1} - K_{P_2} & (K_{P_1} \leq K_0 \leq K_{P_2}) \\
K_{P_2} - K_P & (K_{P_2} \leq K_0)
\end{cases}
\]

\[
\mu_{B_1}(K_i) = \begin{cases} 
K_i - K_{I_1} & (K_i \leq K_{I_1}) \\
K_{I_1} - K_{I_2} & (K_{I_1} \leq K_i \leq K_{I_2}) \\
K_{I_2} - K_P & (K_{I_2} \leq K_i)
\end{cases}
\]

\[
\mu_{C_2}(K_i) = \begin{cases} 
K_i - K_{I_1} & (K_i \leq K_{I_1}) \\
K_{I_1} - K_{I_2} & (K_{I_1} \leq K_i \leq K_{I_2}) \\
K_{I_2} - K_P & (K_{I_2} \leq K_i)
\end{cases}
\]

\[
\mu_{C_3}(K_i) = \begin{cases} 
K_i - K_{I_1} & (K_i \leq K_{I_1}) \\
K_{I_1} - K_{I_2} & (K_{I_1} \leq K_i \leq K_{I_2}) \\
K_{I_2} - K_P & (K_{I_2} \leq K_i)
\end{cases}
\]

\[
\mu_{D_1}(K_d) = \begin{cases} 
K_d - K_{D_1} & (K_d \leq K_{D_1}) \\
K_{D_1} - K_{D_2} & (K_{D_1} \leq K_d \leq K_{D_2}) \\
K_{D_2} - K_P & (K_{D_2} \leq K_d)
\end{cases}
\]

\[
\mu_{D_2}(K_d) = \begin{cases} 
K_d - K_{D_1} & (K_d \leq K_{D_1}) \\
K_{D_1} - K_{D_2} & (K_{D_1} \leq K_d \leq K_{D_2}) \\
K_{D_2} - K_P & (K_{D_2} \leq K_d)
\end{cases}
\]

\[
\mu_{D_3}(K_d) = \begin{cases} 
K_d - K_{D_1} & (K_d \leq K_{D_1}) \\
K_{D_1} - K_{D_2} & (K_{D_1} \leq K_d \leq K_{D_2}) \\
K_{D_2} - K_P & (K_{D_2} \leq K_d)
\end{cases}
\]

where \( K_{P_i}, K_{I_i}, K_{D_i} \) are the gains of PID1.

The rule base consists of three rules:

- \( R^1 \): If the \( P \) is \( A_1 \), then the \( K_0 \) is \( B_1 \), \( K_i \) is \( C_1 \) and \( K_d \) is \( D_1 \).
- \( R^2 \): If the \( P \) is \( A_2 \), then the \( K_0 \) is \( B_2 \), \( K_i \) is \( C_2 \) and \( K_d \) is \( D_2 \).
- \( R^3 \): If the \( P \) is \( A_3 \), then the \( K_0 \) is \( B_3 \), \( K_i \) is \( C_3 \) and \( K_d \) is \( D_3 \).

Once the aggregated fuzzy set representing the fuzzy output variable has been determined, an actual crisp control decision must be made. The process of decoding the output to produce an actual...
value for the control signal is referred to as defuzzification. Here, a fuzzy logic controller-based center-average defuzzifier is implemented (Rubaai et al., 2008). The three outputs of FLC are given by Eqs. (14)–(16):

\[
K_P(P) = \frac{\sum_{i=1}^{3} \mu_{B_i} \mu_{A_i}}{\sum_{i=1}^{3} \mu_{A_i}}
\]

(14)

\[
K_I(P) = \frac{\sum_{i=1}^{3} \mu_{C_i} \mu_{A_i}}{\sum_{i=1}^{3} \mu_{A_i}}
\]

(15)

\[
K_D(P) = \frac{\sum_{i=1}^{3} \mu_{D_i} \mu_{A_i}}{\sum_{i=1}^{3} \mu_{A_i}}
\]

(16)

The composed output of fuzzy-PID was derived as Eq. (17):

\[
u(t) = K_P(P)e(t) + \int_0^t K_I(P)e(\tau)d\tau + K_D(P)e(t)
\]

(17)

3.2. Quadratic optimization by genetic algorithm

The output error \(e(t)\) between the fuzzy-PID and 5-PID control systems at the power levels 40% and 80% is written as Eq. (18). It is used to set up the objective function for the genetic algorithm:

\[
e(t) = y_1 + y_2 - (y_3 + y_4)
\]

(18)

where \(y_1\) and \(y_2\) denote the outputs at power levels 40% and 80% while taking fuzzy-PID as controller. \(y_3\) and \(y_4\) denote the outputs at power 40% and 80% while taking PID2 and PID4 as controller.

Generally, selecting the appropriate parameters to be optimized is very important in using genetic algorithm. In order that the fuzzy sets can cover the whole universe of discourse and the optimized FLC is of good performance, the overlap factor \(\alpha_i\) between adjacent two fuzzy sets is chosen as the parameter to be optimized within the range \(0.3 < \alpha_i < 0.7\) (Chen et al., 2008). As to the trapezoid-type MF, the other parameter to be optimized is the length of upper side \(\beta_i\). In order to increase the chromosome length, the fuzzy sets are axial symmetrical about \(\alpha_i\). Fig. 4(1)–(3) shows the parameters to be optimized for three kinds of MF. The fuzzy sets of three kinds of MF can be expressed by \(\alpha_i\) and \(\beta_i\) in MATLAB (version 6.5). For example, the fuzzy sets of trapezoid-type MF of \(K_P\) are expressed by MATLAB code:

\[
\begin{align*}
& a = \text{addvar}(a, 'output', 1, 'PB', [K_{p1}, K_{p3}, K_{p5}]); \\
& b = \text{addmf}(a, 'output', 1, 'PM', [K_{p1}, K_{p3}, K_{p5} + \beta_1, K_{p5} + (K_{p5} - K_{p1})/(1 - \alpha_1)]); \\
& c = \text{addmf}(a, 'output', 1, 'PS', [K_{p1}, K_{p3}, K_{p5} + (K_{p5} - K_{p1})/2 - \alpha_1 \beta_1]/(1 - \alpha_1), K_{p5} - \beta_1, K_{p5} + \beta_2, K_{p5} + (K_{p5} - K_{p1})/2 - \alpha_2 \beta_2]/(1 - \alpha_2)); \\
& d = \text{addmf}(a, 'output', 1, 'PB', [K_{p3} - (K_{p5} - K_{p1})/2 - \alpha_2 \beta_2]/(1 - \alpha_2), K_{p5} - \beta_2, K_{p5} + \beta_2, K_{p5}]);
\end{align*}
\]

where "a" is a fuzzy inference system in MATLAB (version 6.5). Thus the gauss-type MF and triangle-type MF have the same 8 parameters to be optimized. Therefore, the code of genetic algorithm consists of the following three modules:

(1) Initialization: The population size, generation number, gene length, chromosome length, crossover ratio and mutation ratio are set.

(2) Optimization: Three kinds of MF are, respectively, optimized to get respective optimal shape and the optimum fitness values of the objective function.

(3) Selection: The MF of the maximal fitness values is selected as the optimal MF.

4. Simulation

The fuzzy-PID control strategy is applied to H.B. ROBINSON nuclear plant (Kerlin et al., 1976) reactor power control system to analyze its performance by MATLAB (version 6.5). The reactor power is modeled using the point kinetics equations with six groups of delayed neutrons and two thermal feedbacks due to changes in fuel temperature and coolant temperature. The core heat transfer model is composed of one fuel node and two coolant nodes. The point kinetics dynamic linearized equations are given by (19)–(22). The heat transfer linearized equations from fuel to coolant are given...
Fig. 4. (1) Triangle-type MF. (2) Gauss-type MF. (3) Trapezoid-type MF.

by (23)–(26):

\[
\frac{d\Delta P}{dt} = -\frac{\beta}{A} \Delta P + \sum_{i=1}^{6} \lambda_i \Delta C_i + \frac{P_0}{A} \Delta \rho_r + \frac{\alpha_f P_0}{A} F_f \Delta T_f + \frac{\alpha_c P_0}{A} (F_c_1 \Delta T_c_1 + F_c_2 \Delta T_c_2)
\]

\[
\frac{d\Delta C_i}{dt} = \frac{\beta_i}{A} \Delta P - \lambda_i \Delta C_i \quad i = 1, 2, \ldots, 6
\]

\[
\frac{d\Delta \rho_r}{dt} = G_r z_r
\]

\[
\Delta \rho = \Delta \rho_r + \alpha_f \Delta T_f + \frac{1}{2} \alpha_c \Delta T_c_1 + \frac{1}{2} \alpha_c \Delta T_c_2
\]

\[
\frac{d\Delta T_f}{dt} = \frac{Q_f}{(MC_p)_f} \Delta P - \frac{UA_f}{(MC_p)_f} (\Delta T_f - \Delta T_c_1)
\]

\[
\frac{d\Delta T_c_1}{dt} = \left( \frac{UA_f}{MC_p} \right) \Delta P - \frac{2}{\tau} (\Delta T_c_1 - \Delta T_c_2)
\]

\[
\frac{d\Delta T_c_2}{dt} = \left( \frac{UA_f}{MC_p} \right) \Delta P - \frac{2}{\tau} (\Delta T_c_2 - \Delta T_c_1)
\]

where,

- \( \Delta P \) is the deviation of reactor power from initial steady-state value;
- \( P_0 \) is the initial steady-state power level;
- \( \lambda_i \) is the delayed neutron decay constant for ith delayed neutron group;
- \( \Delta C_i \) is the deviation of normalized precursor concentration from its steady-state value;
- \( \Delta \rho_r \) is the reactivity due to control rod movement;
- \( \alpha_f \) is the fuel temperature coefficient of reactivity;
- \( \alpha_c \) is the coolant temperature coefficient of reactivity;
- \( \Delta \rho \) is the total reactivity;
- \( \Delta T_f \) is the deviation of fuel temperature from its steady-state value;
- \( \Delta T_c_1 \) is the deviation of average coolant temperature in first coolant node from its steady-state value;
- \( \Delta T_c_2 \) is the deviation of outlet coolant temperature in second coolant node from its steady-state value;

Table 1

<table>
<thead>
<tr>
<th>The type of the MF</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle-type MF</td>
<td>1/0.005</td>
</tr>
<tr>
<td>Gauss-type MF</td>
<td>1/0.003</td>
</tr>
<tr>
<td>Trapezoid-type MF</td>
<td>1/0.009</td>
</tr>
</tbody>
</table>

Fig. 5. The 5-PID control strategy simulation results.

\[
\frac{d\Delta T_c_2}{dt} = \left( \frac{UA_f}{MC_p} \right) \left( \Delta T_f - \Delta T_c_1 \right) - \frac{2}{\tau} (\Delta T_c_2 - \Delta T_c_1)
\]

\[
\Delta T_c_1 = \frac{1}{2} (\Delta T_c_2 + \Delta T_c_1)
\]

Fig. 6. The fuzzy-PID control strategy simulation results.
Finally, simulation results for beginning of reactor start-down and the end of reactor start-up operation with a ramp of ±12% per minute have been shown in Fig. 7. PID controllers switch time at 5-PID control strategy is on power levels 90%, 70%, 50%, 30% and 20%. Fig. 7 gives the desired power trajectory, 5-PID control strategy output, FLC control strategy output and fuzzy-PID control strategy output. In Fig. 7, there are large errors between the 5-PID control strategy output and the desired power trajectory except on the power levels 20%, 40%, 60%, 80% and 100%. The FLC control strategy has large error at the whole power range. The fuzzy-PID control strategy output nearly tracks desired power track. As is stated in Section 2, the FLC tunes the gains of PID controller to adapt the model changing with the power, so the fuzzy-PID control strategy has better tracking performance than the 5-PID and FLC control strategies on the whole power levels.

5. Conclusion

The fuzzy-PID controller for nuclear reactor power control system has been designed and optimized by the genetic algorithm. The simulation results show its good performance. The process of designing and optimizing suggests that this method is simple and practical for complex and nonlinear nuclear reactor power control system.

As mentioned in Section 3.1, if the more models are indentified, the designed fuzzy-PID controller will get better performances. So if there are future works to improve the fuzzy-PID controller performance, the more models are supposed to be gained.

References


and Control Technology: Optoelectronic Technology and Instruments, Control Theory and Automation, and Space Exploration, p. 71291J.


