Generalized Predictive Control with Constraints for Ship Autopilot

Geng Tao[1], Zhao Jin[2]

1. Academy of Physics and Electroics, Henan University, Kaifeng 475004
E-mail: gengtao@henu.edu.cn

2. Academy of Control Science and Engineering, Huazhong University of Science and Technology (HUST), Wuhan 430074
E-mail: jinzhao617@mail.hust.edu.cn

Abstract: Predictive control has applied to variable fields successfully. By the limit of amplitude and rate, the rudder, as the autopilot's actuator, has effects on the course control. And, ship autopilot is always exacerbated by environment, i.e. the effects of wind, sea-state, and tide. Autopilot should suppression of fast rudder motions which have hardly any positive effect on the steering behavior. This paper describes an autopilot that adapts the generalized predictive control (GPC) with constraints which takes the physic limits into account. ZONE control limiting the course in a bound improves the control performance of autopilot, disturbed by waves. The autopilot performs well which is valid in semi-physical platform experiment.

Key Words: course control, generalize predictive control, zone control

1 INTRODUCTION

So a great performance does predictive control have that have been applied to variable fields successfully, such as course control[2]. Adaptive generalized predictive control for ship autopilot has been studied. Based on weigh and speed, products a gain shielding for autopilot, order to achieve the dream that course turning control[3]. Then ship course keeping and turning had already been focused on. The problem how to choose and switch has been discussed[4]. Predictive control for ship course keeping has been given[5]. Generalized predictive control for ship autopilot without constrains has been deliberated by Geng Tao[1]. However, the problem constraints in ship control have never been talked about.

This paper creatively applies predictive control in ship autopilot, which handling the constraints in the rudder. Furthermore, to improve the course control performance in waves, a ZONE control mode of GPC with output constraints is adapted to limit the course in the bound. Under this control mode, a new autopilot is proposed.

2 Problem describtion

We derive the none-linear equation of motion with assumption that the vessel is rigid and the NED frame is inertial.

motion in the x-direction

$$\begin{aligned} m(\dot{u}-vr) = X_{\dot{u}}\dot{u} + X_{uu}u^2 + X_{vv}v^2 + \\ X_{rr}r^2 + X_{vr}vr + X_{\delta_r\delta_r}\delta_r^2 + X_T \end{aligned} \tag{1}$$

motion in the y-direction

$$\begin{aligned} m(\dot{v}+ur) = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_v v + Y_r r + Y_{v|v|}v|v| \\ + Y_{v|r|}v|r| + Y_{\delta_r}\delta_r + Y_{|r|\delta_r}|r|\delta_r \end{aligned} \tag{2}$$

motion in the z-direction

$$\begin{aligned} I_z\dot{r} = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_v v + N_r r + N_{v|v|}v|v| \\ + N_{v|r|}v|r| + N_{r|r|}r|r| + N_{\delta_r}\delta_r + N_{|r|\delta_r}|r|\delta_r \end{aligned} \tag{3}$$

rotation about the z-axis

$\dot{\psi} = r$

where,

X(·), Y(·): kinetics derivatives ;

N(·): hydrodynamic derivatives;

u: surge velocity(m/s);

v: sway velocity(m/s);

r: rate of turn or course-angular velocity (m/s);

Ψ: course angle or heading (rad);

δ_r: rudder angle(rad);

m: mass of vessel(kg);

I_Z: moment of inertia with respect to the z-axis (N·m^2);

A simple linear model has been suggested by Davidson and Schiff [6], 1946.

$$m(\dot{v}+ur) = Y_{\dot{v}}\dot{v} + Y_v v + Y_{\dot{r}}\dot{r} + Y_r r + Y_{\delta_r}\delta_r \tag{4}$$

$$I_z\dot{r} = N_{\dot{v}}\dot{v} + N_v v + N_{\dot{r}}\dot{r} + N_r r + N_{\delta_r}\delta_r \tag{5}$$

3 GPC with constraints

GPC adapts the model so-called Controlled auto regressive integrated moving average (CARIMA) model.

$$A(z^{-1})\mathbf{y} = B(z^{-1})\mathbf{u} + \frac{T(z^{-1})}{\Delta}\mathbf{e} \tag{6}$$

This work is supported by National Nature Science Foundation under Grant 60874074

Where, $T(z^{-1})$ is the model of noise, but it is commonplace to treat $T(z^{-1})$ as a design parameter. Because it has direct effects on loop sensitivity and so better closed-loop performance will be got with a $T(z^{-1})$.

Then, a general form of future predictions is

$$\underset{\rightarrow}{\mathbf{y}} = H\Delta\underset{\rightarrow}{\mathbf{u}} + \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} + \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \quad (7)$$

And the followed can be derived

$$H = C_D^{-1}C_B \quad (8)$$

Where, $D(z^{-1}) = \Delta A(z^{-1})$

$$\begin{cases} \tilde{P} = C_T P - HH_T \\ \tilde{Q} = C_T Q + H_T \end{cases} \quad (9)$$

C_T is the toeplitz matrix of $T\left(z^{-1}\right)$ and H_T is the hankel matrix of $T\left(z^{-1}\right)$ [7]. Constraints on process inputs and outputs make the controller and consequently the entire closed-loop, nonlinear.

Use the cost function and optimization

$$J = \sum_{j=1}^{N_y} \left\| r(k+j) - y(k+j \mid k) \right\|_2^2 W_y(j) + \sum_{j=1}^{N_u} \left\| \Delta u(k+j \mid k) \right\|_2^2 W_u(j) \quad (10)$$

Where, $r(k+j)$ is j step ahead reference. N_y is receding horizon and N_u is control step. W_y, W_u is the output and input weight factor. Described in matrix form, the GPC with constraints is converted to be the optimization problem which is

$$\min_{\Delta\underset{\rightarrow}{\mathbf{u}}} J = \Delta\underset{\rightarrow}{\mathbf{u}} S \Delta\underset{\rightarrow}{\mathbf{u}}^T + 2\mathbf{f}^T \Delta\underset{\rightarrow}{\mathbf{u}}$$
$$\text{s.t. } \mathrm{C}\Delta\underset{\rightarrow}{\mathbf{u}} - d_k \le 0 \quad (11)$$

Where, J is subject of the following constraints. Where S is positive definite and $\mathbf{f}$, d_k are time varying (dependent on the current state).

$$S = H^T W_y H + W_u,\ \mathbf{f} = -H^T W_y \left(\underset{\rightarrow}{\mathbf{r}} - \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} - \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \right) \quad (12)$$

The equ.(11) is a standard quadratic programming with constraints. The constraints in GPC will be described as the following.

Input move constraints

$\Delta\underline{\mathbf{u}}$ is the lower bounds of input move constraints, and $\Delta\bar{\mathbf{u}}$ is the upper bounds of input move constraints.

$$\begin{bmatrix} \Delta\underline{\mathbf{u}} \\ \Delta\underline{\mathbf{u}} \\ \vdots \\ \Delta\underline{\mathbf{u}} \end{bmatrix} \le \begin{bmatrix} \Delta\mathbf{u}_k \\ \Delta\mathbf{u}_{k+1} \\ \vdots \\ \Delta\mathbf{u}_{k+Nu-1} \end{bmatrix} \le \begin{bmatrix} \Delta\bar{\mathbf{u}} \\ \Delta\bar{\mathbf{u}} \\ \vdots \\ \Delta\bar{\mathbf{u}} \end{bmatrix}$$

Which can be described in vector form.

$$\Delta\underline{U} \le \Delta\underset{\rightarrow}{\mathbf{u}} \le \Delta\bar{U} \quad (13)$$

And satisfy the matrix inequality

$$\begin{bmatrix} I \\ -I \end{bmatrix} \Delta\underset{\rightarrow}{\mathbf{u}} - \begin{bmatrix} \Delta\bar{U} \\ -\Delta\underline{U} \end{bmatrix} \le 0$$

Input constraints

$\underline{\mathbf{u}}$ is the lower bounds of input constraints, and $\bar{\mathbf{u}}$ is the upper bounds of input constraints.

$$\underset{\rightarrow}{\mathbf{u}} = C_{I/\Delta}\Delta\underset{\rightarrow}{\mathbf{u}} + \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} \mathbf{u}_{k-1}$$

where, $C_{I/\Delta} = \begin{bmatrix} I & 0 & \cdots & 0 \\ I & I & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots \\ I & \cdots & I & I \end{bmatrix}$

$$\begin{bmatrix} \underline{\mathbf{u}} \\ \underline{\mathbf{u}} \\ \vdots \\ \underline{\mathbf{u}} \end{bmatrix} \le \begin{bmatrix} \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \vdots \\ \mathbf{u}_{k+Nu-1} \end{bmatrix} \le \begin{bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{u}} \\ \vdots \\ \bar{\mathbf{u}} \end{bmatrix}$$

$$\underline{U} \le C_{I/\Delta}\Delta\underset{\rightarrow}{\mathbf{u}} + L\mathbf{u}_{k-1} \le \bar{U} \quad (14)$$

The corresponding linear inequalities are:

$$\begin{bmatrix} C_{I/\Delta} \\ -C_{I/\Delta} \end{bmatrix} \Delta\underset{\rightarrow}{\mathbf{u}} - \begin{bmatrix} \bar{U} - L\mathbf{u}_{k-1} \\ -\underline{U} + L\mathbf{u}_{k-1} \end{bmatrix} \le 0$$

Output constrains

The output of plants always needs to be constrained in the bounds, as demand of process requirements. And they always are treated as soft constraints. $\underline{Y}$ is the lower bounds of output constraints, and $\bar{Y}$ is the upper bounds of output constraints.

$$\underline{Y} \le \mathbf{y} \le \bar{Y}$$
$$\underline{Y} \le H\Delta\underset{\rightarrow}{\mathbf{u}} + \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} + \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \le \bar{Y} \quad (15)$$

$$\begin{bmatrix} H \\ -H \end{bmatrix} \Delta\underset{\rightarrow}{\mathbf{u}} - \begin{bmatrix} \bar{Y} - \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} - \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \\ -\underline{Y} + \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} + \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \end{bmatrix} \le 0$$

If all constraints are satisfied, the Equ.(11) can be described as

$$C = \begin{bmatrix} I \\ -I \\ C_{I/\Delta} \\ -C_{I/\Delta} \\ H \\ -H \end{bmatrix},\quad d_k = \begin{bmatrix} \Delta\bar{U} \\ -\Delta\underline{U} \\ \bar{U} - L\mathbf{u}_{k-1} \\ -\underline{U} + L\mathbf{u}_{k-1} \\ \bar{Y} - \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} - \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \\ -\underline{Y} + \tilde{P}\Delta\underset{\leftarrow}{\tilde{\mathbf{u}}} + \tilde{Q}\underset{\leftarrow}{\tilde{\mathbf{y}}} \end{bmatrix} \quad (16)$$

qpOASES (quadratic program Online Active SEt Strategy) is an open-source implementation of the recently proposed online active set strategy, which was inspired by important observations from the field of parametric quadratic programming[8][9]. The standard form is

$$\min_{\Delta\underset{\rightarrow}{\mathbf{u}}} J = \Delta\underset{\rightarrow}{\mathbf{u}}^T S \Delta\underset{\rightarrow}{\mathbf{u}} + 2\Delta\underset{\rightarrow}{\mathbf{u}}^T \mathbf{f}$$
$$\text{s.t. } lb \le \Delta\underset{\rightarrow}{\mathbf{u}} \le ub$$
$$lbA \le A\Delta\underset{\rightarrow}{\mathbf{u}} \le ubA$$

The equ. (11) can be converted to be in the qpOASES form. And the followed can be derived from Equ. (13), (14), (15)

$$\underbrace{\Delta \underline{U}}_{lb} \le \Delta \underline{\mathbf{u}} \le \underbrace{\Delta \bar{U}}_{ub}$$

$$\left(\underline{U} - L\mathbf{u}_{k-1}\right) \le C_{I/\Delta} \Delta \underline{\mathbf{u}} \le \left(\bar{U} - L\mathbf{u}_{k-1}\right)$$

$$\left(\underline{Y} - \tilde{P}\Delta\tilde{\underline{\mathbf{u}}} - \tilde{Q}\tilde{\underline{\mathbf{y}}}\right) \le H\Delta\underline{\mathbf{u}} \le \left(\bar{Y} - \tilde{P}\Delta\tilde{\underline{\mathbf{u}}} - \tilde{Q}\tilde{\underline{\mathbf{y}}}\right)$$

And we can get

$$\underbrace{\begin{bmatrix} \underline{U} - L\mathbf{u}_{k-1} \\ \underline{Y} - \tilde{P}\Delta\tilde{\underline{\mathbf{u}}} - \tilde{Q}\tilde{\underline{\mathbf{y}}} \end{bmatrix}}_{lbA} \le \underbrace{\begin{bmatrix} C_{I/\Delta} \\ H \end{bmatrix}}_{A} \Delta\underline{\mathbf{u}} \le \underbrace{\begin{bmatrix} \bar{U} - L\mathbf{u}_{k-1} \\ \bar{Y} - \tilde{P}\Delta\tilde{\underline{\mathbf{u}}} - \tilde{Q}\tilde{\underline{\mathbf{y}}} \end{bmatrix}}_{ubA}$$

Algorithm - GPC with constraints

(1). Firstly, we can identify the plant $A\left(z^{-1}\right)$, $B\left(z^{-1}\right)$[1], and H, $\tilde{P}$, $\tilde{Q}$ can be calculated from equ. (8), (9), Specify the factor W_y, W_u, $T\left(z^{-1}\right)$, receding horizon N_y, control step N_u, bound of input $\underline{U}$, $\bar{U}$, bound of input rate $\Delta\underline{U}$, $\Delta\bar{U}$, bound of output $\underline{Y}$, $\bar{Y}$, calculate the lb, ub, A, initialize QProblem object（qpOASES）.

(2). Sample the output, update $\underline{\mathbf{y}}$.

(3). update lbA, ubA and QProblem object, return the optimization value $\Delta\underline{\mathbf{u}}$.

(4). update $\Delta\underline{\mathbf{u}}$, output $\Delta\mathbf{u}_k$.

(5). go to (2).

4 GPC autopilot with input constraints

In this and followed section, the GPC with constraints autopilot are issued whose block diagram is shown as Fig. 1.

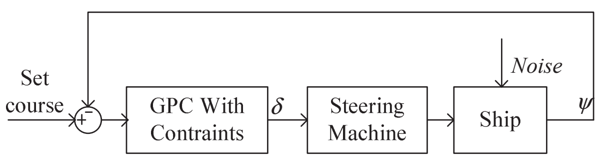

Fig. 1 Block diagram of GPC with constraints autopilot

In autopilot, the rudder is driven by hydraulic power which is limited by maximum rudder speed and deflection. As the bound of input, the GPC with constraints handles this as we talk above to improve the effect of autopilot.

In order to verify the performance of the proposed autopilot, the experiment is taken on semi-physic platform as Fig. 2 shown. Autopilot is based on PC104 (64k RAM, 300MHz clock) embedded VxWorks OS. The rudder is driven by hydraulic power whose maximum rudder deflection is about 30° to both sides and maximum rudder rate is 5°/s. The ship motion simulator simulates the ship motion with nonlinear equation described in section 2. The entire GPC with constraints algorithm is described with C++ language.

We will consider a container ship with data given in the Matlab GNC toolbox version2.3[10]. The vessel model can be found in the container.m. Tab. 1 describes the autopilot parameters setting.

An autopilot is realized by GPC with constraints described in section 3. As shown in Fig. 3 and Fig. 4, it can be seen that the ship course is controlled as desired. When altering the course, the overshoot is very small. Generally speaking, the overshoot is zero which is very ideal.

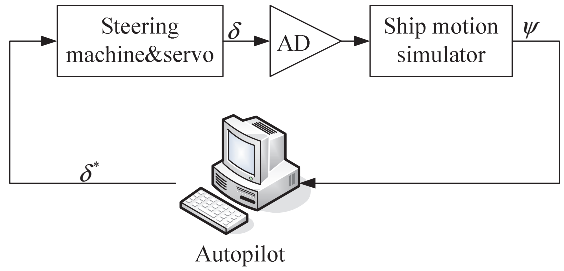

Fig. 2 semi-physic experiment platform

TABLE I Autopilot parameters setting

Parameters setting	Quantity	Symbol	Value
	sample time	T_s	4s
	yaw accuracy		0.1°
Actuator	maximum rudder speed		5°/s
	maximum rudder deflection		30°
	pedictive horizon	N_y	35
	control horizon	N_u	4
	weighting facor	W_y	0.1
	observer polynomial	$T(z^{-1})$	$(1{+}0.8z^{-1})^2$
Course	Bound of course ZONE		±10°

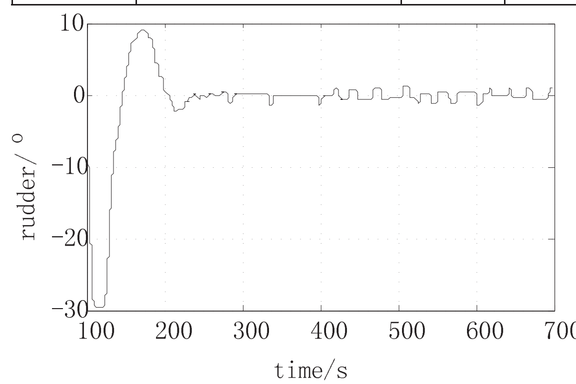

Fig. 3 Motion of rudder

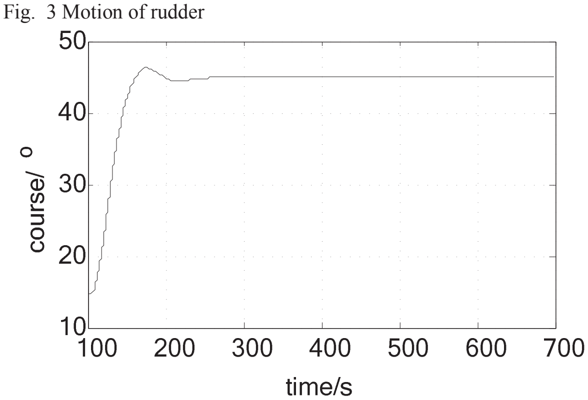

Fig. 4 Changing of course

In practice, the output of some plant can't to be controlled in tight. And always the output should be constrained in the bound. The upper and lower bound is specialized to be constant. This control model is called ZONE control[11]. In the ZONE model of MPC, the factor Wy is always set to be

0. The course of ship control in the wave is typical stochastic control. Compared with the noise in industry, the wave is larger which disturbs the control more seriously. In this paper, the course is limited in the ZONE realized by the GPC with output constraints.

5 GPC autopilot with output constraints

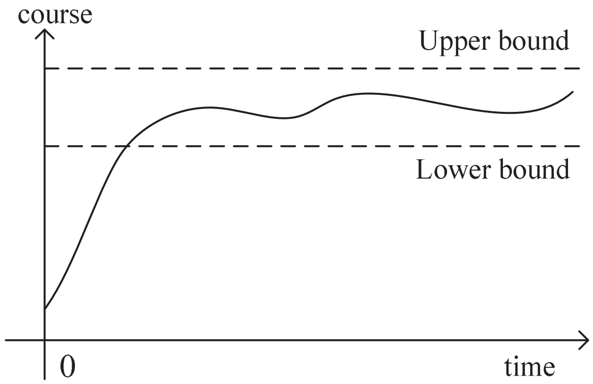

Fig. 5 Nonlinear ZONE control of course

GPC with constraints in the feasible region is equivalent to sub-model control system. As the constraints altering, the GPC transports from one feasible region to another which is equivalent to a multi-model control. The controller is immune for disturbance rejection by waves, as the value of system gain became smaller near the feasible region; the controller is ability of tracking power, as the value of system gain became bigger caused by the boundary constraints. ZONE control makes an effective balance between the noise suppression and tracking capability for design, and this trade-off is in the ZONE control mode. So under the interference of the waves, better heading control effect can be got.

Specializing the ZONE boundary $[\psi_d - 10°, \psi_d + 10°]$, other parameters setting shown in Tab. 1. The course control is shown in Fig. 6 and Fig. 7. We can see that the motion of rudder is smooth and the course is maintained to course demand.

6 CONCLUSION

Handing constrains as the essence of predictive control, compared with heuristic handing constrains, the control system of amplitude and incremental constrains is solved. What's more, the problem that control object indirect output constraints can be realized to. GPC with constraints take the rudder limits to consideration suppress the extra rudder induced motions effectively. The semi-physical experiment is presented to illustrate the effectiveness of the proposed GPC with autopilot.

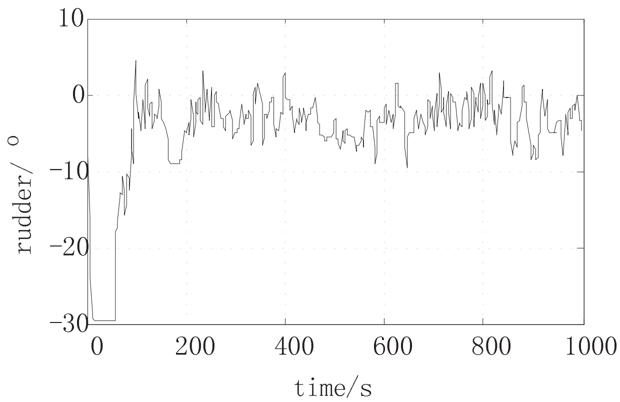

Fig. 6 Motion of rudder disturbed by wave

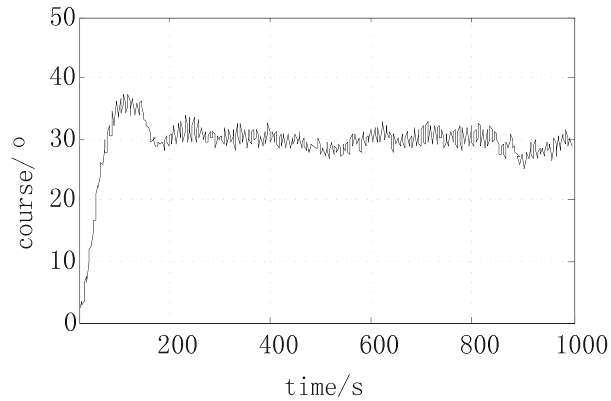

Fig. 7 Changing of course disturbed by wave

REFERENCE

[1] Tao Geng, Jin Zhao. Adaptive Generalized Predictive Control for Ship Autopilot. Chinese Control and Decision Conference, 2011: 2595~2601;

[2] Qin S. Joe, Badgwell Thomas A. A survey of industrial model predictive control technology. Control Engineering Practice, 2003, 11: 733~764;

[3] Garcia R. Ferreiro, Martinez X. C. Pardo. Adaptive GPC algorithm on ship heading control. In: Proceeding on applied simulation and modelling, 2003.

[4] Hu Yaohua, Jiang Dandong, Jia Xinle. The Generalized Predictive Control for Ship Course-keeping. CONTROL AND DECISION, 1998, 13(2): 156~159;

[5] Hu Yaohua, Xu Suwu. A Direct Generalized Predictive Controller for Ship Course Keeping. In: Proceedings of the 15th mediterranean conference on control & automation, Athens, Greece: 2007. 1~4;

[6] Davidson K. S. M., Schiff L. I. Turning and course keeping qualities. Trans. SNAME, 1946, 54;

[7] ROSSITER J. A. Model-Based Predictive Control-A Practical Approach. Washington: CRC PRESS, 2003;

[8] Ferreau H. J. qpOASES (quadratic program Online Active SEt Strategy) version 2.0. http://www.qpoases.org/, 2009;

[9] Ferreau H. J., Bock H. G., Diehl M. An Online Active Set Strategy for Fast Parametric Quadratic Programming in MPC Applications. Proceedings of the IFAC Workshop on Nonlinear Model Predictive Control for Fast Systems, 2006: 21~30;

[10] Fossen T. I. Guidance and Control of Ocean Marine Vehicles. New York: John Wiley and Sons Ltd., 1994;

[11] Maciejowski J. M. Predictive Control with Constraints. Prentice Hall, 2000;