A Unified Model for the Analysis of FACTS Devices in Damping Power System Oscillations—Part III: Unified Power Flow Controller

HaiFeng Wang, Member, IEEE

Abstract—This paper presents the establishment of the linearized Phillips–Heffron model of a power system installed with a Unified Power Flow Controller (UPFC). Two applications based on the Phillips–Heffron model are demonstrated: 1) Study on the effect of UPFC DC voltage regulator on power system oscillation stability; 2) Selection of damping control signal for the design of UPFC damping controller.

Index Terms—FACTS, Phillips–Heffron model, power system dynamic stability, power system modeling, UPFC.

I. INTRODUCTION

THE UNIFIED Power Flow Controller (UPFC) was proposed [1] for the Flexible AC Transmission Systems (FACTS), which is a multiple-functional FACTS controller with primary duty to be power flow control. The secondary functions of the UPFC can be voltage control, transient stability improvement, oscillation damping [2]–[4], etc. In [2] and [3], it is demonstrated by examples that the UPFC can be very effective to damp power system oscillations. So far, however, the damping function of the UPFC has not been investigated thoroughly. In this paper, the linearized Phillips–Heffron model [8], [9] of a power system installed with a UPFC is derived which turns out to be of the exactly same form as that of the unified model presented in [10] and [11] for SVC, TCSC and TCPAR. Thus methods proposed in [10] and [11] based on the Phillips–Heffron model can be directly applied for the analysis and design of UPFC damping control function, which is not to be repeated in the paper. Instead, two applications based on the Phillips–Heffron model which are special to UPFC are demonstrated: 1) Study on the effect of UPFC DC voltage regulator on power system oscillation stability; 2) Selection of damping control signal for the design of UPFC damping controller.

II. SINGLE-MACHINE INFINITE-BUS POWER SYSTEMS

Fig. 1 is a single-machine infinite-bus power system installed with a UPFC which consists of an excitation transformer (ET), a boosting transformer (BT), two three-phase GTO based voltage source converters (VSC’s) and a DC link capacitor. In Fig. 1, \( m_E \), \( m_B \), \( \delta_E \) and \( \delta_B \) are the amplitude modulation ratio and phase angle of the control signal of each VSC respectively, which are the input control signals to the UPFC.

By applying Park’s transformation on the three-phase dynamic differential equations of the UPFC and ignoring the resistance and transients of the transformers, the dynamic model of the UPFC is [3]

\[
\begin{align*}
\dot{V}_E &= \frac{m_E v_{dc}}{2} e^{j\delta_E}, \quad \dot{V}_B = \frac{m_B v_{dc}}{2} e^{j\delta_B} \quad (2.1) \\
\frac{d\delta_{dc}}{dt} &= \frac{3m_E}{4C_{dc}} \left[ \cos \delta_E \sin \delta_E \right] [i_{Ed}] \frac{i_{Eq}}{i_{Bq}} + \frac{3m_B}{4C_{dc}} \left[ \cos \delta_B \sin \delta_B \right] [i_{Bd}] [2.2]
\end{align*}
\]

From Fig. 1 we can have

\[
\begin{align*}
\dot{V}_t &= j\omega t_1 i_t + V_E \\
\dot{V}_{Ex} &= \dot{V}_{Bx} + j\omega t_1 i_{Bx} + V_b
\end{align*}
\]

from which we can obtain

\[
\begin{align*}
i_{Ed} &= \frac{x_{BB} E_y}{x_{dc}} - \frac{m_E\sin(\delta_{EYdc}x_{Ed})}{2x_{dc}} + \frac{x_{de}}{x_{dc}} \left( V_b \cos \delta + \frac{m_B \sin(\delta_{BYdc})}{2} \right) \\
i_{Eq} &= \frac{m_E \cos(\delta_{EYdc}x_{Eq})}{2x_{dc}} - \frac{x_{de}}{x_{dc}} \left( \frac{m_B \cos(\delta_{BYdc})}{2} + V_b \sin \delta \right) \\
i_{Bd} &= -\frac{x_{dt}}{x_{dc}} \left( V_b \cos \delta + \frac{m_B \sin(\delta_{BYdc})}{2} \right) + \frac{x_{de} m_E \sin(\delta_{EYdc})}{2x_{dc}} - \frac{x_{Ed} E_y}{x_{dc}}
\end{align*}
\]
where
\[
x_{qE} = (x_q + \Delta x_E + x_E)(x_B + x_{BV}) + x_E(x_q + \Delta x_E),
\]
\[
x_{qE} = x_q + \Delta x_E, \quad x_{Bq} = x_B + x_{BV}, \quad x_q + \Delta x_E,
\]
\[
x_{qt} = x_q + \Delta x_E + x_E
\]
\[
x_{tE} = (x_q + \Delta x_E + x_E)(x_B + x_{BV}) + x_E(x_q + \Delta x_E),
\]
\[
x_{tE} = x_q + \Delta x_E + x_E
\]
\[
x_{BB} = x_B + x_{BV}, \quad x_{dt} = x_q + \Delta x_E + x_E
\]

By use of \(i_{E_d} = i_{Ed} + i_{Bd}, i_{Q_t} = i_{E_d} + i_{Bq}\) and linearizing (2.2), (2.3) and the nonlinear dynamic differential equations of the power system [10], we can obtain the linearized model to be

\[
\begin{bmatrix}
\Delta \delta \\
\Delta \omega \\
\Delta \dot{E}_q \\
\Delta \dot{E}_{fd}
\end{bmatrix} = 
\begin{bmatrix}
0 & \omega_0 & 0 & 0 \\
-\frac{K_1}{M} & -\frac{D}{M} & -\frac{K_2}{M} & 0 \\
\frac{K_1}{T_{do}} & \frac{K_2}{T_{do}} & 0 & \frac{1}{T_{do}} \\
-\frac{K_{A_k}K_{vd}}{T_A} & -\frac{K_{A_k}K_{vd}}{T_A} & -\frac{K_{A_k}K_{vd}}{T_A} & -\frac{K_{A_k}K_{vd}}{T_A}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \omega \\
\Delta \dot{E}_q \\
\Delta \dot{E}_{fd}
\end{bmatrix}
+ \begin{bmatrix}
\frac{0}{M} \\
\frac{K_{pd}}{T_{do}} \\
\frac{0}{T_{do}} \\
\frac{0}{T_{do}}
\end{bmatrix}
\Delta u_{dc}
\]

From Fig. 2 we can see that the linearized model is of the exactly same form of the unified Phillips–Heffron model for SVC, TCSC and TCPAR [10].

III. MULTI-MACHINE POWER SYSTEMS

Without loss of generality, we assume that in an \(n\)-machine power system, a UPFC is installed between nodes 1 and 2 as shown by Fig. 3. Thus we have

\[
\begin{align*}
\bar{Y}_{11} \bar{V}_1 + I_{1f} + \bar{Y}_{13} \bar{V}_g = 0 \\
\bar{Y}_{22} \bar{V}_2 - I_{2f} + \bar{Y}_{23} \bar{V}_g = 0 \\
\bar{Y}_{31} \bar{V}_1 + \bar{Y}_{32} \bar{V}_2 + \bar{Y}_{33} \bar{V}_g = -\bar{I}_g
\end{align*}
\]

(3.1)

From Fig. 3 we can have

\[
\bar{V}_1 = jx_{1E}I_{1E} + \bar{V}_{Et}
\]

\[
\bar{V}_{Et} = jx_{E2}I_{E2} + \bar{V}_{Bt} + \bar{V}_2, \quad I_E = I_{1E} - I_{E2}
\]

(3.2)

From (3.2) and (2.1) we can obtain

\[
\begin{bmatrix}
\bar{I}_{1E} \\
\bar{I}_{E2}
\end{bmatrix} = \frac{1}{x_{\Sigma}} \begin{bmatrix}
-j(x_{E2} + x_{E2} + x_B) & jx_{E2} \\
-jx_E & j(x_{E2} + x_E)
\end{bmatrix}
\begin{bmatrix}
\bar{V}_1 \\
\bar{V}_{E2}
\end{bmatrix}
\]

(3.3)

where

\[
x_{\Sigma} = (x_{1E} + x_E)(x_{E2} + x_{E2} + x_B) - x_E^2
\]

Then by substituting (3.3) into (3.1) we can have

\[
\bar{I}_g = CV_g + F_E \bar{V}_E + F_B \bar{V}_B
\]

(3.4)
where

\[
C = Y_{33} - [Y_{31} \ Y_{31}] Y_t^{-1} \begin{bmatrix} Y_{13} \\ Y_{23} \end{bmatrix}
\]

\[
F_E = -[Y_{31} \ Y_{31}] Y_t^{-1} \begin{bmatrix} j(x_E x_B + x_E B) \\ x_E \\ j x_E x_B \\ x_E \\ j x_E x_B \\ x_E \\ x_E x_B \\ x_E \\ x_E x_B \\ x_E \end{bmatrix}
\]

\[
F_B = -[Y_{31} \ Y_{31}] Y_t^{-1} \begin{bmatrix} j(x_1 + x_B) \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \end{bmatrix}
\]

\[
Y_t = \begin{bmatrix} Y_{11} + j(x_E x_B + x_E B) \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \\ x_1 x_B \\ x_1 \end{bmatrix}
\]

Substituting (3.6) into the linearized dynamic equations of the \( n \)-machine power system [11] we can obtain (3.7) at the bottom of the next page. If we denote

\[
\Delta f_k = [\Delta v_k \ \Delta \delta_k] \quad K_P = \begin{bmatrix} M^{-1} K_{px} \\ M^{-1} K_{ux} \end{bmatrix},
\]

\[
K_Q = \begin{bmatrix} T_{d0}^{-1} K_{q0} \\ T_{d0}^{-1} K_{uq} \end{bmatrix}, \quad K_V = \begin{bmatrix} T_A^{-1} K_A K_{vd} \\ T_A^{-1} K_A K_{vu} \end{bmatrix}
\]

the linearized model of (3.7) can be shown by Fig. 4, where \( u_k \) could be \( m_E, m_B, \delta_E \) or \( \delta_B \). Obviously, the model is of the exactly same form as that of the unified model for SVC, TCSC and TCPAR presented in [11].

IV. EFFECT OF UPFC DC VOLTAGE REGULATOR ON POWER SYSTEM OSCILLATION STABILITY

For the continuous and effective operation of a UPFC, the DC voltage across the UPFC link capacitor must be kept constant, which can be achieved by installing a DC voltage regulator \( u_k = T_{DC}(s)(v_{dec} - v_{dc}) \) in the UPFC [2], [3]. The DC voltage regulator functions by controlling the exchange of active power between the UPFC and the power system. Hence its influence upon power system oscillation damping should be expected and can be investigated based on the Phillips–Heffron model developed above.

For example, if it is assumed that the active power input to the UPFC installed in the single-machine infinite-bus power system of Fig. 1 is \( P_{UPFC} = u_k l_{dc}. \) The power balance equation of the power system should be \( P_m - P_e = P_{acc} + P_{UPFC} \), where \( P_{m}(\text{constant}) \) is the mechanical power input to the generator, \( P_e \) the electric power output from the generator and \( P_{acc} \) the accelerating power to the rotor movement of the generator. At steady-state operation, \( P_{m} - P_{e} = 0 \) so that \( P_{acc} = 0 \) and \( P_{UPFC} = v_{dc} l_{dc} = 0 \). During the dynamic process, the power balance is achieved to ensure \( \Delta P_e + \Delta P_{acc} + \Delta P_{UPFC} = 0 \). Thus \( \Delta P_{UPFC} \) varies in opposition to that of \( \Delta P_e \) as \( \Delta P_{acc} \) does so that the active power is kept in balance. Therefore, \( \Delta P_{UPFC} \) is opposite to \( \Delta P_e \) in phase and thus leads \( \Delta \delta \) by 90 degrees. Since \( C_{dc} l_{dc} = I_{dc} \) [3], we can have \( \Delta P_{UPFC} = v_{dc} l_{dc} + I_{dc} \Delta \delta = \Delta \delta l_{dc} = sC_{dc} l_{dc} \Delta \delta \). So \( \Delta v_k \) lags \( \Delta P_{UPFC} \) by 90 degrees in phase.
and hence in the same phase with $\Delta \omega$, which can be expressed as $\Delta u_{dc} = K_{DC} \Delta \omega$.

Therefore, from the unified model of the power system installed with the UPFC of Fig. 2, we can obtain the “direct electric torque” [10] contribution from the UPFC DC voltage regulator to the electromechanical oscillation loop of the generator to be $\Delta T_{E_{DC}} = -K_{P_{dc}} T_{DC}(j\omega) \Delta u_{dc} = -K_{P_{dc}} T_{DC}(j\omega) K_{DC} \Delta \omega$. Hence, if the DC voltage regulator is a PI controller and $K_{P_{dc}} > 0$, the proportional control of the DC voltage regulator will provide the power system with negative damping torque. Whilst the integral control of the DC voltage regulator contributes no damping to system oscillations.

Parameters of an example single-machine infinite-bus power system to be installed with a UPFC are given in the Appendix. It is used to confirm the simple analysis above and demonstrate the negative effect of the UPFC DC voltage regulator on power system oscillation stability. The UPFC is equipped with power flow, voltage control function and a DC voltage regulator, 

\[
(P): \quad u_k = m_{E} = K_{P_{f}}(P_{ref} - P), \quad \text{power flow controller}
\]

\[
(AC): \quad u_k = m_{E} = K_{V_{E_{DC}}}(v_{E_{ref} - v_{E}}), \quad \text{AC voltage regulator}
\]

\[
(DC): \quad u_k = \delta_{E} = K_{DC}(v_{dc_{ref}} - v_{dc}), \quad \text{DC voltage regulator}
\]

From the Phillips–Heffron model of the power system we obtain $K_{P_{dc}} = 1.1503 > 0$, which indicates that the UPFC DC voltage regulator damages system oscillation stability. Table I presents the results of Damping Torque (DT) and oscillation mode computation from system Phillips–Heffron model, from which we can see that

1) UPFC DC voltage regulator supplies negative damping torque and reduces the damping of system oscillation mode, as indicated by the simple analysis;

2) UPFC power flow and voltage controller have little influence on system oscillation damping.

Fig. 5 gives the confirmation from the nonlinear simulation where a three-phase to-earth short circuit occurs on the transmission line between the UPFC bus and the infinite bus at 1.0 second and is cleared after 100 ms. Fig. 6 shows the variation of various signals which confirms that $\Delta u_{dc}$ varies in the same

\[
\begin{align*}
\dot{X} &= AX + Bu + D\Delta u \\
\Delta u &= [\Delta m_E \quad \Delta \delta_E \quad \Delta m_B \quad \Delta \delta_B]^T
\end{align*}
\]

TABLE I

<table>
<thead>
<tr>
<th>Study Case</th>
<th>DT</th>
<th>Oscillation Mode</th>
<th>Damping Ratio ($\zeta$)</th>
<th>Damping Frequency ($\omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No UPFC</td>
<td>0.0</td>
<td>-0.1834±j4.6964</td>
<td>$\zeta = 0.04, \omega = 0.74$</td>
<td></td>
</tr>
<tr>
<td>With (P)</td>
<td>0.0</td>
<td>-0.1721±j4.4505</td>
<td>$\zeta = 0.04, \omega = 0.71$</td>
<td></td>
</tr>
<tr>
<td>With (AC)</td>
<td>0.0</td>
<td>-0.1850±j4.3361</td>
<td>$\zeta = 0.04, \omega = 0.69$</td>
<td></td>
</tr>
<tr>
<td>With (DC)</td>
<td>-3.1</td>
<td>-0.0550±j4.4588</td>
<td>$\zeta = 0.01, \omega = 0.71$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5. Non-linear simulation to test the effect of UPFC control on system oscillation damping.
phase with $\Delta \omega$ and $\Delta P_{\text{UPFC}}$ is opposite to $\Delta P_e$ as indicated by the simple analysis.

V. SELECTION OF UPFC INPUT CONTROL SIGNALS FOR APPLYING DAMPING CONTROL

To introduce a damping function into the UPFC, the output control signal from the damping controller can be selected among $m_E$, $m_B$, $\delta_E$ and $\delta_B$ so that a most effective damping control can be achieved (to obtain the satisfactory damping control at minimum control cost, i.e., lowest gain value of the damping controller). Based on the Phillips–Heffron model, the selection can be made as follows.

The state equation of the power system of (2.4) and (3.7) can be arranged to be

$$
\begin{bmatrix}
\Delta \delta_j \\
\Delta \omega_j \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_0 & 0 \\
-k_j & -d_j & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_j \\
\Delta \omega_j \\
x
\end{bmatrix} +
\begin{bmatrix}
0 \\
B_2 \\
B_3
\end{bmatrix} \Delta u_k
$$

Then it can be proved that $b_i(\lambda_i) = K_{bi}(\lambda_i)w_{i2}$ [13], where $K_{bi}(\lambda_i) = B_2 + A_{23}(\lambda_i I - A_{33})^{-1}B_3$, $b_i(\lambda_i)$ is the controllability index of the UPFC damping controller on the oscillation mode $\lambda_i$ of interest and $w_{i2}$ is the $i$th element of the left eigenvector of system state matrix associated with $\lambda_i$. From (2.4) and (3.7) it can be seen that for different input control signals of the UPFC, only $b_i(\lambda_i)$ changes. Therefore, to the oscillation mode of interest $\lambda_i$, $m_E$, $m_B$, $\delta_E$ and $\delta_B$ have same value of $w_{i2}$ and modal observability index $c_i(\lambda_i)$. Therefore, $|K_{bi}(\lambda_i)|$ can be used for the selection of the most effective input control signal among $m_E$, $m_B$, $\delta_E$ and $\delta_B$. In the following, this application based on the Phillips–Heffron model of the power system installed with the UPFC will be demonstrated by an example three-machine power system as shown by Fig. 7.

Table II gives the calculation results based on the Phillips–Heffron model for $u_k$ to be $m_E$, $m_B$, $\delta_E$ and $\delta_B$ is calculated respectively from system Phillips–Heffron model. The results are given in Table II. From Table II it can be seen that the calculation of $|K_{bi}(\lambda_i)|_{uk}$ predicts that the most effective signal is $m_E$ for the design of the damping controller.

To confirm the prediction, a pure-gain damping controller is designed and installed. To achieve an improvement of the damping ratio of the oscillation mode to around 0.1, the gain value of the damping controller, $K$, has to be set to be 1) $K = 4.0$ when $u_k = m_E$; 2) $K = 6.0$ when $u_k = m_B$; 3) $K = 10.0$ when $u_k = \delta_E$; 4) The oscillation mode is uncontrollable when $u_k = \delta_B$. The assignment of the oscillation mode by the damping controller is shown in the third column in Table II.

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VI. CONCLUSIONS

The major contributions of this paper are

1) Establishment of the linearized Phillips–Heffron model of single-machine and multi-machine power systems
installed with a UPFC, which adds the UPFC into the category of FACTS controllers for their unified model [10], [11].

2) The applications of the Phillips-Heffron model are demonstrated by 1) studying the effect of UPFC DC voltage regulator on power system oscillation stability; 2) selecting the most effective damping control signals for the design of the UPFC damping controller.

**APPENDIX**

Example single-machine infinite-bus power system:

\[ H = 4.0 \text{ s}, \quad D = 0.0, \quad T_{\delta 0} = 5.044 \text{ s}, \quad x_d = 1.0, \quad x_q = 0.6, \quad x_d' = 0.3, \quad x_T = 0.03, \quad x_{E T} = x_{EB} = 0.3, \quad K_A = 10.0, \quad T_A = 0.01 \text{ s}, \quad \alpha_{\delta 0} = 10 \text{ kV}, \quad K_{PF} = 10.0, \quad K_{V ET} = 10.0, \quad K_{DC} = 2.0 \text{ p.u.}, \quad P_e0 = 1.0, \quad V_{BO} = V_{B0} = 1.0, \quad T_{DC} \text{ (converter time constant)} = 0.01 \text{ s}. \]

Example three-machine power system:

\[ H_1 = H_2 = 20.09 \text{ s}, \quad H_3 = 11.8 \text{ s}, \]
\[ D_1 = D_2 = D_3 = 0.0, \quad T_{\delta 01} = T_{\delta 02} = 7.5 \text{ s}, \quad T_{\delta 03} = 4.7 \text{ s}, \quad x_d1 = x_d2 = 0.19, \quad x_d3 = 0.41, \]
\[ x_{d1} = x_{d2} = 0.163, \quad x_{d3} = 0.33, \]
\[ x_{d1}' = x_{d2}' = 0.0765, \]
\[ x_{d3}' = 0.173, \quad K_{A1} = K_{A3} = 20.0, \]
\[ K_{A2} = 100, \quad T_{A1} = T_{A3} = 0.05 \text{ s}, \quad T_{A2} = 0.01 \text{ s}, \quad Z_{13} = j0.6 \text{ (double lines),} \]
\[ Z_{23} = j0.1, \quad Z_{T2} = Z_{T3} = j0.03, \quad V_{31} = 1.0\angle14^\circ, \quad V_{21} = 1.0\angle5^\circ, \quad V_{31} = 1.0\angle0^\circ, \quad L_3 = 1.07 + j1.0, \quad K_{DC} = 5.0, \quad T_{DC} = 0.01 \text{ s}. \]

**REFERENCES**


HaiFeng Wang is a Lecturer in University of Bath, U.K. His research interests are power system analysis and control. He is a Chartered Engineer.