Formal Derivation of Direct Torque Control for Induction Machines

Zakdy Sorchini, *Student Member, IEEE*, and Philip T. Krein, *Fellow, IEEE*

***Abstract*—Direct torque control (DTC) is an induction motor control technique that has been successful because it explicitly considers the inverter stage and uses few machine parameters, while being more robust to parameter uncertainty than field-oriented control (FOC). This paper presents a formal derivation of DTC based on singular perturbation and nonlinear control tools. The derivation elaborates an explicit relationship between DTC performance and machine characteristics; low-leakage machines are expected to perform better under DTC. It is shown that DTC is a special case of a sliding-mode controller based on the multiple time-scale properties of the induction machine. The known troublesome machine operating regimes are predicted and justified. Explicit conditions to guarantee stability are presented. DTC is shown to be a suboptimal controller because it uses more control effort than is required for flux regulation. Finally, compensation strategies that extend DTC are discussed. The derivation does not require space vector concepts thus, it is established that the traditional link between DTC and space vectors is not fundamental.**

***Index Terms*—AC motors, direct torque control (DTC), induction motor drives, singularly perturbed systems, vector control.**

I. Introduction

FIELD-ORIENTED control (FOC) [1] and direct torque control (DTC) [2], [3] are common for induction motor control. DTC is attractive because it uses few machine parameters and explicitly considers the inverter stage. Both techniques were originally derived from heuristic field geometric arguments. Initially it was a challenge to understand their principles and limitations. Since then, FOC has been extensively studied and its properties are well understood [4], [5]. A link between FOC and feedback linearization has been established [6], [7]. Significant work has been done to improve DTC [8] and the heuristic derivation of DTC has been clarified [9], but rigorous derivations are still scarce. In [10], geometric control tools are used to formally derive DTC. In that reference, a decoupling property of DTC is revealed. It is shown that performance will be as predicted at low speed under light load, and an improved DTC strategy using correction terms is proposed.

Manuscript received October 9, 2005; revised July 1, 2006. This work was supported by the Grainger Center for Electric Machinery and Electromechanics. This work was presented inpart at the Power Electronics Specialists Conference, Recife, Brazil, 2005

Z. Sorchini was with the Grainger Center for Electric Machinery and Electromechanics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: zsorchini@ieee.org). He is now with Delphi Electronics and Safety One Corporate Center, Kokomo, IN 46904-9005 USA.

P. T. Krein is with with the Grainger Center for Electric Machinery and Electromechanics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: krein@uiuc.edu)

Digital Object Identifier 10.1109/TPEL.2006.882086

The objective of this paper is to improve the theoretical understanding of DTC, particularly of the flux and torque regulating mechanisms, and the implications of flux angle quantization (i.e., sector condition) and the power electronics inverter. An alternative formal derivation of DTC is presented, based on singular perturbation techniques rather than space-vector concepts. The torque and flux magnitude dynamics are considered explicitly and a sliding-mode controller for flux regulation and torque tracking is derived. If a quantized version of the flux angle is used and only the voltages available from the inverter are considered, the resulting control is conceptually equivalent to DTC. The analysis confirms the main results of [10]. In addition, motor parameters are linked to good performance. Also, it is shown that DTC is not an optimal controller since it uses more control effort than is required for flux regulation. Finally, compensation strategies that extend DTC are discussed. For simplicity, full state measurements are assumed here.

II. Singular Perturbation Theory Background

A singularly-perturbed dynamic system is one that exhibits multiple time-scale behavior, i.e., the evolution of some variables is "slow," while for others it is "fast" [11]–[13]. For systems in the so-called standard singular perturbation form

$$\begin{aligned} \dot{x} &= f(x, z, \varepsilon, u) \\ \varepsilon \dot{z} &= g(x, z, \varepsilon, u) \end{aligned} \tag{1}$$

a small parameter ε exists such that in the limit as the parameter goes to zero, the order of the model is reduced. For this form to be a valid singularly-perturbed model, certain technical conditions have to be satisfied [12]. The theory of singular perturbations is useful for systems with distinct time scales and formalizes the $\varepsilon = 0$ approximation. The typical application is to separate a complicated system into fast and slow dynamics.

The model (1), in which x and z are the slow and fast variables, respectively, is said to be the system representation for the slow (or normal) time scale t. The system representation in the fast time scale, defined as $t_f = t/\varepsilon$, is given by

$$\begin{aligned} \frac{dx}{dt_f} &= \varepsilon f(x, z, \varepsilon, u) \\ \frac{dz}{dt_f} &= g(x, z, \varepsilon, u) \end{aligned} \tag{2}$$

where the slow variables x are treated as parameters in the state equation for z. The equilibrium points of the fast system move slowly according to the value of the slow variables; this behavior is known as quasi- or dynamic steady state [12]. For both the slow and fast representation of the system, the corresponding reduced models are obtained by setting $\varepsilon = 0$ in (1) and (2), respectively.

An important tool of the singular perturbation method is composite feedback control [12], [13], which decomposes the control signal as $u = u_{\text{slow}} + u_{\text{fast}}$, with the slow controller being only a function of the slow variables. Control of singularly-perturbed systems is simplified because the slow and fast dynamics can be considered independently. When a controller based on the multiple time-scale properties of the system is used, the behavior of the actual system will remain "ε-close" to that predicted by the analysis and design.

III. Sliding-Mode Control Background

Sliding-mode control is a specific type of variable structure control [14], [15]. For nonlinear systems of the formh

$$\dot{x} = h(x) + g(x)u \tag{3}$$

where h and g are unknown functions, sliding-mode control can achieve high robustness. This is done by forcing the system's dynamic behavior through switching. Formally, a sliding surface or manifold $s = 0$ is defined such that the dynamic behavior of the system is forced while on the manifold. The design process guarantees no matter what is the initial condition the states will reach the sliding manifold in finite time. Once on the manifold, the states "slide" independently of the systems dynamics as dictated by the control objective. A typical control for a sliding-mode design is of the form

$$u = -\beta(x)\text{sgn}(s) \tag{4}$$

where β is a function that bounds the uncertainty of the system; commonly, the function β is just a large gain k [15].

Sliding mode control has been considered for singularly-perturbed systems [16], in which controllers are designed for slow and fast subsystems. Because of the discontinuous nature of the slow controller, the time-scale separation argument does not hold during switching. Therefore, additional technical conditions have to be satisfied to guarantee stability [16].

IV. Induction Machine Model

The well-known, two-phase equivalent model of a symmetrical, balanced, squirrel-cage induction machine in the stationary reference frame and with rotor quantities referred to the stator is given by [4]

$$\begin{aligned}
\frac{d\omega}{dt} &= \frac{1}{J}(T_e - T_L)\\
\frac{\psi_{sa}}{dt} &= -R_s i_{sa} + u_{sa}\\
\frac{\psi_{sb}}{dt} &= -R_s i_{sb} + u_{sb}\\
\frac{\psi_{ra}}{dt} &= -R_r i_{ra} - n_p\omega\psi_{rb}\\
\frac{\psi_{rb}}{dt} &= -R_r i_{rb} + n_p\omega\psi_{ra}
\end{aligned} \tag{5}$$

where ω, ψ, i, and u, denote mechanical speed, flux linkage, current, and voltage, respectively; J, n_p, M, L, R, T_e, and T_L denote inertia, number of pole pairs, mutual inductance, self inductance, resistance, electromagnetic torque, and load torque, respectively; subscripts s and r stand for stator and rotor; and (a, b) denote the components of a vector in the stationary reference frame. For simplicity, the terms flux linkage and flux will be used interchangeably. The electromagnetic torque of the machine is given by

$$T_e = \frac{3}{2}\frac{n_p M}{L_r}(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}). \tag{6}$$

Currents and fluxes are related by nonlinear magnetic equations. If magnetic saturation is not present, the relationship can be approximated linearly as

$$\begin{aligned}
\psi_{sa} &= L_s i_{sa} + M i_{ra}\\
\psi_{sb} &= L_s i_{sb} + M i_{rb}\\
\psi_{ra} &= L_r i_{ra} + M i_{sa}\\
\psi_{rb} &= L_r i_{rb} + M i_{sb}.
\end{aligned} \tag{7}$$

In terms of stator fluxes and currents the model is

$$\begin{aligned}
\frac{d\omega}{dt} &= \frac{1}{J}(T_e - T_L)\\
\frac{d\psi_{sa}}{dt} &= -R_s i_{sa} + u_{sa}\\
\frac{d\psi_{sb}}{dt} &= -R_s i_{sb} + u_{sb}\\
\sigma\frac{di_{sa}}{dt} &= \frac{R_r}{L_s L_r}\psi_{sa} + \frac{n_p}{L_s}\omega\psi_{sb} - \sigma n_p\omega i_{sb}\\
&\quad - \frac{\gamma}{L_s} i_{sa} + \frac{1}{L_s}u_{sa}\\
\sigma\frac{di_{sb}}{dt} &= \frac{R_r}{L_s L_r}\psi_{sb} - \frac{n_p}{L_s}\omega\psi_{sa} + \sigma n_p\omega i_{sa}\\
&\quad - \frac{\gamma}{L_s} i_{sb} + \frac{1}{L_s}u_{sb}
\end{aligned} \tag{8}$$

where $\sigma = 1 - M^2/(L_s L_r)$ and $\gamma = L_s R_r/L_r + R_s$. The electromagnetic torque is given by

$$T_e = \frac{3}{2}n_p(\psi_{sa} i_{sb} - \psi_{sb} i_{sa}). \tag{9}$$

The model (8) is in the standard singular perturbation form, with speed and fluxes as slow variables and currents as fast variables. The leakage factor σ is the perturbation parameter.

Consistent with the control objectives, it is natural to define a (parameter-independent) state transformation

$$\begin{aligned}
\tau &= \psi_{sa} i_{sb} - \psi_{sb} i_{sa}\\
\phi &= \psi_{sa}^2 + \psi_{sb}^2\\
\rho &= \arctan(\psi_{sb}/\psi_{sa})\\
\eta &= \psi_{sa} i_{sa} + \psi_{sb} i_{sb}
\end{aligned} \tag{10}$$

where τ is a normalized torque, ϕ is the flux magnitude squared, and ρ is the flux angle. The definition of η follows from the choice of the other states. Although the transformation is not defined for zero flux, this limitation is common in induction motor control and is not a critical concern. If desired, flux can be initialized to a nonzero value.

Given stator input voltages u_{sa} and u_{sb}, input transformations based on the flux angle are useful and yield equivalent inputs based on

$$\begin{bmatrix} u_\phi \\ u_\tau \end{bmatrix} = \begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix}. \tag{11}$$

By using (10) and (11), (8) can be transformed into

$$\begin{aligned}
\frac{d\omega}{dt} &= \frac{1}{J}\left(\frac{3}{2}n_p\tau - T_L\right)\\
\frac{d\phi}{dt} &= -2(R_s\eta - \sqrt{\phi}u_\phi)\\
\frac{d\rho}{dt} &= -R_s\frac{\tau}{\phi} + \frac{1}{\sqrt{\phi}}u_\tau\\
\sigma\frac{d\tau}{dt} &= -\frac{\gamma}{L_s}\tau - \frac{n_p}{L_s}\omega\phi + \frac{1}{L_s}\sqrt{\phi}u_\tau\\
&\quad + \sigma\left(n_p\omega\eta + \frac{1}{\sqrt{\phi}}(-\eta u_\tau + \tau u_\phi)\right)\\
\sigma\frac{d\eta}{dt} &= -\frac{\gamma}{L_s}\eta + \frac{R_r}{L_sL_r}\phi + \frac{1}{L_s}\sqrt{\phi}u_\phi\\
&\quad - \sigma\left(n_p\omega\tau + R_s\left(\frac{\eta^2+\tau^2}{\phi}\right)\right.\\
&\qquad \left. + \frac{1}{\sqrt{\phi}}(-\eta u_\phi - \tau u_\tau)\right).
\end{aligned} \tag{12}$$

The model (12) is in the standard singular perturbation form with σ as the perturbation parameter. Speed, flux magnitude squared and flux angle are the slow variables while normalized torque and η are the fast ones. Given control voltages u_ϕ and u_τ, the actual instantaneous voltages that need to be applied to the machine can be obtained by using

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \cos\rho & -\sin\rho \\ \cos\left(\rho - \frac{2\pi}{3}\right) & -\sin\left(\rho - \frac{2\pi}{3}\right) \\ \cos\left(\rho + \frac{2\pi}{3}\right) & -\sin\left(\rho + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} u_\phi \\ u_\tau \end{bmatrix} \tag{13}$$

which is the inverse of (11) combined with a two-phase to three-phase transformation [4].

V. Alternative DTC Derivation

The state equation for ϕ in (12) clearly shows that to drive the flux magnitude in a desired direction, u_ϕ must be made sufficiently positive or negative. It is not immediately clear from (12) how to control the torque. Recalling that for a well-designed machine σ is typically small, a sufficiently positive or negative u_τ should be able to drive torque in the desired direction. Instantaneous input voltages can then be obtained from u_ϕ and u_τ using (13), and depend on flux position. This is the heuristic justification behind DTC. It has been shown [17], that DTC can be derived as a quantized approximation to a continuous-time controller. A more rigorous derivation which confirms that DTC is a sliding-mode controller is presented below.

A. Sliding-Mode Controller for Flux Regulation and Torque Tracking

Details about the formal procedure to design a sliding-mode controller can be found in [15]. To derive the controller for flux regulation, the slow, reduced model is obtained from (12) by setting $\sigma = 0$

$$\begin{aligned}
\frac{d\omega}{dt} &= \frac{1}{J}\left(\frac{3}{2}n_p\tau - T_L\right)\\
\frac{d\phi}{dt} &= -2(R_s\eta - \sqrt{\phi}u_{\phi,\text{slow}})\\
\frac{d\rho}{dt} &= -R_s\frac{\tau}{\phi} - \frac{1}{\sqrt{\phi}}u_{\tau,\text{slow}}\\
0 &= -\frac{\gamma}{L_s}\tau - \frac{n_p}{L_s}\omega\phi + \frac{1}{L_s}\sqrt{\phi}u_{\tau,\text{slow}}\\
0 &= -\frac{\gamma}{L_s}\eta + \frac{R_r}{L_sL_r}\phi + \frac{1}{L_s}\sqrt{\phi}u_{\phi,\text{slow}}.
\end{aligned} \tag{14}$$

Only the slow part of the control signals has to be considered [12]. From the second algebraic relationship, the fast variable η can be expressed in terms of the slow variables as

$$\eta = \frac{1}{\gamma}\left(\frac{R_r}{L_r}\phi + \sqrt{\phi}u_{\phi,\text{slow}}\right). \tag{15}$$

Only flux magnitude is of interest in this time scale. The dynamic equation of speed is immaterial, since torque control is going to be performed. The flux angle dynamics represent the zero dynamics of the system [18], and do not affect stability of (14) since they are stable and periodic [10]. The angle can be viewed as an external variable that determines the control inputs u_ϕ and u_τ according to (11). Therefore the slow control for torque can be set to zero by design. By using (15), the state equation for flux magnitude squared can be written as

$$\begin{aligned}
\frac{d\phi}{dt} &= -\frac{2R_sR_r}{\gamma L_r}\phi + 2\sqrt{\phi}\left(\frac{\gamma - R_s}{\gamma}\right)u_{\phi,\text{slow}}\\
&= h_\phi(\phi) + g_\phi(\phi)u_{\phi,\text{slow}}.
\end{aligned} \tag{16}$$

For flux regulation, a sliding surface is defined as

$$s_\phi = \phi - \phi_{\text{ref}} = e_\phi = 0 \tag{17}$$

which has a time derivative equal to

$$\frac{ds_\phi}{dt} = \frac{d\phi}{dt} = h_\phi(\phi) + g_\phi(\phi)u_\phi \tag{18}$$

since the flux reference is constant. By defining a minimum controller gain [15]

$$\begin{aligned} k_{\phi,\min} &:= \frac{2R_s}{L_s}\sqrt{\phi_{\text{rated}}} \\ &> \frac{R_s}{L_s}\sqrt{\phi} = \left|\frac{h_\phi(\phi)}{g_\phi(\phi)}\right| \end{aligned} \tag{19}$$

where ϕ_{rated} is the squared, rated flux linkage of the machine, a sliding-mode controller for flux regulation is proposed as

$$u_\phi = -(k_{\phi,\min}+\delta)\text{sgn}(s_\phi) = -(k_{\phi,\min}+\delta)\text{sgn}(e_\phi) \tag{20}$$

for any $\delta > 0$. Stability of the flux controller is proved by considering the Lyapunov function $V_\phi = (1/2)s_\phi^2$ [15], as is conventional in sliding mode control analysis. The time derivative of V_ϕ is given by

$$\begin{aligned} \frac{dV_\phi}{dt} &= s_\phi \frac{ds_\phi}{dt} \\ &= s_\phi h_\phi(\phi) + g_\phi(\phi) s_\phi u_\phi \\ &\le g_\phi(\phi) k_{\phi,\min}|s_\phi| - g_\phi(\phi)(k_{\phi,\min}+\delta)|s_\phi| \\ &= -\delta g_\phi(\phi)|s_\phi| \\ &\le -2\delta\left(\frac{R_s+\gamma}{\gamma}\right)\sqrt{\phi_0}|s_\phi| \end{aligned} \tag{21}$$

where ϕ_0 is the remanent flux of the machine. Therefore, provided that there is remanent flux in the machine, flux regulation is achieved for any initial condition.

The torque tracking controller can be derived in a similar way. (Regulation in the fast time scale implies tracking in the normal time scale.) First, the fast, reduced model is considered

$$\begin{aligned} &\frac{d\omega}{dt_f} = \frac{d\phi}{dt_f} = \frac{d\rho}{dt_f} = 0 \\ &\frac{d\tau}{dt_f} = -\frac{\gamma}{L_s}\tau - \frac{n_p}{L_s}\omega\phi + \frac{1}{L_s}\sqrt{\phi}(u_{\tau,\text{slow}}+u_{\tau,\text{fast}}) \\ &\frac{d\eta}{dt_f} = -\frac{\gamma}{L_s}\eta + \frac{R_r}{L_s L_r}\phi + \frac{1}{L_s}\sqrt{\phi}(u_{\phi,\text{slow}}+u_{\phi,\text{fast}}) \end{aligned} \tag{22}$$

which results by scaling time as $t_f = t/\sigma$ and by setting $\sigma = 0$. Both the slow and fast parts of the control inputs are used. The slow variables are treated as parameters. Since only the torque equation is of interest, it is rewritten as

$$\begin{aligned} \frac{d\tau}{dt_f} &= -\frac{\gamma}{L_s}\tau - \frac{n_p}{L_s}\omega\phi + \frac{1}{L_s}\sqrt{\phi}u_{\tau,\text{fast}} \\ &= h_\tau(\tau,\omega,\phi) + g_\tau(\phi)u_{\tau,\text{fast}} \end{aligned} \tag{23}$$

where the fact that the slow control for torque is zero is used. A sliding surface is then defined as

$$s_\tau = \tau - \tau_{\text{ref}} = e_\tau = 0 \tag{24}$$

whose derivative with respect to (w.r.t.) the fast time variable is given by

$$\frac{ds_\tau}{dt_f} = \frac{d\tau}{dt_f} = h_\tau(\tau,\omega,\phi) + g_\tau(\phi)u_{\tau,\text{fast}} \tag{25}$$

where it is assumed that the derivative of the torque reference w.r.t. the fast time variable is zero, i.e., the variation of τ_{ref} is slow in the sense of the fast time scale. By defining a minimum controller gain [15]

$$\begin{aligned} k_{\tau,\min} &:= 2\gamma\frac{|\tau_{\max}|}{\sqrt{\phi_{\min}}} + 2n_p|\omega|\sqrt{\phi_{\text{rated}}} \\ &< \gamma\frac{|\tau|}{\sqrt{\phi}} + n_p|\omega|\sqrt{\phi} = \left|\frac{h_\tau(\phi)}{g_\tau(\phi)}\right| \end{aligned} \tag{26}$$

where $\tau_{\max}$ and ϕ_{min} are the maximum dynamic torque and minimum flux level to be requested, respectively, the sliding-mode controller for torque tracking is proposed as

$$u_{\tau,\text{fast}} = -(k_{\tau,\min}+\delta)\text{sgn}(s_\tau) = -(k_{\tau,\min}+\delta)\text{sgn}(e_\tau) \tag{27}$$

for any $\sigma > 0$. Stability of the torque controller is proved by considering the Lyapunov function $V_\tau = (1/2)s_\tau^2$ [15]. The derivative of V_τ w.r.t. the fast time variable is given by

$$\begin{aligned} \frac{dV_\tau}{dt_f} &= s_\tau \frac{ds_\tau}{dt_f} \\ &= s_\tau h_\tau(\phi) + g_\tau(\phi)s_\tau u_\tau \\ &\le g_\tau(\phi)k_{\tau,\min}|s_\tau| - g_\tau(\phi)(k_{\tau,min}+\delta)|s_\tau| \\ &= -\delta g_\phi(\phi)|s_\phi| \\ &\le -\frac{\delta}{L_s}\sqrt{\phi_0}|s_\phi|. \end{aligned} \tag{28}$$

Again, provided that there is a remanent flux in the machine, torque tracking (in the sense of the slow time scale) is achieved. Finally, the actual controller is obtained from the slow and fast parts and is given by

$$\begin{aligned} u_\phi &= u_{\phi,\text{slow}} + u_{\phi,\text{fast}} = -(k_{\phi,\min}+\delta)\text{sgn}(e_\phi) \\ u_\tau &= u_{\tau,\text{slow}} + u_{\tau,\text{fast}} = -(k_{\tau,\min}+\delta)\text{sgn}(e_\tau). \end{aligned} \tag{29}$$

Because of (13), the actual instantaneous voltages that need to be applied to the machine depend on the flux angle.

Since flux control is based on the slow reduced model, it is important that the state η converges to the slow manifold, i.e., that (15) holds after the fast transient. To simplify the analysis, define an auxiliary variable

$$\xi = \eta - \frac{1}{\gamma}\left(\frac{R_r}{L_r}\phi + \sqrt{\phi}u_{\phi,\text{slow}}\right) \tag{30}$$

which is equal to the difference between η and its slow manifold. The derivative of ξ w.r.t. the fast time variable is given by

$$\frac{d\xi}{dt_f} = \frac{d\eta}{dt_f} - \frac{R_r}{\gamma L_r}\frac{d\phi}{dt_f} - \frac{1}{\gamma}\left(\sqrt{\phi}\frac{du_{\phi,\text{slow}}}{dt_f} + u_{\phi,\text{slow}}\frac{d\sqrt{\phi}}{dt}\right). \quad (31)$$

By using (22) the expression simplifies to

$$\frac{d\xi}{dt_f} = -\frac{\gamma}{L_s}\xi + \frac{1}{L_s}\sqrt{\phi}u_{\phi,\text{fast}} - \frac{1}{\gamma}\sqrt{\phi}\frac{du_{\phi,\text{slow}}}{dt_f}. \quad (32)$$

In a standard singular-perturbation controller, the derivative of the slow control, the last term in (32), would be zero. However, here $u_{\phi,\text{slow}}$, slow is a discontinuous signal. The derivative must be considered and can be viewed as an impulsive disturbance [16]. As discussed there, for zero fast control and zero disturbance, ξ is exponentially stable, therefore $u_{\phi,\text{fast}}$, fast can be set to zero by design. If it is assumed that the disturbance is fast w.r.t. the fast time scale, then the response observed in ξ due to this disturbance will be small. In essence, the discontinuity in the slow controller causes η to move away instantaneously from the slow manifold. Then the state quickly converges back to the manifold. Controller stability is not compromised [16].

The minimum gains specified by (19) and (26) are given in terms of the parameters of the machine. Typically machine parameters are not perfectly known. By considering bounds on the machine parameters, new gains can be determined. Robustness is then obtained by increasing the controller gains beyond the minimum values [15]. Therefore, the controller can then be redefined as

$$\begin{aligned} u_\phi &= -\,k_\phi \text{sgn}(e_\phi) \\ u_\tau &= -\,k_\tau \text{sgn}(e_\tau) \end{aligned} \quad (33)$$

where $k_\phi > k_{\phi,\min}$ and $k_\tau > k_{\tau,\min}$ are obtained by taking into account the parametric uncertainty of the model.

B. Flux Angle Quantization and Inverter Considerations

Without loss of generality, only positive and negative values of (33) can be used (identically-zero-errors are almost impossible in an actual implementation). Fig. 1 shows the machine voltages as a function of the flux angle for all combinations of the signs of u_ϕ and u_τ, i.e., for the case of unity gains. The control law jumps between sets of voltages (which evolve continuously as a function of the flux angle) according to the sign of the errors.

Now, consider the case in which only quantized information about the flux angle is available, with the quantizer given by

$$\rho_q = \frac{2\pi}{n}\text{round}\left(\frac{n\rho}{2\pi}\right) \quad (34)$$

where n defines the number of quantization steps. There is one restriction on n: for a given u_ϕ and u_τ, the resulting phase voltages should preserve the sign information of the control voltages. It is not difficult to show, by using (13) and its inverse

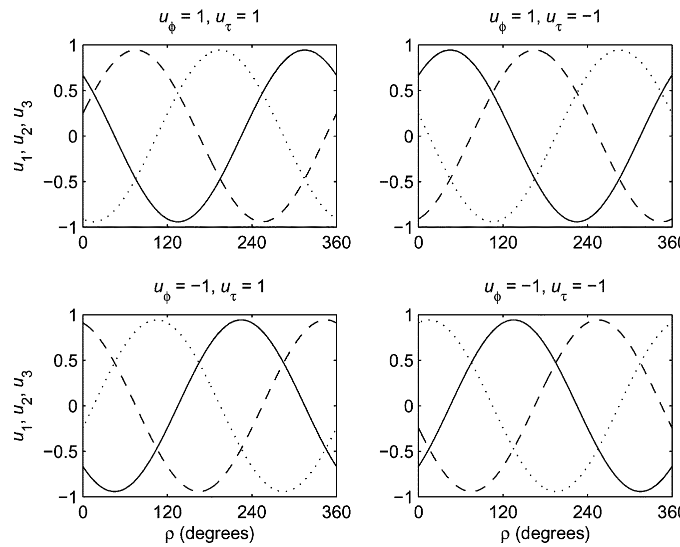

Fig. 1. Machine voltages as a function of the flux angle for given control signals.

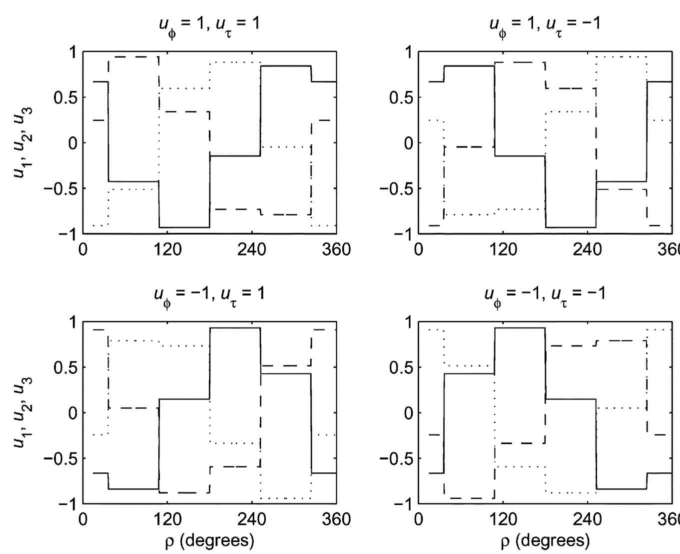

Fig. 2. Machine voltages as a function of the flux angle for given control signals, when the flux angle is quantized with $n = 5$.

transformation, that the smallest number that satisfies the restriction is $n = 5$. The resulting phase voltages are shown in Fig. 2.

Using the quantized flux angle instead of the actual angle implies that the controller gain will change. Since a minimum gain is required to guarantee stability, the actual controller gain has to be increased by a factor k_q which depends on the number of quantization steps. Because of the nonlinear nature of (13) w.r.t. the flux angle, it is easier to find the minimum gain by numeric analysis. For $n = 5$, a gain $k_q = 5$ will preserve the minimum gain requirement. For any larger n this gain will decrease. Therefore the controllers can be redefined to compensate for quantization as

$$\begin{aligned} u_\phi &= -\,k_q k_\phi \text{sgn}(e_\phi) \\ u_\tau &= -\,k_q k_\tau \text{sgn}(e_\tau) \end{aligned} \quad (35)$$

but now

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \cos\rho_q & -\sin\rho_q \\ \cos\left(\rho_q - \frac{2\pi}{3}\right) & -\sin\left(\rho_q - \frac{2\pi}{3}\right) \\ \cos\left(\rho_q + \frac{2\pi}{3}\right) & -\sin\left(\rho_q + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} u_\phi \\ u_\tau \end{bmatrix} \quad (36)$$

must be used to obtain the machine voltages.

In Fig. 2, the machine voltages resulting from quantization do not form a balanced, three-phase set. By increasing n the quality of the waveforms can be improved. Under normal operation this is perhaps immaterial since the control law jumps between voltage sets relatively fast. Only when the errors do not change sign for a significant amount of time (i.e., for several cycles of the flux angle) the waveform quality might be important. For $n = 5$ the voltage waveforms are not ideal; $n = 6$ makes the waveforms equal (displaced by 120°), and $n = 12$ makes the waveforms a balanced, three-phase set. In general, multiples of $n = 6$ will provide good quality waveforms.

Since an inverter is used to actuate the machine, the phase voltages given by (33) or (35) have to be approximated, e.g., with PWM. If an approximation is not used, then the instantaneous behavior of the inverter has to be considered. A three-phase inverter feeding a balanced load can only provide instantaneous phase voltages equal to $\pm 2/3 V_{\mathrm{dc}}$, $\pm 1/3 V_{\mathrm{dc}}$ and 0, where V_{dc} is the dc-bus voltage. These values cannot be obtained arbitrarily per phase, since the voltages are not independent. At any given instant in time, an inverter state, i.e., which switches are on or off, has to be selected in such a way as to minimize the error between the phase voltages determined by the control law and the available voltages. Thus, the inverter can be viewed as a nonlinear gain stage with discrete-valued outputs. It should be pointed out that this fixed output resolution implies that increasing the quantization of the flux angle does not have a significant impact for large n.

The sliding-mode controller only requires that the effective gains be greater than the minimum gains needed for stability. Therefore, the nonlinear gain effect of the inverter can be neglected as long as a lower bound for the inverter gain can be determined. Fig. 3 shows the phase voltages from the inverter for the case of $n = 5$, $V_{\mathrm{dc}} = 1$ and unity controller gains. By comparing it to Fig. 2, the nonlinear gain of the inverter is evident. By using the difference between the desired voltages and the inverter voltages, it can be shown that a gain of three (i.e., $V_{\mathrm{dc}} = 3$) is required to make the instantaneous inverter voltage more positive or negative than the desired voltage. In fact, this gain can be used regardless of n. Therefore the use of an inverter can be modeled as applying the control voltages

$$\begin{aligned} u_\phi &= -\frac{1}{3} V_{\mathrm{dc}} \mathrm{sgn}(e_\phi) \\ u_\tau &= -\frac{1}{3} V_{\mathrm{dc}} \mathrm{sgn}(e_\tau) \end{aligned} \quad (37)$$

to the machine. This means that if the conditions

$$V_{\mathrm{dc}} > 3k_q k_\phi, \quad V_{\mathrm{dc}} > 3k_q k_\tau \quad (38)$$

are satisfied, stability is guaranteed when using (37) and (36), i.e., when the discrete inverter voltages are considered and only

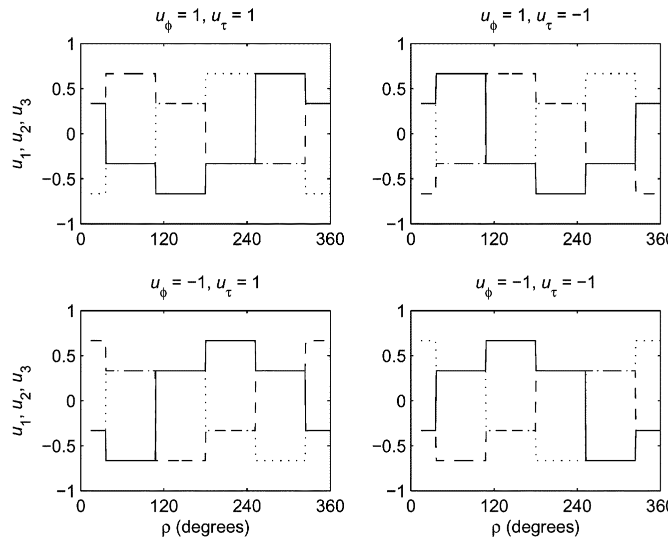

Fig. 3. Machine voltages as a function of the flux angle for given control signals, when flux angle is quantized with $n = 5$, and an inverter with $V_{\mathrm{dc}} = 1$ is considered.

the sign of the errors and the quantized flux angle are used to determine the inverter state. By defining an auxiliary vector w as

$$w = \begin{bmatrix} \cos\rho_q & -\sin\rho_q \\ \cos\left(\rho_q - \frac{2\pi}{3}\right) & -\sin\left(\rho_q - \frac{2\pi}{3}\right) \\ \cos\left(\rho_q + \frac{2\pi}{3}\right) & -\sin\left(\rho_q + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} -\mathrm{sgn}(e_\phi) \\ -\mathrm{sgn}(e_\tau) \end{bmatrix} \quad (39)$$

then all the information needed to actuate the machine is contained in this vector. The sign of an element of w directly dictates which switch (positive—upper, negative—lower) will be active for the corresponding inverter leg. Because only a finite combination of quantized flux angle values and error signs exists, (39) can be easily computed offline. Then, for given flux position and flux magnitude and torque errors, the corresponding inverter state can be read from a look-up table.

Table I shows the required inverter state as a function of flux position and the flux and torque errors as dictated by (39), for the case $n = 6$. This level of quantization represents a good compromise, as already mentioned. An element of the table determines which switch in the corresponding inverter leg will be on (1-upper, −1-lower). Table II shows, with the notation used here, the switching strategy for standard DTC as defined in [3] (the errors in that reference are the negative of the errors used here). It can be seen that the elements in Table I match exactly with the corresponding elements in Table II. Therefore DTC has just been derived.

VI. DTC Stability and Performance Considerations

It was already mentioned that if the conditions (38) are satisfied, stability is guaranteed. DTC makes an implicit assumption that these conditions are satisfied for all operating conditions. By using typical values in (19), it can be seen that flux regulation is almost always guaranteed. On the contrary, from (26) it can be seen that for rated load and rated speed conditions, the conditions are almost guaranteed to be violated. Only when the

TABLE I
SWITCHING STRATEGY FOR THE QUANTIZED SLIDING MODE CONTROLLER FOR $n = 6$

		$\rho \in(-\pi/6,\pi/6)$	$\rho \in(\pi/6,\pi/2)$	$\rho \in(\pi/2,5\pi/6)$	$\rho \in(5\pi/6,-5\pi/6)$	$\rho \in(-5\pi/6,-\pi/2)$	$\rho \in(-\pi/2,-\pi/6)$
$e_\phi > 0$	$e_\tau > 0$	(–1,–1,1)	(1,–1,1)	(1,–1,–1)	(1,1,–1)	(–1,1,–1)	(–1,1,1)
	$e_\tau < 0$	(–1,1,–1)	(–1,1,1)	(–1,–1,1)	(1,–1,1)	(1,–1,–1)	(1,1,–1)
$e_\phi < 0$	$e_\tau > 0$	(1,–1,1)	(1,–1,–1)	(1,1,–1)	(–1,1,–1)	(–1,1,1)	(–1,–1,1)
	$e_\tau < 0$	(1,1,–1)	(–1,1,–1)	(–1,1,1)	(–1,–1,1)	(1,–1,1)	(1,–1,–1)

TABLE II
SWITCHING STRATEGY FOR STANDARD DTC AS IN [3]

		$\rho \in(-\pi/6,\pi/6)$	$\rho \in(\pi/6,\pi/2)$	$\rho \in(\pi/2,5\pi/6)$	$\rho \in(5\pi/6,-5\pi/6)$	$\rho \in(-5\pi/6,-\pi/2)$	$\rho \in(-\pi/2,-\pi/6)$
	$e_\tau > 0$	(–1,–1,1)	(1,–1,1)	(1,–1,–1)	(1,1,–1)	(–1,1,–1)	(–1,1,1)
$e_\phi > 0$	$e_\tau = 0$	(–1,–1,–1)	(1,1,1)	(–1,–1,–1)	(1,1,1)	(–1,–1,–1)	(1,1,1)
	$e_\tau < 0$	(–1,1,–1)	(–1,1,1)	(–1,–1,1)	(1,–1,1)	(1,–1,–1)	(1,1,–1)
	$e_\tau > 0$	(1,–1,1)	(1,–1,–1)	(1,1,–1)	(–1,1,–1)	(–1,1,1)	(–1,–1,1)
$e_\phi < 0$	$e_\tau = 0$	(1,1,1)	(–1,–1,–1)	(1,1,1)	(–1,–1,–1)	(1,1,1)	(–1,–1,–1)
	$e_\tau < 0$	(1,1,–1)	(–1,1,–1)	(–1,1,1)	(–1,–1,1)	(1,–1,1)	(1,–1,–1)

machine is running under light load at low speed can stability be guaranteed. This confirms and explains the erratic behavior of DTC observed by practitioners under high-speed, high-load operation [9]. It should be pointed out that the gain (26) guarantees stability for all operating regimes, i.e., motoring, generation and braking. For motoring operation, the gain might be too conservative. Also, (37) is an approximation that considers only the minimum gain that the inverter can provide at all times. Because of this, (38) is in general conservative on an instantaneous basis.

The derivation presented is based on the reduced models that explicitly use the $\sigma = 0$ approximation. On one hand, the singular-perturbation analysis implies that, for a small enough σ, stability is guaranteed [12], [16]. The actual behavior of the machine will be dictated by the full dynamics. In particular, the dynamic coupling between torque and η plays a significant role. Because of the nonlinear dynamics and switched nature of the inputs, analysis is particularly challenging and is beyond the scope of this paper. (The analysis can be done from the point of view of hybrid systems by considering state-dependent switching [19]. It is tractable since only four combinations of the signs of the errors exist.) On the other hand, the use of $\sigma = 0$ as a design basis has an important implication. Since σ is the leakage factor of the machine, low-leakage machines are expected to perform better. Low-leakage machines are commonly regarded as high-performance machines [5]. Therefore DTC is well suited for control of high-performance induction machines.

A. Performance Improvement

The same conditions that guarantee stability of the sliding-mode controller also make evident the main problem of DTC. From (37) it is clear that DTC uses the same control effort to regulate flux as it does to control torque. But from (38), (19), and (26), flux regulation requires less control effort. By using the proper weights for the control efforts, performance can, in principle, be improved.

It has already been mentioned that DTC can also be derived from a continuous control law [17]. The performance achievable with that control law is, in principle, comparable to that of DTC. Using a continuous controller has significant advantages, particularly since the dynamic behavior and stability properties are well-defined and easier to understand. Because of the practical significance of standard DTC in its sliding-mode control form, it is important to point how its performance can be improved.

It is known that sliding-mode control trades off robustness with ripple amplitude [15]. This is particularly true for DTC, since the structure of the induction machine dynamics is almost completely ignored. By using information about the system structure, it is possible to improve the performance while reducing ripple [15]. Since only a nominal model is required, robustness is not compromised. What this implies in the case of DTC is that speed and torque compensation are required to improve the dynamic behavior; the analysis confirms this well-known fact [10], [17].

In the derivation of DTC the use of the quantized flux angle is arbitrary. Since the flux angle is available as a continuous variable, it is reasonable to use all the information. Similarly, it was shown that taking into account the instantaneous behavior of the inverter provides no advantage and, in fact, is the reason why standard DTC does not have a constant switching frequency. In [10] it was shown that the average behavior of DTC is what dictates dynamic behavior. By using a standard sliding-mode control design, it should be possible to improve the dynamic behavior of DTC. For example, following the design procedure of [15], an alternative controller is given by

$$\begin{aligned} u_\phi &= -k_\phi \operatorname{sgn}(e_\phi) \\ u_\tau &= \frac{1}{\sqrt{\phi}}(\gamma\tau + n_p\omega\phi) - k_\tau \operatorname{sgn}(e_\tau) \end{aligned} \tag{40}$$

for which the machine voltages are now determined using (13), and the voltages have to be approximated, e.g., with PWM. (Alternatively, the sign of those voltages can be used to directly control the inverter switches.) The controller effectively uses torque and speed compensation to improve DTC performance, and is essentially the improved DTC strategy proposed in [10], where experimental results are shown. Additional considerations for a sliding-mode controller, beyond the scope of this paper, can further improve the behavior [14], [15].

TABLE III
CHARACTERISTICS OF THE MACHINE USED FOR SIMULATION

Parameter	Symbol	Value
Rated power (kW)	–	2.24
Rated voltage (V)	–	220
Rated current (A)	–	5.8
Rated torque (N·m)	$T_{e,rated}$	12.5
Rated flux linkage (Wb)	$\phi_{s,rated}^{1/2}$	0.48
Rated speed (rad/s)	–	180
Stator resistance (W)	R_s	0.435
Rotor resistance (W)	R_r	0.816
Stator inductance (mH)	L_s	71.31
Rotor inductance (mH)	L_r	71.31
Mutual inductance (mH)	M	69.31
Number of pole pairs	n_p	2

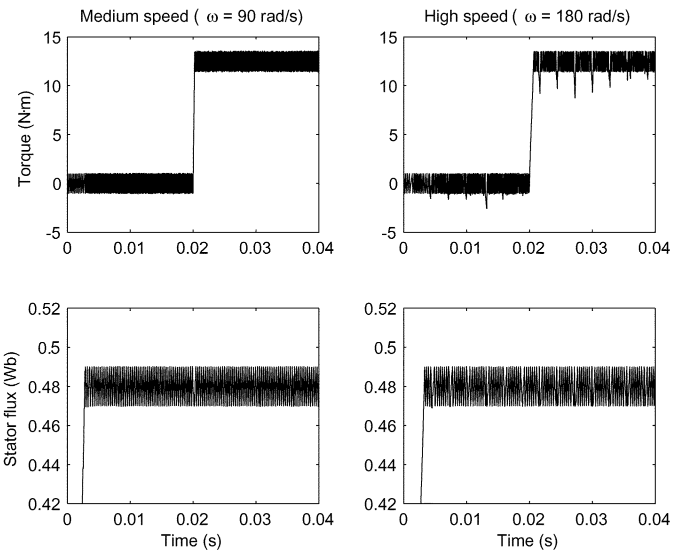

Fig. 4. Torque and flux response (top and bottom) for standard DTC, at medium and high speed (left and right). Torque response at high speed shows the erratic behavior of DTC.

VII. SIMULATION

The analysis presented in this paper shows that DTC can behave erratically under high speed and high load, and confirms the behavior observed by practitioners. To exemplify this behavior, simulations were performed for an induction machine (taken from [4]) whose characteristics are shown in Table III. Standard DTC and the sliding-mode controller (40) were simulated. For the sliding-mode controller, the signs of the machine voltages obtained from (13) are used to determine the inverter state. An extensive simulation study of DTC can be found in [20].

A 400-V bus voltage is used for the simulations. The flux and torque hysteresis bands were set to ± 0.01 Wb and ± 1 N $\cdot$ m, respectively. For the sliding-mode controller the flux and torque gains where set to 100 and 150, respectively. A large inertia was assumed; speed was held constant at 90 and 180 rad/s. At $t =$ 0 s, flux is commanded to its rated value, and at $t = 0.02$ s, torque is commanded to its rated value. Fig. 4 shows results for standard DTC. The torque response at high speed is erratic, but torque control is not lost; only a slightly lower average torque is obtained. Notice that this behavior is not due to a lack of voltage, since the inverter can achieve a phase voltage of at least 245 V (at 67 Hz) [21], which is above rated machine voltage. Fig. 5 shows the results for the sliding-mode controller. The erratic torque behavior is not present, but the flux behavior is slightly different.

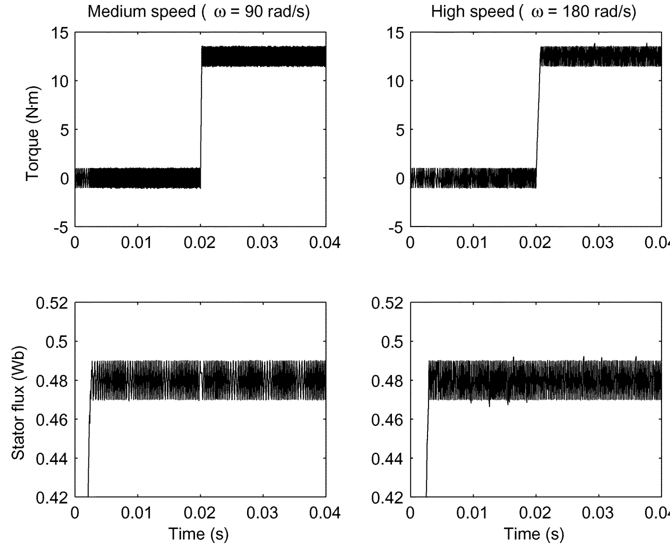

Fig. 5. Torque and flux response (top and bottom) for the sliding-mode controller (40), at medium and high speed (left and right). Torque response at high speed is improved.

VIII. CONCLUSION

A derivation of DTC that avoids heuristic approximations has been presented. The derivation is based on the multiple time-scale properties of the induction machine. It is shown that DTC is a sliding-mode controller on top of a singular perturbation structure. The derivation is not tied to space-vector concepts. The stability and dynamic behavior of DTC can be explained from the analysis. Explicit conditions necessary for stability are presented. The troublesome machine-operating regimes observed in practice are predicted and justified. DTC applies the same control effort to regulate flux as it does for torque; this is not required and sacrifices performance. Since DTC is a sliding mode controller, it can be improved by applying the control literature for variable structure systems. By considering a nominal model of the induction machine, performance can be improved by using torque and speed compensation. Finally, an explicit relationship between DTC performance and machine characteristics has been revealed. This relationship can be exploited to improve DTC performance by proper design of an induction motor.

APPENDIX

In this paper, the matrices used to transform the original three-phase variables into the two-phase representation [4] are given by

$$\Gamma^s_{123\to ab} = \frac{2}{3}\begin{bmatrix} 1 & \cos\left(\frac{2\pi}{3}\right) & \cos\left(\frac{2\pi}{3}\right) \\ 0 & \sin\left(\frac{2\pi}{3}\right) & -\sin\left(\frac{2\pi}{3}\right) \end{bmatrix} \tag{41}$$

and

$$\Gamma^r_{123\to ab} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta+\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{2\pi}{3}\right) \end{bmatrix} \tag{42}$$

for the stator and rotor quantities, respectively. The angle θ is the rotor position.

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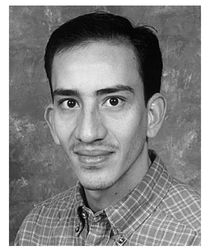

Zakdy Sorchini (S'00) was born in Mexico. He received the B.S. degree in electrical engineering from the Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM), Monterrey, Mexico, in 2000 and the M.S. and Ph.D. degrees in electrical engineering from the University of Illinois at Urbana-Champaign (UIUC), Urbana, IL, in 2003 and 2006, respectively.

He was an Engineer at Symtx de México during 2001. He is currently an Engineer at Delphi Electronics and Safety, Kokomo, IN. His research interests include educational aspects of power electronics and application of nonlinear analysis and control techniques to electric machines and power electronics.

Philip T. Krein (S'76–M'82–SM'93–F'00) received the B.S. degree in electrical engineering and the A.B. degree in economics and business from Lafayette College, Easton, PA, and the M.S. and Ph.D. degrees in electrical engineering from the University of Illinois, Urbana.

He was an Engineer with Tektronix, Beaverton, OR, then returned to the University of Illinois. At present, he holds the Grainger Endowed Director's Chair in Electric Machinery and Electromechanics as Professor and Director of the Grainger Center for Electric Machinery and Electromechanics.He published an undergraduate textbook *Elements of Power Electronics* (Oxford, UK: Oxford Univ. Press, 1998). In 2001, he helped initiate the International Future Energy Challenge (a major student competition involving fuel cell power conversion and energy efficiency). He holds nine U.S. patents with additional patents pending. His research interests address all aspects of power electronics, machines, drives, and electrical energy, with emphasis on nonlinear control approaches.

Dr. Krein was a Senior Fulbright Scholar at the University of Surrey, Surrey, UK, from 1997 to 1998, was recognized as a University Scholar by the University of Illinois in 1999, and received the IEEE William E. Newell Award in Power Electronics in 2003. From 1999 to 2000, he served as President of the IEEE Power Electronics Society. He is a Registered Professional Engineer in Illinois and in Oregon.