



# Saturated Induction Machine Modelling Based on High Frequency Signal Injection

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**Abstract** – The classical induction motor model is not able to analyze the interaction between the low frequency main field and the field generated by high frequency signal injection. To remedy this problem we introduce in this paper a new saturated induction motor model adopts to describe the phenomena produced by adding high frequency signal. This model contains saturation which is considered as the difference between unsaturated and saturated magnetizing flux. The machine is modelling with two different rotor circuits because due to the saturation the air gap flux contains the fundamental and the third harmonic. In order to describe the effects produced by injection of high frequency signal, a rotating voltage signal is added to normal supply of saturated induction motor. The simulation tests are used to verify the validity and accuracy of proposed model.

**Keyword-** Induction Machine, Modelling, Third Harmonic, Signal Injection

## I. INTRODUCTION

Induction machines are widely used in industrial applications involving electromechanical energy conversion due to their favourable attributes like low cost, reasonable size, robustness, and low maintenance. Many of these applications require the use of adjustable speed drives and a suitable control system that provides a large operating range and good dynamic performance. The most commonly used techniques are field oriented and direct torque controls [1]. The implementation of both control algorithms requires the knowledge of rotor position and flux angle which are generally obtained by a mechanical shaft sensor. The cost and reliability advantages of eliminating mechanical sensors and cabling for the measurement of position velocity and flux has led to active research into what is commonly termed “sensorless” or “self-sensing” control of induction machine [2]. Sensorless vector control in low and zero speed region cannot be performed by using the information obtained from a conventional model of the machine excited at the stator frequency. At frequency near to zero, the voltage drop on the stator resistance cannot be neglected while the back emf becomes lower and lower, vanishing any possibility of having continuous low or zero speed operation and limits the application of conventional flux observers [3]. Injecting high frequency signals is one of the most studied and analyzed techniques for sensorless control of Ac machine in the zero speed range. Using the additional signal that feeds

the machine with a persistent high frequency excitation ( $5\div 1\text{kHz}$ ), higher than the fundamental it overcomes the problems present by sensorless in zero speed range and parameter dependant [4]. The types of persistent excitation that have been proposed can be classified into three main categories. The first one is injection high frequency carrier signal (rotating carrier injection current or voltage, pulsating carrier injection current or voltage) the second injection a transient signal (test voltage vector injection superimposed on fundamental PMW) the third standard PMW switching exploit the switching of fundamental PMW waveforms [5]. Add the high frequency signal to the fundamental excitation creates an additional harmonics in the stator currents such harmonics were then used to monitor the machine saliency produced by saturation of main flux and therefore to calculate the position of air gap flux [4]. In addition to the interaction between the main field and the high frequency field creates the modulation of saturation level along the path of magnetizing flux. To analyze this interaction an improved saturated model is proposed in this paper, where the convectional saturated model is modified by introducing the saturation factor through the difference between the unsaturated magnetizing flux and saturated one. Modelling the variation of magnetizing flux is not sufficient to describe the saturation, because due to the saturation the air gap include two components the fundamental and the third harmonic flux. So to achieve an accurately model we must introduce the third harmonic component in this model.

## II. SATURATED INDUCTION MACHINE MODEL

A saturation phenomenon in ac electrical machines, especially in induction machines, has received considerable attention in the past few years. Many types of saturated models are presented in literature one of these models is based on a small signal linearization around an operating point like is described in [6]. The other used the concept of reorientation of the d-q axis in order to incorporate the effects of spatially dependent saturation into the main flux path proposed in [7]. In both models the harmonic due to the saturation do not be described. The saturated model of ac machine with taking account of harmonic components of air gap flux is presented in [8], The model is generated from classical model of ac machine with adapted modification to take a consideration of the saturation. This modification depends on making the air gap length as a function of the

air gap flux position and amplitude. It is shown that as a consequence of saturation a third harmonic flux component exists and the response of the rotor cage to this component is a third harmonic current which will create a ripple in total torque. The modelling approach proposed in this paper combines with the presence of third harmonic component and saturation concept to elaborate the model able to present the effects produced by adding high frequency signal. The stator and rotor voltages equations in stationary reference frame are:

$$\bar{V}_s = R_s \bar{I}_s + \frac{d\bar{\lambda}_s}{dt} \quad (1)$$

$$\bar{V}_r = R_r \bar{I}_r + \frac{d\bar{\lambda}_r}{dt} - jw\bar{\lambda}_r \quad (2)$$

$$\bar{V}_{r3} = R_r \bar{I}_{r3} + \frac{d\bar{\lambda}_{r3}}{dt} - j3w\bar{\lambda}_{r3} \quad (3)$$

Where

$\bar{\lambda}_s, \bar{\lambda}_r$  are the flux linkages of the stator and rotor.

$\bar{\lambda}_{r3}$  is the third harmonic rotor flux linkages.

The  $\bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_{r3}$  can be replaced by the modified flux linkages  $\bar{\Psi}_s, \bar{\Psi}_r, \bar{\Psi}_{r3}$  having units of flux linkages per second or volts as described in[9]:

$$\bar{\Psi}_s = w_b \bar{\lambda}_s, \bar{\Psi}_r = w_b \bar{\lambda}_r, \bar{\Psi}_{r3} = w_b \bar{\lambda}_{r3}$$

Where

$w_b$  is base reference angular velocity.

So the voltages equations (1), (2) and (3) become:

$$\bar{V}_s = R_s \bar{I}_s + \frac{1}{w_b} \frac{d\bar{\Psi}_s}{dt} \quad (4)$$

$$\bar{V}_r = R_r \bar{I}_r + \frac{1}{w_b} \frac{d\bar{\Psi}_r}{dt} - j \frac{w}{w_b} \bar{\Psi}_r$$

$$\bar{V}_{r3} = R_r \bar{I}_{r3} + \frac{1}{w_b} \frac{d\bar{\Psi}_{r3}}{dt} - j3 \frac{w}{w_b} \bar{\Psi}_{r3}$$

The flux linkages equations are written in terms of stator and rotor current components and equivalent reactances as:

$$\bar{\Psi}_s = X_s \bar{I}_s + \bar{\Psi}_{m,sat} \quad (5)$$

$$\bar{\Psi}_r = X_r \bar{I}_r + \bar{\Psi}_{m,sat} \quad (6)$$

$$\bar{\Psi}_{r3} = X_r \bar{I}_{r3} + \bar{\Psi}_{m3} \quad (7)$$

$\bar{\Psi}_{m,sat}$  and  $\bar{\Psi}_{m3}$  are respectively the magnetizing flux and third harmonic magnetizing flux.

The currents obtained are:

$$\bar{I}_s = \frac{\bar{\Psi}_s - \bar{\Psi}_{m,sat}}{X_s} \quad (8)$$

$$\bar{I}_r = \frac{\bar{\Psi}_r - \bar{\Psi}_{m,sat}}{X_r}$$

$$\bar{I}_{r3} = \frac{\bar{\Psi}_{r3} - \bar{\Psi}_{m3}}{X_r} \quad (9)$$

Substituting (8) and (9) in (4), the flux linkage equations become:

$$\frac{\bar{\Psi}_s}{w_b} = \frac{1}{s} \left( \bar{V}_s - \frac{R_s}{X_s} [\bar{\Psi}_s - \bar{\Psi}_{m,sat}] \right)$$

$$\frac{\bar{\Psi}_r}{w_b} = \frac{1}{s} \left( \bar{V}_r - \frac{R_r}{X_r} [\bar{\Psi}_r - \bar{\Psi}_{m,sat}] \right) + j \frac{w}{w_b} \bar{\Psi}_r \quad (10)$$

$$\frac{\bar{\Psi}_{r3}}{w_b} = \frac{1}{s} \left( \bar{V}_{r3} - \frac{R_r}{X_r} [\bar{\Psi}_{r3} - \bar{\Psi}_{m3}] \right) + j3 \frac{w}{w_b} \bar{\Psi}_{r3}$$

Where

$\bar{\Psi}_{m,sat}$  and  $\bar{\Psi}_{m3}$  are generated functions of  $\bar{\Psi}_{m,unsat}$

The Fig.1 shows the difference of magnetizing flux  $\Delta\bar{\Psi}_m$  as function of  $\bar{\Psi}_{m,unsat}$ . This difference is a non linear function of unsaturated magnetizing flux, it obtained from experimental tests.

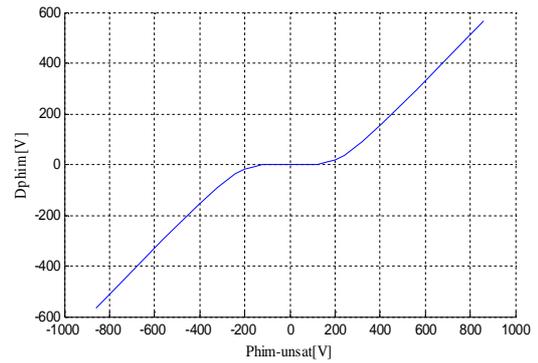


Fig.1 Curve  $\Delta\bar{\Psi}_m = f(\bar{\Psi}_{m,unsat})$

The magnetizing flux  $\bar{\Psi}_{m,sat}$  can be obtained from this expression:

$$\bar{\Psi}_{m,sat} = \bar{\Psi}_{m,unsat} - \Delta\bar{\Psi}_m \quad (11)$$

To obtain unsaturated magnetizing flux  $\bar{\Psi}_{m,unsat}$  substituting (8) in following expressions:

$$\bar{\Psi}_{m,unsat} = X_m (\bar{I}_s + \bar{I}_r) \quad (12)$$

So the expression of  $\bar{\Psi}_{m,unsat}$  can be written as:

$$\bar{\Psi}_{m,unsat} = \frac{X_{mm}}{X_s} \bar{\Psi}_s + \frac{X_{mm}}{X_r} \bar{\Psi}_r + X_{mm} X \Delta\bar{\Psi}_m \quad (13)$$

Where

$$X_{mm} = \frac{1}{\frac{1}{X_m} + \frac{1}{X_s} + \frac{1}{X_r}}, X = \left( \frac{1}{X_s} + \frac{1}{X_r} \right)$$

Two torque components arise from this analysis. One of these components is related to the interaction between the stator flux linkage and saturated magnetizing flux. Whereas the other torque term is related to the interaction between the third harmonic rotor flux linkage and the difference between the stator flux linkage and saturated magnetizing flux. The expression of these two components torque is given by:

$$T_{e,1} = -\frac{p}{wbX_s} (\bar{\Psi}_s \otimes \bar{\Psi}_{m,sat}) \quad (14)$$

$$T_{e,3} = \frac{pkr}{wbX_s} (\bar{\Psi}_{r,3} \otimes \bar{\Psi}_s - \bar{\Psi}_{r,3} \otimes \bar{\Psi}_{m,sat}) \quad (15)$$

The equation related the electrical system to the mechanical system is:

$$T_e - T_l = J \frac{d\Omega}{dt} \quad (16)$$

Where

$T_e$  is the addition of two components  $T_{e,1}$  and  $T_{e,3}$ . The models which are given by (10) and (11),(13),(16) can be described the saturation phenomenon with the presence of third harmonic.

### III. HIGH FREQUENCY SIGNAL INJECTION

When we inject the high frequency voltage signal  $V_{shf}$  to the standard stator voltage, a high frequency field  $F_{hf}$  generates, as shown in Fig.2. The forms of  $F_{hf}$  dependent to the nature of set of injected voltage added to the normal supply.  $F_{hf}$  is pulsating if  $V_{shf}$  is injected only in a phase or consists of three identical voltages. Rotating if  $V_{shf}$  consists of a balanced set voltage. In both case the interaction between the high frequency field  $F_{hf}$  and the main field  $F_m$  produces an oscillation of machine saturation level. In particular, saturation will be increased when the high frequency field is aligned and in phase with the main field, it will be decrease when the high frequency field is opposite, while no variation of saturation when the two fields are orthogonal.

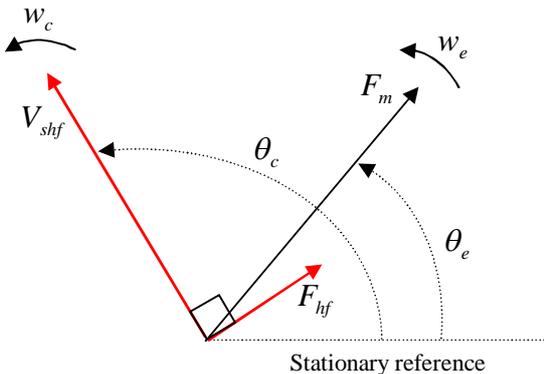


Fig.2 Low and high frequency fields' relation

To describe the effects produced by injection of high frequency signal, a rotating voltage is added to the standard stator voltage, the expression of total voltage supply in stationary frame is:

$$\bar{V}_s = \bar{v}_s + \bar{V}_{s-c} \quad (17)$$

$\bar{v}_s$  a standard stator voltage

$\bar{V}_{s-c}$  a balanced set voltage rotating at the high frequency  $w_c$

So

$$\bar{V}_s = V_{\max} \begin{bmatrix} \cos(w_e t) \\ \sin(w_e t) \end{bmatrix} + V_{hf} \begin{bmatrix} \cos(w_c t) \\ \sin(w_c t) \end{bmatrix} \quad (18)$$

Where

$w_e$  is angular frequency of the main magnetizing field.

The main flux resulting from the interaction of the two fields contains an additional term whose amplitude is high frequency modulated. The angular frequency of this modulation is the relative angular frequency between two fields [10].

$$\lambda_e = \Lambda_e \cos(w_e t) + \Lambda_{e,hf} \cos(w_e t) \quad (19)$$

Where

$$\Lambda_{e,hf} = \Delta \Lambda_{e,hf} \cos(w_c - w_e)t \quad (20)$$

is the additional component due to the variation of saturation.

Substituting equation (20) in (19) the main flux becomes:

$$\lambda_e = \Lambda_e \cos(w_e t) + \Delta \Lambda_{e,hf} \cos(w_c - w_e)t \cos(w_e t) \quad (21)$$

The simplification of (21) gives:

$$\lambda_e = \Lambda_e \cos(w_e t) + \frac{\Delta \Lambda_{e,hf}}{2} \cos(w_e t) + \frac{\Delta \Lambda_{e,hf}}{2} \cos(w_c - 2w_e)t \quad (22)$$

The main flux  $\lambda_e$  includes two high frequencies harmonics, these harmonics pulsate at  $w_c$  and  $w_c - 2w_e$ . However if the main flux amplitude considers the high frequency harmonics will appear in main flux pulsate at  $w_c - w_e$ .

As a result of add high frequency signal to the voltage set of the stator phases, the main flux includes high frequency harmonic component pulsate at  $w_c - w_e$  due to the interaction between hf field and the saturation due to the low frequency field. Furthermore there is another harmonics component due to the interaction of low frequency field with the modulation of saturation. These harmonics pulsate at  $w_c$  and  $w_c - 2w_e$ .

To study the effects due to the injection of high frequency signal, the model was given by (18) is replaced in (10).

#### IV. SIMULATION RESULTS

For evaluating the performance of saturated induction machine with and without injection of high frequency signal the modelling simulation tests are accomplished. The induction machine parameters are given in Table I.

TABLE I  
MACHINE PARAMETERS

machine parameter	value	machine parameter	Value
Rated Power	4 [KW]	$\ell_s$	0.156 [H]
Pole number	4	$\ell_r$	0.156 [H]
$R_s$	1.2[Ω]	$M$	0.15 [H]
$R_r$	1.8[Ω]	$J$	0.05 [kgm <sup>2</sup> ]

##### A- Saturated Induction Machine without High Frequency Signal Injection

To verify the accuracy of saturated model including third harmonic component proposed by (10), the induction machine is fed by  $27,5V_{rms}$  balanced set stator voltage pulsate at 5 Hz. It runs at no-load condition. Fig.3 and Fig.4 show the speed and electromagnetic torque responses, it is evident from these responses that the presence of third harmonic air gap flux component causes a ripple in electromagnetic torque and oscillation in speed.

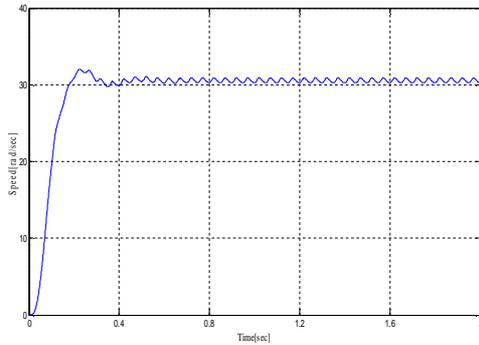


Fig.3 Speed response without HF signal injection

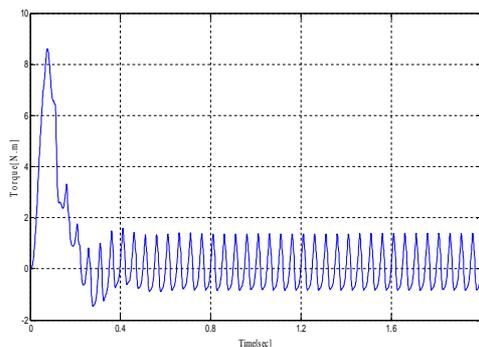


Fig.4 Electromagnetic Torque response without HF signal injection

Fig.5 gives the response current, which had slightly affected by the presence of the third harmonic component. This affect can be seen clearly in trajectory current presented by Fig.6.

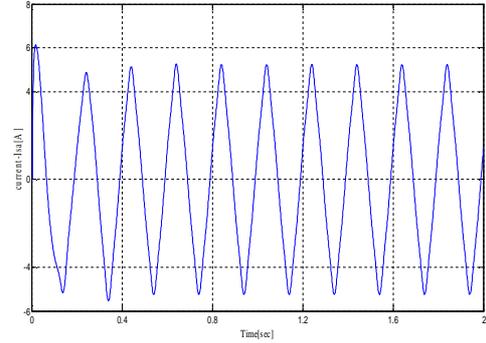


Fig.5 Phase stator current without HF signal injection

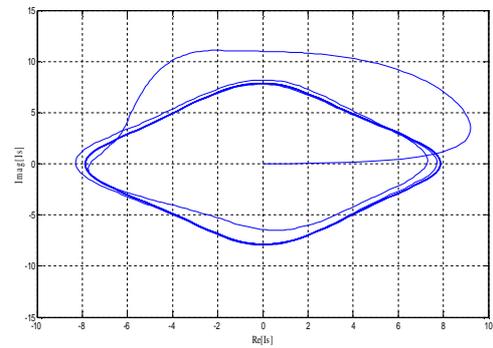


Fig.6 Phase stator current trajectory

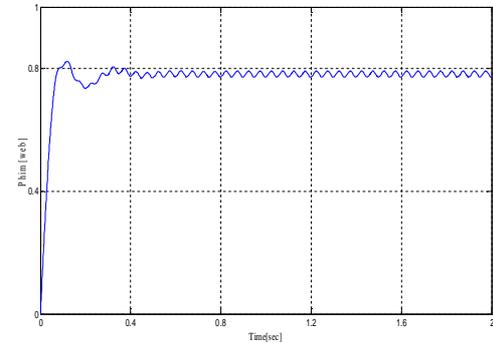


Fig.7 Magnetizing flux without HF signal injection

It can be seen from Fig.7 that the magnetizing flux presents the oscillation due to the presence of third harmonic component in the air gap, but this oscillation does not modify the main value of magnetizing flux in steady state.

##### B- Saturated Induction Machine with High Frequency Signal Injection

In this case the machine is supplied by high frequency rotating voltage superimposed to its normal stator voltage supply. The induction machine is operated at no load with,  $20V_{rms}$ ,  $500\text{ Hz}$  as high frequency rotating voltage superimposed to  $27,5V_{rms}$ ,  $5\text{ Hz}$  normal supply stator voltage. Fig.8 shows the supply stator voltages represented in stationary reference frame.

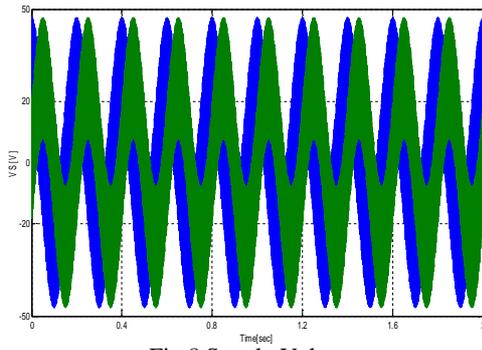


Fig.8 Supply Voltages

The injection of high frequency voltage signal creates the harmonics which appear in both current and flux responses. Generally the harmonics presented in response current are using to extract the position of rotor speed in sensorless control. In our case the harmonics present in the main magnetizing flux are sufficient to clear the modulation of saturation. Fig.9 shows the amplitude of magnetizing flux, it can be observed from this figure a ripple appears which is considered as a consequence of injected signal. The additional signal does not affect the amplitude value of magnetizing flux but it produces the ripple in magnetizing flux.

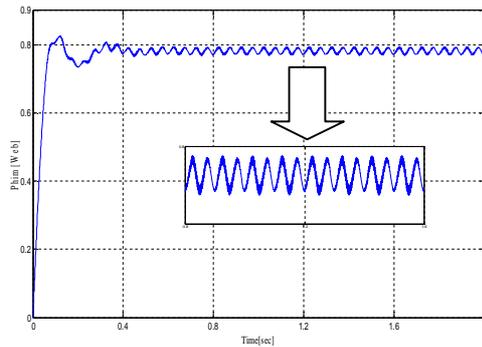


Fig.9 Magnetizing flux with HF signal injection

This ripple can be clearly seen in spectrum presentation of magnetizing flux Fig.10. The harmonic which appears is pulsated at :  $f_1 = \omega_c - \omega_e = 495 \text{ Hz}$ .

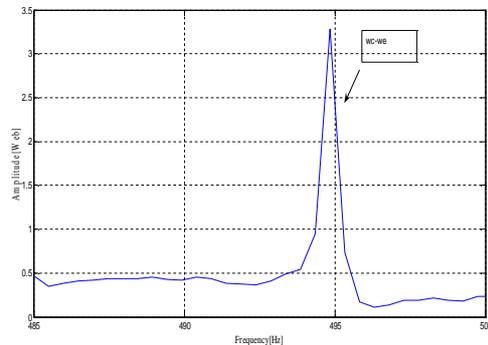


Fig.10 Spectrum of Magnetizing flux with HF signal injection

It signifies the presence of modulation of saturation due to the interaction between the high frequency field and main field. The harmonics frequencies differ when we do not consider the magnetizing flux amplitude as shown in Fig.11, the harmonics frequencies are:

$$f_1 = \omega_c - 2\omega_e = 490 \text{ Hz}, f_2 = 500 \text{ Hz}$$

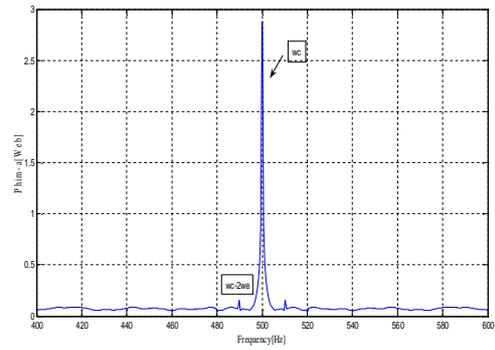


Fig.11 Spectrum of Magnetizing flux  $\lambda_{m-a}$  with HF signal injection

The Fig.12 proves that the speed does not change by adding high frequency signal to normal supply stator voltage. On contrary the stator current is affected by the injection of high frequency signal because of the additional high frequency signal induces high frequency current in stator. This high frequency current creates a ripple in current curve as shown in Fig.13.

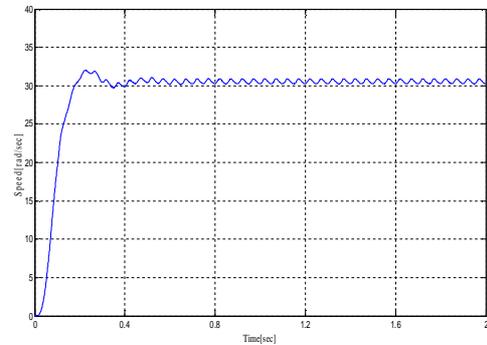


Fig.12 Speed response with HF signal injection

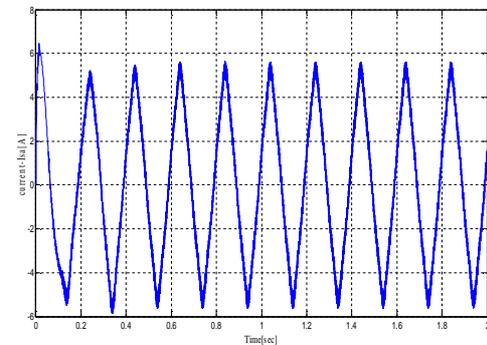


Fig.13 Phase stator current with HF signal injection

The trajectory of stator current creates an elliptical shape as consequence of asymmetric created by adding the high frequency rotating voltage seen Fig.14.

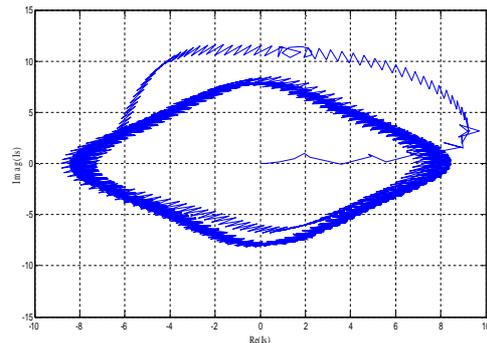


Fig.14 Phase stator current trajectory

The electromagnetic torque is also affected by addition of high frequency signal as shown in Fig.15. This effect appears as ripple in curve of electromagnetic torque whereas its steady state value does not change.

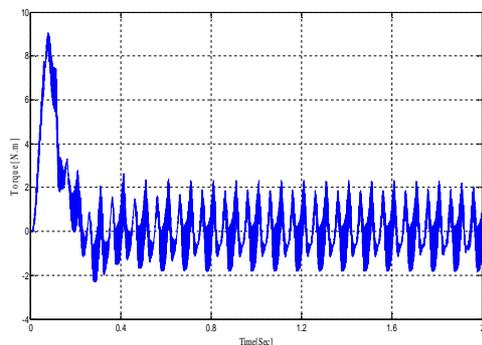


Fig.15 Electromagnetic Torque response with HF signal injection

## V. CONCLUSION

A new saturation model for an induction machine adopted to high frequency signal injection has been developed and presented in this paper. In this model the concept of saturation is introduced by considered the difference between unsaturated magnetizing flux and saturated one. Due to saturation the space harmonic components of flux are appeared in air gap the most dominated component is the third harmonic component which is considered in this model. From these two considerations we can elaborate an improved model able to describe and analyze the phenomena produced by adding the high frequency signal. The injection of high frequency rotating voltage to the normal voltage supply created a modulation of saturation level which is confirmed by simulation tests. The effects of introducing of high frequency signal injection to the standard supply of induction machine are appeared clearly in stator current and magnetizing flux responses. In concept of sensorless control the harmonics presented in magnetizing flux can be used to extract the flux position which tracked by high frequency signal injection.

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