Fractional Derivative Filter for Image Contrast Enhancement with Order Prediction

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PRANAMS AT HIS LOTUS FEET
Leibniz: “Can the meaning of derivatives with integer order be generalised to derivatives with non-integer order?”

L’Hospital: “What if $n=1/2$?”

Leibniz: “Thus it follows that $d^{\frac{1}{2}}x$ will be equal to $x\sqrt{dx:x}$, an apparent paradox, from which one day useful consequences will be drawn.”

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DEFINITIONS*

1730 – L. Euler - “If n is a positive integer, d^n can be found by continued differentiation. Such a way, however is not evident if n is a fraction. But the matter may be expedited with the help of the interpolation of series ..”

... 1819 - S. F. LaCroix - n^{th} derivative of v^m by induction \( \frac{d^{1/2} v}{dy^{1/2}} = \frac{2\sqrt{v}}{\sqrt{\Pi}} \)

... 1835 – J. Liouville – fractional derivative as infinite series \( \frac{d^n v}{dx^u} = \sum A_n e^{\alpha x} m^u \)

... 1847 – B. Riemann – generalisation of Taylor’s series expansion to derive the expression for fractional integration \( \frac{d^{-r}}{dx^{-r}} u(x) = \frac{1}{\Gamma(r)} \int_c^{x} (x-k)^{-r-1} u(k)dk \)

... 1867-68 – A.K. Grunwald & A.V. Letnikov \( D^r f(x) = \lim_{h \to 0} \frac{\sum_{m=0}^{\infty} (-1)^m r C_m f(x + (r-m)h)}{h^r} \)

... 1966-67 – M.Caputo  
\[ D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \]

POTENTIAL APPLICATIONS

- Modelling rugged surface of a malignant breast cell nucleus\(^1\)
- Utilising *Fractional order PID controller* in industrial control systems
- Employing *fractional discriminant functions* to improve perceptual quality of image\(^2\)
- Fractional Calculus based *image and signal processing*\(^3\)

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PROPOSED SCHEME

1. **Blur Estimation**
   - Actual Blur

2. **Statistical Model Estimation**

3. **Order Prediction**
   - Desired Blur
   - Fractional Order

4. **Derivative Mask Computation**
   - Mask Size

5. **Contrast Enhancement**
   - Output Image
Compute $h$ – the inter-pixel distances, say, between P and D1

Use it in the GL formulation for a specified fractional order ‘r’

$$D^r f(x) = \lim_{h \to 0} \sum_{0 \leq m < \infty} (-1)^m C^r_m f(x + (r - m)h)$$
The challenge here is to predict the fractional order to achieve the desired performance of contrast improvement / blur reduction.

Radial Basis Function networks with Greens function as processing element have shown to be best multi-dimensional interpolators.

Therefore, RBF network is used to learn the values of r from an ensemble of training images and their performance over a range of fractional order filters.
PREDICTION ARCHITECTURE
ALGORITHM

Aim
- Given a test image, based on its statistics, initial blur and the desired blur value, we should be able to predict the appropriate fractional order.

Training for the prediction network
- Input to the network:
  - Image statistics
  - Initial blur, fractional order, resulting blur

Performance Validation
- Feed in the training set (seen samples) and compute the deviation of the predicted value from the desired value.
ALGORITHM - VALIDATION

Desired blur
Obtained blur
Given any arbitrary image, compute
+ first four order statistics
+ the original blur
+ the desired blur

Use trained RBF network to predict r.

For this r,
+ compute the multi-directional mask
+ convolve the image
+ compute the resulting blur value
+ compare with desired blur reduction
RESULTS

Original
Blur=0.2575

Desired
Blur=0.1803

Enhanced
Blur=0.1859, r=0.35
RESULTS

Original
Blur=0.2685

Desired
Blur=0.1880

Enhanced
Blur=0.2002, r=0.36
RESULTS

Original
Blur=0.2541

Desired
Blur=0.1780

Enhanced
Blur=0.1771, r=0.346
SUMMARY

Strengths of the proposed system

- Controllability in respect of blur reduction
- Automated prediction of the fractional order

Future work

- Improve the blur metric
- Local adaptation of filter
- Extend to color
Thank You