Leader-Follower Formation Control Using Artificial Potential Functions: A Kinematic Approach

Kishorekumar H Kowdiki,
Department of Electrical Engg.,
Jadavpur University,
Kolkata-700032
kkowdiki@yahoomail.com

Ranjit Kumar Barai,
Department of Electrical Engg.,
Jadavpur University,
Kolkata-700032
ranjit.k.barai@gmail.com

Samar Bhattacharya,
Department of Electrical Engg.,
Jadavpur University,
Kolkata-700032
samar_bhattacharya@ee.jdvu.ac.in

Abstract- **This paper presents a novel formation control technique of a group of differentially driven wheeled mobile robots employing artificial potential field based navigation and leader-follower formation control scheme. In the proposed method, the leader robot of the group determines its path of navigation by an artificial potential field and the other robots in the group follow the leader maintaining a particular formation employing the $(l-\psi)$ control. As the leader robot navigates itself by artificial potential field it can easily avoid the collisions with the obstacles and can follow an optimal path while reaching to the goal position. The follower robots adapt their formation by suitably controlling the desired separation distance and the bearing angle. Thus, the original formation can be regained even if the formation is temporarily lost due to passage through narrow opening / path. Therefore, the overall formation control scheme results into a robust and adaptive formation control for a group autonomous differentially driven wheeled mobile robots. The effectiveness of the proposed formation control technique has been verified in simulation.**

Index Terms**—Artificial Potential Field based Navigation, Formation Control, Leader-Follower approach, Tracking Control.**

I. INTRODUCTION

Various creatures exhibit swarming behavior in biological world, viz. flocks of birds, schools of fish, herds of animals, and colonies of bacteria [1]. Swarm robotics, that mimics the these group behaviors of the biological world into the control and coordination of multiple autonomous robots to achieve one or more tasks in a cooperative manner, has been an interesting research topic for last two decades. Multi-robot systems can often deal with tasks that are difficult, to be accomplished by an individual robot. A team of robots may contribute cooperatively to solve the assigned task, or they may perform the assigned task in a more reliable, faster, or cheaper way beyond what is possible with single robots [2].

Formation control of multi-robot is a methodology of keeping multi-robot system in the scheduled formation and adapting to the environment while moving to the target. By formation control, we simply mean the problem of controlling the relative positions and orientations of robots in a group, while allowing the group to move as a whole [6]. Formation control is one of the active research issues in swarm robots.

Robot motion design can be approached at three levels: a) A geometric level, where the trajectory in the configuration space is designed in the presence of obstacles; b) A kinematic level, where the velocity profiles are designed; and c) A dynamics level, where robot forces are designed as well [7] [9]. A good robot motion planning method must lead to a robot trajectory with desirable geometrical features (e.g. robot moves to target along a short path while keeping a good safety distance from obstacles), desirable kinematical features (e.g. robot maintains a reasonably uniform and brisk speed while traveling but slows down in tight spots), and desirable dynamic features (e.g. robot forces are reasonable and easy to compute). The artificial potential field (APF) approach is a widely adopted approach to mobile robot navigation and control and which claims to address all three levels either directly or indirectly [7]. The artificial potential field (APF) approach is a widely adopted approach to mobile robot navigation and control and which claims to address all three levels either directly or indirectly. The main advantage of using the APF approach is that it is easy in computation; real time computations can be done and can handle the dynamics of the robot.

There are various control strategies which are implemented to leader-follower formation scheme which include input-output linearization [4], Backstepping Based [11], graph theoretic [16] [17], Direct Lyapunov method kinematic control [18], switching strategy [19] and many other. In this method, each robot takes another neighboring robot as a reference point to determine its motion. The referenced robot is called a leader, and the robot following it called a follower.

In the all the approaches of leader-follower formation control it has been assumed that the leader robot knows its path of navigation. That actually makes the fully autonomous

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operation during the formation control of the swarm difficult. In our approach, we are allowing the leader robot to autonomously plan its path using APF and then the follower robots will track the leader's path by kinematically controlling the desired separation distance l_{12}^d and the bearing angle Ψ_{12}^d. Therefore, the formation control problem then becomes a tracking control problem. To the best of the knowledge of the authors, leader-follower formation control employing APF for the autonomous path planning of the leader and then tracking the leader's path by kinematic principles has never been addressed in the literature.

This paper is organized as follows. In section II the kinematic model is given, section III will describe the artificial potential functions for leader's path planning, section IV deals with the tracking control. In section V simulation results are presented and finally conclusion and the future work to be carried out are shown in section VI.

II. KINEMATIC MODEL

We are considering the kinematic model described by [4], in which two scenarios for feedback control within the formation are described. In the first scenario, one robot follows another by controlling the relative distance and orientation between the two $(l-\psi)$ and in another scenario, a robot maintains its position in the formation by maintaining a specified distance from two robots, or from one robot and an obstacle in the environment $(l-l)$.

We have considered the $l-\psi$ formation scheme. In the $l-\psi$ control of the two mobile robots, the aim is to maintain a desired length (separation distance), l_{12}^d and a desired relative angle (bearing angle) ψ_{12}^d between the two robots. The kinematic equations for the system of two mobile robots shown in Fig. 1 is given by

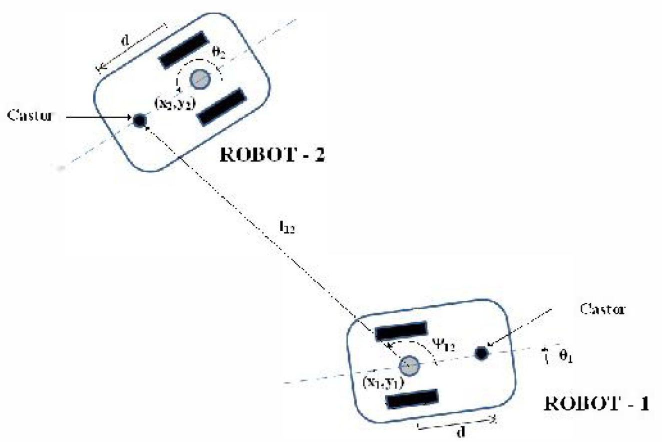

Fig. 1: Notation for l - ψ control

$$\dot{x}_i = v_i \cos\theta_i$$
$$\dot{y}_i = v_i \sin\theta_i$$
$$\dot{\theta}_i = \omega_i \qquad (1)$$

The model for leader-follower formation using $l-\psi$ formation scheme is given as:

$$\dot{l}_{12} = v_2 \cos\gamma_1 - v_1 \cos\psi_{12} + d\omega_2 \sin\gamma_1$$
$$\dot{\psi}_{12} = \frac{1}{l_{12}}\{v_1 \sin\psi_{12} - v_2 \sin\gamma_1 + d\omega_2 \cos\gamma_1 - l_{12}\omega_1\}$$
$$\dot{\theta}_2 = \omega_2 \qquad (2)$$

where, $\gamma_1 = \theta_1 + \psi_{12} - \theta_2$ and $v_i, \omega_i (i = 1,2)$, are the linear and angular velocities at the center of the axle of each robot. In order to avoid collisions between robots, we will require that $l_{12} > d$ .where d is the distance between the castor wheel and the centre of rear wheels.

III. ARTIFICIAL POTENTIAL FUNCTION FOR PATH PLANNING OF THE LEADER ROBOT

We consider the simple artificial potential function described by [8], In this method, a robot is modeled as a moving particle inside an artificial potential field that is generated by superposing an attractive potential that pulls the robot to a goal configuration and a repulsive potential that pushes robot away from obstacles. The negative gradient of the generated global potential field is interpreted as an artificial force acting on the robot and dictating its motion.

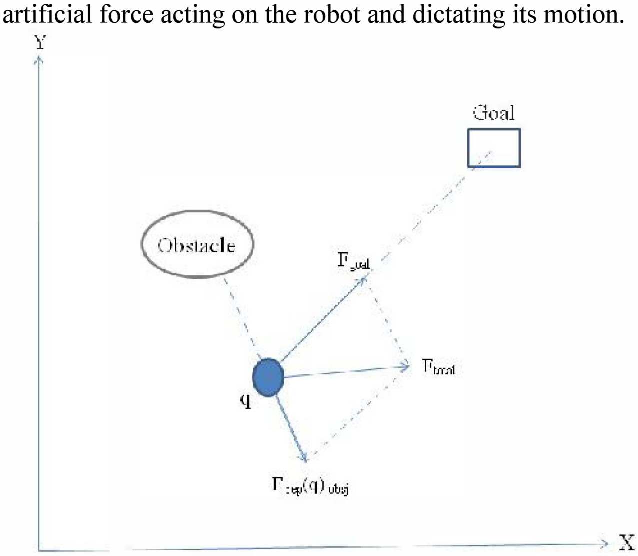

Fig. 2: Moving direction of robot in Artificial Potential Field

Artificial potential field (APF) two has two kinds of potential sources: gravitation pole and repulsion pole. The target is the gravitation pole, and the obstacle is the repulsion pole. They jointly produce the artificial potential field. As shown in Fig. 2, the negative gradient of the APF is the moving direction of the robot in the system.

The target gravitation and obstacle repulsion in APF are defined as:

Let q be the position of the robot, $\rho(q,g)$ be the distance between the robot and the target g, the gravitational potential field U_g and gravitation F_g at robot q are defined as:

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$$U_g(q) = \frac{1}{2}\xi\rho^2(q,g) \tag{3}$$

$$F_g(q) = \xi\rho(g,q) \tag{4}$$

Let q_{obsj} (j=1,..,m) be the position of the j[th] obstacle, $\rho(q,q_{obsj})$ be the distance between robot q and the q_{obsj}, the repulsive potential field $U_{rep}(q)$ and gravitation $F_{rep}(q_{obs})$ at the robot q are defined as:

$$U_{rep}(q) = \begin{cases} \frac{1}{2}\zeta\left(\frac{1}{\rho(q,q_{obsj})} - \frac{1}{\rho_s}\right)^2 & if\, \rho(q,q_{obsj}) \le \rho_s \\ 0 & if\, \rho(q,q_{obsj}) > \rho_s \end{cases} \tag{5}$$

$$F_{rep}(q_{obs}) = \begin{cases} \zeta\left(\frac{1}{\rho(q,q_{obsj})} - \frac{1}{\rho_s}\right)^2 \frac{1}{\rho^2(q,q_{obsj})}\nabla\rho(q,q_{obsj}) \\ 0 \end{cases}$$

$$if\, \rho(q,q_{obsj}) \le \rho_s$$
$$if\, \rho(q,q_{obsj}) > \rho_s \tag{6}$$

So the resultant force of robot q in the APF is:

$$F_{total}(q) = F_g(q) + \sum_{j=1}^{m} F_{rep}(q_{obsj}) \tag{7}$$

A fundamental problem in the application of potential field method is how to deal with the local minima that may occur in a potential field environment [9].

IV. TRACKING CONTROL

The robots are considered as a point mass. The leader's path is defined by the artificial potential functions, depending upon location of static obstacles and the goal position. The follower robot will follow the leader by keeping the separation distance l_{ij} and bearing angle Ψ_{ij}.

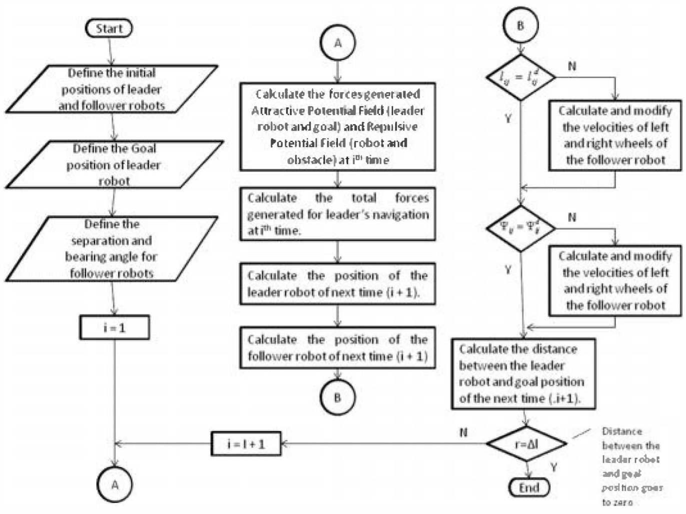

Fig. 3: Flow Chart for Formation Control.

The formation control scheme is described in the flow chart as shown in Fig. 3. Since, the next position and orientation of the leader robot are dictated by the artificial potential field, we have (*x*, *y*) i.e. position and θ i.e. orientation of the leader robot. l_{ij} and Ψ_{ij} i.e. the separation distance and the bearing angle respectively, which the follower robot has to maintain while following the leader robot. We will have to develop the control algorithm for deriving the velocities of the left and right wheels of the follower robot.

To avoid collisions, separation distances are measured from the back of the leader to the front of the follower, and the kinematic equations for the front of the j^{th} follower robot can be written as:

$$\dot{q} = \begin{bmatrix} \dot{x}_j \\ \dot{y}_j \\ \dot{\theta}_j \end{bmatrix} = S_j(q_j)v_j = \begin{bmatrix} \cos\theta_j & -d\sin\theta_j \\ \sin\theta_j & -d\cos\theta_j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \tag{8}$$

Where d is the distance from the rear axle to the front of the robot,

Consider the tracking controller error system presented in [8] used to control a single robot as

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{jr} - x_j \\ y_{jr} - y_j \\ \theta_{jr} - \theta_j \end{bmatrix} \tag{9}$$

$$\dot{x}_{jr} = v_{jr}\cos\theta_{jr}, \dot{y}_{jr} = v_{jr}\sin\theta_{jr}, \dot{\theta}_{jr} = \omega_{jr},$$

$$\dot{q}_{jr} = \begin{bmatrix} \dot{x}_{jr} & \dot{y}_{jr} & \dot{\theta}_{jr} \end{bmatrix}^T \tag{10}$$

Where x_j, y_j, and θ_j are actual position and orientation of the robot, and x_{jr}, y_{jr}, and θ_{jr} are the positions and orientation of a virtual reference cart robot *j* seeks to follow [10].

The basic tracking control problems can be extended to formation control as follows. The virtual reference cart is replaced with a physical mobile robot acting as the leader *i*, and x_{jr} and y_{jr} are defined as points at a distance l_{ijd} and a desired angle Ψ_{ijd} from the lead robot. Now basic navigation problems can be introduced for leader-follower formation control as.

Let there be a leader *i* for follower *j* such that

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -d\sin\theta_i \\ \sin\theta_i & d\cos\theta_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} \tag{11}$$

$$x_{jr} = x_i - d\cos\theta_i + l_{ij}^d\cos\left(\Psi_{ij}^d + \theta_i\right)$$

$$y_{jr} = y_i - d\sin\theta_i + l_{ij}^d\sin\left(\Psi_{ij}^d + \theta_i\right)$$

$$\theta_{jr} = \theta_i \tag{12}$$

And

$$v_{jr} = \left[|v_i| \quad |\omega_i|\right]^T \tag{13}$$

Then the actual position and orientation of the follower *j* with respect to leader *i* can be defined as.

$$x_j = x_i - d\cos\theta_i + l_{ij}\cos\left(\Psi_{ij} + \theta_i\right)$$

$$y_j = y_i - d\sin\theta_i + l_{ij}\sin\left(\Psi_{ij} + \theta_i\right)$$

$$\theta_j = \theta_i \tag{14}$$

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Where l_{ij} and Ψ_{ij} is the actual separation and bearing of the follower *j*.

Using (12), (14) and simple trigonometric identities, the error system (9) for follower robot *j* with respect to leader robot *i* can be rewritten as

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} (\begin{bmatrix} l_{ij}^d \cos\left(\Psi_{ij}^d + \theta_i\right) - l_{ij}\cos\left(\Psi_{ij} + \theta_i\right) \\ l_{ij}^d \sin\left(\Psi_{ij}^d + \theta_i\right) - l_{ij}\sin\left(\Psi_{ij} + \theta_i\right) \\ \theta_i - \theta_j \end{bmatrix} \tag{15}$$

And after further simplification (14) becomes

$$e_j = \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} l_{ij}^d \cos\left(\Psi_{ij}^d + e_{j3}\right) - l_{ij}\cos\left(\Psi_{ij} + e_{j3}\right) \\ l_{ij}^d \cos\left(\Psi_{ij}^d + e_{j3}\right) - l_{ij}\cos\left(\Psi_{ij} + e_{j3}\right) \\ \theta_i - \theta_j \end{bmatrix} \tag{16}$$

This transformed error system now acts as a formation tracking controller which not only seeks to remain at a fixed desired distance l_{ij}^d with a desired angle Ψ_{ij}^d relative to the leader robot 1, but also achieves the same orientation as the leader robot which is desirable when $\omega_i = 0$

In order to calculate the dynamics of the error system (16), it is necessary to calculate the derivatives of l_{ij} and Ψ_{ij}, and it is considered that the desired separation distance l_{ij}^d and the desired bearing angle Ψ_{ij}^d are constant. Consider the two robot formation as shown in fig. 1. The x and y components of l_{ij} can be defined as

$$\begin{aligned} l_{ijx} &= x_{irear} - x_{jfront} = x_i - d\cos\theta_i - x_j \\ l_{ijy} &= y_{irear} - y_{jfront} = y_i - d\sin\theta_i - y_j \end{aligned} \tag{17}$$

And the derivatives of the x and y components of l_{ij} can be found as

$$\begin{aligned} \dot{l}_{ijx} &= v_i\cos\theta_i - v_j\cos\theta_j + d\omega_j\sin\theta_j \\ \dot{l}_{ijy} &= v_i\sin\theta_i - v_j\sin\theta_j + d\omega_j\cos\theta_j \end{aligned} \tag{18}$$

Noting that $l_{ij} = \sqrt{l_{ijx}^2 + l_{ijy}^2}$ and $\Psi_{ij} = \arctan\left(\frac{l_{ijy}}{l_{ijx}}\right) - \theta_i + \pi$, It can be shown that the derivatives of separation and bearing are similar as the kinematic equation described in (2).

$$\begin{aligned} \dot{l}_{ij} &= v_j\cos\gamma_j - v_i\cos\Psi_{ij} + d\omega_j\sin\gamma_j \\ \dot{\Psi}_{ij} &= \frac{1}{l_{ij}}\left(v_i\sin\Psi_{ij} - v_j\sin\gamma_j + d\omega_j\cos\gamma_j - l_{ij}\omega_i\right) \end{aligned} \tag{19}$$

Where $\gamma_j = \Psi_{ij} + e_{j3}$.

Now, using derivative of (16), equation (19) and applying simple trigonometric identities, the error dynamics becomes

$$\begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = \begin{bmatrix} -v_j + v_i\cos e_{j3} + \omega_j e_{j2} - \omega_i l_{ij}^d \sin\left(\Psi_{ij}^d + e_{j3}\right) \\ -\omega_j e_{j1} + v_i\sin e_{j3} - d\omega_j + \omega_i l_{ij}^d \cos\left(\Psi_{ij}^d + e_{j3}\right) \\ \omega_i - \omega_j \end{bmatrix} \tag{20}$$

Examining (20) and the error dynamics of a tracking controller for a single robot in [10], it can be seen that dynamics of a single follower with a leader is similar to [10], except additional terms are introduced as a result of (8) and (19).

We have considered only ($l-\Psi$) control of leader-follower formation schemes, this can be also implemented for $(l-l)$ control. In this paper we will consider only the velocity of the robots as the control input as we have proposed kinematic control of the robot swarm to maintain the desired ($l-\Psi$) while the follower robots are tracking the path traversed by the leader robot. Here, we have assumed that the operating velocities of the robots are not much high, therefore, the dynamic effects can be neglected while calculating the control inputs. The main objective of this work is to investigate the effectiveness of incorporating autonomous navigation strategy of the leader robot using artificial potential field for robust formation control in the leader-follower framework and to get a better insight of how the formation can be maintained even in the face of obstacles or how the original formation can be regained even if the formation is temporarily lost due to passage through narrow opening / path. With this aim we have further neglected other factors like nonlinearities or disturbances. Therefore, a simple proportional control like kinematic controller is sufficient provided the velocity terms can be calculated properly to form the required control input.

V. SIMULATION RESULTS

A triangular formation of three identical mobile robots is considered where the leader's path /navigation is dictated by the artificial potential field and are considered as the desired formation trajectory, and simulations are been carried out in MATLAB with various cases. Simulation cases of formation control with one, two and three obstacles in the environment.

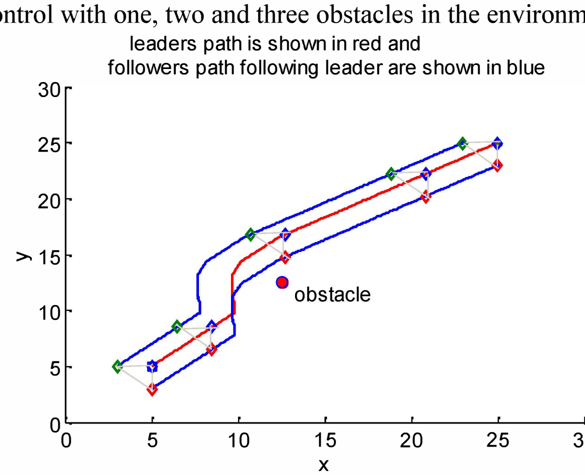

Fig. 4: Simulation of triangular formation with one obstacle.

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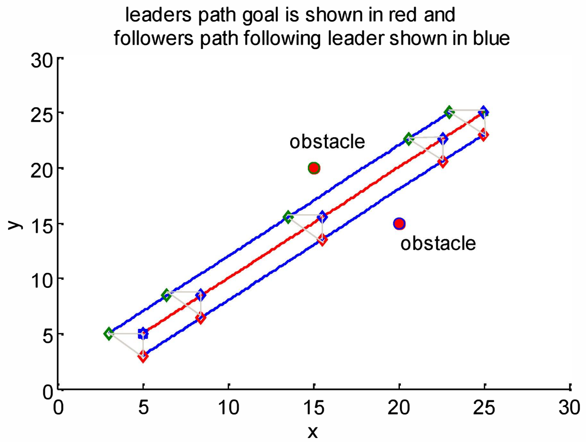


Fig. 5: Simulation of triangular formation with two obstacles.

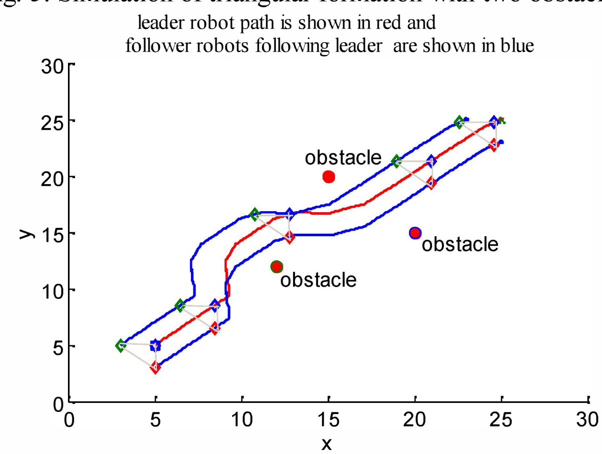
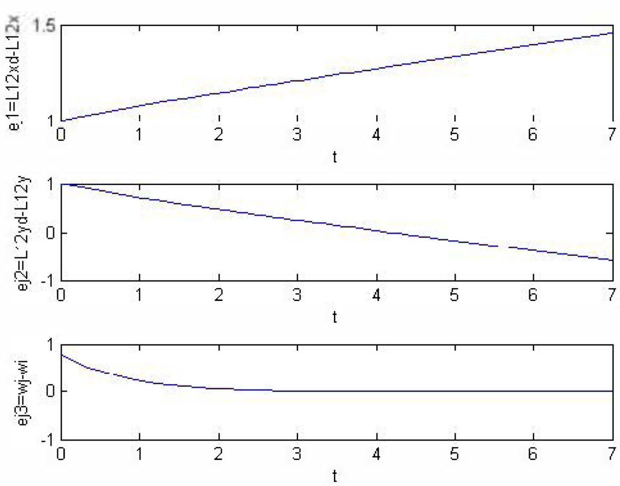


Fig.6:Simulation of triangular formation with three obstacles.

Fig. 4, 5 and 6 shows the triangular formation with one, two and three obstacles in the environment respectively. The initial positions of the leader and two followers are defined. The goal/target position for the leader is defined. The positions of the obstacles are defined. The leader's path is dictated by the artificial potential fields as in section III shown in red line. The followers (robots) follow the leader as described in section IV and are shown in blue lines. Robots positions after regular time intervals are also shown.

We have assigned the linear velocity and angular velocity as in [18] and the simulation results are as follows



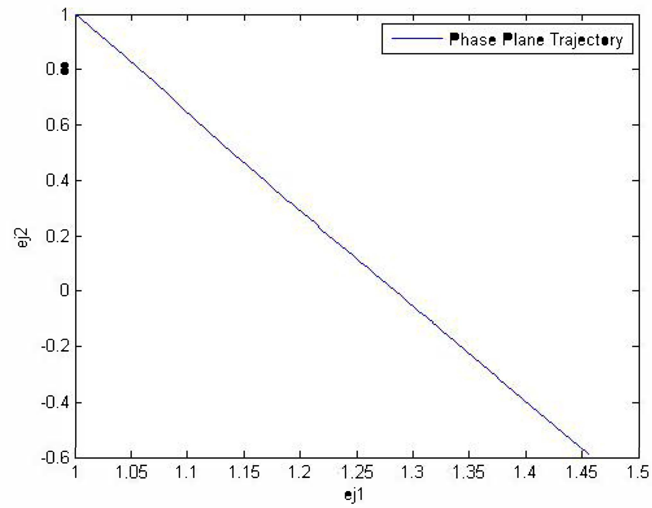
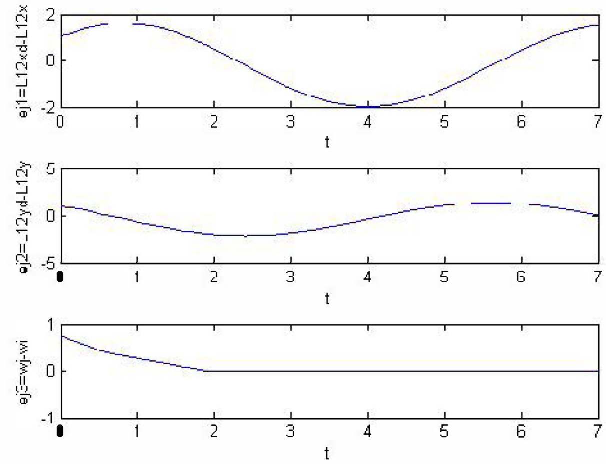


Figure 7: Simulation Results when the linear velocity of leader =1 and angular velocity of leader = 0



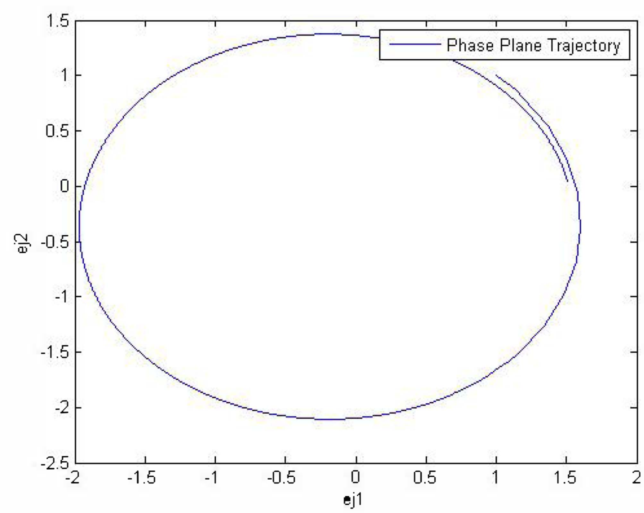


Figure 8: Simulation Results when the linear velocity of the leader =1 and angular velocity of the leader =1

VI. CONCLUSION AND FUTURE WORK

Based on the simulation results, it can be seen that as the leader robot navigates itself by artificial potential field, its

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locomotion control is stable and robust against collision while reaching to the goal position and the followers are following the leader's path effectively.

Thus, from the simulation results, we can see that the desired formation control using leader-follower scheme is effective. In future work, a suitable controller will be selected to show that the error dynamics described in this paper will tend to zero as time tends to infinity. And in this work we have considered only the kinematics of the differential-drive mobile robot, so in future we will include the dynamics as it is well known that due to non-holonomic constraint of the differential-drive mobile robot, the perfect velocity tracking will not hold, we will have to consider the torque as well.

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