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Fuzzy bang-bang relay controller for satellite attitude control system

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Abstract

Two level bang-bang controllers are generally used in conjunction with the thrust reaction actuator for spacecraft/satellite attitude control. These controllers are fast acting and dispense time dependent; full or no thrust-power to control the satellite attitude in minimum time. A minimum time-fuel attitude control system extends the life of a satellite and is the main focus of this paper. Fuzzy controllers are favored for satellite control due to their simplicity and good performance in terms of fuel saving, absorbing non-linearities and uncertainties of the plant. A fuzzy controller requires a soft fuzzy engine, and a hardware relay to accomplish bang-bang control action. The work in this paper describes a new type of fuzzy controller in which the hardware relay action is configured in the soft fuzzy engine. The new configuration provides fuzzy decision-making flexibility at the inputs with relay like two-level bang-bang output. The new fuzzy controller is simulated on a three-axis satellite attitude control platform and compared with conventional a fuzzy controller, sliding mode controller and linear quadratic regulator. The result shows that the proposed controller has minimum-time response compared to other controllers. Inherent chattering associated with a two-level bang-bang controller produces undesirable low amplitude frequency limit cycles. The chattering can be easily stopped in the proposed fuzzy bang-bang relay controller, hence adding multi-functionality to its simple design.

Keywords: Bang-bang; Slide mode control; Fuzzy logic control; State dependent Riccati equation

1. Introduction

Minimum response time of a control system complements energy-saving measures especially in embedded control systems. A satellite attitude control system is one such example where fuel saving is highly desirable. Likewise, remotely controlled submersibles, deep space exploration probes can also benefit from such measures but to a lesser extent than communication satellites due to their high launching cost. Minimum response time also ensures that satellite orientation error can be efficiently removed without degrading the performance of the satellite.

Actuators such as reaction wheels, magnetic torques, or reaction thrusters control satellite attitude. The attitude control is described as fast or slow depending upon the slew angle ranges of the mission. Fast attitude control systems employ thrusters, which gives large control torques to counter large slew angle disturbances arising due to Earth's gravitation, airflow, magnetic fields, and solar winds. Thrusters can work as proportional (analog) or two-level switching bang-bang

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controls. Proportional controllers are precise but problematic and costlier than bang-bang controllers. The variable jet nozzles of proportional controllers easily get clogged with dirt, debris and ice, resulting in failed operation. On the other hand, a bang-bang controller uses large and constant area nozzles. Full discharging bang-bang thrusters can open the spring-loaded flaps covering the nozzles from the elements [12].

In a bang-bang control system the valves can be reliably operated to stay open for as little as a few milliseconds. The full opening of the valves for a finite time changes the discrete angular velocity with each actuation. As a result it is not possible to get zero residual angular velocity. To prevent opposing thrusters fighting each other, a deadband is introduced between the on–off control, and the controller is shut down in this deadband region. As a result the controlled system coasts to the equilibrium point (origin) with reduced velocity or increased damping. This generates low frequency (and amplitude) limit-cycles, and hence dissipates the thruster force.

Simple hardware relays or hard limiter programming functions are used for implementation of the bang-bang control system. In the absence of a deadband in bang-bang control, hysteresis is commonly used to compensate for the deadband action. Hysteresis provides a neat solution, but lacks robustness as it is fixed or chosen to optimize performance for one particular nominal plant operating condition.

One solution is to use flexible hysteresis for bang-bang control action. Mazari and Mekri [2] show how the flexible fuzzy hysteresis band (HB) is used to optimize the pulse width modulator (PWM) for reduction of harmonic current pollution. Nowadays, digital processors are readily available to implement software based fuzzy and neural network controllers. Fuzzy controllers are flexible, simple to build and provide robustness to the bang-bang controller. In a detailed literature review Lee [10] reported a wide range of non-linear systems controlled by fuzzy logic. Berenji et al. [3] proposed a fuzzy control scheme for attitude stabilization of a space shuttle. Bang-bang fuzzy logic controllers have been developed in the past. A bang-bang fuzzy controller developed by Chiang and Jang [16], made its debut in Cassini spacecraft's deep space exploration project. The controller proves its superiority over the conventional bang-bang controller. Other applications include minimum-time fuzzy satellite attitude controller [3], crane hoisting/lowering operation [13]. However, these applications are not well suited to the standard linear control design methodologies. The applications described in [3,13] use fuzzy bang-bang control. The defuzzified outputs in these works were based on center of gravity (COG), center of area (COA) or centroid methods. These methods yield crisp analog outputs, which are converted to two level-control using hard limiting devices.

The idea of fuzzy bang-bang relay is not new. Kendal and Zhang [1] and Keichert and Mamdani [21] first pointed out that fuzzy controllers are identical to multilevel relays, when using mean of maxima (MOM) defuzzification technique. Fuzzy relay in power control application was first presented by Panda and Mishra [8], where the relay was used in conjecture with fuzzy rules. Hard limiter was used in this work to convert the defuzzified output to two level controls.

New controllers are not acceptable to the control community, unless their stability is proven using established techniques. In the case of a fuzzy controller, the heuristic approach of fuzzy rules results in partitioning of the decision space (phase plane) into two semi-planes by a *sliding* (*switching*) *line*. Similarity between a fuzzy bang-bang controller and a sliding mode controller (SMC) can be used to redefine the diagonal form of an FLC in terms of an SMC, with boundary limits, to verify the stability of the proposed bang-bang controller [5,15]. SMC is a robust control method [19] and its stability can be proven using Lyapunov's direct method. So in lieu of SMC, the fuzzy bang-bang control stability can be easily established.

SMC requires a non-linear representation of the system to design the controller. It has an undesirable characteristic of producing high frequency control signals, which can shake the system vigorously. This drawback could be removed by using fuzzy logic controllers [14]. A fuzzy logic controller results in a highly non-linear control scheme [22] in which thrusters are active for a given duration. The controller has to evaluate the time and sign of the switch of each thruster. Three MISO fuzzy controllers are used for attitude stabilization of a small satellite in a low earth orbit, proving that fuzzy controller solves the control constraint problem by choosing the appropriate duration and polarity of the thrusters.

The minimum-time control is highly desirable for bang-bang controller design, especially for satellite attitude control [20]. Minimum control discussion is not complete without mentioning the celebrated Riccati equation. LQR methods are applicable to linear systems; however, state-dependent Riccati equation (SDRE) [6] is a promising technique for controlling non-linear systems, such as the six-degree of freedom satellite attitude control system considered in this paper. In SDRE, the state matrix and control law are updated and used as linear approximations at every sample time. The advantage of SDRE is its efficiency in converging to a solution and is a primary source of spacecraft tracking control systems [7].

In this paper a new configuration of fuzzy controller is proposed. This controller directly produces a two-level bang-bang crisp output. The inputs to the controller are configured on standard fuzzy sets based on Mamdani inference. Here, the Takagi–Sugeno–Kang (TSK) inference is not suitable as the weighted average of rules gives a singleton output, which is linear or has a constant level. However, the bang-bang controller has only two output levels. The controller does not require any further saturation or hard limiting devices. The fuzzy rules' consequents parts have only two linguistic values, while the premise parts are chosen freely. The fuzzy bang-bang controller, which works like a two-level relay and is referred to as a *fuzzy bang-bang relay controller* (FBBRC). Due to fixed two level bang-bang output the FBBRC decision making capability is limited to its inputs only. Further, being software based the FBBRC is switched off once the desired angle is achieved within the specified deadband limit. The FBBRC is also computationally less demanding than an LQR controller. In the next section, satellite kinematics and dynamics are defined and modeled in state space form. In Section 3 the proposed controller, FBBRC, is developed. An SDRE and SMC controllers are described in Section 5 and Appendix B, respectively, followed by simulations and comparison of controllers' performance in Section 6.

2. Three degree of freedom satellite state space model

Three degree of freedom (3-dof) rigid satellite model is presented in this section. The attitude motions about the three principal axes are nearly uncoupled, and stabilization about each principal axis may be treated separately. Axes X_B, Y_B, and Z_B define the satellite's body axis frame, and the axis system is considered centered at the center of gravity as shown in Fig. 1. Thrusters are available to produce torques about each of the three principal axes. The physical interpretation of the Euler angles [18] for a micro-satellite platform is illustrated in Fig. 1. The roll (ϕ), pitch (θ), and yaw (ψ) angles are defined by successive rotations around the coordinate axes X_B, $Y_{B,}$ and Z_B in the body fixed frame. For large angles and position control [7,4] of spacecraft, quaternion rotation is used. This is more suitable as it avoids singularity functions and gimbal lock [9].

Euler angular moment ($\dot{H}$) [18] which performs satellite attitude-rotational motion in space is expressed in the form of a matrix as follows:

$$
\begin{aligned}
&M = \frac{dH}{dt} + [\omega \times H] \\
&\frac{dH}{dt} = M - [\omega \times H] \quad \text{because } H = I \cdot \omega \\
&I \cdot \frac{d\omega}{dt} = M - [\omega \times I \cdot \omega] \\
&\frac{d\omega}{dt} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = I^{-1} \cdot M - [I^{-1} \cdot \omega] \times [I \cdot \omega]
\end{aligned}
\tag{1}
$$

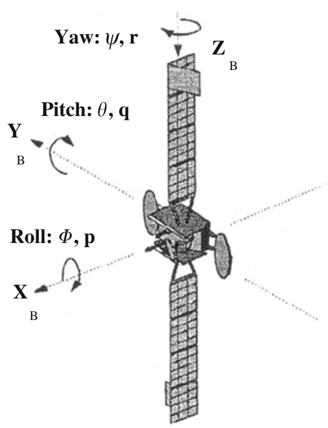

Fig. 1. Satellite reference and body coordinates.

where vector M, the applied moment (thrusters), is the input u, vector ω is the angular rate and matrix $\boldsymbol{I}$ is the inertia around the body principal axes X_B, Y_B, and Z_B. These are given by

$$u = [M_x, M_y, M_z]^T$$

$$\omega = [p, q, r]^T$$

$$\boldsymbol{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{2}$$

Solving (1) and (2)

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{M_x - q \cdot I_{zz} \cdot r + r \cdot I_{yy} \cdot q}{I_{xx}} \\ \dfrac{M_y - r \cdot I_{xx} \cdot p + p \cdot I_{zz} \cdot r}{I_{yy}} \\ \dfrac{M_z - p \cdot I_{yy} \cdot q + q \cdot I_{zz} \cdot p}{I_{zz}} \end{bmatrix} \tag{3}$$

The relationship between body rates $\omega_b = [p_b, q_b, r_b]^T$ of the satellite and angular rates $\omega = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ is defined by the following transformation [18]:

$$\begin{aligned} \begin{bmatrix} \dot{\phi}_I \\ \dot{\theta}_I \\ \dot{\psi}_I \end{bmatrix} &= \begin{bmatrix} 1 & \dfrac{\sin(\phi)\sin(\theta)}{\cos(\theta)} & \dfrac{\sin(\phi)\sin(\theta)}{\cos(\theta)} \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \cdot \begin{bmatrix} p_b \\ q_b \\ r_b \end{bmatrix} \\ &= \begin{bmatrix} P + \dfrac{(q\ \sin(\phi) + r\ \cos(\phi))\sin(\theta)}{\cos(\theta)} \\ q\ \cos(\phi) - r\ \sin(\phi) \\ \dfrac{q\ \sin(\phi) + r\ \cos(\phi)}{\cos(\theta)} \end{bmatrix} \end{aligned} \tag{4}$$

The non-linear state model of the satellite can be derived by partial derivatives of the model states $x = [p_b, q_b, r_b, \phi_I, \theta_I, \psi_I]^T$

$$\begin{bmatrix} \dot{p}_b \\ \dot{q}_b \\ \dot{r}_b \\ \dot{\phi}_I \\ \dot{\theta}_I \\ \dot{\psi}_I \end{bmatrix} = f(x, u) = \begin{bmatrix} \dfrac{M_x - q \cdot I_{zz} \cdot r + r \cdot I_{yy} \cdot q}{I_{xx}} \\ \dfrac{M_y - r \cdot I_{xx} \cdot p + p \cdot I_{zz} \cdot r}{I_{yy}} \\ \dfrac{M_z - p \cdot I_{xx} \cdot r + q \cdot I_{yy} \cdot p}{I_{zz}} \\ p + \dfrac{(q\ \sin(\phi) + r\ \cos(\phi))\sin(\theta)}{\cos(\theta)} \\ q\ \cos(\phi) - r\ \sin(\phi) \\ \dfrac{q\ \sin(\phi) + r\ \cos(\phi)}{\cos(\theta)} \end{bmatrix}$$

Table 1
Satellite parameters.

Parameters	Description	Value
I_{xx}	Moment of inertia (x-axis)	$1.928\,\mathrm{kg\,m^2}$
I_{yy}	Moment of inertia (y-axis)	$1.928\,\mathrm{kg\,m^2}$
I_{zz}	Moment of inertia (z-axis)	$4.953\,\mathrm{kg\,m^2}$
Thruster	Input to satellite (M_x, M_y, M_z)	$1\,\mathrm{kg\,m^2}$
Φ_0	Initial roll Euler angle	0.362 rad
θ_0	Initial pitch Euler angle	0.524 rad
Ψ_0	Initial yaw Euler angle	−0.262 rad
p	Body pitch roll rate	0 rad/s
q	Body yaw rate	0 rad/s
r	Body roll rate	0 rad/s
α	Deadband	±10 mrad

The non-linear state space model can be found by taking the Jacobians $\boldsymbol{A} = Jacobian(f(x), x)$ and $\boldsymbol{B} = Jacobian(f(x), u)$:

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{pmatrix} 0 & \dfrac{-rI_{zz}+rI_{yy}}{I_{xx}} & \dfrac{-qI_{zz}+qI_{yy}}{I_{xx}} & 0 & 0 & 0 \\ \dfrac{-rI_{xx}+rI_{zz}}{I_{yy}} & 0 & \dfrac{-pI_{xx}+pI_{zz}}{I_{yy}} & 0 & 0 & 0 \\ \dfrac{-qI_{yy}+qI_{xx}}{I_{zz}} & \dfrac{-pI_{yy}+pI_{xx}}{I_{zz}} & 0 & 0 & 0 & 0 \\ 1 & \dfrac{\sin\phi\sin\theta}{\cos\theta} & \dfrac{\cos\phi\sin\theta}{\cos\theta} & \dfrac{q\cos\phi - r\sin\phi\sin\theta}{\cos\theta} & q\sin\phi + r\cos\phi + \dfrac{(q\sin\phi + r\cos\phi)(\sin^2\theta)}{\cos^2\theta} & 0 \\ 0 & \cos\phi & -\sin\phi & -q\sin\phi - r\cos\phi & 0 & 0 \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} & \dfrac{q\cos\phi - r\sin\phi}{\cos\theta} & q\sin\phi + \dfrac{r\cos\phi\sin\theta}{\cos^2\theta} & 0 \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} \dfrac{1}{I_{xx}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{I_{yy}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{I_{zz}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{5}$$

where matrix $\boldsymbol{A}(\boldsymbol{x})$ and $\boldsymbol{B}(u)$ are Jacobian matrix of non-linear function $f(x)$with respect to state vector $x = [p\ \ q\ \ r\ \ \phi\ \ \theta\ \ \psi]^T$ and input vector $u = [M_x\ \ M_y\ \ M_z\ \ 0\ \ 0\ \ 0]^T$, respectively. The satellite parameters in Table 1 are based on the model referenced in Thongchet and Kuntanapreeda [20].

3. Fuzzy bang-bang relay controller

A bang-bang fuzzy relay controller is developed in this section. The controller is developed for only roll-axis and is identical for the other two axes. The controller takes advantage of largest of maxima defuzzification (LOM) technique to yield bang-bang output *directly*. For any fuzzy controller it is necessary to determine the ranges of its state, input and output variables. These should be a reasonable representation of all the situations the controller will face, and should yield to stability and optimality conditions. The following ranges are selected for simulation purposes: $\Phi(t) = [-3, 3]$ rad, $\dot{\Phi}(t) = [-3, 3]$ rad/s and control signal $u_r = [-M_x, +M_x]$.

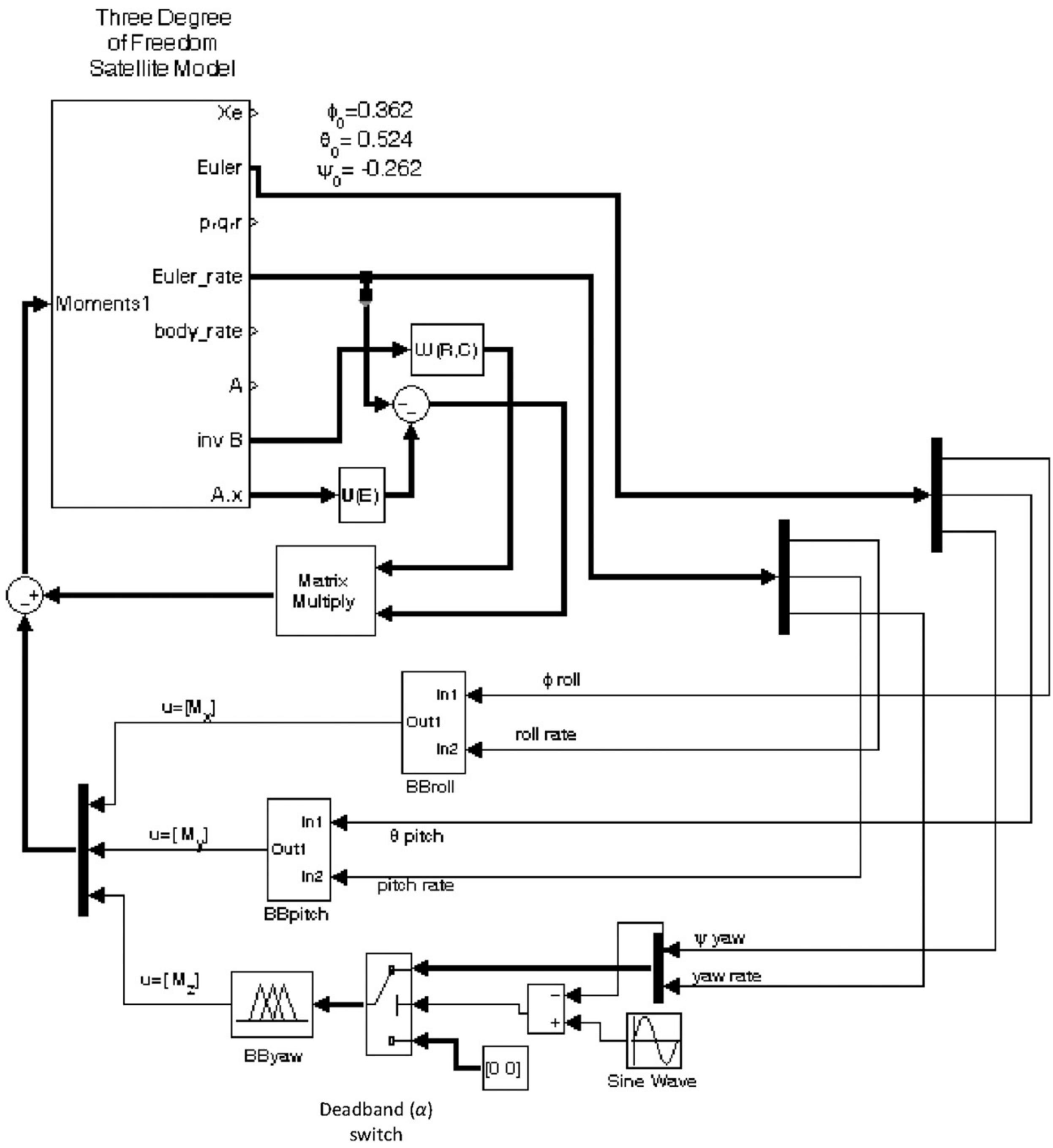

Fig. 2. Three degree of freedom satellite model and fuzzy bang-bang relay controllers.

3.1. Linguistic description

The input and output variables of the fuzzy controllers are as shown in Fig. 2. The input and output parameters, partitions and spread of the controller membership functions are selected to match the dynamic response of the satellite. The inputs $x_i \in \chi_i$, where χ_i, $i = 1, 2$ is the universe of discourse of the two inputs. For linguistic input variable, $\tilde{x}_1$ = "error angle", the universe of discourse, $\chi_1 = [-3, 3]$ rad, represents the angle of perturbation angle from the

Table 2
Fuzzy rules for FBBRC.

$\dot{\theta}$	$\boldsymbol{\theta}$				
	LN	**SN**	**Z**	**SP**	**LP**
LN	$+M_x$	$+M_x$	$+M_x$	$+M_x$	
SN	$+M_x$	$+M_x$	$+M_x$		$-M_x$
Z	$+M_x$	$+M_x$		$-M_x$	$-M_x$
SP	$+M_x$		$-M_x$	$-M_x$	$-M_x$
LP		$-M_x$	$-M_x$	$-M_x$	$-M_x$

Table 3
Fuzzy rules for standard FLC.

$\dot{\theta}$	$\boldsymbol{\theta}$				
	LN	**SN**	**Z**	**SP**	**LP**
LN	PL*	PL	PL	PS	OFF
SN	PL	PL	PS	OFF	NS
Z	PL	PS	OFF	NS	NL
SP	PS	OFF	NS	NL	NL
LP	OFF	NS	NL	NL	NL

*PL, positive large; PS, positive small; NL, negative large.

zero reference and for linguistic input variable $\tilde{x}_2$ =“error angle rate,” the universe of discourse is $\chi_2 = [-3, 3]$ rad/s. The output universe of discourse $\gamma = [-M_x, +M_x]$ represents the bang-bang output $\tilde{\gamma} \in \gamma$.

The set $\tilde{A}_i^j$ defines the *j*th linguistic value of linguistic variable $\tilde{x}_i$, which is defined over the universe of discourse χ_i. The control level of the system operation can be adequately defined for input $\tilde{x}_1$ by the following linguistic values:

$$\tilde{A}_{i=1}^j = [\tilde{A}_1^1 = LN, \tilde{A}_1^2 = SN, \tilde{A}_1^3 = Z, \tilde{A}_1^4 = SP, \tilde{A}_1^5 = LP]$$

Similar linguistic values are selected for input $\tilde{x}_2$; i.e., $\tilde{A}_2^j \equiv \tilde{A}_1^j$. The set $\tilde{B}_1^j$ denotes the linguistic values for output linguistic variable $\tilde{y}_1$ and is defined by

$$\tilde{B}_i^j = [\tilde{B}_1^1 \to J2, \tilde{B}_1^2 \to J1]$$

where $J1$ and $J2$ are on/off-firing command for the thrusters.

3.2. Fuzzy rules

The fuzzy linguistic rules assembled in this work reset the angle $\Phi = 0°$. The rules are based on two input variables, each with five linguistic values. Thus there are at most 25 possible rules. The rules are described in matrix form in Tables 2 and 3. In later sections it will be shown that the rules can be interpreted from sliding mode control. The main diagonal entry in the rule Table 2 is not used. The rules-partitions are heuristically chosen to reset the beam smoothly over the universe of discourse.

The rules matrix symmetry is no surprise and arises from the symmetry of the system dynamics. The decomposition of linguistic rules from the controller's input side to the output is

$$\mu_{\tilde{B}_{i_g}^j}(y) = \min\{\mu_{\tilde{A}_{1_g}^j}(x_1),\ \mu_{\tilde{A}_{2_g}^j}(x_2)\} \tag{6}$$

The index g is for the number of rules used in the implication.

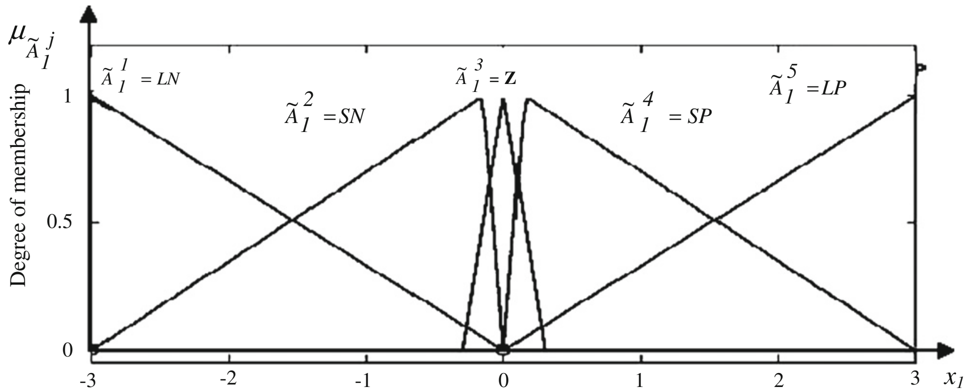

Fig. 3. Membership functions of input $\theta \Rightarrow \tilde{x}_1 =$ "error angle" and linguistic values $\tilde{A}_1^j$ for both FBBRC and standard FLC.

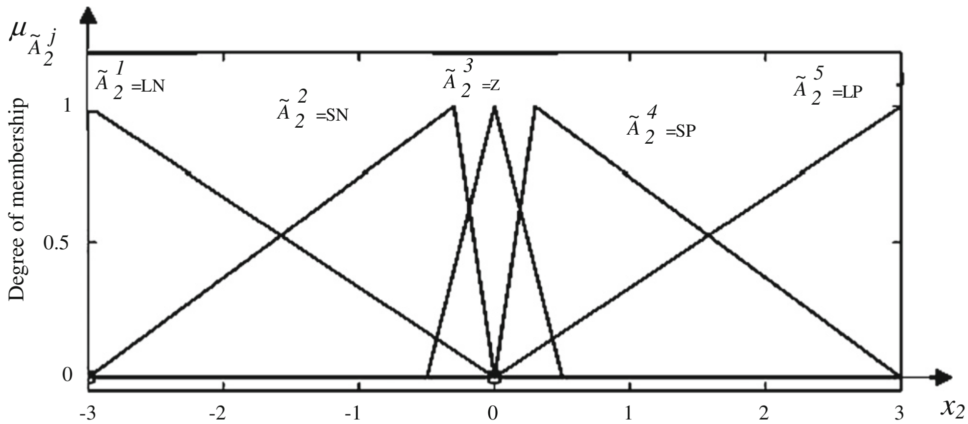

Fig. 4. Membership functions of input $\dot{\theta} \Rightarrow \tilde{x}_2 =$ "error angle rate" and linguistic values $\tilde{A}_2^j$ for both FBBRC and standard FLC controller.

3.3. Fuzzy set membership functions

The input linguistic variables and values assigned to fuzzy set membership functions are shown in Figs. 3 and 4. Triangular shape membership functions are used in this work, which are sensitive to small changes in the vicinity of their centers. A small change across the central membership function, $\tilde{A}_i^3$, located at the origin, can produce abrupt switching. This effect is countered by asymmetrical (smaller) spread of $\tilde{A}_1^3$. Center width narrow (CWN) membership function with the narrow width near the origin is used to prevent overshoot [17], Figs. 3 and 4. Adjusting $\tilde{A}_1^3$ overlaps with the adjacent membership functions control its degree of sensitivity. The input state near the origin quickly contributes to the switching of control command u between $+ve$ and $-ve$ half of universe of discourse, producing chattering. The overlapping of the central membership function $\tilde{A}_1^3$ with the neighboring membership functions $\tilde{A}_1^2$ and $\tilde{A}_1^4$ provides insensitivity—fuzzy hysteresis to bang-bang control action. The hysteresis controls the switching time delay between $J1$ and $J2$.

Triangular membership functions in Figs. 3 and 4 are based on mathematical characteristics given in Table 4. Smooth transition between adjacent membership functions is achieved with higher percentage of overlap, which is commonly set at 50%. The output membership function for standard FLC is shown in Fig. 5 and for FBBRC see Fig. 6. In FBBRC note the absence of a central membership function at the origin of the output universe of discourse. This is also reflected by the absence of diagonal rules in Table 2. Standard FLC uses the same input membership function as shown in Figs. 3 and 4.

Table 4
Mathematical characterization of triangular membership functions.

Linguistic value	*Triangular membership functions*
$\tilde{A}_i^1$, $c^L = -3$	$\mu_{\tilde{A}_i^1=\tilde{B}_1^1}(x) = \begin{cases} 1, & x = -3 \\ \max\left\{0, 1 + \dfrac{x - c^L}{-3}\right\}, & -2.9 \le x \le 0 \end{cases}$
$\tilde{A}_i^2$, $c^{IL} = -0.3$	$\mu_{\tilde{A}_i^2}(x) = \begin{cases} \max\left\{0, 1 + \dfrac{x - c^{IL}}{27}\right\}, & -3 \le x \le -0.4 \\ \max\left\{0, 1 + \dfrac{x - c^{IL}}{-0.3}\right\}, & -0.3 \le x \le 0 \end{cases}$
$\tilde{A}_i^3$, $c = 0$	$\mu_{\tilde{A}_i^3}(x) = \begin{cases} \max\left\{0, 1 + \dfrac{x - c}{3}\right\}, & -0.3 \le x \le 0 \\ \max\left\{0, 1 + \dfrac{x - c}{-0.3}\right\}, & 0 \le x \le 0.3 \end{cases}$
$\tilde{A}_i^4$, $c^{IR} = 3$	$\mu_{\tilde{A}_i^4}(x) = \begin{cases} \max\left\{0, 1 + \dfrac{x - c^{IR}}{3}\right\}, & 0 \le x \le 0.3 \\ \max\left\{0, 1 + \dfrac{x - c^{IR}}{-2.7}\right\}, & 0.4 \le x \le 3 \end{cases}$
$\tilde{A}_i^5$, $c^R = 3$	$\mu_{\tilde{A}_i^5=\tilde{B}_1^2}(x) = \begin{cases} \max\left\{0, 1 + \dfrac{x - c^R}{3}\right\}, & 0 \le x \le 2.9 \\ 1, & x = 3 \end{cases}$

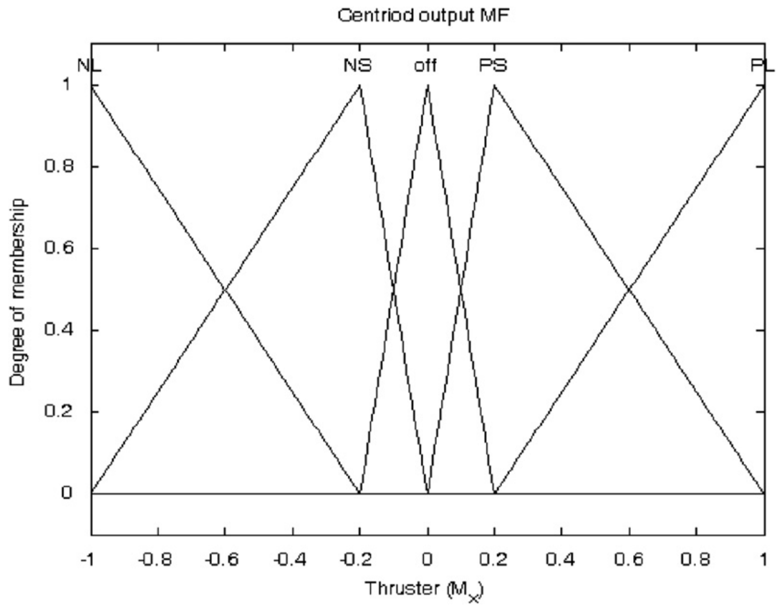

Fig. 5. Output membership functions of FLC, min–max aggregation and centroid.

3.4. Largest of maximum (LOM) aggregation

The selection of membership functions in Fig. 6a and LOM aggregation formulates the fuzzy bang-bang relay controller. Any input perturbation of the beam from the zero reference acts on the output membership functions according to the rules matrix in Table 2.

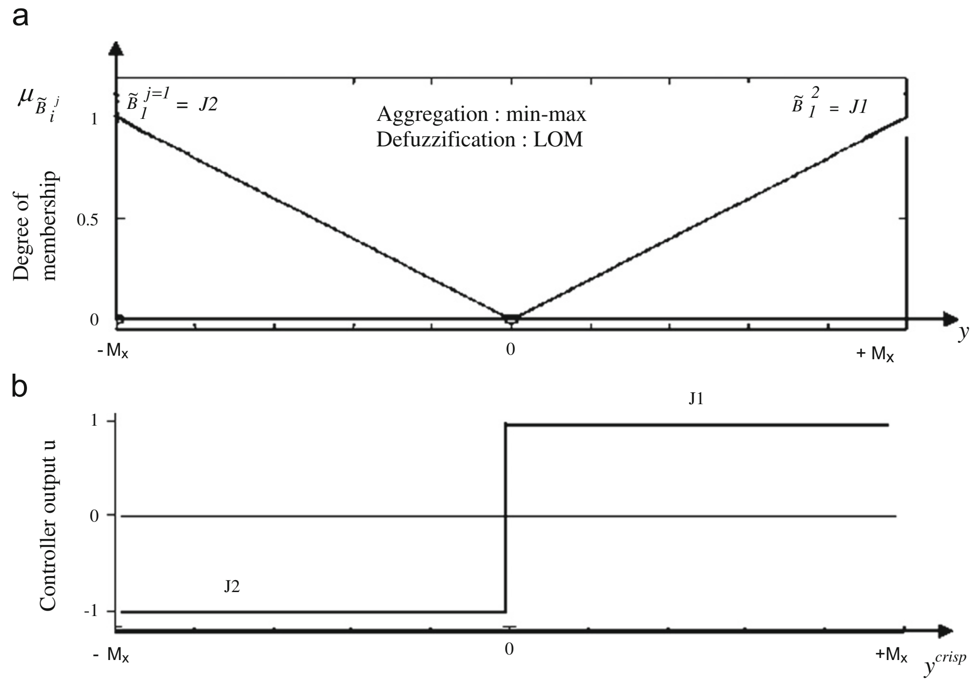

Fig. 6. (a) FBBBRC output y membership functions and linguistic values $\tilde{B}_1^j$. (b) FBBRC two level bang-bang y^{crisp} output.

Let $\mu_{overall}$ be the membership function of the overall implied fuzzy rules $g = 1, 2 \ldots G$. Then, it is obtained by taking the maximum [10] as follows:

$$\mu_{overall}(y) = \max_g \{\mu_{\tilde{B}_{1_g}^1}(y), \mu_{\tilde{B}_{2_g}^2}(y), \ldots, \mu_{\tilde{B}_{i_g}^j}(y)\} \tag{7}$$

The defuzzified crisp output y^{crisp} is obtained as

$$s y^{crisp} = \arg\sup\left\{\mu_{overall}(y)\right\} \tag{8}$$

The supremum in (8) is the largest of maximum (LOM) value and will occur at the extremes of the output of the universe of discourse $\gamma = [-M_x, M_x]$. The argument arg(sup(μ)) returns $y^{crisp} = [-M_x, M_x]$. The results of firing action of membership functions $J2$ and $J1$ are shown in Fig. 6b.

4. Fuzzy sliding mode controller

The fuzzy rules described in Table 2 can be systematically constructed on the basis of sliding mode control and hitting condition described by Eq. (B.13) in Appendix B. Appendix B provides detailed derivation of the control law and stability condition of the SMC and the Simulink simulation block diagram.

The rules in Table 2 can be deduced from Eq. (A.8). Multiplying it with s yields

$$s\dot{s} = f(\theta; t)s + b(\theta; t)us + \lambda\dot{\theta}s \tag{9}$$

For $b > 0$, if $s < 0$, then increasing $u(+M_x)$ will result in decreasing $s\dot{s}$; and if $s > 0$, then decreasing $u\,(-M_x)$ will result in decreasing $s\dot{s}$. The control value u should be selected so that $s\dot{s} < 0$ for $0 < s > 0$. The slope of sliding line is represented by λ.

Considering s as θ and $\dot{s}$ as $\dot{\theta}$, $M_x = 1$, $u = [-1, +1]$, the fuzzy rules in Tables 2 and 3 and the membership functions shown in Fig. 6a agree with the sliding mode condition, Appendix B.3.

5. State dependent Riccati equation (SDRE)

Linear quadratic regulator (LQR) is an established technique for satellite attitude control which uses the celebrated Riccati equation for the solution of minimum control signal. Linear quadratic regulator requires a linear system or a non-linear system linearized at an operating point for solving using Riccati equation. However, SDRE [16] can be used for non-linear systems such as satellite attitude control [17] and the non-linear state dependent matrix $\boldsymbol{A}(\boldsymbol{x})$ in (5) is updated at every sample time. The non-linear state model (1)–(5) can be written in a linear state dependent coefficient (SDC) form of

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x}) \cdot \boldsymbol{x} + \boldsymbol{B} \cdot \boldsymbol{u} \tag{10}$$

Consider satellite attitude as a non-linear regulator problem for minimizing, in infinite time, the performance criteria J given by

$$\boldsymbol{J} = \frac{1}{2}\int_0^\infty [\boldsymbol{y}^T \boldsymbol{Q}_{3\times3}(\boldsymbol{x})\boldsymbol{y}_{3\times1} + \boldsymbol{u}^T \boldsymbol{R}_{3\times3}(\boldsymbol{x})\boldsymbol{u}_{3\times1}]\, dt$$

The stability condition requires that $\boldsymbol{Q}$, a state dependent weighting matrix is positive semi-definite while $\boldsymbol{R}$, the control weight matrix is positive definite. Solving the following Riccati equation for state dependent matrix $\boldsymbol{P}(\boldsymbol{x}) > 0$

$$\boldsymbol{A}^T(\boldsymbol{x})\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}(\boldsymbol{x}) - \boldsymbol{P}\boldsymbol{B}(\boldsymbol{x})\boldsymbol{R}^{-1}(\boldsymbol{x})\boldsymbol{B}^T(\boldsymbol{x})\boldsymbol{P} + \boldsymbol{Q}(\boldsymbol{x}) = 0 \tag{11}$$

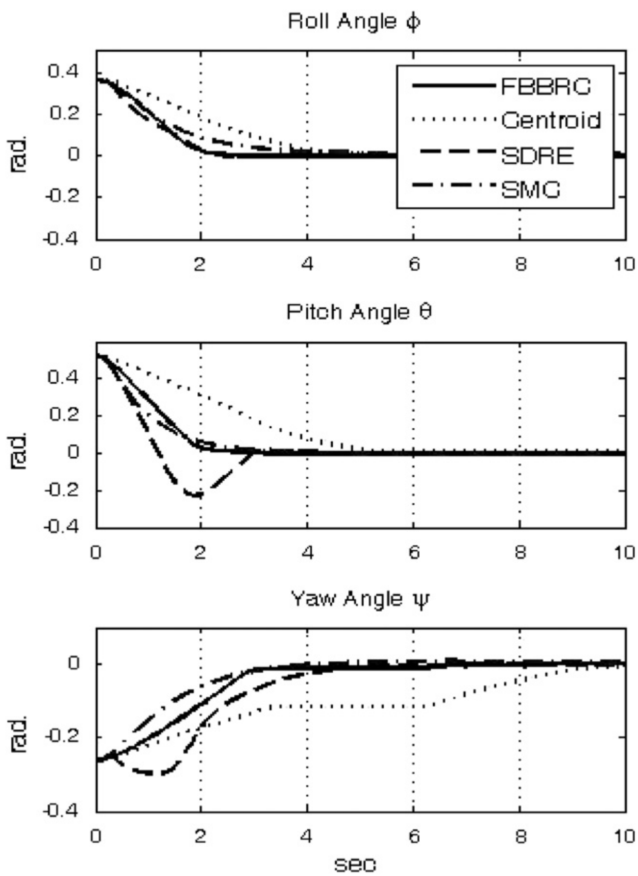

Fig. 7. Euler angles reset comparison between three techniques from different initial positions.

Gives the non-linear state variable feedback control law u:

$$\boldsymbol{K} = -\boldsymbol{R}^{-1}(\boldsymbol{x})\boldsymbol{B}^T(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})$$

$$\boldsymbol{u} = -\boldsymbol{K} \cdot \boldsymbol{x} \tag{12}$$

Then $u = [M_x \ M_y \ M_z \ 0 \ 0 \ 0]^T$ in (12) is injected to the system model and new states $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{p}} \ \tilde{\boldsymbol{q}} \ \tilde{\boldsymbol{r}} \ \tilde{\phi} \ \tilde{\theta} \ \tilde{\psi}]^{\boldsymbol{T}}$ are obtained. The state dependent system matrix $\boldsymbol{A}(\tilde{\boldsymbol{x}})$ is updated and (12) is solved again and the cycle is repeated until the states reach the desired values. For satellite attitude reset the output u used here is

$$u = -\boldsymbol{K} \cdot \boldsymbol{x}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}) + \boldsymbol{x}(\phi, \theta, \psi) \tag{13}$$

Simulation diagram for SDRE is provided in Appendix C.

6. Simulation response

The satellite model in simulation is based on Eqs. (1)–(5) and the parameters are given in Table 1 and Fig. 2. The simulated performance of the proposed controller is compared with other controllers.

The simulation response of the FBBRC, the centroid FLC, SMC and SDRE are shown in Fig. 7. Thruster strength and cycle comparison of the controllers are shown in Fig. 8.

Both fuzzy controllers use the same input membership functions and initial conditions. However, the output membership functions are different.

The FBBRC has the output membership function as shown in Fig. 6 and standard FLC has centroid output membership function shown in Fig. 5. Angle reset demonstrated in Fig. 7 shows that SRDE and FBBRC have close response but the control effort u of SDRE are five time more in thruster strength and cycle in comparison to FBBRC.

An important feature of the controllers is shutting down of bang-bang action within the deadband, $\alpha = \pm 0.01$ rad as shown in Fig. 2. This prevents the controllers from chattering, avoiding high frequency vibration and fuel expending at zero states. The output universe of discourse for both controllers is $\gamma = [-1, +1]$. These results are expected because the bang-bang control system is inherently a time optimal controller [19,11].

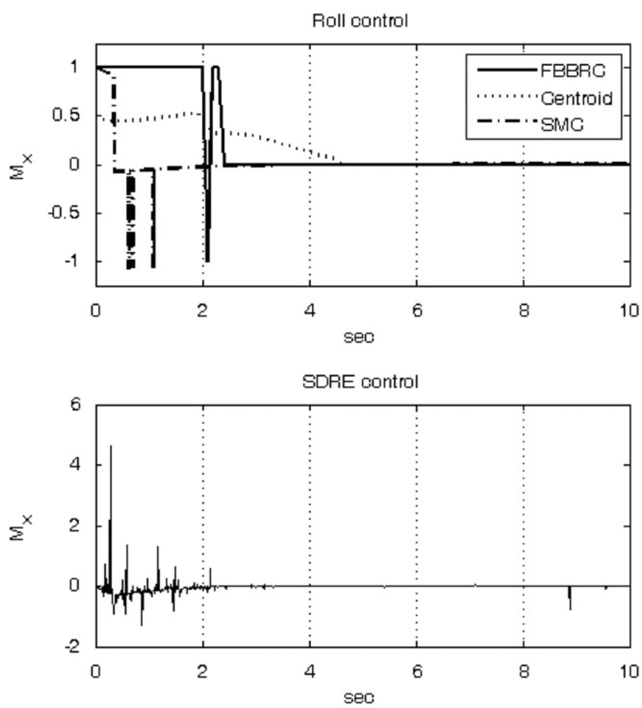

Fig. 8. Control signal $u = [-M_x + M_x]$ comparison among the various techniques for roll axis with deadband.

Any proposed control strategy should be supported by stability analysis for acceptance by the control system community. Fuzzy bang-bang relay controller is no exception. As discussed earlier, conventional bang-bang control system has firm stability ground via sliding mode control, which uses Lyapunov like function to satisfy the stability criteria, which is discussed in Appendix B.1.

7. Conclusion

This paper proposes an integrated fuzzy bang-bang relay controller for satellite attitude control. Its capabilities were demonstrated on simulated model and compared with other optimal control technique (LQR). The simulation model, representing a non-linear satellite model without uncertainties, was used for the development, stability, and optimal response. Fuzzy controllers are known for absorbing the non-linearity of the systems and, as the results show, it works well for the real system.

The bang-bang control is inherently time-optimal, and this property is an important feature of the FBBRC. Comparison between the FBBRC and the standard FLC showed that FBBRC could reset the initial condition in optimal time and with a smaller overshoot. FBBRC uses fewer output membership functions in comparison to the standard FLC and thus simplifying the unification process of the fuzzy rule matrix. In the FBBRC, the largest of maxima defuzzification (LOM) method yields direct bang-bang output from the fuzzy controller. The bang-bang output is similar to conventional hardware relay output. However, unlike fixed relay control/SMC inputs the FBBRC inputs are flexible to meet the changing demands of the non-linear system dynamics. The stability and optimality of FBBRC were satisfied by the SMC-Lyapunov criterion and optimal bang-bang control theory, respectively.

The FBBRC performs better than LQR optimal control using Riccati equation and SMC. For similar conditions the LQR method requires higher thruster power and more firing cycle than the FBBRC technique, thus reducing the size and fuel consumption.

Appendix A. Glossary

Symbols and abbreviations

$\boldsymbol{A}(\boldsymbol{x})$	Jacobian $(f(x), x)$
$\tilde{A}$	fuzzy input set
$\tilde{A}_i^j$	jth linguistic value of input linguistic variable $\tilde{x}_i$
$B(u)$	Jacobian $(f(x), u)$
$\tilde{B}$	fuzzy output set
$\tilde{B}_1^j$	linguistic values for output linguistic variable $\tilde{y}_i$
α	deadband
β	bounded condition for control gain
COA	center of area
COG	center of gravity
CWN	center width narrow
c	center of triangular membership function
c^L	center left
c^{IL}	center inner left
c^R	center right
c^{IR}	center inner right
e	state error
η	reaching condition $+ve$ constant
FBBRC	fuzzy bang-bang relay controller
FLC	fuzzy logic controller (standard)

$f(\boldsymbol{x})$	non-linear states function of 3 dof satellite system
g	fuzzy rule index
γ	output universe of discourse
G_d	derivative gain
G	number of fuzzy rules
HB	hysteresis band
$\dot{H}$	angular momentum
i	input index
I	moment of inertia
J	performance index
j	linguistic value index
J	control action bounds
$J1$	decomposition of linguistic rules $\mu_{\tilde{B}_1^2}$—on/off command
$J2$	decomposition of linguistic rules $\mu_{\tilde{B}_1^1}$—on/off command
K	constant, sliding gain
λ	slope of sliding line
LOM	largest of maxima
LQR	linear quadratic regulator
M	thruster momentum
μ	decomposed value of fuzzy rule
$\mu_{overall}$	decomposition of overall implied fuzzy rules
MISO	multi input single output
MOM	mean of maxima
P	state dependent matrix
p	body pitch rate
Φ	roll angle
ψ	yaw angle
Q	state dependent weight matrix
q	body yaw rate
R	control weight matrix
R^m	dimensional states span
r	body roll rate
s	sliding switching line
$\dot{s}$	rate of sliding line
SDC	state differential coefficient
SDRE	state dependent Ricatti equation
SMC	sliding mode controller
θ	pitch angle, output of interest
u	thrusters input vector
ω	Euler angular rate vector
ω_b	body angular rate vector
$\boldsymbol{x}$	states vector
$\tilde{x}$	LQR states
$\tilde{x}_i$	linguistic input variable
X_B	body roll axis
Y_B	body pitch axis
Z_B	body yaw axis
χ_i	input universe of discourse
y	defuzzified output
$\tilde{y}_i$	linguistic output variable
y^{crisp}	defuzzified crisp output

Appendix B. Sliding mode control (SMC)

A general 2nd order non-linear single-input–single-output (SISO) control system could be described [12] as

$$\dot{\omega}(t) = f(\theta; t) + b(\theta; t)u \tag{B.1}$$

where $\theta(t)$ is the output of interest, $u(t)$ is the scalar input, and $\theta = [\Phi, \dot{\Phi}]^T$ is state vector. In general, $f(\theta; t)$ is not precisely known, but upper bounded by a known continuous function of θ. Similarly $b(\theta; t)$ is not known, but is of known sign and is bounded by a known continuous function of x as

$$\begin{aligned} &|f - \hat{f}| \le F(\theta; t) \\ &\frac{1}{\beta(\theta; t)} \le \frac{\hat{b}}{b} \le \beta(\theta; t) \end{aligned} \tag{B.2}$$

where $\hat{f}$ and $\hat{b}$ are nominal values of f and b, respectively, without the function argument for brevity purpose.

Comparing Eqs. (1) and (B.1), the system becomes

$$\begin{aligned} &f(\theta; t) = -\frac{1}{I}\dot{\theta}(t) \\ &b(\theta; t)u = \frac{M}{I}u(t) \quad \text{because } b(\theta; t) = \frac{M}{I} \end{aligned} \tag{B.3}$$

where $u(t)$ is a unit step input.

The control problem is to get the state $\boldsymbol{\theta}$ to track $\boldsymbol{\theta}_d = [\theta_d \;\; \dot{\theta}_d]^T$ in minimum time and in the presence of imprecise friction. The initial θ_d should be the following in view of finite control u:

$$\theta_d(0) = \theta(0) \tag{B.4}$$

The tracking error between the actual and desired state would be

$$e = \theta - \theta d \doteq [e \;\; \dot{e}]^T \tag{B.5}$$

A sliding-switching line $s(\theta, t)$ in the second order state space $\boldsymbol{R}^2$ is defined such that e follows the line $s(\boldsymbol{\theta}, t) = 0$. The sliding line $s(\boldsymbol{\theta}, t)$ is determined by

$$s(\theta, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{B.6}$$

Eq. can be expanded with binomial expansion and λ is positive constant. For $n = 2$

$$s = \dot{e} + \lambda e \quad \text{because } \dot{e} = \dot{\theta} \text{ and } e = \theta \tag{B.7}$$

Then from Eq. (B.1)

$$\dot{S} = f(\theta; t) + b(\theta, t)u + \lambda\dot{e} \tag{B.8}$$

B.1. SMC control law

Let u_{eq} be the equivalent control law that will keep the states on the sliding trajectory, computed by $\dot{s} = 0$ for $u \equiv u_{eq,}$ then from Eqs. (B.4), (B.5) and (B.7)

$$\begin{aligned} &s = \dot{\theta} + \lambda\theta \\ &\dot{s} = \ddot{\theta} + \lambda\dot{\theta} \end{aligned} \tag{B.9}$$

Then from Eq. (B.8) with uncertainties

$$\dot{s}|_{u=u_{eq}} = \hat{f}(\theta; t) + \hat{b}(\theta, t)u_{eq} + \lambda\dot{e} = 0$$

Solving the above equation

$$u_{eq} = \hat{b}^{-1}\hat{u} \tag{B.10}$$

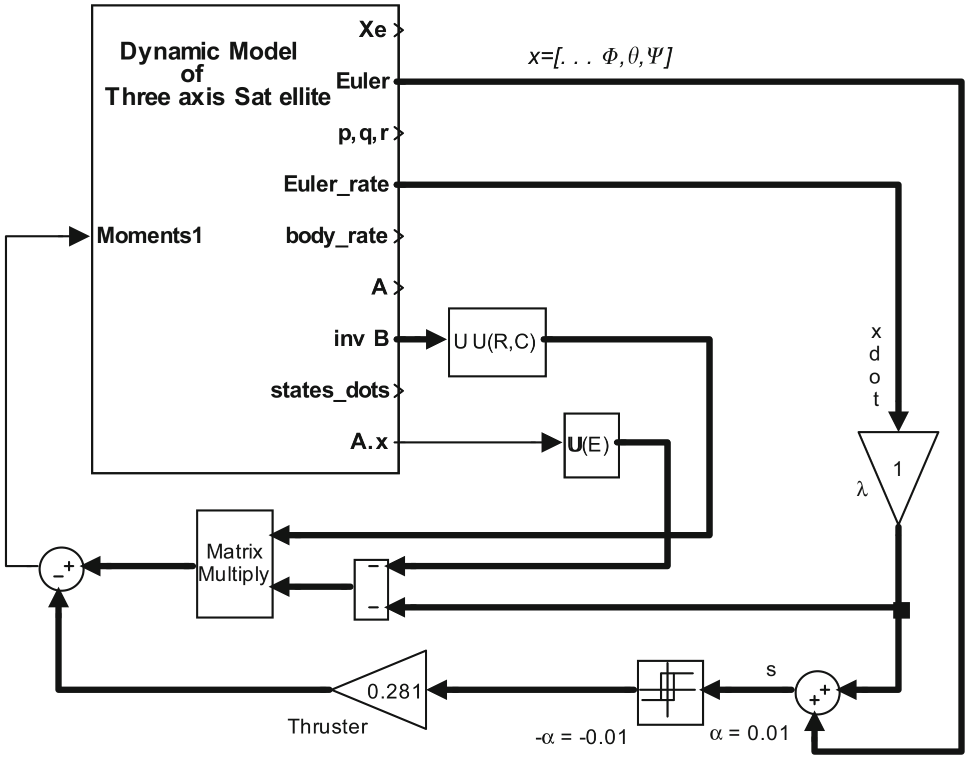

Fig. B1. Sliding mode controller described in Eq. (B.15).

where

$$\hat{u} = [-\hat{f}(\theta; t) - \lambda\dot{e}]$$

or

$$\lambda\dot{e} = -\hat{f}(\theta; t) - \hat{u} \tag{B.11}$$

is the nominal control input in presence of uncertainties.

The control input u to get the state $\boldsymbol{\theta}$ to track $\boldsymbol{\theta}_d$ is then made to satisfy the Lyapunov-like function $V = (1/2)s^2$, if there exist $\eta > 0$ and by the following sliding condition [12]:

$$\frac{1}{2}\frac{d}{dt}s^2(\theta, t) \leq -\eta|s| \tag{B.12}$$

which is reduced to the so-called sliding mode '*reaching condition*' for Eq. (1)

$$\dot{s} \cdot \operatorname{sgn}(s) \leq -\eta|s|, \quad \eta > 0 \tag{B.13}$$

The control law that satisfies the sliding mode reaching conditions (Figs. B1 and B2) Eq. (B.13) can be obtained as

$$u = u_{eq} + u_S \tag{B.14}$$

where

$$u_s = -K\ \operatorname{sgn}(s) \tag{B.15}$$

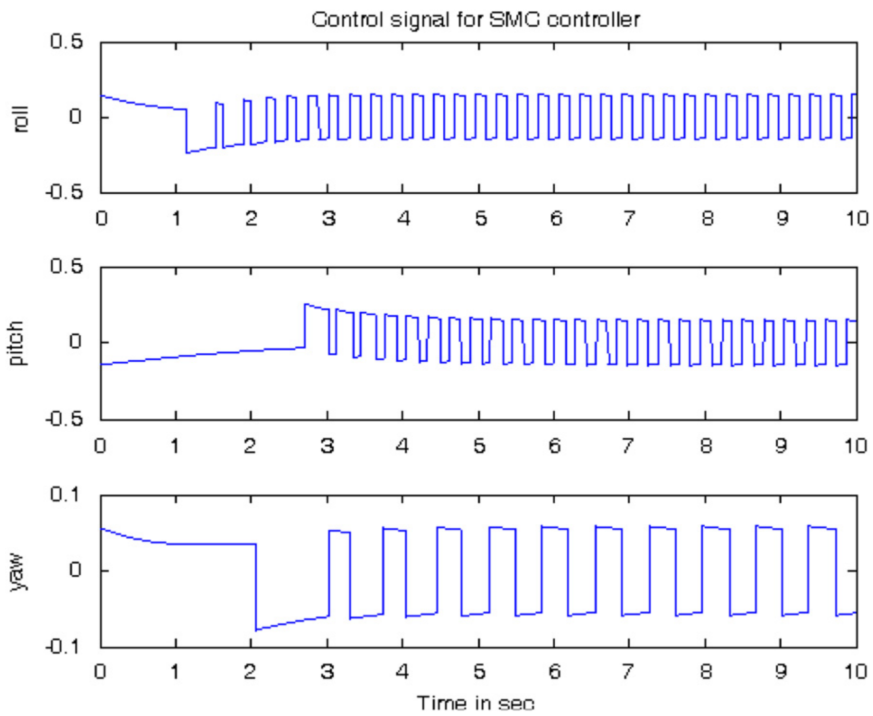

Fig. B2. Typical chattering control signal of sliding mode controller without deadband.

and

$$\mathrm{sgn}(s) = \begin{cases} +1 & \text{if } s > 0 \\ -1 & \text{if } s < 0 \end{cases}$$

Substituting Eqs. (B.1) and (B.8) in Eq. (B.13)

$$s\dot{s} = s(f + bu + \lambda\dot{e}) \le -\eta|s|$$

Note: Here we have dropped the function argument for brevity purpose. Then equivalently we can write

$$s\dot{s} = \mathrm{sgn}(s)(f + \lambda\dot{e}) + bu\,\mathrm{sgn}(s) \le -\eta|s| \tag{B.16}$$

To satisfy (B.16) control signal u is chosen as

$$u = \frac{1}{B}(-A \cdot x - \lambda \cdot x) - K sign(s) \tag{B.17}$$

Substituting Eqs. (B.14) and (B.15) into Eq. (B.16)

$$s\dot{s} = \mathrm{sgn}(s)(f + \lambda\dot{e}) + b[u_{eq} + \hat{b}^{-1}K\,\mathrm{sgn}(s)]\mathrm{sgn}(s) \le -\eta|s|$$

Substituting from Eqs. (B.10) and (B.11) in above, we get

$$s\dot{s} = \mathrm{sgn}(s)\,[f + (-\hat{f} - \hat{u})] + b\,[\hat{b}^{-1}\hat{u} + \hat{b}^{-1}K\,\mathrm{sgn}(s)]\mathrm{sgn}(s) \le -\eta|s|$$

simplifying we get

$$\mathrm{sgn}(s)\,(f - \hat{f}) + \left[\frac{b}{\hat{b}} - 1\right]\hat{u}\,\mathrm{sgn}(s) - \frac{b}{\hat{b}}K \le -\eta|s| \tag{B.18}$$

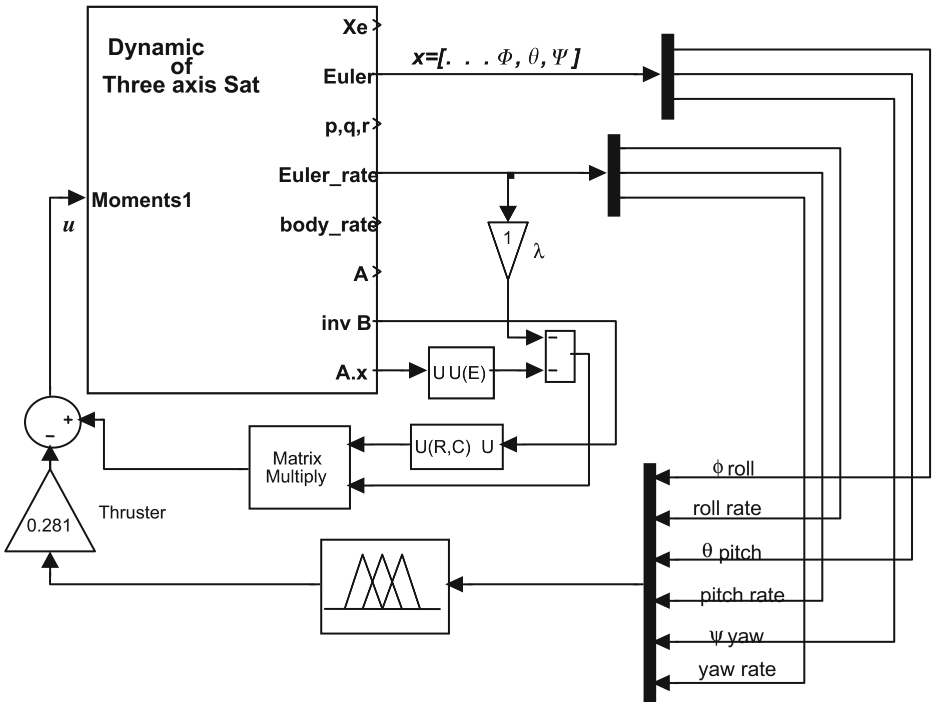

Fig. B3. Fuzzy logic controller formulated in Eq. (B.22).

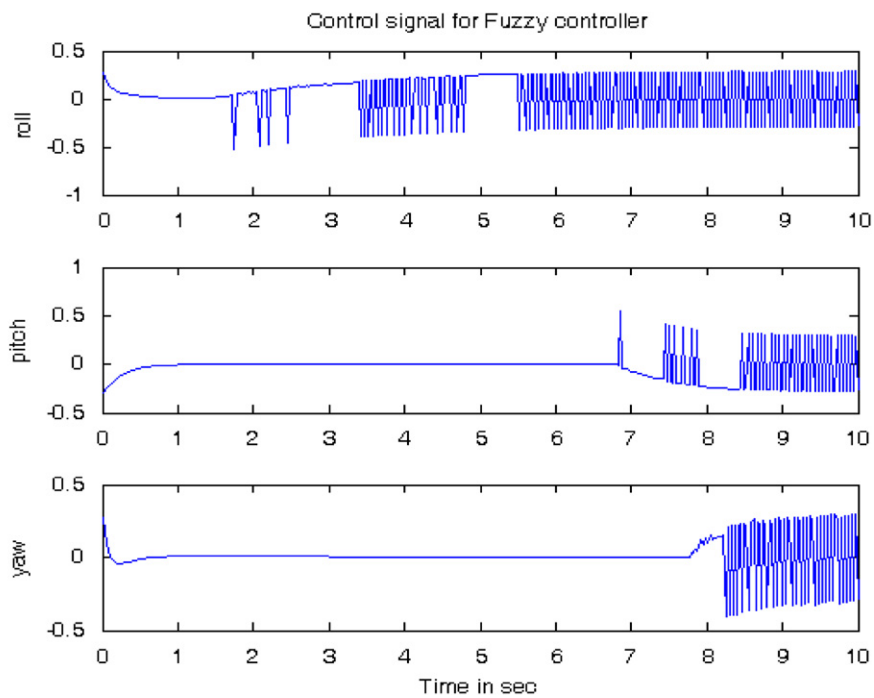

Fig. B4. Bang-bang output of fuzzy controller without deadband.

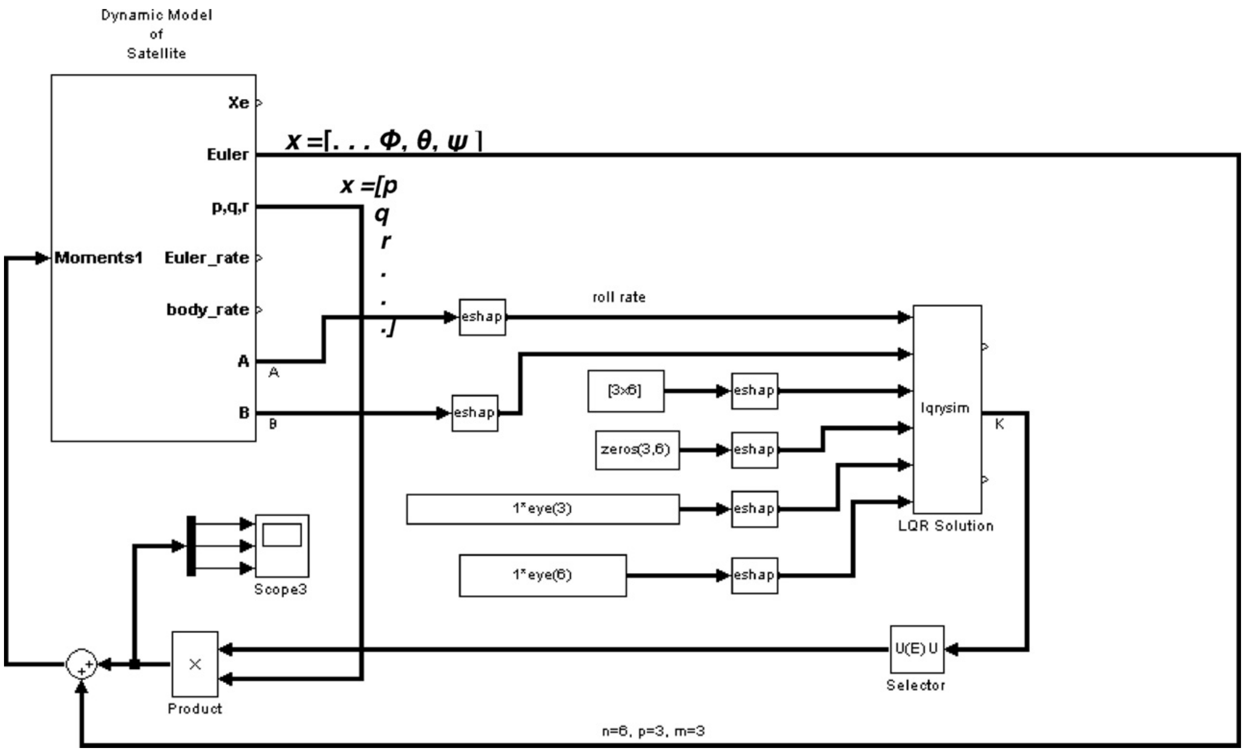

Fig. C1. State dependent Riccati controller from Eqs. (11)–(13).

Then for upper bounds from Eq. (B.1) need

$$K \geq \beta[(F+\eta)+(\beta-1)|\hat{u}|] \tag{B.19}$$

to satisfies the reaching or hitting condition.

Appendix B.2. Fuzzy sliding mode controller

Let $K_{fuzz}(x, \dot{x}, \lambda)$ be the absolute value of the control output of the FC. Then the controller term in (B.17) is

$$-\boldsymbol{K}\,\boldsymbol{sign}(\boldsymbol{s}) = -\boldsymbol{K}_{fuzz}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{\lambda}) \cdot \overline{\boldsymbol{sign}}(\boldsymbol{s}) \tag{B.20}$$

Using the analogy of SMC, an important thing to observe here is that the strength of output u is constant as the distance $|s|$ from the diagonal $s = 0$ increases, positive large—PL or negative large—NL. The bang-bang system requires switching $\overline{\boldsymbol{sign}}(\boldsymbol{s})$ and $\boldsymbol{K}_{fuzz}(|\boldsymbol{s}|)$ equal to thruster output . Then the control law simplifies to

$$\begin{aligned} &-\boldsymbol{k}\,\boldsymbol{sign}\{\boldsymbol{s}(\boldsymbol{x})\} = -\boldsymbol{K}_{fuzz}(|\boldsymbol{s}|)\overline{\boldsymbol{sign}}(\boldsymbol{s}) \\ &\boldsymbol{K}_{fuzzy}(|\boldsymbol{s}|) = 0.281\,\boldsymbol{Nm}\,(\boldsymbol{Thruster}) \end{aligned} \tag{B.21}$$

Putting (B.21) in (B.17) completes the control law (Figs. B3 and B4)

$$\boldsymbol{u} = (\lambda\boldsymbol{B})^{-1}[-\lambda\dot{\boldsymbol{x}} - \boldsymbol{A}\cdot\boldsymbol{x})] - \boldsymbol{K}_{fuzz}(|\boldsymbol{s}|)\cdot\overline{\boldsymbol{sign}}(\boldsymbol{s}) \tag{B.22}$$

Appendix C. State dependent Riccati regulator

See Figs. C1 and C2.
Controllers response describe in Appendices B and C (See Figs. C3–C5).

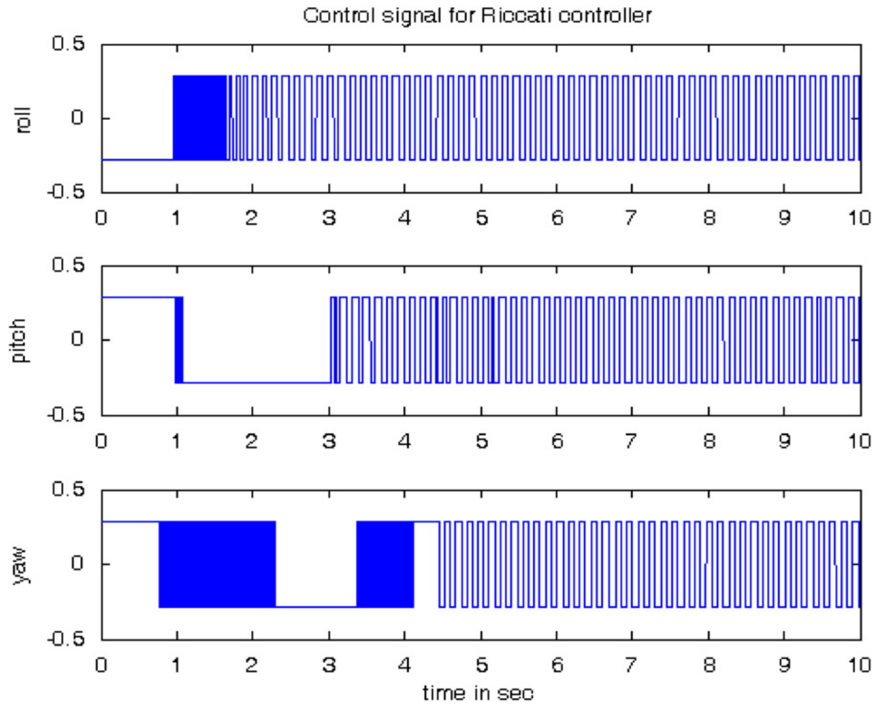

Fig. C2. Bang-bang output from SDRE controller without deadband.

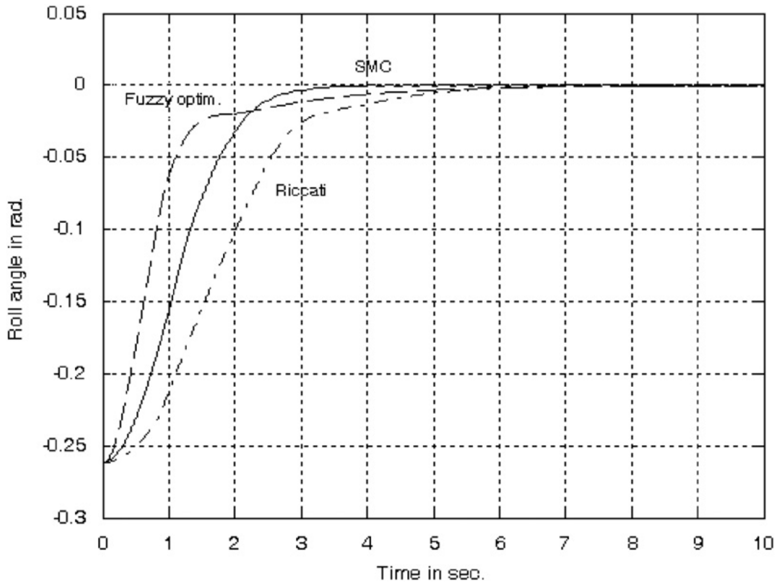

Fig. C3. Roll angle comparison of SDRE, SMC and fuzzy techniques.

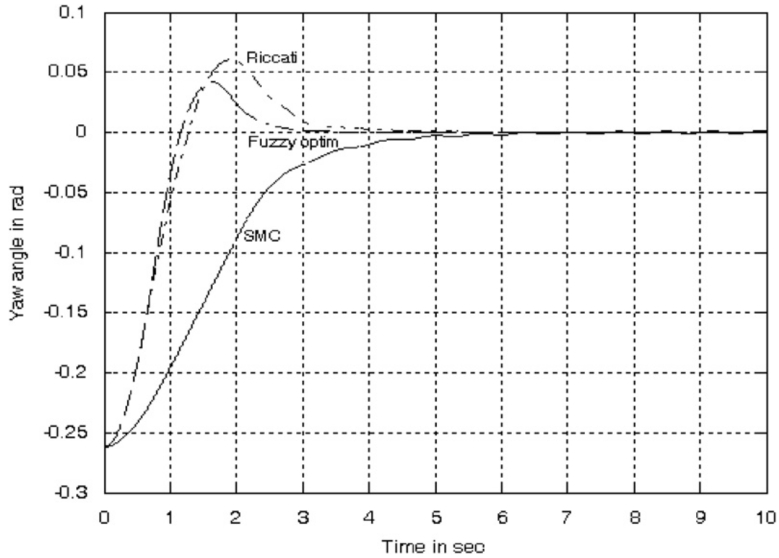

Fig. C4. Yaw angle comparison of three techniques.

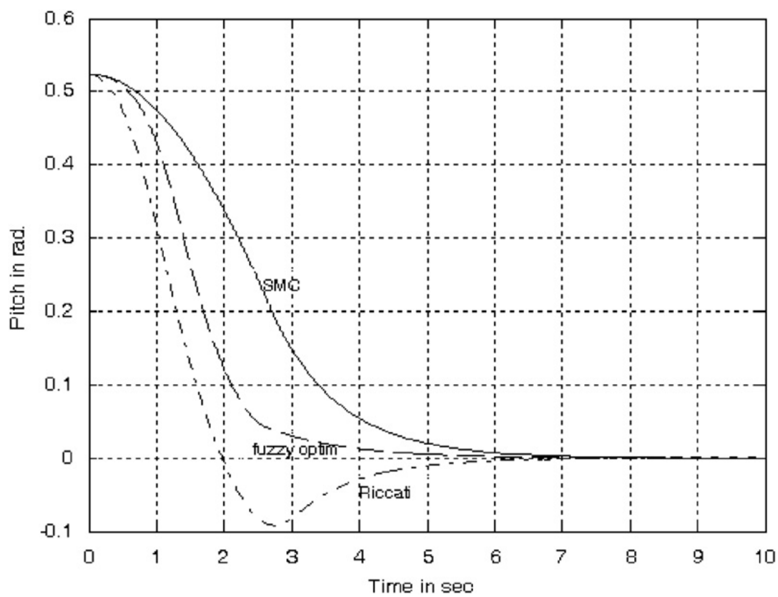

Fig. C5. Pitch angle comparison of three techniques.

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