Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Chaos, Solitons and Fractals 25 (2005) 579–584
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Chaos synchronization of a chaotic system via nonlinear control

Ju H. Park *

Robust Control and Nonlinear Dynamics Laboratory, Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea

Accepted 23 November 2004

Abstract

In this letter, the problem of chaos synchronization of a chaotic system which is proposed by Lü et al. [Int J Bifurcat Chaos 2004;14:1507] is considered. A novel nonlinear controller is designed based on the Lyapunov stability theory. The proposed controller ensures that the states of the controlled chaotic slave system asymptotically synchronizes the states of the master system. A numerical example is given to illuminate the design procedure and advantage of the result derived.

1. Introduction

Chaos is very interesting nonlinear phenomenon and has been intensively studied in the last three decades [1–20]. It is found to be useful or has great potential in many disciplines [1]. Especially, the subject of chaotic synchronization has received considerable attentions since 1990. For the state of the art, see two books [2,3]. Also, numerous applications to synchronization have been widely explored in a variety of fields including physical, chemical and ecological systems, secure communications, etc. [10–20]. Recently, the chaos synchronization of linearly coupled chaotic systems is investigated by Li et al. [17], Lü et al. [18], and Park [19]; the synchronization problem via nonlinear control scheme is studied by Chen and Han [13] and Chen [20].

On the other hand, more recently, Lü et al. [21] introduced a new chaotic system of three-dimensional quadratic autonomous ordinary differential equations, which can display two 1-scroll chaotic attractors simultaneously with only three equilibria, and two 2-scroll chaotic attractors simultaneously with five equilibria.

In this letter, the chaotic synchronization of the chaotic system devised by Lü et al. [21] is investigated. A class of novel nonlinear control scheme for the synchronization is proposed, and the synchronization is achieved by the Lyapunov stability theory.

The organization of this letter is as follows. In Section 2, the problem statement and master-slave synchronization scheme are presented for the chaotic system. In Section 3, we provide an numerical example to demonstrate the effectiveness of the proposed method. Finally concluding remark is given.

* Fax: + 82 53 8104629.
E-mail address: jessie@yu.ac.kr

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.11.038

2. Chaos synchronization

Consider the following three-dimensional autonomous system, which has been devised by Lü et al. [21] and display two chaotic attractors simultaneously:

$$\begin{cases} \dot{x} = -\frac{ab}{a+b}x - yz + c, \\ \dot{y} = ay + xz, \\ \dot{z} = bz + xy, \end{cases} \tag{1}$$

where a,b,c are real constants and x,y,z are status variables.

This system is found to be chaotic in a wide parameter range and has many interesting complex dynamic behaviors. The system is chaotic for the parameters $a = -10$, $b = -4$ and $|c| < 19.2$. For instance, when the parameters are $a = -10$, $b = -4$, $c = 0$, the orbits of three state of the system (1) for two initial conditions, $(x(0), y(0), z(0)) = (3, -4, 2)$ and $(x(0), y(0), z(0)) = (3.1, -4.1, 2.1)$ are given in Fig. 1 and it shows that the system is actually chaotic. Also, it displays the chaotic attractor as shown in Fig. 2. For details of other dynamic properties of the system, see the paper [21].

Now, our goal is to make synchronization for two identical chaotic systems of the form (1) based on the Lyapunov method. For the chaotic systems (1), the master and slave systems are defined below, respectively,

$$\begin{cases} \dot{x}_m = -\frac{ab}{a+b}x_m - y_m z_m + c, \\ \dot{y}_m = ay_m + x_m z_m, \\ \dot{z}_m = bz_m + x_m y_m, \end{cases} \tag{2}$$

and

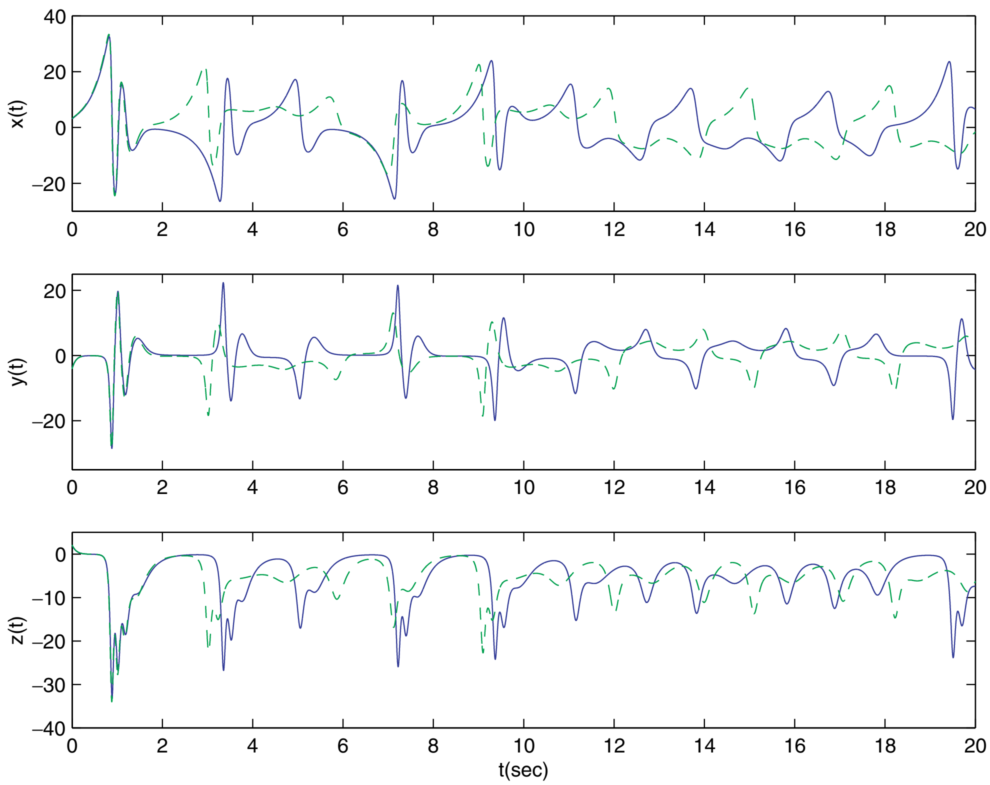

Fig. 1. Chaotic response.

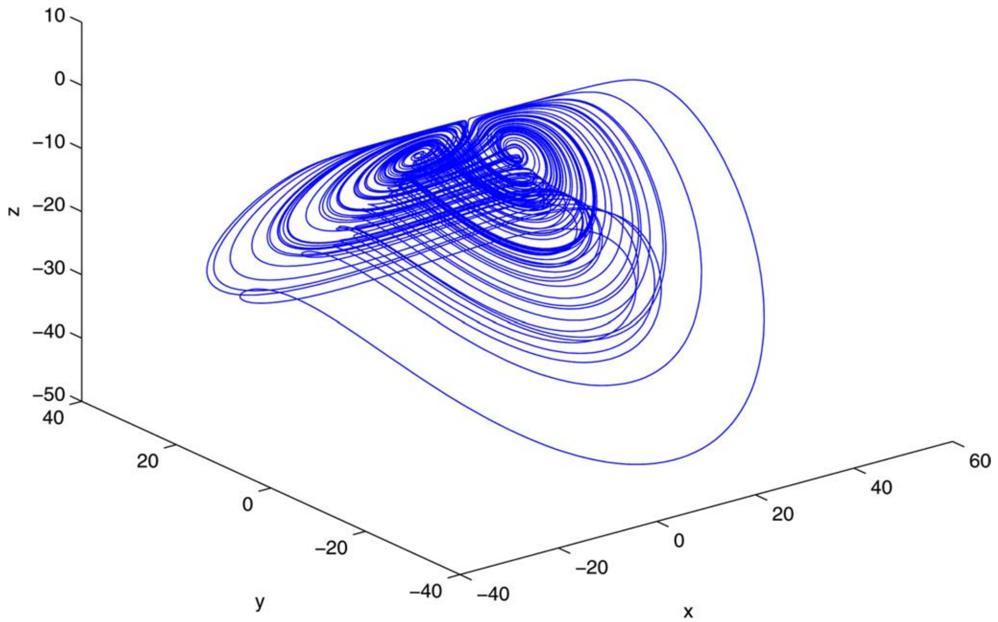

Fig. 2. Chaotic attractor ($a = -10,\ b = -4,\ c = 0$).

$$\begin{cases} \dot{x}_s = -\frac{ab}{a+b}x_s - y_s z_s + c + u_1, \\ \dot{y}_s = a y_s + x_s z_s + u_2, \\ \dot{z}_s = b z_s + x_s y_s + u_3, \end{cases} \quad (3)$$

where the lower scripts m and s stand for the master (or drive) systems, the slave (or response) one, respectively, u_1,u_2 and u_3 are the nonlinear controller such that two chaotic systems can be synchronized.

Define the error signal as

$$\begin{cases} e_1(t) = x_s(t) - x_m(t), \\ e_2(t) = y_s(t) - y_m(t), \\ e_3(t) = z_s(t) - z_m(t), \end{cases} \quad (4)$$

which gives that

$$\begin{cases} -x_m z_m + x_s z_s = z_m e_1 + x_s e_3, \\ -x_m y_m + x_s y_s = y_m e_1 + x_s e_2, \\ y_m z_m - y_s z_s = -y_s e_3 - z_m e_2. \end{cases} \quad (5)$$

From Eqs. (4)-(5), we have the following error dynamics:

$$\begin{aligned} \dot{e}_1(t) &= -\beta e_1 - y_s e_3 - z_m e_2 + u_1, \\ \dot{e}_2(t) &= a e_2 + z_m e_1 + x_s e_3 + u_2, \\ \dot{e}_3(t) &= b e_3 + y_m e_1 + x_s e_2 + u_3 \end{aligned} \quad (6)$$

where $\beta = ab/(a + b)$.

For two identical the chaotic systems without control ($u_i = 0$), if the initial condition $(x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate controller scheme. For this end, we propose the following control law for the system (3):

$$\begin{aligned} u_1 &= (\beta - 1)e_1 + e_2 e_3, \\ u_2 &= -(a + 1)e_2 - 2x_s e_3, \\ u_3 &= -(b + 1)e_3. \end{aligned} \quad (7)$$

Then, we have the following theorem.

Theorem 1. *The controlled chaotic systems (2) and (3) will approach synchronization for any initial conditions* $(x_m(0), y_m(0), z_m(0))$ *and* $(x_s(0), y_s(0), z_s(0))$ *by the control law (7).*

Proof. Construct a Lyapunov function

$$V = (1/2)\left(e_1^2 + e_2^2 + e_3^2\right). \tag{8}$$

The differential of the Lyapunov function along the trajectory of system (4) is

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \\ &= e_1(-\beta e_1 - y_s e_3 - z_m e_2 + u_1) + e_2(a e_2 + z_m e_1 + x_s e_3 + u_2) + e_3(b e_3 + y_m e_1 + x_s e_2 + u_3) \\ &= -\beta e_1^2 + a e_2^2 + b e_3^2 - y_s e_1 e_3 + 2 x_s e_2 e_3 + y_m e_1 e_3 + e_1 u_1 + e_2 u_2 + e_3 u_3, \\ &= -\beta e_1^2 + a e_2^2 + b e_3^2 - e_1 e_2 e_3 + 2 x_s e_2 e_3 + e_1 u_1 + e_2 u_2 + e_3 u_3.\end{aligned} \tag{9}$$

Substituting Eq. (7) into Eq. (9) gives that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -e_1^2 - e_2^2 - e_3^2 < 0, \tag{10}$$

which gives asymptotic stability of the system by Lyapunov stability theory. This means that the controlled chaotic systems (2) and (3) is synchronized for any initial conditions. This completes our proof. □

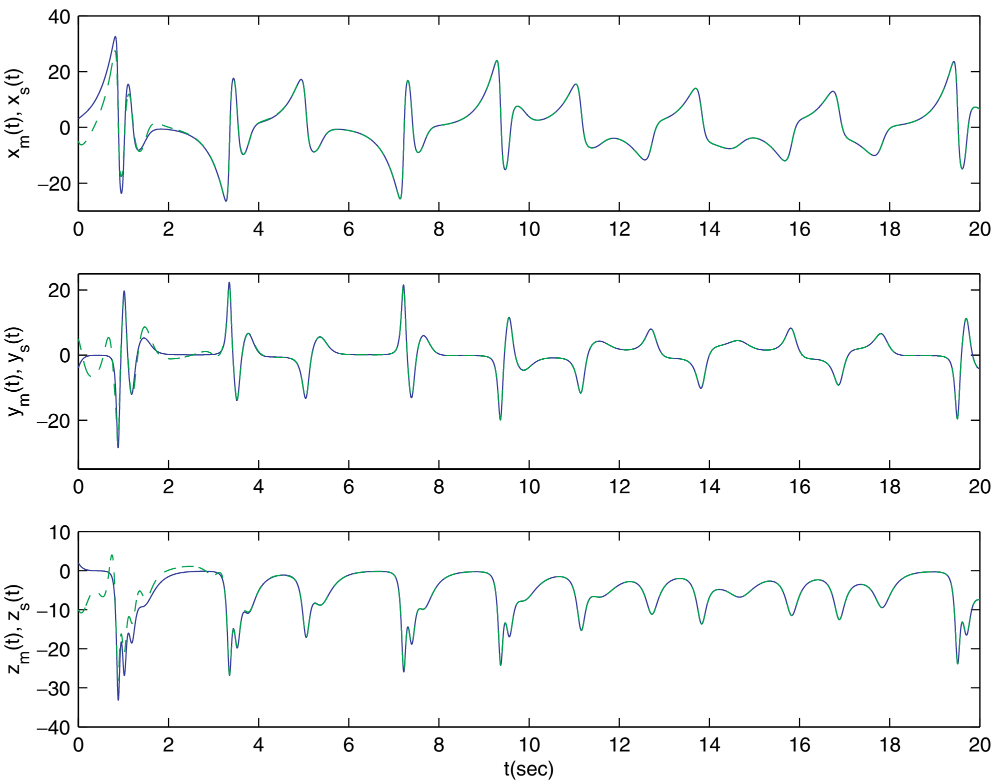

Fig. 3. The orbits of three states of master and slave systems.

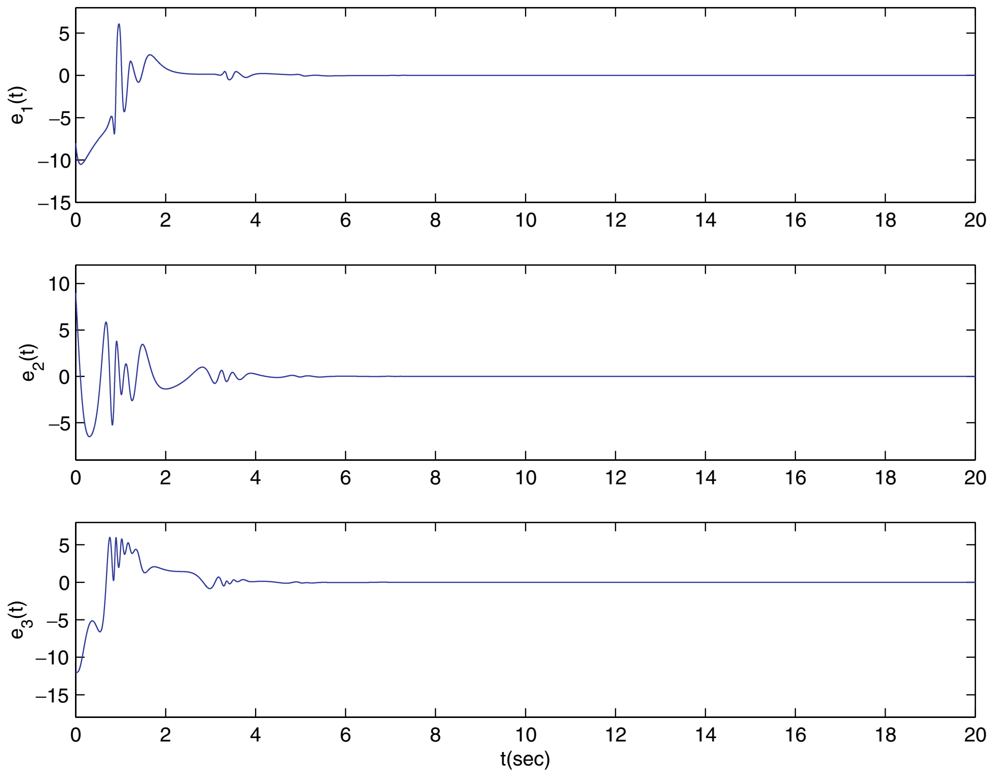

Fig. 4. Synchronization errors.

3. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for synchronization of the chaotic systems (2) and (3). In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001.

For this numerical simulation, we assume that the initial condition, $(x_m(0), y_m(0), z_m(0)) = (3, -4, 2)$, and $(x_s(0), y_s(0), z_s(0)) = (-5, 5, -10)$ is employed. Figs. 3 and 4 display the state response and synchronization errors of systems (2) and (3). It can be seen that the synchronization errors converge to zero rapidly.

4. Concluding remark

In this letter, we investigate the synchronization of controlled unified chaotic systems. We have proposed a novel nonlinear control scheme for asymptotic chaos synchronization using the Lyapunov method. Finally, a numerical simulation is provided to show the effectiveness of our method.

References

[1] Chen G, Dong X. From chaos to order. Singapore: World Scientific; 1998.
[2] Mosekilde E, Mastrenko Y, Postnov D. Chaotic synchronization. Applications for living systems. Singapore: World Scienctific; 2002.
[3] Pikovsky A, Rosenblum M, Kurths J. Synchronization, a universal concept in nonlinear science. Cambridge: Cambridge University Press; 2001.
[4] Ott E, Grebogi C, Yorke JA. Controlling chaos. Phys Rev Lett 1990;64:1196–9.
[5] Pyragas K. Continuous control of chaos by self-controlling feedback. Phys Lett A 1992;170:421–8.

[6] Park JH, Kwon OM. LMI optimization approach to stabilization of time-delay chaotic systems. Chaos, Solitons & Fractals 2005;23:445–50.
[7] Park JH. Controlling chaotic systems via nonlinear feedback control. Chaos, Solitons & Fractals 2005;23:1049–54.
[8] Hwang CC, Hsieh JY, Lin RS. A linear continuous feedback control of Chua's circuit. Chaos, Solitons & Fractals 1997;8:1507–15.
[9] Lu JH, Lu JA. Controlling uncertain Lü system using linear feedback. Chaos, Solitons & Fractals 2003;17:127–33.
[10] Pecora LM, Carroll TL. Synchronization in chaotic systems. Phys Rev Lett 1990;64:821–4.
[11] Carroll TL, Pecora LM. Synchronizing chaotic circuits. IEEE Trans Circ Syst I 1991;38:453–6.
[12] Chua LO, Itah M, Kosarev L, Eckert K. Chaos synchronization in Chua's circuits. J Circuits Syst Comput 1993;3:93–108.
[13] Chen M, Han Z. Controlling and synchronizing chaotic Genesio system via nonlinear feedback control. Chaos, Solitons & Fractals 2003;17:709–16.
[14] Lü J, Chen G, Cheng D, Celikovsky S. Bridge the gap between the Lorenz system and the Chen system. Int J Bifurcat Chaos 2002;12:2917–26.
[15] Lu J, Wu X, Lü J. Synchronization of a unified chaotic system and the application in secure communication. Phys Lett A 2002;305:365–70.
[16] Wu X, Lu J. Parameter identification and backstepping control of uncertain Lü system. Chaos, Solitons & Fractals 2003;18:721–9.
[17] Li D, Lu JA, Wu X. Linearly coupled synchronization of the unified chaotic systems and the Lorenz systems. Chaos, Solitons & Fractals 2005;23:79–85.
[18] Lü J, Zhou T, Zhang S. Chaos synchronization between linearly coupled chaotic systems. Chaos, Solitons & Fractals 2002;14:529–41.
[19] Park JH. Stability criterion for synchronization of linearly coupled unified chaotic systems. Chaos, Solitons & Fractals 2005;23:1319–25.
[20] Chen HK. Global chaos synchronization of new chaotic systems via nonlinear control. Chaos, Solitons & Fractals 2005;23:1245–51.
[21] Lü J, Chen G, Cheng D. A new chaotic system and beyond: the generalized Lorenz-like system. Int J Bifurcat Chaos 2004;14:1507–37.