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Parametric convergence and control of chaotic system using adaptive feedback linearization

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Abstract

Adaptive feedback linearization control technique for chaos suppression in a chaotic system is proposed. The dynamics of the system is altered so that the closed loop model matches with a specified linear reference model. The controller parameters are assumed to be unknown and are evolved using an adaptation law that aims to drive these parameters towards their ideal values so as to achieve perfect matching between the reference and the system model. A common external forcing signal to both chaotic Genesio system and reference system is considered and adaptation laws are derived considering Lyapunov function based stability. Simulation results show that the chaotic behavior is suppressed effectively with proposed controller. Analysis of linear and nonlinear parametric convergence is also shown through simulation, both with and without excitation using suitable forcing function.

1. Introduction

Chaotic systems and their behavior have been of intensive interest since last few decades. The chaotic behavior is undesirable due to its sensitivity to initial conditions, which restricts the operation of physical plants. Because of the difficulty of accurate prediction of a chaotic system behavior, chaos may cause system instability or degradation in performance and it should be eliminated in many cases. Chaos control is necessary because erratic oscillations can be undesirable in physical devices. Suppressing chaos can avoid damages to physical systems as the oscillations induced by chaos may result in resonance. Moreover, chaos control can be exploited to deal with control problems like chaos suppression in dc–dc converters, regulation of the fluid dynamics, designing systems for secure communication and some problems in biomedical sciences. A lot of work has been done in order to handle chaotic systems effectively.

Chaotic system control has been approached in several ways. One of the earlier approaches to control chaos is suggested in [1], wherein the main goal was to stabilize unstable periodic orbits by means of small time-depending change in an accessible system parameter, as a special case of pole placement techniques. Some traditional methods for controlling chaotic behavior to have desirable performance from them are discussed in [2,3]. Many chaos-controlling methods

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are based on cancellation of nonlinear terms of the chaotic systems in order to impose a desired behavior. Under the above philosophy many control techniques have been successfully employed [4–6]. Robust tracking control of uncertain chaotic system with time varying parameters has been proposed in [7].

In recent papers, suboptimal control law using state dependent Riccatti equation has been proposed for control of Duffing, Lorenz and Chen chaotic systems in [8]. Linear feedback for controlling chaos and Routh–Hurwitz criteria based stability analysis has been done in [9]. Tian et al. [10] solved an optimal control problem for the control of chaos with constrained control input. Chang and Yan [11] designed a robust adaptive PID controller for chaotic systems. Considerable effort has been also made to design control systems using feedback linearization and backstepping design technique for deterministic as well as uncertain chaotic systems [12–17].

Genesio system as proposed in [18] is another interesting chaotic system. This system is a third order nonlinear differential model, which captures many features of chaotic systems. Backstepping design based synchronization of two Genesio chaotic systems is proposed in [19]. In this paper, adaptive backstepping control law is derived to asymptotically synchronize drive and response systems.

In present paper, chaos suppression of Genesio system is achieved using adaptive feedback linearization based controller. An adaptation law has been derived using Lyapunov theory based stability analysis for achieving parameter estimation in presence of forcing function. In this paper, parameters appearing as coefficients of linear variable terms are referred as linear parameters whereas parameters appearing as coefficients of nonlinear variable terms are referred as nonlinear parameters. Convergence of both linear and nonlinear parameters is studied with and without forcing function. In absence of any forcing function, parameter estimation errors do not converge to zero though perfect tracking and regulation of states is achieved. When system is excited with a suitable forcing function, then linear parameters converge to their true value whereas nonlinear parametric convergence is not achieved. From simulation it has been established that for achieving convergence of m linear parameters to their true value, atleast $m/2$ sinusoids of different frequency should be present in the forcing signal. However no such statement could be made for convergence of nonlinear parameters. Numerical simulations, verifies the above facts for chaotic Genesio system.

Section 2 of the paper discusses about the Genesio system and its chaotic nature in brief. Section 3 gives details about the problem formulation, derivation of adaptation law and stability analysis based on Lyapunov approach. Section 4 gives numerical simulations depicting successful control and tracking behavior of the system states. Section 5 concludes the paper.

2. Genesio–Tesi chaotic system

Genesio–Tesi chaotic system can be represented by following set of nonlinear differential equations [18]:

$$\begin{aligned} \dot{x_1} &= x_2, \\ \dot{x_2} &= x_3, \\ \dot{x_3} &= -cx_1 - bx_2 - ax_3 + dx_1^2 + ku, \end{aligned} \tag{1}$$

where x_1, x_2 and x_3 are state variables, and a, b and c are positive real constants satisfying $ab < c$. For instance, the system is chaotic for the parameters $a = 1.2$, $b = 2.92$ and $c = 6$. Here, a, b and c are linear parameters and d is nonlinear parameter which is taken as one without loss of generality. Constant scalar k is assumed to be known and u is the control input to the model. The initial condition for the states is taken as $x(0) = [0.1, -0.2, 0.2]^{\mathrm{T}}$. The chaotic behavior of the system for the above set of parameters is shown in the Fig. 1. Phase portrait is shown in Figs. 1a and b. The open loop response of the system to initial condition without any control input is shown in Fig. 1c. A block diagram of the adaptive feedback linearization scheme is shown in Fig. 2.

3. Adaptive feedback linearization and stability analysis

Nonlinear dynamics of Genesio system is given in (1). All the states of the system are assumed to be measurable. The feedback linearizing control is selected as follows:

$$u = \theta_1 y_1 + \theta_2 y_2 + \theta_3 y_3 - \theta_4 y_1^2 + r(t). \tag{2}$$

Here controller parameters are to be adapted as they cannot be determined in the absence of knowledge of the system parameters. The term $r(t)$ denotes an external forcing signal. The combined plant-controller system can be written as follows:

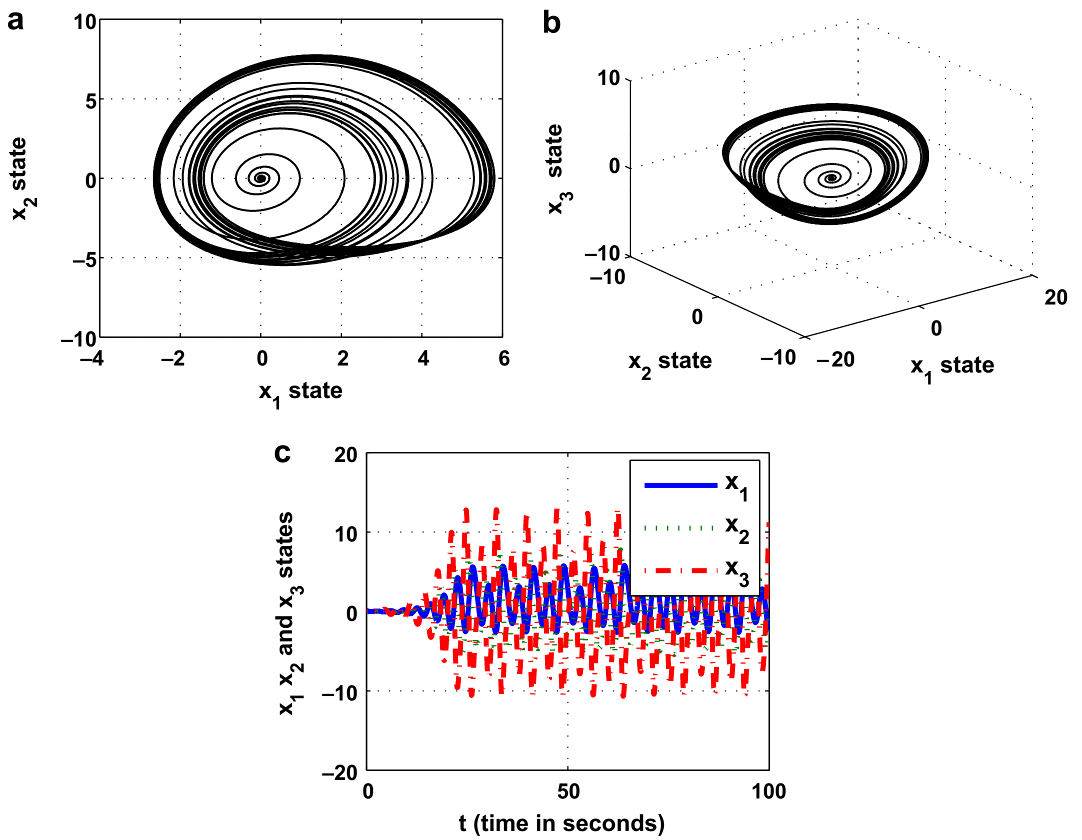

Fig. 1. (a) Phase portrait for Genesio system (with two states only); (b) phase portrait for Genesio system (with all states); (c) variation of state trajectories of Genesio system without controller.

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= x_3,\\ \dot{x}_3 &= (-c + k\theta_1)x_1 + (-b + k\theta_2)x_2 + (-a + k\theta_3)x_3 + (1 - k\theta_4)x_1^2 + kr(t).\end{aligned} \tag{3}$$

Note that the measurement noise terms are not considered for the analysis of adaptive feedback linearization scheme. So output $y_i = x_i$ for $i = 1, 2, 3$. The reference model is taken to be a third order stable linear system, as follows:

$$\begin{aligned}\dot{x}_{m1} &= x_{m2},\\ \dot{x}_{m2} &= x_{m3},\\ \dot{x}_{m3} &= -c_m x_{m1} - b_m x_{m2} - a_m x_{m3} + kr(t).\end{aligned} \tag{4}$$

Let the output of the reference model is given as

$$y_{m1} = x_{m1}; \quad y_{m2} = x_{m2}; \quad y_{m3} = x_{m3}. \tag{5}$$

From (3) and (4), it is clear that combined Genesio system and controller dynamics and reference model dynamics will be identical, if the following equalities hold:

$$(-c + k\theta_1^*) = -c_m; \quad (-b + k\theta_2^*) = -b_m; \quad (-a + k\theta_3^*) = -a_m; \quad (1 - k\theta_4^*) = 0, \tag{6}$$

where θ_1^*, θ_2^*, θ_3^* and θ_4^* are the ideal controller parameters for which the control law in (2) theoretically provides the possibility of perfect matching between combined Genesio system and reference model. In practice if the adaptation law succeeds in evolving the controller parameters to a sufficient good approximation to their ideal values θ_1^*, θ_2^*, θ_3^*

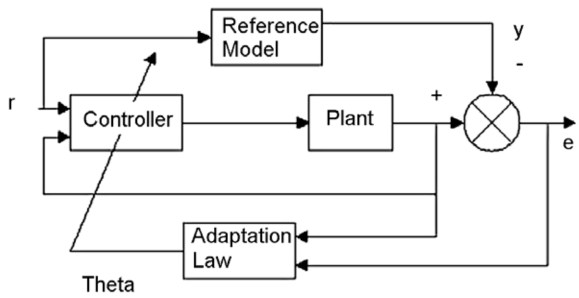

Fig. 2. Block diagram showing adaptive feedback controller for Genesio system.

and θ_4^*, then (6) can be used for estimating the actual system parameters a, b, c and d. Defining the error in the states of actual system and reference system as follows:

$$e_i = y_i - y_{mi} \text{ for } i = 1, 2 \text{ and } 3. \tag{7}$$

We get the error dynamics as follows:

$$\begin{aligned} \dot{e_1} &= e_2, \\ \dot{e_2} &= e_3, \\ \dot{e_3} &= \dot{y_3} - \dot{y_{m3}} = -c_m e_1 - b_m e_2 - a_m e_3 + (c_m - c + k\theta_1)x_1 + (b_m - b + k\theta_2)x_2 + (a_m - a + k\theta_3)x_3 + (1 - k\theta_4)x_1^2. \end{aligned} \tag{8}$$

Note that forcing signal $r(t)$ is not appearing in error dynamics. If we use the definition for θ_i^* as per (6), and define error ψ_i in controller parameters in the following manner:

$$\psi_i = \theta_i^* - \theta_i \text{ for } i = 1, 2 \text{ and } 3 \tag{9}$$

then the tracking error for the system can be represented in matrix form as follows:

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_1} \\ \dot{e_1} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ -c_m & -b_m & -a_m \end{bmatrix} \begin{bmatrix} e_1 \\ e_1 \\ e_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -k\psi_1 x_1 - k\psi_2 x_2 - k\psi_3 x_3 + k\psi_4 x_1^2 \end{bmatrix}. \tag{10}$$

Here we define error system matrix A as

$$A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ -c_m & -b_m & -a_m \end{bmatrix}, \tag{11}$$

with this choice the above error dynamics can be represented in compact form as

$$\dot{e} = Ae + bk\psi^{\mathrm{T}} f(x_{nl}), \tag{12}$$

where $e^{\mathrm{T}} = (e_1\, e_2\, e_3)$ is the vector of tracking errors, A is 3×3 matrix, $b^{\mathrm{T}} = (0\ 0\ 1)$, ψ is the row vector of parameter errors and $f(x_{nl})$ is the column vector given by

$$f(x_{nl}) = (-x_1 - x_2 - x_3\ x_1^2). \tag{13}$$

So we can say that the evolution of error depends on error itself, the parameter error ψ and a function of plant states $f(x_{nl})$, i.e.

$$\dot{e} = f_e(e, \psi, x). \tag{14}$$

In a similar way using (6) and (9), we can represent the closed loop Genesio system dynamics given in (3) as follows:

$$\dot{x} = Ax + bk\psi^T f(x_{nl}) + kr(t), \tag{15}$$

where $x^{\mathrm{T}} = (x_1\ x_2\ x_3)$ represents the actual system state vector whereas other terms have the same meaning as described earlier. The appearance of this equation is exactly similar to that of the error dynamic in (12) and we can represent this equation symbolically as follows:

$$\dot{x} = f_x(\psi, x, r(t)). \tag{16}$$

Further, in this paper the adaptation laws for parametric error are derived. The effect of forcing signal is considered in deriving these laws. As forcing signal does not appear in error dynamics, so it becomes necessary to consider actual system dynamics in the analysis to derive adaptation laws. Main results, i.e. achieving tracking of reference system and parametric estimation are presented in the form of theorem as follows:

Theorem. *Let the forcing signal is selected as per the following function*:

$$r(t) = -2(x^T P_2 b)k \tag{17}$$

and the adaptation or updating law for parameter estimation error is taken as

$$\dot{\psi} = -\frac{1}{k} H^{-1} f(x_{nl})[e^T P_1 + x^T P_2]b. \tag{18}$$

Here P_1 and P_2 are positive definite matrices obtained as solution of Lyapunov equation $A^T P_i + P_i A = -Q_i$; $i = 1, 2$. Matrix A is as defined in (11) and H is a positive definite weighing matrix. With these choices, the controlled system in (3) follows the reference system given in (4) and along with that system state regulation is also achieved.

Proof. Taking into consideration the error and system dynamics given in (12) and (15), respectively, an adaptation law of the following form is required so as to drive the output error e, system states x and the parameter error ψ towards zero:

$$\dot{\psi} = g(e, x). \tag{19}$$

As ideal parameters of the controller are taken to be time invariant, so the parameter update law can be represented by $\dot{\theta} = -g(e, x)$ Note that the adaptation law for ψ cannot be function of ψ as it would require the parameter update law for $\dot{\theta}$ to be a function of θ and θ^*, but ideal parameter vector θ^* is unknown. To derive the stability conditions and the adaptation laws, following set of dynamical equations, is required to be solved:

$$\begin{aligned} \dot{e} &= f_e(e, \psi, x), \\ \dot{x} &= f_x(\psi, x, r(t)), \\ \dot{\psi} &= g(e, x). \end{aligned} \tag{20}$$

The above combined dynamical system is non-autonomous as forcing signal is also present in the set of dynamic equations. For this system $(e, x, \psi) = (0, 0, 0)$ is an equilibrium point if forcing signal $r(t) = 0; t \geqslant t_0$. We need to establish the stability of this equilibrium point so that close loop stability and parameter convergence for the adaptive feedback linearization scheme could be ensured. To derive these results the Lyapunov function is selected as follows:

$$V(e, x, \psi) = e^{\mathrm{T}} P_1 e + x^{\mathrm{T}} P_2 x + k^2 \psi^{\mathrm{T}} H \psi \tag{21}$$

where P_1 and P_2 are symmetric positive definite matrices of appropriate dimension and H is a (4×4) positive definite weighing matrix. So it is certain that $V(0, 0, 0) = 0$ and $V(e, x, \psi) > 0$ for all other values of e, x and ψ.The adaptation law is derived by computing $\dot{V}$, i.e. the time derivative of the Lyapunov function in (21) as

$$\dot{V}(e, x, \psi) = e^{\mathrm{T}} P_1 \dot{e} + \dot{e}^{\mathrm{T}} P_1 e + x^{\mathrm{T}} P_2 \dot{x} + \dot{x}^{\mathrm{T}} P_2 x + k^2 \psi^{\mathrm{T}} H \dot{\psi} + k^2 \dot{\psi}^{\mathrm{T}} H \psi. \tag{22}$$

Using (16) and (19) in the above equation for $\dot{V}$, we obtain the following equation:

$$\dot{V}(e, x, \psi) = e^{\mathrm{T}}(P_1 A + A^{\mathrm{T}} P_1)e + x^{\mathrm{T}}(P_2 A + A^{\mathrm{T}} P_2)x + 2bk\psi^{\mathrm{T}} f(x_{nl})[e^{\mathrm{T}} P_1 + x^{\mathrm{T}} P_2] + 2x^{\mathrm{T}} P_2 bkr(t) + 2\psi^{\mathrm{T}} H \dot{\psi} k^2. \tag{23}$$

Using Lyapunov function based stability theorem, we can reduce it to following:

$$\dot{V}(e, x, \psi) = -e^{\mathrm{T}} Q_1 e - x^{\mathrm{T}} Q_2 x + 2bk\psi^{\mathrm{T}} f(x_{nl})[e^{\mathrm{T}} P_1 + x^{\mathrm{T}} P_2] + 2x^{\mathrm{T}} P_2 bkr(t) + 2\psi^{\mathrm{T}} H \dot{\psi} k^2. \tag{24}$$

To get adaptation law, we compare the terms independent of forcing function $r(t)$ to zero. With this manipulation we get:

$$\dot{\psi} = -\frac{1}{k} H^{-1} f(x_{nl})[e^{\mathrm{T}} P_1 + x^{\mathrm{T}} P_2]b \tag{25}$$

and with this adaptation law the expression for $\dot{V}(e,x,\psi)$ reduces to

$$\dot{V}(e,x,\psi) = -e^{\mathrm{T}}Q_1e - x^{\mathrm{T}}Q_2x + 2x^{\mathrm{T}}P_2bkr(t), \tag{26}$$

where Q_1 and Q_2 are symmetric positive definite matrices of appropriate dimension and are related to P_1 and P_2 by the Lyapunov equation:

$$A^{\mathrm{T}}P_i + P_iA = -Q_i; \quad i = 1,2 \tag{27}$$

as matrix A is a stable matrix. Matrix Q can be taken as identity and the equation can be solved to get elements of matrix P. If we consider forcing signal to be zero, then it is clear from (26) that $\dot{V} \leqslant 0$, i.e. $\dot{V}$ is negative semi-definite function. Hence selected function in equation can be taken as Lyapunov function and results for stability analysis using this function are valid. It follows that e, x and ψ states are bounded and consequently $\dot{e}$ and $\dot{x}$ are bounded as well. It then follows that $\ddot{V}$ is bounded and hence is uniformly continuous. Use of Barbalat's lemma proposed in [20] gives us $\dot{V} \to 0$ as $t \to \infty$ which further implies that both $e \to 0$ and $x \to 0$ as $t \to \infty$. It means that global convergence of output error and system states is established. However equilibrium point $(e,x,\psi) = (0,0,0)$ is not asymptotically stable as convergence of parameter error vector is not established. So above analysis only ensures boundedness of parameter errors but not parametric convergence. If we choose the forcing function as

$$r(t) = -2(x^{\mathrm{T}}P_2b)k. \tag{28}$$

Then from (26), $\dot{V}$ is once again negative semi-definite and previous arguments are valid once more, i.e. global convergence of output error and system states is established. However as before the analysis only shows boundedness of parameter errors but not parametric convergence. Though parametric convergence may possibly be achieved by a suitable persistently exciting function, but this is not guaranteed. The above analysis is useful in the sense that a suitable choice of forcing function $r(t)$ can provide an additional term to negative semi-definite function $\dot{V}$ and when compared with unforced case, this may give faster convergence of output error and system states. It has been shown through numerical simulation in subsequent section that a suitable choice of forcing function $r(t)$ may lead to the convergence of linear parametric errors whereas for nonlinear parameters only boundedness can be assured. □

Remark 1. Suitable choice of forcing function $r(t)$ lowers the value of Lyapunov function V as time $t \to \infty$ and hence better convergence of parametric errors is achieved. Also it may lead to faster convergence of output error and system states.

The procedural steps for obtaining the requisite controller and parameter adaptation laws can be summarized as follows:

(i) Construct the combined plant-controller system dynamics $\dot{x} = Ax + bk\psi^{\mathrm{T}}f(x_{nl}) + kr(t)$ using (2) and formulate the error dynamics $\dot{e} = Ae + bk\psi^{\mathrm{T}}f(x_{nl})$ by choosing a suitable reference model as given in (4).
(ii) Select Q_i suitably and solve $A^{\mathrm{T}}P_i + P_iA = -Q_i$ where $i = 1,2$.
(iii) Select H to be a suitable diagonal matrix and formulate the estimation error laws as $\dot{\psi} = -\frac{1}{k}H^{-1}f(x_{nl})[e^{\mathrm{T}}P_1 + x^{\mathrm{T}}P_2]b$.
(iv) Solve the set of differential equations evolved in step (i) and (iii) simultaneously with suitable initial conditions, by considering the case with and without forcing function $r(t)$.

4. Numerical simulation

For simulation purpose, Q_1 and Q_2 are taken as I and ϵI matrices of size (3×3), respectively. Here ϵ is taken as a positive constant equal to 0.01, constant k is taken as one without loss of generality and H is taken to be a diagonal matrix as

$$H = \mathrm{diag}(0.1, 0.1, 0.01, 0.01) \tag{29}$$

Initial conditions on the system and model states are taken as [0.1 −0.2 0.2] and [0.1 0.2 0.1], respectively, and the estimation errors are considered to be zero initially. Simulation is run for 40 s with step size of 0.01. In 1st part, forcing signal is taken to be zero. Simulations performed show the state regulation and tracking behavior. But the parametric errors do no converge to zero though these errors settle to some finite value. Fig. 3 shows plots of system with adaptive feedback linearization based controller. Fig. 3a shows the phase portrait of Genesio system, Fig. 3b–d show the comparison between actual and reference system states. The plot of error in different states is shown in Fig. 3e. Estimation

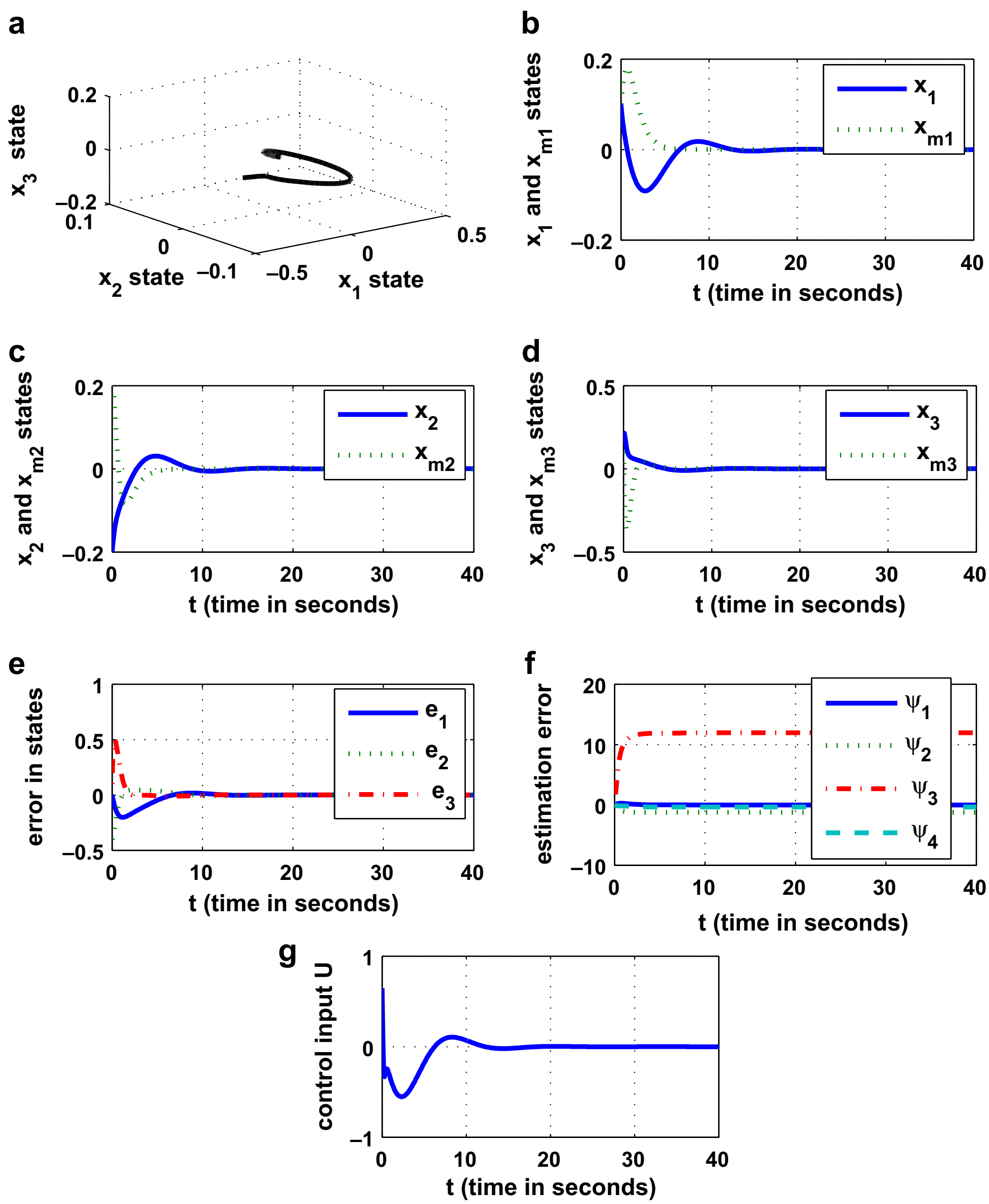

Fig. 3. Response of system with controller, (a) phase portrait; (b), (c) and (d) state trajectory variation for Genesio system (x_1, x_2 and x_3: actual system states and x_{m1}, x_{m2} and x_{m3}: reference system states); (e) variation of error in states; (f) variation of Estimation error in different parameters; and (g) variation of controller input.

error in parameters is shown in Fig. 3f. Here parametric convergence is not assured but boundedness of parametric error is ascertained. The controller input is shown in Fig. 3g.

If system is persistently excited with a suitable forcing function, then linear parameters converge to their true value whereas nonlinear parametric convergence is not achieved. As per (27), for analysis point of view, the suitable forcing function is selected. To achieve the parametric convergence for linear parameters, the forcing signal is taken with suitable number of sinusoids as following:

$$r(t) = r_0 e^{-t/T} \sin(st) + r_0 e^{-t/T} \sin(2st). \tag{30}$$

Here different parameters of the forcing function are taken as $r_0 = 1$, $T = 307$ and $s = 1.82$; Note that this function is a bounded function. Simulation is run for 2000 s with step size of 0.01. Initial conditions for actual system states,

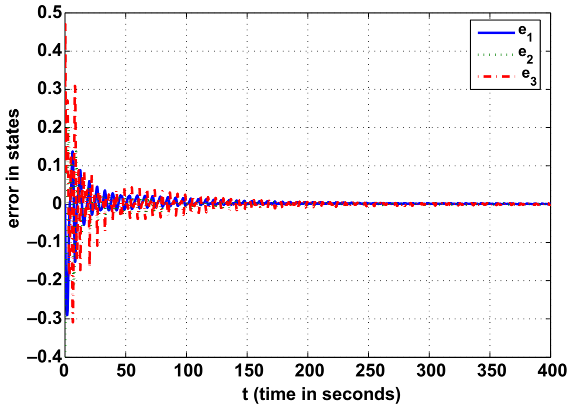

Fig. 4. Variation of error in states of actual system and reference system.

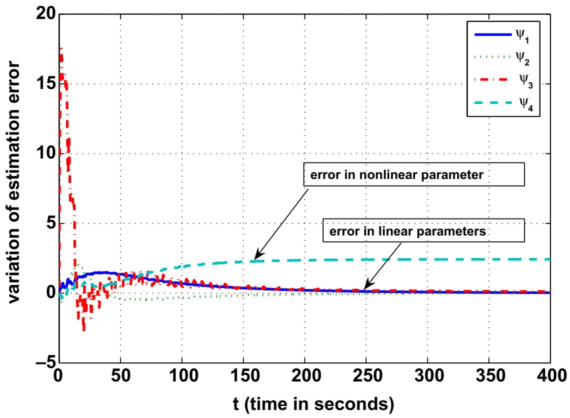

Fig. 5. Variation of estimation error in different parameters of the actual system.

reference system states and parametric errors are taken as that of first part of simulation. The plot of error in different states is shown in Fig. 4. Error convergence is assured using proposed controller with forcing signal taken as in (30).

Variation of estimation error in different parameters of the actual system is shown in Fig. 5. Parametric convergence of linear parameters is assured in this case though it is not true for the nonlinear parameters. But boundedness of nonlinear parametric error is ascertained as is clear from Fig. 5. For showing tracking error and parametric error results are displayed only up to 400 s for better clarity though simulations are run up to 2000 s. Here forcing signal is deliberately taken to be consisting of two different frequency sinusoids, as three linear parameters are involved in the dynamics for which estimation is to be achieved.

Remark 2. Numerical simulations are also performed by considering forcing function $r(t)$ with single sinusoid, but in this case parametric convergence could not be ensured for all the linear parameters.

5. Conclusion

Chaos suppression of Genesio system is successfully achieved using the adaptive feedback linearization based controller. A novel adaptation law has been derived using Lyapunov function based analysis for achieving parameter estimation in presence of forcing function. Simulation shows successful control and tracking behavior of the system states. Simulations show that for achieving convergence of linear parameters to their true value, sufficient number of sinusoids of different frequency should be present in the forcing signal. However no such statement could be made for convergence of nonlinear parameters, though nonlinear parameters converge to some finite value.

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