Modelling & Simulation for Optimal Control of Nonlinear Inverted Pendulum Dynamical System using PID Controller & LQR

Lal Bahadur Prasad, Barjeev Tyagi, Hari Om Gupta

Department of Electrical Engineering, Indian Institute of Technology Roorkee, (Uttarakhand)-247667, India
(erlbprasad@gmail.com, ibpeedee@iitr.ernet.in, harifee@iitr.ernet.in, btyagfee@iitr.ernet.in)

Abstract—**This paper presents the modelling and simulation for optimal control design of nonlinear inverted pendulum-cart dynamic system using Proportional-Integral-Derivative (PID) controller and Linear Quadratic Regulator (LQR). LQR, an optimal control technique, and PID control method, both of which are generally used for control of the linear dynamical systems have been used in this paper to control the nonlinear dynamical system. The nonlinear system states are fed to LQR which is designed using linear state-space model. Inverted pendulum, a highly nonlinear unstable system is used as a benchmark for implementing the control methods. Here the control objective is to control the system such that the cart reaches at a desired position and the inverted pendulum stabilizes in upright position. The MATLAB-SIMULINK models have been developed for simulation of control schemes. The simulation results justify the comparative advantages of LQR control methods.**

Keywords—Inverted pendulum; nonlinear system; PID control; optimal control; LQR

I. INTRODUCTION

The Inverted Pendulum is an inherently open loop & closed loop unstable system with highly nonlinear dynamics. This is a system which belongs to the class of under-actuated mechanical systems having fewer control inputs than degrees of freedom. This renders the control task more challenging making the inverted pendulum system a classical benchmark for the design, testing, evaluating and comparing of different classical & contemporary control techniques.

The inverted pendulum is among the most difficult systems being an inherently unstable system, is a very common control problem, and so being one of the most important classical problems, the control of inverted pendulum has been a research interest in the field of control engineering. Due to its importance this is a choice of dynamic system to analyze its dynamic model and propose a control law. The aim of this case study is to stabilize the Inverted Pendulum (IP) such that the position of the cart on the track is controlled quickly and accurately so that the pendulum is always erected in its inverted position during such movements. Realistically, this simple mechanical system is representative of a class of altitude control problems whose goal is to maintain the desired vertically oriented position at all times [1-4].

In general, the control problem consists of obtaining dynamic models of systems, and using these models to determine control laws or strategies to achieve the desired system response and performance. The simplicity of control algorithm as well as to guarantee the stability and robustness in the closed-loop system is challenging task in real situations. Most of the dynamical systems such as power systems, missile systems, robotic systems, inverted pendulum, industrial processes, chaotic circuits etc. are highly nonlinear in nature. The control of such systems is a challenging task.

The Proportional-Integral-Derivative (PID) control gives the simplest and yet the most efficient solution to various real-world control problems. Both the transient and steady-state responses are taken care of with its three-term (i.e. P, I, and D) functionality. Since its invention the popularity of PID control has grown tremendously. The advances in digital technology have made the control system automatic. The automatic control system offers a wide spectrum of choices for control schemes, even though, more than 90% of industrial controllers are still implemented based around the PID algorithms, particularly at the lowest levels, as no other controllers match with the simplicity, clear functionality, applicability, and ease of use offered by the PID controller.

The performance of the dynamical systems being controlled is desired to be optimal. There are many optimization & optimal control techniques which are present in the literatures for linear & nonlinear dynamical systems [5-7]. The recent development in the area of artificial intelligence (AI), such as artificial neural network (ANN), fuzzy logic theory (FL), and evolutionary computational techniques such as genetic algorithm (GA), and particle swarm optimization (PSO) etc., commonly all these are known as intelligent computational techniques which have given novel solutions to the various control system problems. The intelligent optimal control has emerged as viable recent approach by the application of these intelligent computational techniques [8-18].

There are many literatures present which have taken inverted pendulum-cart dynamical system for implementing the various control schemes [16-21]. Linear quadratic regulator (LQR), an optimal control method, and PID control which are generally used for control of the linear dynamical systems have been used in this paper to control the nonlinear inverted pendulum-cart dynamical system. In recent trends even the various advance control approaches are developing and being tried for many dynamical systems control, the proposed control method is simple, effective, and robust.

This paper is organized in 5 sections. Section I presents the relevance & the general introduction of the paper. Section II describes the mathematical model of the inverted

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pendulum-cart system. In section III the control methods of PID control and optimal control using LQR have been discussed briefly. Section IV presents MATLAB-SIMULINK modeling, and simulation results. In section V conclusion is presented. At the end a brief list of references is given.

II. Mathematical Modelling

A. Inverted Pendulum System Equations

The free body diagram of an inverted pendulum mounted on a motor driven cart is shown in Fig. 1 [1-4, 16-21]. The system equations of this nonlinear dynamic system can be derived as follows [1,3,4,16,20]. It is assumed here that the pendulum rod is mass-less, and the hinge is frictionless. The cart mass and the ball point mass at the upper end of the inverted pendulum are denoted as M and m, respectively. There is an externally x-directed force on the cart, $u(t)$, and a gravity force acts on the point mass at all times. The coordinate system considered is shown in Fig. 1, where $x(t)$ represents the cart position and $\theta(t)$ is the tilt angle referenced to the vertically upward direction.

A force balance in the x-direction gives that the mass times acceleration of the cart plus the mass times the x-directed acceleration of the point mass must equal the external force on the system.

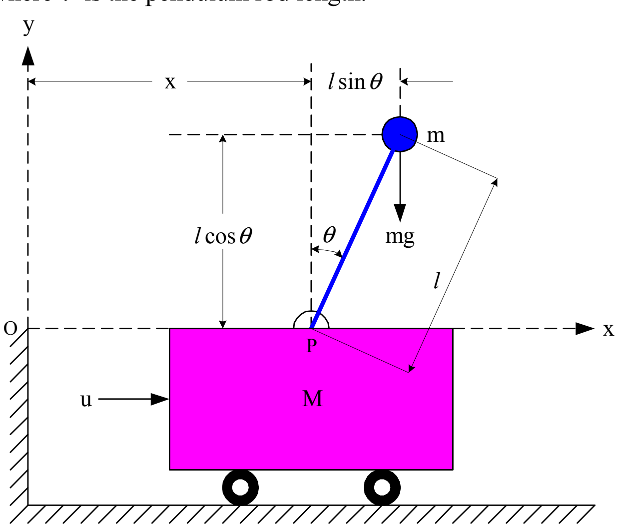
This can be written as

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_G = u \tag{1}$$

where the time-dependent center of gravity (COG) of the point mass is given by the coordinates, (x_G, y_G). For the point mass assumed here, the location of the center of gravity of the pendulum mass is simply

$$x_G = x + l\sin\theta \quad \text{and} \quad y_G = l\cos\theta \tag{2}$$

where l is the pendulum rod length.



Inverted Pendulum - Cart System

Figure 1. Motor Driven Inverted Pendulum-Cart System.

Substitution of eqn. (2) into (1) gives

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}(x + l\sin\theta) = u \tag{3}$$

which gives

$$(M+m)\ddot{x} - ml\sin\theta\dot{\theta}^2 + ml\cos\theta\ddot{\theta} = u \tag{4}$$

In a similar way, a torque balance on the system is performed, where torque is the product of the perpendicular component of the force and the distance to the pivot point (lever arm length, l). In this case, the torque on the mass due to the acceleration force is balanced by the torque on the mass due to the gravity force. The resultant torque balance can be written as

$$(F_x\cos\theta)l - (F_y\sin\theta)l = (mg\sin\theta)l \tag{5}$$

where the force components, F_x and F_y, are determined as

$$F_x = m\frac{d^2}{dt^2}x_G = m\left[\ddot{x} - l\sin\theta\dot{\theta}^2 + l\cos\theta\ddot{\theta}\right] \tag{6}$$

$$F_y = m\frac{d^2}{dt^2}y_G = -m\left[l\cos\theta\dot{\theta}^2 + l\sin\theta\ddot{\theta}\right] \tag{7}$$

Substituting eqns. (6) & (7) into eqn. (5) we have

$$m\ddot{x}\cos\theta - ml\sin\theta\cos\theta\dot{\theta}^2 + ml\cos^2\theta\ddot{\theta} + ml\sin\theta\cos\theta\dot{\theta}^2$$
$$+ml\sin^2\theta\ddot{\theta} = mg\sin\theta$$

or

$$m\ddot{x}\cos\theta + ml\ddot{\theta} = mg\sin\theta \tag{8}$$

Equations (4) and (8) are the defining equations for this system. These two equations definitely represent a nonlinear system which is relatively complicated from a mathematical viewpoint. However, since the goal of this particular system is to keep the inverted pendulum in upright position around $\theta = 0$, the linearization might be considered about this upright equilibrium point. This has been presented in subsection C to compare the linear and nonlinear dynamics of the system. Following subsection B presents the standard state space form of these two nonlinear equations.

B. Nonlinear System Equations of Inverted Pendulum

For numerical simulation of the nonlinear model for the inverted pendulum-cart dynamic system, it is required to represent the nonlinear equations (4) and (8) into standard state space form,

$$\frac{d}{dt}\mathbf{x} = \boldsymbol{f}(\mathbf{x}, u, t) \tag{9}$$

To put eqns. (4) and (8) into this form, firstly these equations are manipulated algebraically to have only a single second derivative term in each equation. From eqn. (8), we have

$$ml\ddot{\theta} = mg\sin\theta - m\ddot{x}\cos\theta$$

and putting this into eqn. (4) gives

$$(M+m)\ddot{x} - ml\sin\theta\dot{\theta}^2 + mg\cos\theta\sin\theta - m\ddot{x}\cos^2\theta = u$$

or

$$(M + m - m\cos^2\theta)\ddot{x} = u + ml\sin\theta\dot{\theta}^2 - mg\cos\theta\sin\theta \tag{10}$$

Similarly, from eqn. (8) we have

$$\ddot{x} = \frac{g\sin\theta - l\ddot{\theta}}{\cos\theta}$$

and putting this into eqn. (4) gives

$$\frac{(M+m)(g\sin\theta - l\ddot{\theta})}{\cos\theta} - ml\sin\theta\dot{\theta}^2 + ml\cos\theta\ddot{\theta} = u$$

or

$$(M+m)(g\sin\theta - l\ddot{\theta}) - ml\cos\theta\sin\theta\dot{\theta}^2$$
$$+ml\cos^2\theta\ddot{\theta} = u\cos\theta$$

and

$$\begin{aligned}&(ml\cos^2\theta - (M+m)l)\ddot{\theta} = u\cos\theta\\&-(M+m)g\sin\theta + ml\cos\theta\sin\theta\dot{\theta}^2\end{aligned} \tag{11}$$

Finally, dividing by the lead coefficients of eqns. (10) and (11) gives

$$\ddot{x} = \frac{u + ml(\sin\theta)\dot{\theta}^2 - mg\cos\theta\sin\theta}{M + m - m\cos^2\theta} \tag{12}$$

$$\ddot{\theta} = \frac{u\cos\theta - (M+m)g\sin\theta + ml(\cos\theta\sin\theta)\dot{\theta}}{ml\cos^2\theta - (M+m)l} \tag{13}$$

Now these equations may be represented into state space form by considering the state variables as following:

$$x_1 = \theta \qquad x_2 = \dot{\theta} = \dot{x}_1 \qquad x_3 = x \qquad x_4 = \dot{x} = \dot{x}_3 \tag{14}$$

Then, the final state space equation for the inverted pendulum system may be written as

$$\frac{d}{dt}\mathbf{x} = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{15}$$

where, $f_1 = x_2$, $f_3 = x_4$, and

$$f_2 = \frac{u\cos x_1 - (M+m)g\sin x_1 + ml(\cos x_1\sin x_1)x_2^2}{ml\cos^2 x_1 - (M+m)l}$$

$$f_4 = \frac{u + ml(\sin x_1)x_2^2 - mg\cos x_1\sin x_1}{M + m - m\cos^2 x_1}$$

This expression is now in the desired form as given in eqn. (9). If both the pendulum angle θ and the cart position x are the variables of interest, then the output equation may be written as

$$\mathbf{y} = \mathrm{C}\mathbf{x} \quad \text{or} \quad \mathbf{y} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \mathrm{C}\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \tag{16}$$

Equations (15) and (16) give a complete state space representation of the nonlinear inverted pendulum-cart dynamic system.

C. Linear System Equations of Inverted Pendulum

The linear model for the system around the upright stationary point is derived by simply linearization of the nonlinear system given in eqn. (15). Since the usual **A** and **B** matrices are zero for this case; and so every term is put into the nonlinear vector function, $\boldsymbol{f}(\mathbf{x},u,t)$, then the linearized form for the system becomes

$$\frac{d}{dt}\delta\mathbf{x} = \mathbf{J}_{\mathbf{x}}(\mathbf{x}_0, u_0)\delta\mathbf{x} + \mathbf{J}_{\mathrm{u}}(\mathbf{x}_0, u_0)\delta\mathrm{u} \tag{17}$$

where, the reference state is defined with the pendulum stationary and upright with no input force. Under these conditions, $\mathbf{x}_0 = 0$, and $\mathrm{u}_0 = 0$.

Since the nonlinear vector function is rather complicated, the components of the Jacobian matrices are determined systemically, term by term. The elements of the first second, third, and fourth columns of $\mathbf{J}_{\mathbf{x}}(\mathbf{x}_0, u_0)$ are given by $\left.\frac{\partial f_i}{\partial x_1}\right|_{\mathbf{x}_0,\mathrm{u}_0}$, $\left.\frac{\partial f_i}{\partial x_2}\right|_{\mathbf{x}_0,\mathrm{u}_0}$, $\left.\frac{\partial f_i}{\partial x_3}\right|_{\mathbf{x}_0,\mathrm{u}_0}$, and $\left.\frac{\partial f_i}{\partial x_4}\right|_{\mathbf{x}_0,\mathrm{u}_0}$ respectively. Thus, combining all these separate terms gives

$$\mathbf{J}_{\mathbf{x}}(\mathbf{x}_0, u_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

For the derivative of the nonlinear terms with respect to u, we have

$$\mathbf{J}_u(\mathbf{x}_0, u_0) = \begin{bmatrix} \partial f_1/\partial u \\ \partial f_2/\partial u \\ \partial f_3/\partial u \\ \partial f_4/\partial u \end{bmatrix}_{\mathbf{x}_0,u_0}$$

$$= \begin{bmatrix} 0 \\ \frac{\cos x_1}{ml\cos^2 x_1 - (M+m)l} \\ 0 \\ \frac{1}{M + m - m\cos^2 x_1} \end{bmatrix}_{\mathbf{x}_0,u_0} = \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} \tag{19}$$

Finally, after all these manipulations eqn. (17) may be written explicitly as

$$\frac{d}{dt}\delta\mathbf{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & 0 \end{bmatrix}\delta\mathbf{x} + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}\delta u \tag{20}$$

This is the open loop linearized model for the inverted pendulum with a cart force, $\delta u(t)$, (written in perturbation form). Thus, LTI system is in standard state space form. The eqn. (20) may be written in general as

$$\frac{d}{dt}\delta\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u \tag{21}$$

Equation (21) along with the output eqn. (16) represents the final linear model of the inverted pendulum-cart system. This is the simplified model which is used to study the system behaviour and LQR design.

III. CONTROL METHODS

To control the nonlinear inverted pendulum-cart dynamical system the following control methods are presented in this paper.

A. PID Control

To stabilize the inverted pendulum in upright position and to control the cart at desired position using PID control approach two PID controllers- angle PID controller, and cart PID controller have been designed for the two control loops of the system. The equations of PID control are given as following:

$$u_p = K_{pp} e_\theta(t) + K_{ip} \int e_\theta(t) + K_{dp} \frac{de_\theta(t)}{dt} \tag{9}$$

$$u_c = K_{pc} e_x(t) + K_{ic} \int e_x(t) + K_{dc} \frac{de_x(t)}{dt} \tag{10}$$

where, $e_\theta(t)$ and $e_x(t)$ are angle error and cart position error. Since the pendulum angle dynamics and cart position dynamics are coupled to each other so the change in any controller parameters affects both the pendulum angle and cart position which makes the tuning tedious. The tuning of controller parameters is done using trial & error method and observing the responses of SIMULINK model to be optimal.

B. Optimal Control using LQR

Optimal control refers to a class of methods that can be used to synthesize a control policy which results in best possible behavior with respect to the prescribed criterion (i.e. control policy which leads to maximization of performance). The main objective of optimal control is to determine control signals that will cause a process (plant) to satisfy some physical constraints and at the same time extremize (maximize or minimize) a chosen performance criterion (performance index (PI) or cost function). The optimal control problem is to find a control which causes the dynamical system to reach a target or follow a state variable (or trajectory) and at the same time extremize a PI which may take several forms [1,4-7].

Linear quadratic regulator (LQR) is one of the optimal control techniques, which takes into account the states of the dynamical system and control input to make the optimal control decisions. This is simple as well as robust [1,4-7].

After linearization of nonlinear system equations about the upright (unstable) equilibrium position having initial conditions as $X_0 = [0,0,0,0]^T$, the linear state-space equation is obtained as

$$\dot{X} = AX + Bu \tag{11}$$

where, $X = [\theta, \dot{\theta}, x, \dot{x}]^T$.

The state feedback control $u = -KX$ leads to

$$\dot{X} = (A - BK)X \tag{12}$$

where, K is derived from minimization of the cost function

$$J = \int (X^T Q X + u^T R u)dt \tag{13}$$

where, Q and R are positive semi-definite and positive definite symmetric constant matrices respectively.

The LQR gain vector K is given by

$$K = R^{-1} B^T P \tag{14}$$

where, P is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic reccatti equation (ARE)

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{15}$$

In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, pendulum angle θ, angular velocity $\dot{\theta}$, cart position x, and cart velocity $\dot{x}$ have been considered available for measurement which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The tuning of the PID controllers which are used here either as PID control method or PID+LQR control methods is done by trial & error method and observing the responses achieved to be optimal.

IV. SIMULATION & RESULTS

The MATLAB-SIMULINK models for the simulation of modelling, analysis, and control of nonlinear inverted pendulum-cart dynamical system have been developed. The typical parameters of inverted pendulum-cart system setup are selected as [16,20]: mass of the cart (*M*): 2.4 kg, mass of the pendulum (*m*): 0.23 kg, length of the pendulum (*l*): 0.36 m, length of the cart track (*L*): ± 0.5 m, friction coefficient of the cart & pole rotation is assumed negligible.

After linearization the system matrices used to design LQR are computed as below:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 29.8615 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.9401 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1.1574 \\ 0 \\ 0.4167 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With the choice of

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix}, \quad \text{and} \quad R = 1,$$

we obtain LQR gain vector as following:

$$K = \begin{bmatrix} -137.7896 & -25.9783 & -22.3607 & -27.5768 \end{bmatrix}$$

Here three control schemes have been implemented for optimal control of nonlinear inverted pendulum-cart dynamical system: 1. PID control method having two PIDs i.e. angle PID & cart PID, 2. Two PIDs (i.e. angle PID & cart PID) with LQR control method, 3. One PID (i.e. cart PID) with LQR control method. Both alternatives of PID+LQR control method are similar in all respect of control techniques but they differ only in number of PID controllers used. The SIMULINK models for these control schemes are shown in Figs. 2, 4, and 6 respectively. The corresponding simulation results are shown in Figs. 3, 5, and 7 respectively. The reference angle has been set to 0 (rad), and reference cart position is set to 0.1 (m). The tuned PID controller parameters of these control schemes are given as in table I.

TABLE I. PID CONTROLLER PARAMETERS

Control Schemes	Angle PID Control			Cart PID Control		
	K_{pp}	K_{ip}	K_{dp}	K_{pc}	K_{ic}	Kdc
PID	-40	0	-8	-1	0	-3
2 PID+LQR	1	1	1	1.5	-7.5	5
1 PID+LQR	---	---	---	1.5	-7.5	5

PID control response is shown in Fig. 3. It is observed here that the pendulum stabilizes in vertically upright position after two small overshoots. The cart position x reaches the desired position of 0.1 (m) quickly & smoothly. The control input u is bounded in range [-0.1 0.1].

The response of optimal control of inverted pendulum system using two PID controllers (angle PID & cart PID) with LQR control method is shown in Fig. 5, and using one PID controller (cart PID) with LQR control method is shown in Fig. 7 respectively. Here for both control methods of PID+LQR the responses of angle θ, angular velocity $\dot{\theta}$, cart position x, cart velocity $\dot{x}$, and control u have been plotted. It is observed that in both control schemes the pendulum stabilizes in vertically upright position quickly & smoothly after two minor undershoots and a minor overshoot The angular velocity approaches 0 (rad/s) quickly. The cart position x reaches smoothly the desired position of 0.1 (m) quickly in approx. 6 seconds, and the cart velocity reaches to zero. The control input u is bounded in range [-0.1 0.1].

Comparing the results it is observed that the responses of both alternatives of PID+LQR control method are better than PID control, which are smooth & fast also. It is also observed that the responses of 2PID+LQR control and cart PID+LQR control are similar. Since 2PID+LQR method has additional degree of freedom of control added by the angle PID controller, this will have overall better response under disturbance input. But the cart PID+LQR control has structural simplicity in its credit. The performance analysis of the control schemes gives that these control schemes are effective & robust.

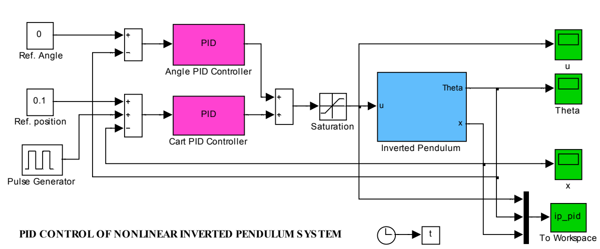


Figure 2. PID Control of Nonlinear Inverted Pendulum-Cart System.

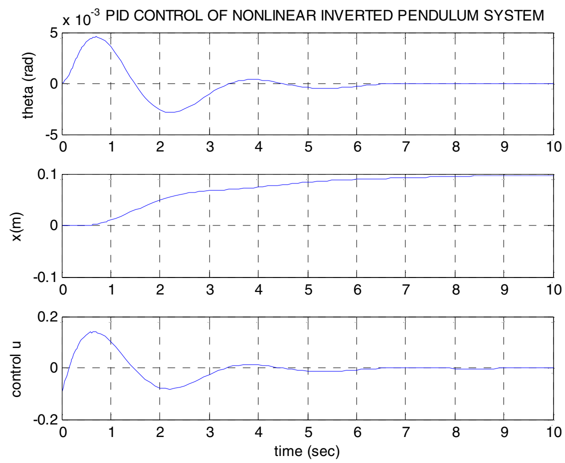


Figure 3. Responses of pendulum angle θ, cart position x, and control force u of nonlinear inverted pendulum system with PID control.

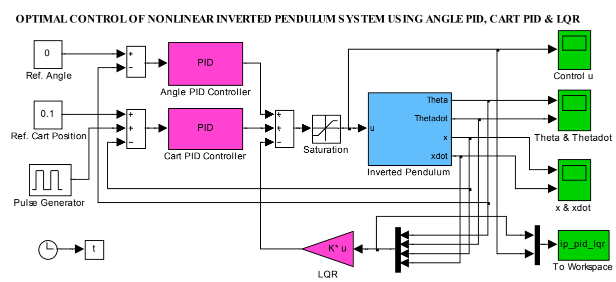


Figure 4. Angle PID, Cart PID & LQR Control of Nonlinear Inverted Pendulum System.

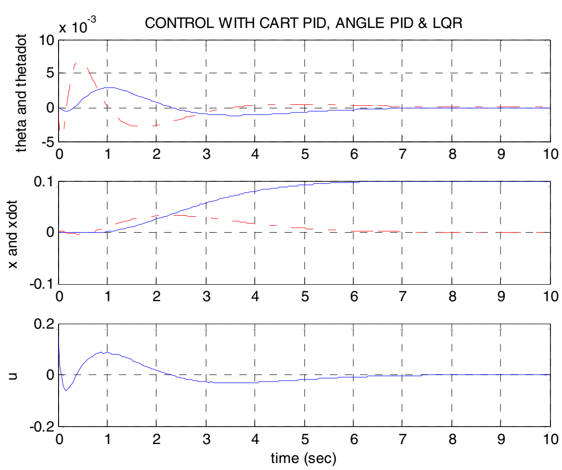


Figure 5. Responses of pendulum angle θ, angular velocity $\dot{\theta}$, cart position x, cart velocity $\dot{x}$, and control force u of nonlinear inverted pendulum system with Angle PID , Cart PID & LQR Control.

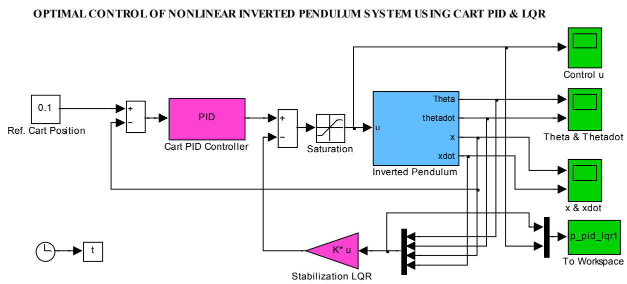


Figure 6. Cart PID & LQR Control of Nonlinear Inverted Pendulum System.

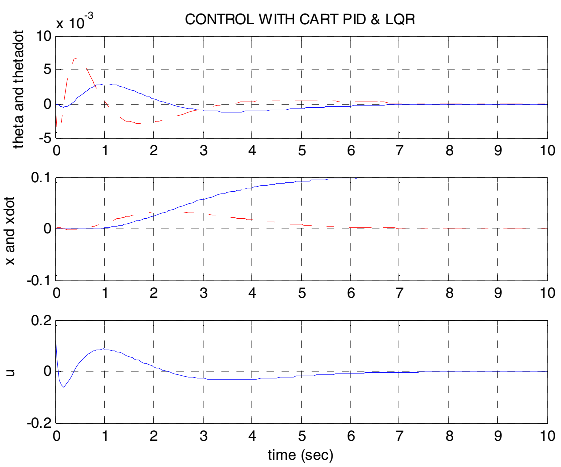


Figure 7. Responses of pendulum angle θ , angular velocity $\dot{\theta}$, cart position x , cart velocity $\dot{x}$,and control force u of nonlinear inverted pendulum system with Cart PID & LQR Control.

V. CONCLUSION

PID control, and LQR, an optimal control technique to make the optimal control decisions, have been implemented to control the nonlinear inverted pendulum-cart system. To compare the results PID control has been implemented. In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, are considered available for measurement, which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The MATLAB-SIMULINK models have been developed for simulation of the control schemes. The simulation results justify the comparative advantages of optimal control using LQR method. The pendulum stabilizes in upright position justify that the control schemes are effective & robust. The performance of PID+LQR control scheme is better than PID control scheme. The performance investigation of this control approach with tuning of PID controller parameters using GA, and PSO instead of trial & error method may be done as a future scope of this work.

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