Available online at www.sciencedirect.com
ScienceDirect
Systems & Control Letters 57 (2008) 123 – 131
SYSTEMS & CONTROL LETTERS
www.elsevier.com/locate/sysconle

Robust static output feedback control for linear discrete-time systems with time-varying uncertainties

Jiuxiang Dong[a], Guang-Hong Yang[a,b,*]

[a]College of Information Science and Engineering, Northeastern University, Shenyang 110004, PR China
[b]Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, PR China

Received 25 April 2006; received in revised form 12 August 2007; accepted 21 August 2007
Available online 4 October 2007

Abstract

This paper is concerned with the problem of designing robust static output feedback controllers for linear discrete-time systems with time-varying polytopic uncertainties. Sufficient conditions for robust static output feedback stabilizing controller designs are given in terms of solutions to a set of linear matrix inequalities, and the results are extended to H_2 and H_∞ static output feedback controller designs. Numerical examples are given to illustrate the effectiveness of the proposed design methods.

Keywords: Discrete-time systems; Static output feedback; Time-varying uncertainty; Robust control; Linear matrix inequalities (LMIs); Parameter dependent Lyapunov function

1. Introduction

Robust control problems for linear systems with polytopic uncertainties have been extensively studied in the past decades, and many important results have been obtained. A popular approach to solve this problem is Lyapunov approach. In [3,10], a single quadratic Lyapunov function approach is used for the analysis and synthesis of linear uncertain systems. However, the obtained results might be conservative. This is due to the fact that the same Lyapunov function is used to assure the robust stability of the system for the entire uncertainty domain. In order to overcome this conservativeness, parameter dependent Lyapunov function methods have been proposed in [8,6,17,19,1,11,21,7,4,20,14,12,2]. In particular, the methods to separate Lyapunov matrix with system matrix by adding a slack variable are noticeable, in [17,19,1,11], respectively, for discrete- and continuous-time systems. By simple modifications to the standard Linear matrix inequality (LMI) which appears in the continuous-time bounded real lemma (BRL), an H_∞ controller design method is proposed for continuous linear systems with time-invariant uncertainties in [21]. Further, for linear systems with time-varying parametric uncertainties, methods of introducing multi-slack variables are developed in [7,4], respectively, for discrete-time and continuous case. Moreover, robust D-stability problems are considered in [20,14], and filter designs are studied in [12,2]. However, the above-mentioned controller design methods of linear uncertain systems are based on the assumption of the states are available for controller implementation, which is not true in many practical cases. When the states of a system are not available, an output feedback control design is necessary. In recent years, there have been some results for output feedback control designs [15,5,22,13,16,18].

Among them, static output feedback control is very useful and more realistic, since it can be easily implemented with low cost. Recently, static output feedback control for linear uncertain systems has been investigated by many researchers see [5,22,13,16,18] and the references therein. In [5], LMI conditions for solving static output feedback control problem of linear continuous- and discrete-time systems are given. A linear parameter dependent approach for designing

* Corresponding author. College of Information Science and Engineering, Northeastern University, Shenyang 110004, PR China.
E-mail addresses: dongjiuxiang@ise.neu.edu.cn, dong_jiuxiang@163.com (J. Dong), yangguanghong@ise.neu.edu.cn, yang_guanghong@163.com (G.-H. Yang).

0167-6911/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2007.08.001

static output feedback controllers [22] is proposed for a linear continuous-time system with H_2 or H_∞ performance requirements, and the results are extended to discrete-time linear systems that contain stochastic white-noise parameter uncertainties in [13]. In [16], a two-step method for designing static or dynamic output feedback controllers is given, where the first step is to obtain a state feedback controller gain whereas the second one is to obtain a static output feedback controller gain. By introducing a parameter-independent slack variable with sub-triangle structure, an LMI-based method of designing robust static output feedback controller for linear systems with time-invariant uncertainties is proposed in [18]. These methods are concerned with designing static output feedback controllers for linear systems with time-invariant uncertainties, and are applicable for the time-varying case when a single quadratic Lyapunov function is exploited. However, the design methods given by [5,22,13,18] require that the system output matrix or input matrix is fixed, i.e., without uncertainties. Although the method of [16] is applicable for the system output matrix and input matrix simultaneously with uncertainties, its design is dependent on the obtained state feedback controller gain in the first step. For the case that the considered system output matrix and input matrix simultaneously are with time-varying uncertainties, few convex methods have been proposed to design robust static output feedback controllers in the literatures.

On the other hand, the approach to separate system matrix and Lyapunov matrix by introducing slack variables has been extensively applied for designing state feedback controllers for linear systems with polytopic uncertainties, and less conservative controller design conditions are obtained, see [17,19,1,11,21,7,4,20,14,12,2]. In this paper, the approach is extended for static output feedback controller designs of linear systems with time-varying uncertainties and new sufficient conditions for static output feedback controller designs are given in terms of solutions to a set of linear matrix inequalities. In contrast to existing approaches, a parameter-dependent slack variable with lower-triangular structure is introduced by considering the properties of input or output matrices, which is helpful for obtaining less conservative results. Moreover, the new technique is also applicable for linear systems with the time-varying polytopic uncertainties, which may simultaneously emerge on system output and input matrices.

The paper is organized as follows. In the next section, system description and some preliminaries are given. In Section 3, sufficient conditions for designing robust static output feedback controllers are proposed, and the results are extended to H_2 and H_∞ static output feedback controller designs. Section 4 presents numerical examples to illustrate the effectiveness of the proposed design methods. Finally, Section 5 concludes the paper.

Notation: The symbol $*$ within a matrix represents the symmetric entries.

2. System description and preliminaries

Consider a linear discrete-time system (1) with time-varying polytopic uncertainties described by the following state-space equations:

$$\begin{aligned} x(k+1) &= A(\alpha(k))x(k) + B_1(\alpha(k))w(k) + B_2(\alpha(k))u(k), \\ z(k) &= C_1(\alpha(k))x(k) + D_{11}(\alpha(k))w(k) + D_{12}(\alpha(k))u(k), \\ y(k) &= C_2(\alpha(k))x(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^m$ is the disturbance input, $u(k) \in \mathbb{R}^p$ is the control input, $y(k) \in \mathbb{R}^r$ is the measured output, $z(k) \in \mathbb{R}^q$ is the controlled output. $\alpha(k) = [\alpha_1(k), \alpha_2(k), \ldots, \alpha_N(k)]^{\mathrm{T}} \in \mathbb{R}^N$ is an unknown but bounded time-varying parameter, satisfying

$$\alpha_i(k) \geqslant 0, \quad \sum_{i=1}^{N} \alpha_i(k) = 1$$

and

$$A(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)A_i, \quad B_1(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)B_{1i},$$

$$B_2(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)B_{2i},$$

$$C_1(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)C_{1i}, \quad C_2(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)C_{2i},$$

$$D_{11}(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)D_{11i}, \quad D_{12}(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k)D_{12i}.$$

The matrices $A_i \in \mathbb{R}^{n\times n}$, $B_{1i} \in \mathbb{R}^{n\times m}$, $B_{2i} \in \mathbb{R}^{n\times p}$, $C_{1i} \in \mathbb{R}^{q\times n}$, $C_{2i} \in \mathbb{R}^{r\times n}$, $D_{11i} \in \mathbb{R}^{q\times m}$ and $D_{12i} \in \mathbb{R}^{q\times p}$, $1 \leqslant i \leqslant N$.

The problem under consideration in this paper is to design a static output feedback controller

$$u(k) = Ky(k), \tag{2}$$

such that the resulting closed-loop system

$$\begin{aligned} x(k+1) &= (A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k)))x(k) \\ &\quad + B_1(\alpha(k))w(k), \\ z(k) &= (C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k)))x(k) \\ &\quad + D_{11}(\alpha(k))w(k) \end{aligned} \tag{3}$$

is robustly stable or satisfies H_2 (or H_∞) performance requirements.

Definition 1 (*Barbosa et al. [2]*). (i) Suppose that system (1) is asymptotically stable. The H_2 norm of system (1) is defined by

$$\lim_{N\longrightarrow\infty} \mathscr{E}\left\{\frac{1}{N}\sum_{k=1}^{N} z^{\mathrm{T}}(k)z(k)\right\}$$

when $x(0) = 0$ and $w(k)$ is a zero-mean white noise with an identity covariance matrix, where in the above $\mathscr{E}$ denotes mathematical expectation.

(ii) (Daafouz and Bernussou [6]). Suppose that system (1) is asymptotically stable and for all $w(k) \in \mathbb{R}^m$, satisfies

$$\sum_{i=0}^{\infty} z^{\mathrm{T}}(k)z(k) < \gamma^2 \sum_{i=0}^{\infty} w^{\mathrm{T}}(k)w(k)$$

$$\text{for all } \sum_{i=0}^{\infty} w^{\mathrm{T}}(k)w(k) > 0 \tag{4}$$

then H_∞ norm of system (1) is said to be less than γ.

The following two lemmas will be used in the sequel.

Lemma 1 (*Barbosa et al.* [2]). (H_2 *performance*). *If there exist symmetric matrices* $P(\alpha(k))$ *and* $Z(\alpha(k+1))$, *such that*

$$\mathrm{trace}(Z(\alpha(k+1))) \leqslant \gamma, \tag{5}$$

$$\begin{bmatrix} -P(\alpha(k)) & * & * \\ P(\alpha(k+1))(A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k))) & -P(\alpha(k+1)) & * \\ C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k)) & 0 & -I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} -Z(\alpha(k+1)) & * & * \\ P(\alpha(k+1))B_1(\alpha(k)) & -P(\alpha(k+1)) & * \\ D_{11}(\alpha(k)) & 0 & -I \end{bmatrix} < 0 \tag{7}$$

then H_2 *norm of system* (1) *is less thans* $\sqrt{\gamma}$.

Lemma 2 (H_∞ *performance*). *If there exists a symmetric matrix* $P(\alpha(k))$, *such that*

$$\begin{bmatrix} -P(\alpha(k)) & * & * & * \\ 0 & -\gamma I & * & * \\ \Gamma_1 & \Gamma_2 & -P(\alpha(k+1)) & * \\ \Gamma_3 & D_{11}(\alpha(k)) & 0 & -\gamma I \end{bmatrix} < 0, \tag{8}$$

where

$$\Gamma_1 = P(\alpha(k+1))(A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k))),$$

$$\Gamma_2 = P(\alpha(k+1))B_1(\alpha(k)),$$

$$\Gamma_3 = C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k))$$

then H_∞ *norm of system* (1) *is less than* γ.

Proof. Let the initial state $x(0)=0$. Choose Lyapunov function $V(k) = x^{\mathrm{T}}(k)\ P(\alpha(k))x(k)$. From (8), we have

$$\begin{bmatrix} -P(\alpha(k)) & * \\ P(\alpha(k+1))(A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k))) & -P(\alpha(k+1)) \end{bmatrix} < 0$$

then system (1) with $w(k) = 0$ is asymptotically stable.

Consider

$$V(k+1) - V(k) + \frac{1}{\gamma} z^{\mathrm{T}}(k)z(k) - \gamma w^{\mathrm{T}}(k)w(k)$$

$$= [x^{\mathrm{T}}(k)\ \ w^{\mathrm{T}}(k)] \begin{bmatrix} \Lambda_{11} & * \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}, \tag{9}$$

where

$$\begin{aligned} \Lambda_{11} = {} & (A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k)))^{\mathrm{T}} P(\alpha(k+1)) \\ & \times (A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k))) - P(\alpha(k)) \\ & + \frac{1}{\gamma}(C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k)))^{\mathrm{T}} \\ & \times (C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k))), \\ \Lambda_{21} = {} & B_1^{\mathrm{T}}(\alpha(k))P(\alpha(k+1))(A(\alpha(k)) + B_2(\alpha(k))KC_2(\alpha(k))) \\ & + \frac{1}{\gamma} D_{11}^{\mathrm{T}}(\alpha(k))(C_1(\alpha(k)) + D_{12}(\alpha(k))KC_2(\alpha(k))), \\ \Lambda_{22} = {} & B_1^{\mathrm{T}}(\alpha(k))P(\alpha(k))B_1(\alpha(k)) \\ & + \frac{1}{\gamma} D_{11}^{\mathrm{T}}(\alpha(k))D_{11}(\alpha(k)) - \gamma I. \end{aligned}$$

Applying Schur complement lemma to (8), we have

$$\begin{bmatrix} \Lambda_{11} & * \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} < 0$$

which can guarantee that (9) is less than zero, for $\begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \neq 0$. Then

$$\sum_{k=0}^{\infty} \left[\left(V(k+1) - V(k) + \frac{1}{\gamma} z^{\mathrm{T}}(k)z(k) - \gamma w^{\mathrm{T}}(k)w(k) \right) \right]$$

$$= V(\infty) - V(0) + \sum_{k=0}^{\infty} \frac{1}{\gamma} z^{\mathrm{T}}(k)z(k) - \sum_{k=0}^{\infty} \gamma w^{\mathrm{T}}(k)w(k)$$

$$= V(\infty) + \sum_{k=0}^{\infty} \frac{1}{\gamma} z^{\mathrm{T}}(k)z(k) - \sum_{k=0}^{\infty} \gamma w^{\mathrm{T}}(k)w(k)$$

$$< 0 \quad \text{for } \sum_{i=0}^{\infty} w^{\mathrm{T}}(k)w(k) > 0,$$

where $V(\infty) = \lim_{k\to\infty} V(k)$. Since $V(\infty) \geqslant 0$, the above inequality implies that (4) holds. Therefore, the H_∞ norm of system (3) is less than γ. □

3. Robust static output feedback controller design

In this section, two methods for designing robust static output feedback controllers are given, respectively, for two cases. The first case is that C_{2i}, $1 \leqslant i \leqslant N$, are of full rank, and the second one is that B_{2i}, $1 \leqslant i \leqslant N$, are of full rank.

3.1. Case I: C_{2i} is of full rank

In this subsection, assume that C_{2i}, $1\leqslant i\leqslant N$, are of full rank, and let invertible matrices T_i, $1\leqslant i\leqslant N$, satisfy

$$C_{2i}T_i=[I\ \ 0]\quad \text{for } 1\leqslant i\leqslant N. \tag{10}$$

Under the assumption, a method for designing robust static output feedback stabilizing controllers is firstly given, and then followed by extensions to H_2 and H_∞ controller designs.

Remark 1. For each C_{2i}, the corresponding T_i generally is not unique. A special T_i can be obtained by the following formula:

$$T_i=[C_{2i}^{\mathrm T}(C_{2i}C_{2i}^{\mathrm T})^{-1}\ \ C_{2i}^{\perp}], \tag{11}$$

where $C_{2i}^{\perp}$ denotes an orthogonal basis for the null space of C_{2i}.

3.1.1. Robust stabilizing controller design

The following lemma is essential to the later development.

Lemma 3. *If there exist symmetric matrices Q_i, $1\leqslant i\leqslant N$ and matrices S_{ij}, $1\leqslant i,j\leqslant N$, satisfying* (12a) (*or* (12b)),

$$\begin{bmatrix} Q_i-S_{ij}-S_{ij}^{\mathrm T} & * \\ (A_i+B_{2j}KC_{2i})S_{ij} & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N, \tag{12a}$$

$$\begin{bmatrix} Q_i-S_{ij}-S_{ij}^{\mathrm T} & * \\ (A_i+B_{2i}KC_{2j})S_{ij} & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N. \tag{12b}$$

then system (3) *with $w(k)=0$ is robustly stable via static output feedback control law* (2).

Proof. By (12a) and considering the block (2, 2) of the left side of (12a), then we have $Q_i>0$. From the block (1, 1) of the left side of (12a), it follows that $Q_i-S_{ij}-S_{ij}^{\mathrm T}<0$. Combining it with $Q_i>0$, we can obtain $S_{ij}+S_{ij}^{\mathrm T}>0$, which implies that S_{ij} is invertible. From $Q_i>0$, then we have

$$(S_{ij}-Q_i)^{\mathrm T}Q_i^{-1}(S_{ij}-Q_i)\geqslant 0$$

which is equivalent to

$$-S_{ij}^{\mathrm T}Q_i^{-1}S_{ij}\leqslant Q_i-S_{ij}-S_{ij}^{\mathrm T}. \tag{13}$$

From (12a) and (13), it follows that

$$\begin{bmatrix} -S_{ij}^{\mathrm T}Q_i^{-1}S_{ij} & * \\ (A_i+B_{2j}KC_{2i})S_{ij} & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N. \tag{14}$$

Let $P_i=Q_i^{-1}$, $1\leqslant i\leqslant N$. Pre- and post-multiplying (14) by

$$\begin{bmatrix} (S_{ij}^{-1})^{\mathrm T} & 0 \\ 0 & P_l \end{bmatrix}$$

and its transpose, then we have

$$\begin{bmatrix} -P_i & * \\ P_l(A_i+B_{2j}KC_{2i}) & -P_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N. \tag{15}$$

Multiplying (15) by $\alpha_i(k)\alpha_j(k)\alpha_l(k+1)$, and summing them, then it follows that

$$\begin{bmatrix} -\sum_{i=1}^N\alpha_iP_i & * \\ \left(\sum_{l=1}^N\alpha_lP_l\right)\left(\sum_{i=1}^N\alpha_iA_i\right)+\left(\sum_{l=1}^N\alpha_lP_l\right)\left(\sum_{j=1}^N\alpha_jB_{2j}\right)K\left(\sum_{i=1}^N\alpha_iC_{2i}\right) & -\sum_{l=1}^N\alpha_lP_l \end{bmatrix}<0, \tag{16}$$

where $\alpha_i=\alpha_i(k)$, $\alpha_j=\alpha_j(k)$, $\alpha_l=\alpha_l(k+1)$.

If we choose a parameter dependent Lyapunov function

$$V(k)=x^{\mathrm T}(k)\left(\sum_{i=1}^N\alpha_i(k)P_i\right)x(k)$$

then from (16), we have $V(k+1)-V(k)<0$ for $x(k)\neq 0$, which implies that system (3) is asymptotically stable. Moreover, by replacing inequality (12a) with (12b), the corresponding proof is easily obtained and omitted. □

Remark 2. If $K=0$ and $S_{ij}=S_i$, then (12) becomes

$$\begin{bmatrix} Q_i-S_i-S_i^{\mathrm T} & * \\ A_iS_i & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,l\leqslant N, \tag{17}$$

which reduces to the robust stability condition given in [7]. In particular, for the following parameter dependent Lyapunov function:

$$V(x(k),\alpha(k))=x^{\mathrm T}(k)\left(\sum_{i=1}^N\alpha_i(k)Q_i^{-1}\right)x(k).$$

Eq. (17) is a necessary and sufficient condition to make system (1) with $u(k)=0$ and $w(k)=0$ robustly stable, see [7, Theorem 3].

Based on Lemma 3, we have the following result for robust static output feedback stabilizing controller designs.

Theorem 1. *If there exist symmetric matrices Q_i, $1\leqslant i\leqslant N$ and matrices $\mathbf{S}_{ij}$, L, $1\leqslant i,j\leqslant N$ with*

$$\mathbf{S}_{ij}=\begin{bmatrix} S_{11} & 0 \\ S_{21}^{ij} & S_{22}^{ij} \end{bmatrix},\quad L=[L_1\ \ 0]$$

satisfying (18a) (*or* (18b)),

$$\begin{bmatrix} Q_i-T_i\mathbf{S}_{ij}-\mathbf{S}_{ij}^{\mathrm T}T_i^{\mathrm T} & * \\ A_iT_i\mathbf{S}_{ij}+B_{2j}L & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N, \tag{18a}$$

$$\begin{bmatrix} Q_i-T_j\mathbf{S}_{ij}-\mathbf{S}_{ij}^{\mathrm T}T_j^{\mathrm T} & * \\ A_iT_j\mathbf{S}_{ij}+B_{2i}L & -Q_l \end{bmatrix}<0,\quad 1\leqslant i,j,l\leqslant N \tag{18b}$$

then system (1) *with $w(k)=0$ is asymptotically stable via the following static output feedback gain*:

$$K=L_1S_{11}^{-1}. \tag{19}$$

Proof. From the structure of $L, \mathbf{S}_{ij}$ and (10), (19), we can obtain

$$L = [KS_{11} \ \ 0] = [K \ \ 0]\begin{bmatrix} S_{11} & 0 \\ S_{21}^{ij} & S_{22}^{ij} \end{bmatrix} = K[I \ \ 0]T_i^{-1}T_i\mathbf{S}_{ij} = KC_{2i}T_i\mathbf{S}_{ij} = KC_{2i}S_{ij},$$

where $S_{ij} = T_i\mathbf{S}_{ij}$. Substituting S_{ij} for $T_i\mathbf{S}_{ij}$, $KC_{2i}S_{ij}$ for L in (18a), then (18a) can be rewritten as follows:

$$\begin{bmatrix} Q_i - S_{ij} - S_{ij}^{\mathrm{T}} & * \\ AS_{ij} + B_{2j}KC_{2i}S_{ij} & -Q_l \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N$$

which implies that (12a) holds. Then, from Lemma 3, it follows that the system (3) with $w(k) = 0$ is asymptotically stable. Moreover, by replacing inequality (18a) with (18b), the corresponding proof is easily obtained and omitted. □

Remark 3. In Theorem 1, two sufficient conditions are proposed for robust static output feedback stabilizing controller designs of linear discrete-time systems with time-invariant uncertainties, and the design conditions are given in terms of solutions to a set of LMIs, which can be effectively solved by using LMI Control Toolbox [9]. If the variables $\mathbf{S}_{ij} = \mathbf{S}_{11}$, $Q_i = Q_1$, $T_i = T_1$, $i, j = 1, \ldots, N$ in Theorem 1, and the output matrices $C_{2i} = C_{21} = [I \ \ 0]$, $i = 1, \ldots, N$, then the condition (with (12b)) of Theorem 1 reduces to the result in [18]. It should be pointed out that the methods given in [5,22,13,16,18] are also applicable for designing static output feedback stabilizing controllers for the time-varying case when a single quadratic Lyapunov function is exploited. The comparisons between the above-mentioned methods and Theorem 1 are illustrated in Section 4.

It should be noted that for each C_{2i}, there may exist different choices of T_i satisfying (10). The following theorem shows that the feasibility of the conditions of Theorem 1 is independent of the choices of T_i.

Theorem 2. *If the condition of Theorem* 1 *is feasible for* T_i *satisfying* (10), *then for each* V_i *satisfying* (10), *the condition of Theorem* 1 *with* $T_i = V_i$ *is also feasible.*

Proof. Since both T_i and V_i satisfy (10),

$$C_{2i}T_i = C_{2i}V_i = [I \ \ 0]$$

which implies that

$$[I \ \ 0]T_i^{-1} = [I \ \ 0]V_i^{-1}. \tag{20}$$

Post-multiplying both sides of (20) by T_i, then we have

$$[I \ \ 0] = [I \ \ 0]V_i^{-1}T_i. \tag{21}$$

Denote $W_i = V_i^{-1}T_i = \begin{bmatrix} W_{11}^i & W_{12}^i \\ W_{21}^i & W_{22}^i \end{bmatrix}$, then from (21), it follows that $W_{11}^i = I$, $W_{12}^i = 0$.

Consider

$$T_i\mathbf{S}_{ij} = V_iV_i^{-1}T_i\mathbf{S}_{ij} = V_iW_i\mathbf{S}_{ij} = V_i\begin{bmatrix} I & 0 \\ W_{21}^i & W_{22}^i \end{bmatrix}\begin{bmatrix} S_{11} & 0 \\ S_{21}^{ij} & S_{22}^{ij} \end{bmatrix} = V_i\begin{bmatrix} S_{11} & 0 \\ \mathscr{S}_{21}^{ij} & \mathscr{S}_{22}^{ij} \end{bmatrix}, \tag{22}$$

where

$$\mathscr{S}_{21}^{ij} = W_{21}^iS_{11} + W_{22}^iS_{21}^{ij}, \quad \mathscr{S}_{22}^{ij} = W_{22}^iS_{22}^{ij}.$$

Let $\mathscr{S}_{ij} = \begin{bmatrix} S_{11} & 0 \\ \mathscr{S}_{21}^{ij} & \mathscr{S}_{22}^{ij} \end{bmatrix}$, then (22) can be rewritten as follows:

$$T_i\mathbf{S}_{ij} = V_i\mathscr{S}_{ij}. \tag{23}$$

Therefore, if (13) holds for T_i, then we have

$$\begin{bmatrix} Q_i - V_i\mathscr{S}_{ij} - \mathscr{S}_{ij}^{\mathrm{T}}V_i^{\mathrm{T}} & * \\ A_iV_i\mathscr{S}_{ij} + B_{2j}L & -Q_l \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N,$$

or $$\begin{bmatrix} Q_i - V_j\mathscr{S}_{ij} - \mathscr{S}_{ij}^{\mathrm{T}}V_j^{\mathrm{T}} & * \\ A_iV_j\mathscr{S}_{ij} + B_{2i}L & -Q_l \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N,$$

where Q_i, L, and S_{11} satisfy the condition of Theorem 1. It follows that the condition of Theorem 1 is feasible for $T_i = V_i$. □

3.1.2. H_2 and H_∞ control

In this subsection, the results for robust static output feedback stabilizing controller design are extended to the cases of H_2 and H_∞ control.

For the case of H_2 control, we have

Theorem 3 (H_2 *performance*). *If there exist symmetric matrices* Q_i, Z_i, $1 \leqslant i \leqslant N$ *and matrices* $\mathbf{S}_{ij}$, L, $1 \leqslant i, j \leqslant N$ *with*

$$\mathbf{S}_{ij} = \begin{bmatrix} S_{11} & 0 \\ S_{21}^{ij} & S_{22}^{ij} \end{bmatrix}, \quad L = [L_1 \ \ 0]$$

satisfying (24a) (*or* (24b)) *and* (25), (26)

$$\begin{bmatrix} Q_i - T_i\mathbf{S}_{ij} - \mathbf{S}_{ij}^{\mathrm{T}}T_i^{\mathrm{T}} & * & * \\ A_iT_i\mathbf{S}_{ij} + B_{2j}L & -Q_l & * \\ C_{1i}T_i\mathbf{S}_{ij} + D_{12j}L & 0 & -I \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N, \tag{24a}$$

$$\begin{bmatrix} Q_i - T_j\mathbf{S}_{ij} - \mathbf{S}_{ij}^{\mathrm{T}}T_j^{\mathrm{T}} & * & * \\ A_iT_j\mathbf{S}_{ij} + B_{2i}L & -Q_l & * \\ C_{1i}T_j\mathbf{S}_{ij} + D_{12i}L & 0 & -I \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N, \tag{24b}$$

$$\begin{bmatrix} -Z_j & * & * \\ B_{1i} & -Q_j & * \\ D_{11i} & 0 & -I \end{bmatrix} < 0, \quad 1 \leqslant i, j \leqslant N, \tag{25}$$

$$Tr(Z_i) < \gamma, \quad 1 \leqslant i \leqslant N, \tag{26}$$

then the static output feedback control law (2) *renders* H_2 *norm of system* (3) *less than* $\sqrt{\gamma}$, *where*

$$K = L_1S_{11}^{-1}.$$

Proof. Let $S_{ij} = T_i\mathbf{S}_{ij}$, $P_i = Q_i^{-1}$, then using the technique of Theorem 1, from (24a), we can obtain

$$\begin{bmatrix} -P_i & * & * \\ P_l(A_i+B_{2j}KC_{2i}) & -P_l & * \\ C_{1i}+D_{12j}KC_{2i} & 0 & -I \end{bmatrix} < 0, \quad 1 \leqslant i, j, l \leqslant N. \quad (27)$$

Multiplying (27) by $\alpha_i(k)\alpha_j(k)\alpha_l(k+1)$ and summing them, then we can have

$$\begin{bmatrix} -\sum_{i=1}^{N}\alpha_i P_i & * & * \\ \left(\sum_{l=1}^{N}\alpha_l P_l\right)\left(\sum_{i=1}^{N}\alpha_i A_i\right) + \left(\sum_{l=1}^{N}\alpha_l P_l\right)\left(\sum_{j=1}^{N}\alpha_j B_{2j}\right) K \left(\sum_{i=1}^{N}\alpha_i C_{2i}\right) & -\sum_{l=1}^{N}\alpha_l P_l & * \\ \left(\sum_{i=1}^{N}\alpha_i C_{1i}\right) + \left(\sum_{j=1}^{N}\alpha_j D_{12j}\right) K \left(\sum_{i=1}^{N}\alpha_i C_{2i}\right) & 0 & -I \end{bmatrix} < 0, \quad (28)$$

where $\alpha_i = \alpha_i(k)$, $\alpha_j = \alpha_j(k)$, $\alpha_l = \alpha_l(k+1)$.

Let $P(\alpha(k)) = \sum_{i=1}^{N}\alpha_i(k)P_i$, then (28) is same to (6).

On the other hand, pre- and post-multiplying (25) by

$$\begin{bmatrix} I & 0 & 0 \\ 0 & P_j & 0 \\ 0 & 0 & I \end{bmatrix}$$

and its transpose, it follows that

$$\begin{bmatrix} -Z_j & * & * \\ P_jB_{1i} & -P_j & * \\ D_{11i} & 0 & -I \end{bmatrix} < 0, \quad 1 \leqslant i, \quad j \leqslant N. \quad (29)$$

Multiplying (29) by $\alpha_i(k)\alpha_j(k+1)$ and summing them, then we can obtain (7) with $Z(\alpha(k+1)) = \sum_{i=1}^{N}\alpha_i(k+1)Z_i$. Similarly, from (26), it follows that (5) holds. Then by virtue of Lemma 1, it further implies that H_2 norm of system (1) is less than $\sqrt{\gamma}$. Moreover, by replacing inequality (24a) with (24b), the corresponding proof is easily obtained and omitted. □

The following theorem presents sufficient conditions for robust H_∞ static output feedback controller designs.

Theorem 4 (*H_∞ performance*). *If there exist symmetric matrices Q_i, $1 \leqslant i \leqslant N$ and matrices $\mathbf{S}_{ij}$, L, $1 \leqslant i, j \leqslant N$ with*

$$\mathbf{S}_{ij} = \begin{bmatrix} S_{11} & 0 \\ S_{21}^{ij} & S_{22}^{ij} \end{bmatrix}, \quad L = [L_1 \ \ 0]$$

satisfying (30a) (*or* (30b))

$$\begin{bmatrix} Q_i - T_i\mathbf{S}_{ij} - \mathbf{S}_{ij}^{\mathrm{T}}T_i^{\mathrm{T}} & * & * & * \\ 0 & -\gamma I & * & * \\ A_iT_i\mathbf{S}_{ij} + B_{2j}L & B_{1i} & -Q_l & * \\ C_{1i}T_i\mathbf{S}_{ij} + D_{12j}L & D_{11i} & 0 & -\gamma I \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N, \quad (30a)$$

$$\begin{bmatrix} Q_i - T_j\mathbf{S}_{ij} - \mathbf{S}_{ij}^{\mathrm{T}}T_j^{\mathrm{T}} & * & * & * \\ 0 & -\gamma I & * & * \\ A_iT_j\mathbf{S}_{ij} + B_{2i}L & B_{1i} & -Q_l & * \\ C_{1i}T_j\mathbf{S}_{ij} + D_{12i}L & D_{11i} & 0 & -\gamma I \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N \quad (30b)$$

then the static output feedback control law (2) *renders H_∞ norm of system* (3) *less than γ where*

$K = L_1S_{11}^{-1}$.

Proof. It can be completed by using Lemma 2 and the arguments similar to those for Theorem 3, and the details are omitted here. □

Remark 4. Similar to Theorem 2, the feasibility of the conditions of Theorems 3 and 4 also is independent of the choices of T_i. The proof is omitted here.

3.2. Case: B_{2i} is of full rank

In this subsection, assume that B_{2i}, $1 \leqslant i \leqslant N$, are of full rank, and let invertible matrices U_i, $1 \leqslant i \leqslant N$, satisfy

$$U_iB_{2i} = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad \text{for } 1 \leqslant i \leqslant N \quad (31)$$

and under the assumption, sufficient conditions for robust static output feedback controller designs are presented.

Remark 5. For each B_{2i}, the U_i satisfying (31) generally is not unique. A special U_i can be obtained by the following formula:

$$U_i = \begin{bmatrix} (B_{2i}^{\mathrm{T}}B_{2i})^{-1}B_{2i}^{\mathrm{T}} \\ H_i^{\mathrm{T}} \end{bmatrix}, \quad (32)$$

where H_i denotes an orthogonal basis for the null space of B_{2i}^{T}.

Similar to Lemma 3, and Theorems 1 and 2, the following result is given.

Lemma 4. *If there exist symmetric matrices P_i, $1 \leqslant i \leqslant N$ and matrices R_{ij}, $1 \leqslant i, j \leqslant N$ satisfying* (33a) (*or* (33b))

$$\begin{bmatrix} -P_i & * \\ R_{ij}(A_i + B_{2i}KC_{2j}) & P_l - R_{ij} - R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N. \quad (33a)$$

$$\begin{bmatrix} -P_i & * \\ R_{ij}(A_i + B_{2j}KC_{2i}) & P_l - R_{ij} - R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N \tag{33b}$$

then system (3) *with* $w(k)=0$ *is robustly stable via static output feedback control law* (2).

Proof. If (33a) holds, then

$$-R_{ij}\left(\sum_{l=1}^{N}\alpha_l P_l\right)^{-1} R_{ij}^{\mathrm{T}} \leqslant \sum_{l=1}^{N}\alpha_l P_l - R_{ij} - R_{ij}^{\mathrm{T}}. \tag{34}$$

Multiplying (33a) by $\alpha_l(k+1)$ and summing them, then

$$\begin{bmatrix} -P_i & * \\ R_{ij}(A_i + B_{2i}KC_{2j}) & \sum_{l=1}^{N}\alpha_l P_l - R_{ij} - R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, j \leqslant N. \tag{35}$$

From (34) and (35), it follows that

$$\begin{bmatrix} -P_i & * \\ R_{ij}(A_i + B_{2i}KC_{2j}) & -R_{ij}\left(\sum_{l=1}^{N}\alpha_l P_l\right)^{-1} R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, \quad j \leqslant N. \tag{36}$$

Pre- and post-multiplying (36) by

$$\begin{bmatrix} I & 0 \\ 0 & R_{ij}^{-1} \end{bmatrix}$$

and its transpose, then we have

$$\begin{bmatrix} -P_i & * \\ A_i + B_{2i}KC_{2j} & -\left(\sum_{l=1}^{N}\alpha_l P_l\right)^{-1} \end{bmatrix} < 0, \quad 1 \leqslant i, \quad j \leqslant N. \tag{37}$$

Multiplying (37) by $\alpha_i(k)\alpha_j(k)$, and summing them, then the conclusion follows using the arguments similar to those in the proof of Lemma 3. Moreover, by replacing inequality (33a) with (33b), the corresponding proof is easily obtained and omitted. □

Based on Lemma 4, we have

Theorem 5. *If there exist symmetric matrices* P_i, $1 \leqslant i \leqslant N$ *and matrices* $\mathbf{R}_{ij}$, M, $1 \leqslant i, j \leqslant N$ *with*

$$\mathbf{R}_{ij} = \begin{bmatrix} R_{11} & R_{12}^{ij} \\ 0 & R_{22}^{ij} \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ 0 \end{bmatrix}$$

satisfying (38a) (*or* (38b))

$$\begin{bmatrix} -P_i & * \\ R_{ij}U_iA_i + MC_{2j} & P_l - R_{ij}U_i - U_i^{\mathrm{T}}R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N, \tag{38a}$$

$$\begin{bmatrix} -P_i & * \\ R_{ij}U_jA_i + MC_{2i} & P_l - R_{ij}U_j - U_j^{\mathrm{T}}R_{ij}^{\mathrm{T}} \end{bmatrix} < 0,$$
$$1 \leqslant i, j, l \leqslant N, \tag{38b}$$

then system (1) *with* $w(k)=0$ *is asymptotically stable via the following static output feedback gain*:

$$K = R_{11}^{-1}M_1.$$

Similar to Theorem 2, the following theorem shows that the feasibility of the conditions of Theorem 5 is independent of the choices of U_i.

Theorem 6. *If the condition of Theorem* 5 *is feasible for* U_i *satisfying* (31), *then for each* V_i *satisfying* (31), *the condition of Theorem* 5 *with* $U_i = V_i$ *is also feasible*.

Proof. Since U_i and V_i satisfy (31),

$$U_iB_{2i} = V_iB_{2i} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

which implies that

$$U_iV_i^{-1}\begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{39}$$

then $U_iV_i^{-1} = \begin{bmatrix} I & \bullet \\ 0 & \bullet \end{bmatrix}$, Consider

$$\mathbf{R}_{ij}U_i = \mathbf{R}_{ij}U_iV_i^{-1}V_i = \begin{bmatrix} R_{11} & R_{12}^{ij} \\ 0 & R_{22}^{ij} \end{bmatrix}\begin{bmatrix} I & \bullet \\ 0 & \bullet \end{bmatrix}$$
$$V_i = \begin{bmatrix} R_{11} & \mathscr{R}_{12}^{ij} \\ 0 & \mathscr{R}_{22}^{ij} \end{bmatrix} V_i = \mathscr{R}_{ij}V_i, \tag{40}$$

where $\mathscr{R}_{ij} = \begin{bmatrix} R_{11} & \mathscr{R}_{12}^{ij} \\ 0 & \mathscr{R}_{22}^{ij} \end{bmatrix}$, the rest of the proof is similar to that of Theorem 2, and the details are omitted here. □

4. Examples

Example 1. Consider system (1) which belongs to the 2-polytopic convex polyhedron with

$$A_1 = \begin{bmatrix} 1+\delta & 0 \\ 1 & 0.7 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.3 & -0.65 \\ 0.3 & -0.9 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1.4 \\ 1 \end{bmatrix},$$

$$C_{21} = C_{22} = [1 \;\; 0],$$

where $\delta \leqslant \delta^*$. Since $C_{21} = C_{22}$, the methods in [5,22,13,16,18], and Theorems 1 and 5 are applicable for designing robust static output feedback stabilizing controllers. Now applying these methods to design robust static output feedback stabilizing controllers such that δ^* is maximized. The obtained δ^* and corresponding static output feedback gains are shown in Table 1.

Table 1
δ^* and K

	δ^*	K
[5]	Infeasible	–
[22,13]	0.43	−0.5263
[16]	0.42	−0.5360
[18]	0.24	−0.5038
Theorem 1 with (18a)	0.23	−0.5067
Theorem 1 with (18b)	0.26	−0.4947
Theorem 5 with (38a)	0.53	−0.5366
Theorem 5 with (38b)	0.52	−0.5341

Table 2
δ^* and K

	δ^*	K
[16]	0.23	−0.9809
Theorem 1 with (18a)	0.40	−1.3336
Theorem 1 with (18b)	0.36	−1.2176
Theorem 5 with (38a)	0.33	−1.1161
Theorem 5 with (38b)	0.34	−1.2052

From Table 1, the computational results show that the methods given by Theorems 1 and 5 can result in different designs, and also justify that the condition of Theorem 1 with (18b) is less conservative than the result in [18] (see Remark 3). And it can be seen that Theorem 5 can give less conservative results than other methods.

Example 2. Consider system (1) which belongs to the 2-polytopic convex polyhedron with

$$A_1 = \begin{bmatrix} 1.3+\delta & -1 \\ 0.2 & 0.6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & -1.5 \\ 0.1 & 0.3 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_{21} = [1 \;\; 1.5], \quad C_{22} = [1 \;\; 1],$$

where $\delta \leqslant \delta^*$. Since $C_{21} \neq C_{22}$ and $B_{21} \neq B_{22}$, the methods given in [5,22,13,18] are not applicable. But, the methods given in [16], and Theorems 1 and 5 are applicable. Applying the technique in [16], a state feedback gain $K_0 = [-1.0392 \;\; 1.1961]$ is easily obtained in the first step, then using Theorem 4.1 in [16] with $P_i = P_1$, the maximal δ^* can be obtained as 0.23, and the corresponding static output feedback gain is $K = -0.9809$. The design results by using the method given in [16], and Theorems 1 and 5 are shown in Table 2.

From Table 2, it can be seen that the new methods can give less conservative results.

Example 3. Consider system (1) which belongs to the 2-polytopic convex polyhedron with

$$A_1 = \begin{bmatrix} 1.6000 & 0 \\ 1.0000 & 0.7000 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.1000 & -0.3000 \\ 0 & 0.1000 \end{bmatrix},$$

Table 3
H_2 performance index

	Theorem 3 with (24a)	Theorem 3 with (24b)
$\sqrt{\gamma}$	0.8226	0.8226
K	−0.8629	−0.8631

Table 4
H_∞ performance index

	Theorem 4 with (30a)	Theorem 4 with (30b)
γ	1.6562	1.6559
K	−0.9537	−0.9531

$$B_{11} = \begin{bmatrix} 0.6000 \\ 0.3000 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0.5000 \\ 0.4000 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1.5000 \\ 1.0000 \end{bmatrix},$$

$$C_{11} = [1.0000 \;\; 0.3000], \quad C_{12} = [1.2000 \;\; 0.5000],$$

$$C_{21} = [1.0000 \;\; -0.1000], \quad C_{22} = [1.0000 \;\; 0.1000],$$

$$D_{111} = 0.5000, \quad D_{122} = 0.6000, \quad D_{121} = 1,$$

$$D_{122} = 0.9000.$$

By (11), we can obtain

$$T_1 = \begin{bmatrix} 0.9901 & 0.0995 \\ -0.0990 & 0.9950 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.9901 & -0.0995 \\ 0.0990 & 0.9950 \end{bmatrix},$$

satisfying (10).

By using Theorems 3 and 4, the optimal H_2 performance $\sqrt{\gamma}$ and H_∞ performance γ as well as the corresponding static output feedback gain K are obtained, and shown in Tables 3 and 4, respectively.

5. Conclusion

In this paper, the problem of designing robust static output feedback controllers for linear discrete-time systems with time-varying polytopic uncertainties has been studied. New sufficient conditions for robust static output feedback stabilizing controller designs are given in terms of solutions to a set of linear matrix inequalities, and the results are extended to H_2 and H_∞ static output feedback controller designs. Numerical examples have shown the effectiveness of the proposed design methods.

Acknowledgment

This work was supported in part by Program for New Century Excellent Talents in University (NCET-04-0283), the Funds for Creative Research Groups of China (No. 60521003), Program for Changjiang Scholars and Innovative Research Team in University (No. IRT0421), the State Key Program of National Natural Science of China (Grant No. 60534010), the Funds of National Science of China (Grant No. 60674021) and the Funds of PhD program of MOE, China (Grant No. 20060145019).

References

[1] P. Apkarian, T.H. Duong, J. Bernussou, Continuous-time analysis, eigenstructure assignment and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations, IEEE Trans. Automat. Control 46 (2001) 1941–1946.

[2] K.A. Barbosa, C.E. de Souza, A. Trofino, Robust H2 filtering for discrete-time uncertain linear systems using parameter-dependent Lyapunov functions, in: Proceedings of the American Control Conference, Anchorage, AK, 2002, pp. 3224–3229.

[3] J. Bernussou, J.C. Geromel, P.L.D. Peres, A linear programming oriented procedure for quadratic stabilization of uncertain systems, Systems Control Lett. 13 (1) (1989) 65–72.

[4] Y.Y. Cao, Z. Lin, A descriptor system approach to robust stability analysis and controller synthesis, IEEE Trans. Automat. Control 49 (11) (2004) 2081–2084.

[5] C.A.R. Crusius, A. Trofino, Sufficient LMI conditions for output feedback control problems, IEEE Trans. Automat. Control 44 (5) (1999) 1053–1057.

[6] J. Daafouz, J. Bernussou, Poly-quadratic stability and H_∞ performance for discrete systems with time varying uncertainties, in: Proceedings of the 40th IEEE Conference on Decision and Control, 2001, Orlando, Florida, USA, pp. 267–272.

[7] J. Daafouz, J. Bernussou, Parameter dependent Lyapunov functions for discrete time systems with time varying parametric uncertainties, Systems Control Lett. 43 (2001) 355–359.

[8] P. Gahinet, P. Apkarian, M. Chilaly, Affine parameter dependent Lyapunov functions and real parametric uncertainty IEEE Trans. Automat. Control 41 (3) (1996) 436–442.

[9] P. Gahinet, A. Nemirovski, A.J. Laub, M. Chilali, LMI Control Toolbox, The MathWorks, Natick, MA, 1995.

[10] F. Garofalo, G. Celentano, L. Glielmo, Stability robustness of interval matrices via Lyapunov quadratic forms, IEEE Trans. Automat. Control 38 (2) (1993) 281–284.

[11] J.C. Geromel, P. Colaneri, Robust stability of time varying polytopic systems, Systems Control Lett. 55 (2006) 81–85.

[12] J.C. Geromel, M.C. de Oliveira, J. Bernussou, Robust filtering of discrete-time linear systems with parameter dependent Lyapunov functions, SIAM J. Control Optim. 41 (3) (2002) 700–711.

[13] E. Gershon, U. Shaked, Static H_2 and H_∞ output-feedback of discrete time LTI systems with state multiplicative noise, Systems Control Lett. 55 (3) (2006) 232–239.

[14] V.J.S. Leite, P.L.D. Peres, An improved LMI condition for robust D-stability of uncertain polytopic systems, IEEE Trans. Automat. Control 48 (3) (2003) 500–504.

[15] L. Lu, R. Yang, L. Xie, Robust H_2 and H_∞ control of discrete-time systems with polytopic uncertainties via dynamic output feedback, in: Proceedings of the American Control Conference, Portland, OR, USA, 2005, pp. 4315–4319.

[16] D. Mehdi, E.K. Boukas, O. Bachelier, Static output feedback design for uncertain linear discrete time systems, IMA J. Math. Control Inform. 21 (3) (2004) 1–13.

[17] M.C. de Oliveira, J. Bernussou, J.C. Geromel, A new discrete-time robust stability condition, Systems Control Lett. 37 (1999) 261–265.

[18] M.C. de Oliveira, J.C. Geromel, J. Bernussou, Extended H_2 and H_∞ characterizations and controller parameterizations for discrete-time systems, Int. J. Control 75 (9) (2002) 666–679.

[19] M.C. de Oliveira, J.C. Geromel, A class of robust stability conditions where linear parameter dependence of the Lyapunov function is a necessary condition for arbitrary parameter dependence, Systems Control Lett. 54 (2005) 1131–1134.

[20] D. Peaucelle, D. Arzelier, O. Bachelier, J. Bernussou, A new robust D-stability condition for real convex polytopic uncertainty, Systems Control Lett. 40 (2000) 21–30.

[21] U. Shaked, Improved LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty, IEEE Trans. Automat. Control 46 (2001) 652–656.

[22] U. Shaked, An LPD approach to robust H_2 and H_∞ static output-feedback design, IEEE Trans. Automat. Control 48 (5) (2003) 866–872.