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Brief Paper

Parallel structure and tuning of a fuzzy PID controller☆

Jian-Xin Xu*, Chang-Chieh Hang, Chen Liu

Department of Electrical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore

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Abstract

In this paper, a parallel structure of fuzzy PID control systems is proposed. It is associated with a new tuning method which, based on gain margin and phase margin specifications, determines the parameters of the fuzzy PID controller. In comparison with conventional PID controllers, the proposed fuzzy PID controller shows higher control gains when system states are away from equilibrium and, at the same time, retains a lower profile of control signals. Consequently, better control performance is achieved. With the proposed formula, the weighting factors of a fuzzy logic controller can be systematically selected according to the plant under control. By virtue of using the simplest structure of fuzzy logic control, the stability of the nonlinear control system can be analyzed and a sufficient BIBO stability condition is given. The superior performance of the proposed controller is demonstrated through an experimental example. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Fuzzy PID control; PID tuning; Gain and phase margins; BIBO stability

1. Introduction

In recent years, fuzzy logic controllers (FLC), especially fuzzy proportional-integral-derivative (PID) controllers have been widely used for industrial processes owing to their heuristic nature associated with simplicity and effectiveness for both linear and nonlinear systems. In fact, for single-input single-output systems, fuzzy logic controllers are essentially the fuzzy PD type, fuzzy PI type or fuzzy PID type associated with nonlinear gains. Because of the nonlinear property of control gains, fuzzy PID controllers possess the potential to improve and achieve better system performance over the conventional PID controller if the nonlinearity can be suitably utilized. On the other hand, due to the existence of nonlinearity, it is usually difficult to conduct theoretical analyses to explain why fuzzy PID can achieve better performance. Consequently, it is important, from both the theoretical and practical points of view, to explore the essential nonlinear control properties of fuzzy PID and find out appropriate design methods which will assist the control engineers to confidently utilize the nonlinearity of the fuzzy PID controllers to improve the closed-loop performance.

To fulfill the above target, we need to answer the following three fundamental questions: (1) what is the suitable structure for fuzzy PID controllers? (2) how to systematically tune the fuzzy PID controller parameters? and (3) how to analyze and evaluate the designed fuzzy PID controllers?

To answer the first question, let us investigate the existing fuzzy PID controllers. There are many design factors in a fuzzy logic controller determining its structure, such as membership functions, input space partition by fuzzy rules, various types of fuzzy inference mechanisms, defuzzification schemes, etc. They may appear either highly nonlinear or approximately linear. Nevertheless, to perform proportional, integral and derivative control modes, the structure of a fuzzy logic controller has to be in some way analogous to a normal PID controller. Although various types of fuzzy PID (including PI and PD) controllers have been proposed (Zhao, Tomizuka & Isaka, 1993; Ying, 1993; Qin & Borders, 1994; Malki, Li & Chen, 1994; Xu, Liu & Hang, 1998; Li, 1998), they can be classified into two major categories according to the way of construction.

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* Corresponding author. Tel.: 65-8742566; fax: 65 7791103.
E-mail address: elexujx@nus.edu.sg (J.-X. Xu)

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Nomenclature

y	plant output
r	reference input
r_0	set-point change
e	error
Δe	change of error
w_e, w_{e1}, w_{e2}	scaling coefficients of errors
$w_{\Delta e}, w_{\Delta e1}, w_{\Delta e2}$	scaling coefficients of change of errors
$w_u, w_{\Delta u}$	scaling coefficients of FLC outputs
$\Delta_{u_1}^{(F)}$	fuzzy PI control
$u_2^{(F)}$	fuzzy PD control
$u_{PID}^{(F)}$	fuzzy PID control
$\Delta u_{PID}^{(F)}$	increment of fuzzy PID control
μ, μ_P, μ_N	membership functions
$P, P_e, P_{\Delta e}$	positive fuzzy labels
$N, N_e, N_{\Delta e}$	negative fuzzy labels
$K_c^{(F)}$	fuzzy proportional gain
$T_i^{(F)}$	fuzzy integral constant
$T_d^{(F)}$	fuzzy derivative constant
K_c	proportional gain
T_i	integral constant
T_d	derivative constant
Δt	sampling period
s	Laplace operator
z	delay operator
$G_c(s)$	PID controller
$G(s)$	plant
τ_1, τ_2	time delay constants
L	dead time
A_m	gain margin (dB)
Φ_m	phase margin
ω_p	phase crossover frequency
$A_m^{(F)}$	fuzzy gain margin
α_0	equivalent gain/phase contour
$K_{c\alpha_0}^{(F)}$	FLC gain on α_0 contour
$A_{m\alpha_0}^{(F)}$	fuzzy gain margin on α_0 contour
$q(t)$	tank 1 inlet
$H_2(t)$	water level of tank 2

One category of fuzzy PID controllers is composed of the conventional PID control system in conjunction with a set of fuzzy rules (knowledge base) and a fuzzy reasoning mechanism. The PID gains are tuned on-line in terms of the knowledge base and fuzzy inference, and then the PID controller generates the control signal. One such design was given by (Zhao et al., 1993). By virtue of the gain scheduling property, this type of fuzzy PID controllers can adapt themselves to varying environments. The main difficulty in using this category of fuzzy PID controllers is that the analysis task is relatively tough, as it is hard to acquire the equivalent nonlinearity of the fuzzy knowledge base. Besides, associating three PID gains adaptively with the system responses requires ad hoc expertise which may not be so straightforward for a user or designer to extract.

Another category of fuzzy PID controllers is a typical fuzzy logic controller constructed as a set of heuristic control rules, hence the control signal is directly deduced from the knowledge base and the fuzzy inference. These heuristic control rules can be very simple and primitive, hence easy to derive. They are referred to as fuzzy PID controllers because, from the viewpoint of input–output relationship, their structure is analogous to that of the conventional PID controller. Once the structure is fixed, the nonlinear property of the fuzzy PID controller is uniquely determined. When a large number of fuzzy rules are involved in such a fuzzy logic controller and unevenly distributed in the fuzzy input space, it will be again difficult to do any analysis. On the other hand, if a relatively simple fuzzy rule base is used, it is possible to capture the essential nonlinear model of the fuzzy PID controller. Thus numerous model-based controller analysis tools and controller parameter setting methods can be used. A considerable amount of work has been done in this particular area of FLC structure analysis.

In this paper we propose a new type of the fuzzy PID controller of the second category. It has the simplest structure: only two fuzzy labels are used for each fuzzy input variable and three fuzzy labels for the fuzzy control output variable. The considerations behind our selection are as follows. First, from the practical point of view, it seems that the heuristic knowledge of the second category is more analogous to that of human operator or expert, therefore it is easier to be acquired. Second, owing to the similarity of the input–output relationship between the fuzzy and conventional PID controllers, it is possible for us to borrow conventional PID tuning methods to decide the fuzzy PID controller parameters. Third, with the simplest structure of the dynamics of the fuzzy PID, it is convenient for us to conduct further theoretical analysis and evaluation.

It should be pointed out that, for fuzzy PID controllers, normally a 3-D rule base is required. This is difficult to obtain since 3-D information is usually beyond the sensing capability of a human expert. To obtain all of proportional, integral and derivative control action, it is intuitive and convenient to combine PI and PD type fuzzy logic controllers together to form a fuzzy PID controller. Therefore, in the proposed fuzzy control system there are only four control rules for the fuzzy PI and fuzzy PD control channels, respectively, and the two channels are combined in parallel.

After determining the structure, we are ready to answer the second and third questions. There are two ways of determining fuzzy logic controller parameters, depending on how much is known about the process under control. Without knowing the process dynamics or its approximation, the FLC parameters can only be tuned through trial and error. On the other hand, it is well known that for most industrial control problems, the effective tuning of conventional PID controllers is based on estimating and approximating the process dynamics, whether linear or nonlinear, by a simple linear first or second order system with dead time. There exist many tuning or auto-tuning algorithms such as Ziegler–Nichols tuning formula, internal model control tuning method (Åström & Hägglund, 1995), optimization based tuning (Åström, Panagopoulos & Hägglund, 1998), etc. Because a fuzzy controller of the second category is essentially a PD type, PI or PID-type controller with nonlinear gains, it is possible to borrow the standard estimation method, for instance the relay test, and tuning methods of a conventional PID controller to design the fuzzy PID controller. Gain margin and phase margin are important measures of the closed-loop system characteristics and they offer a convenient means for designing control systems. The auto-tuning method for PI/PID controllers to satisfy a pair of specified gain margin and phase margin has been proven to be effective (Ho, Hang & Cao, 1994). In this paper, we introduce a tuning method, which is similarly based on gain margin and phase margin, to determine the parameters of the proposed fuzzy PID controller. The auto-tuning formula is applied here to decide the fuzzy PID parameters (the weighting coefficients for error, the change of error and controller output) with respect to a selected equivalent gain/phase margin contour on which both fuzzy and conventional PID controllers possess the same gain and phase margins. This ensures the required gain and phase margins, which give the local stability, of the fuzzy PID controller on the selected contour. The fine-tuning of the error coefficient, which does not affect the given gain and phase margins, is based on heuristic knowledge.

As for the analysis and evaluation of the fuzzy PID controllers, we will focus on an important issue: the stability problem. Although fuzzy logic controllers have been adopted in many engineering applications, their performance is not guaranteed since there is a lack of stability analysis. Note that the concept of local stability based on the gain and phase margins is essentially from the linear control systems. Therefore, more general stability analysis methods which can incorporate the nonlinear nature of the fuzzy PID controller are preferred. In this paper, the well-known small-gain theorem is employed to evaluate the bounded-input/bounded-output stability condition of the proposed fuzzy PID control system. Through analysis we will show another important property of the new fuzzy PID controller: it possesses higher control gains but yields less control efforts than the conventional PID controllers.

This paper is organized as follows. The fuzzy PID controller constructed by the parallel combination of fuzzy PI and fuzzy PD controllers and its tuning formula is first presented. The stability condition and the property of the proposed fuzzy PID controller are then studied. Experiments are carried out which demonstrate better control performance of the proposed fuzzy PID controller, thereby showing the validity of the proposed control method.

2. Design of fuzzy PID control system

2.1. Fuzzy PID controller with parallel structure

Usually, a fuzzy controller is either a PD or a PI type depending on the output of fuzzy control rules. A fuzzy PID controller may be constructed by introducing the third information besides error and change in error, which is either rate of change in error or sum of error, with a 3-D rule base. Such a fuzzy PID controller with a 3-D rule base is difficult to construct because: (1) for the case of using rate of change in error, a human expert can hardly sense the third dimension of information, for instance, the acceleration besides position and velocity in a motion control system, and thus it is difficult to obtain the control rules; (2) for the case of using sum of error, it is difficult to quantitate its linguistic value since a different plant needs different integral gain and steady-state value of sum of error; (3) a 3-D rule base can be very complex when the number of quantizations of each dimension increases; in this situation, the control rule number increases cubically with the number of quantizations.

In this paper, we propose a parallel combination of a fuzzy PI controller and a fuzzy PD controller to achieve a fuzzy PID controller. The overall structure is shown in Fig. 1.

Simplest structures are used in each FLC. There are only two fuzzy labels (Positive and Negative) used for the fuzzy input variables and three fuzzy labels (Positive, Zero and Negative) for the fuzzy output variable. There are two main reasons which motivate us to choose this type of FLC: (1) theoretical analysis is possible owing to its simplicity and (2) the nonlinearity of the simplest fuzzy controller is the strongest (Buckley & Ying, 1989). Therefore, we can expect better control performance from this simplest structure controller as long as we can correctly use its nonlinear property.

First of all, the error and the change of error are defined as

$$e(k) = r(k) - y(k),$$
$$\Delta e(k) = e(k) - e(k-1). \quad (1)$$

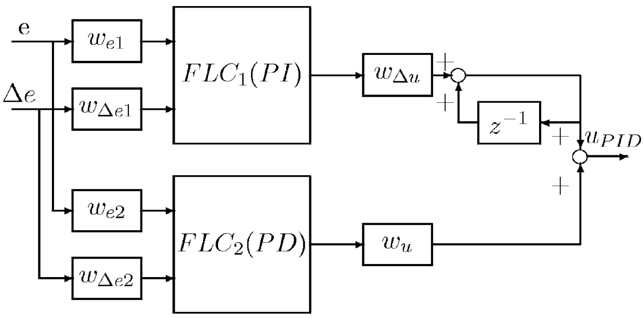

Fig. 1. The overall structure of fuzzy PID controller.

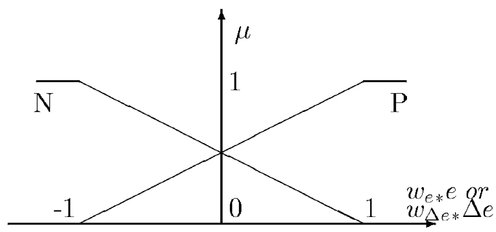

Fig. 2. Membership functions of e & Δe.

The inputs of the fuzzy controller are normalized error ($w_{e*}e$) and normalized change of error ($w_{\Delta e*}\Delta e$) where w_{e*} and $w_{\Delta e*}$ are weighting factors. The notation $*(* = \{1, 2\})$ denotes different types of FLCs. The membership functions $\mu(\bullet)$ of fuzzified inputs are defined in Fig. 2.

According to this kind of triangular-shape membership functions, there are four fuzzy labels $P_e, P_{\Delta e}, N_e$ and $N_{\Delta e}$ for the two fuzzy input variables and the corresponding membership functions are described as

$$\mu_{P_{e*}} = \begin{cases} 0, & w_{e*} \cdot e < -1, \\ \frac{1}{2} + \frac{1}{2} w_{e*} \cdot e, & -1 \le w_{e*} \cdot e < 1, \\ 1, & w_{e*} \cdot e \ge 1, \end{cases} \tag{2}$$

$$\mu_{N_{e*}} = \begin{cases} 1, & w_{e*} \cdot e < -1, \\ \frac{1}{2} - \frac{1}{2} w_{e*} \cdot e, & -1 \le w_{e*} \cdot e < 1, \\ 0, & w_{e*} \cdot e \ge 1, \end{cases} \tag{3}$$

$$\mu_{P_{\Delta e*}} = \begin{cases} 0, & w_{\Delta e*} \cdot \Delta e < -1, \\ \frac{1}{2} + \frac{1}{2} w_{\Delta e*} \cdot \Delta e, & -1 \le w_{\Delta e*} \cdot \Delta e < 1, \\ 1, & w_{\Delta e*} \cdot \Delta e \le 1, \end{cases} \tag{4}$$

$$\mu_{N_{\Delta e*}} = \begin{cases} 1, & w_{\Delta e*} \cdot \Delta e < -1, \\ \frac{1}{2} - \frac{1}{2} w_{\Delta e*} \cdot \Delta e, & -1 \le w_{\Delta e*} \cdot \Delta e < 1, \\ 0, & w_{\Delta e*} \cdot \Delta e \ge 1. \end{cases} \tag{5}$$

Consequently, there are only four simple fuzzy control rules used in each FLC (see Table 1). The fuzzy labels of control outputs are singletons defined as $P = 1$, $Z = 0$ and $N = -1$. By using Larsen's product inference method with Zadeh fuzzy logic AND and Lukasiewicz fuzzy logic OR, using the center-of-gravity defuzzification method, and for simplicity choosing $w_{e1} = w_{e2} = w_e$ and $w_{\Delta e1} = w_{\Delta e2} = w_{\Delta e}$, the control output of each FLC can be obtained, in the universe of discourse (Ying, 1993), as

$$\Delta u_1^{(F)} = \frac{w_{\Delta u1}}{4 - 2\max(w_{e1}|e|, w_{\Delta e1}|\Delta e|)}(w_{e1}e + w_{\Delta e1}\Delta e)$$

$$= \frac{w_{\Delta u}}{4 - 2\alpha}(w_e e + w_{\Delta e}\Delta e), \tag{6}$$

$$u_2^{(F)} = \frac{w_{u2}}{4 - 2\max(w_{e2}|e|, w_{\Delta e2}|\Delta e|)}(w_{e2}e + w_{\Delta e2}\Delta e)$$

$$= \frac{w_u}{4 - 2\alpha}(w_e e + w_{\Delta e}\Delta e), \tag{7}$$

where

$$\alpha = \max(w_{e1}|e|, w_{\Delta e1}|\Delta e|) = \max(w_{e2}|e|, w_{\Delta e2}|\Delta e|)$$

$$= \max(w_e|e|, w_{\Delta e}|\Delta e|).$$

The overall fuzzy control output will be

$$u_{PID}^{(F)} = \sum_0^k \Delta u_1^{(F)} + u_2^{(F)}$$

$$= \sum_0^k \frac{w_{\Delta u} w_{\Delta e}}{4 - 2\alpha}\left(\Delta e + \frac{\Delta t}{w_{\Delta e}/w_e \Delta t} e\right)$$

$$+ \frac{w_u w_e}{4 - 2\alpha}\left(e + \frac{w_{\Delta e}\Delta t}{w_e}\frac{\Delta e}{\Delta t}\right). \tag{8}$$

If we choose

$$K_c^{(F)} = \frac{w_{\Delta u} w_{\Delta e}}{4 - 2\alpha},$$

$$T_i^{(F)} = \frac{w_{\Delta e}}{w_e}\Delta t, \tag{9}$$

$$K_c^{(F)} \frac{T_d^{(F)}}{T_i^{(F)}} = \frac{w_u w_e}{4 - 2\alpha},$$

then the fuzzy control output in (8) can be rewritten as

$$u_{PID}^{(F)} = \sum_0^k K_c^{(F)}\left(\Delta e + \frac{\Delta t}{T_i^{(F)}} e\right)$$

$$+ K_c^{(F)} \frac{T_d^{(F)}}{T_i^{(F)}}\left(e + T_i^{(F)} \frac{\Delta e}{\Delta t}\right). \tag{10}$$

Now assume that the time constants of the plant are sufficiently large compared with the sampling interval,

Table 1
Fuzzy control rules (FLC_1 and FLC_2). N: negative; P: positive; Z: zero

PI Part FLC_1	Rule 1	If error is N and change of error is N, change in control action is N
	Rule 2	If error is N and change of error is P, change in control action is Z
	Rule 3	If error is P and change of error is N, change in control action is Z
	Rule 4	If error is P and change of error is P, change in control action is P
PD Part FLC_2	Rule 1	If error is N and change of error is N, control action is N
	Rule 2	If error is N and change of error is P, control action is Z
	Rule 3	If error is P and change of error is N, control action is Z
	Rule 4	If error is P and change of error is P, control action is P

which is common and reasonable in process control, such that

$$\dot{e} \approx \frac{\Delta e}{\Delta t},$$

then the overall control output can be approximated as

$$\begin{aligned} u_{\mathrm{PID}}^{(\mathrm{F})} &\approx \int_0^{k\cdot\Delta t} K_{\mathrm{c}}^{(\mathrm{F})}\left(\mathrm{d}e + \frac{e}{T_{\mathrm{i}}^{(\mathrm{F})}}\,\mathrm{d}t\right) \\ &\quad + K_{\mathrm{c}}^{(\mathrm{F})}\frac{T_{\mathrm{d}}^{(\mathrm{F})}}{T_{\mathrm{i}}^{(\mathrm{F})}}\left(e + T_{\mathrm{i}}^{(\mathrm{F})}\frac{\mathrm{d}e}{\mathrm{d}t}\right) \\ &= \int_0^{k\cdot\Delta t} K_{\mathrm{c}}^{(\mathrm{F})}\dot{e}\,\mathrm{d}t + \int_0^{k\cdot\Delta t}\frac{K_{\mathrm{c}}^{(\mathrm{F})}}{T_{\mathrm{i}}^{(\mathrm{F})}}e\,\mathrm{d}t \\ &\quad + \frac{K_{\mathrm{c}}^{(\mathrm{F})}T_{\mathrm{d}}^{(\mathrm{F})}}{T_{\mathrm{i}}^{(\mathrm{F})}}(e + T_{\mathrm{i}}^{(\mathrm{F})}\dot{e}). \end{aligned} \tag{11}$$

Note that the linear PID controller in series form is

$$G_{\mathrm{c}}(s) = \frac{K_{\mathrm{c}}}{T_{\mathrm{i}}s}(1 + sT_{\mathrm{i}})(1 + sT_{\mathrm{d}})$$

or

$$u = \int_0^t K_{\mathrm{c}}\dot{e}\,\mathrm{d}t + \int_0^t \frac{K_{\mathrm{c}}}{T_{\mathrm{i}}}e\,\mathrm{d}t + \frac{K_{\mathrm{c}}T_{\mathrm{d}}}{T_{\mathrm{i}}}(e + T_{\mathrm{i}}\dot{e}),$$

in time domain. Comparing (11) with the above formula we can conclude that the fuzzy PID controller (8) is a nonlinear PID controller with variable proportional gain.

Remark 1. By adopting the simplest FLC in Fig. 1, its nonlinear structure and the inherent relationships between its components and their functioning can be made transparent to the designer. With the formulas (6), (7) and (11), the FLC is in essence a *nonlinear* PID-type controller because its structure is analogous to that of a common linear PID controller. Moreover, the equivalent proportional control gain $K_{\mathrm{c}}^{(\mathrm{F})}$, integral time $T_{\mathrm{i}}^{(\mathrm{F})}$ and derivative time $T_{\mathrm{d}}^{(\mathrm{F})}$ are composed of FLC parameters $w_e, w_{\Delta e}, w_u$, and $w_{\Delta u}$ as shown in Eq. (9). This greatly facilitates the property analysis and setting of control parameters, as will be shown subsequently.

Remark 2. It is worthwhile pointing out that the fuzzy PID control system has six control parameters free for design, whereas the conventional PID only has three. In this paper we choose $w_{e1} = w_{e2} = w_e$ and $w_{\Delta e1} = w_{\Delta e2} = w_{\Delta e}$ to reduce the undetermined fuzzy PID control parameters. The purpose is to facilitate the application of conventional PID tuning algorithms to the fuzzy PID controller. It is clear that if all the control parameters are used, we actually have more degrees of freedom in designing fuzzy PID to achieve multiple control targets such as robustness or adaptation. However, in this paper we will not pursue any further discussions in this direction.

Remark 3. Note here that the series form of conventional PID controller is used in the comparison. This structure is also implemented in many commercial controllers and can be easily transformed to the parallel structure (Åström & Hägglund, 1995). Although this kind of PID controllers cannot introduce complex zeros, it is sufficient for the purpose of process control.

2.2. Tuning of the fuzzy PID controller

Suppose that a process can be modeled by a second-order plus dead-time structure which has the transfer function of

$$G(s) = \frac{K_p \mathrm{e}^{-sL}}{(1 + s\tau_1)(1 + s\tau_2)}, \quad \tau_1 \geq \tau_2 \tag{12}$$

and a pair of gain margin and phase margin $(A_{\mathrm{m}}, \Phi_{\mathrm{m}})$ is given as the closed-loop performance specification. The tuning formulae of a conventional PID controller can be obtained as (Ho et al., 1994)

$$\omega_p = \frac{A_m\Phi_m + \frac{1}{2}\pi A_m(A_m - 1)}{(A_m^2 - 1)L},$$

$$K_c = \frac{\omega_p\tau_1}{A_m K_p}, \tag{13}$$

$$T_i = \frac{1}{2\omega_p - 4\omega_p^2 L/\pi + 1/\tau_1},$$

$$T_d = \tau_2,$$

where ω_p is the resultant phase crossover frequency.

Remark 4. In many practical control problems, ranging from the level control in chemical industry to the servo tracking control in disk drive industry, it is sufficient and sometimes also necessary to use a simple model such as (12) to design a controller. It is sufficient because a PI/PID controller based on the linearized model can work well even though the original process has high nonlinearities and uncertainties (Åström & Hägglund, 1995; Seborg, Edgar & Mellichamp, 1989). It is also necessary because other advanced control methods may either be too complicated to be implemented, or require too many measurements which are not available. For instance, consider the level control of a coupled-tank system, the complete model takes a nonlinear cascaded form. Since the liquid level of the 1st tank is usually not available, many advanced control methods such as backstepping-approach-based robust adaptive control methods, which are dedicated to this kind of nonlinear systems, are not applicable. However PID, despite its simplicity, gives satisfactory control performance.

Comparing (13) with (9), let

$$K_c^{(F)} = K_c, \qquad T_i^{(F)} = T_i, \qquad T_d^{(F)} = T_d.$$

It is easy to derive

$$w_{\Delta e} = \frac{w_e}{(2\omega_p - 4\omega_p^2 L/\pi + 1/\tau_1)\Delta t}$$

$$w_{\Delta u} = \frac{\omega_p\tau_1(2\omega_p - 4\omega_p^2 L/\pi + 1/\tau_1)}{A_m K_p w_e}(4 - 2\alpha)\Delta t \tag{14}$$

$$w_u = \frac{\omega_p\tau_1\tau_2(2\omega_p - 4\omega_p^2 L/\pi + 1/\tau_1)}{A_m K_p w_e}(4 - 2\alpha).$$

Thus, we have three independent equations with four undetermined control parameters. Usually, the system output has a working range which is highly related to the changing range of set point. If the working range is large, w_e should be relatively small and vice versa. Such a suitable w_e ensures the normalized error fitted into the interval of $[-1,1]$. This is reasonable because the quantization values of the fuzzy linguistic variables e or Δe are dependent on what range the system is working in. Thus the normalizing factor of error should be proportional to the reciprocal of the working range, or specifically in our study, the set-point changing range, i.e.

$$w_e = \frac{\chi}{r_0}. \tag{15}$$

Based on extensive numerical studies, we choose $\chi = 0.2$ to make possible a compromise among rise time, overshoot and settling time, where r_0 is the set-point change. For a fuzzy PI controller, this w_e can be used to approximately minimize the ITAE to set-point response (Xu, Liu & Hang, 1996). With Eqs. (14) and (15), the coefficients of the fuzzy PID controller (8) can be uniquely determined with respect to any plant in the form of (12).

In the tuning algorithm, α can be interpreted as an equivalent gain/phase contour in the sense of the gain and phase margins. When determining the fuzzy PID parameters in terms of gain and phase margins, we need to assign a fixed value to the quantity α. Let $\alpha = \alpha_0$ where $\alpha_0 \in [0,1]$. α_0 actually specifies a particular contour on the normalized $e/\Delta e$ plane such that, on this contour the gain and phase margins (which are measures of the local stability of the closed-loop system) satisfy the specifications (Fig. 3).

According to the tuning formulae (14) and (15), when a particular α_0 is selected, the weighting factors $w_e, w_{\Delta e} w_{\Delta u}$ and w_u will be fixed. This we have

$$T_i^{(F)}(e, \Delta e) = \frac{w_{\Delta e}\Delta t}{w_e},$$

$$T_d^{(F)}(e, \Delta e) = \frac{w_u}{w_{\Delta u}}\Delta t,$$

$$\begin{aligned} K_c^{(F)}(e, \Delta e) &= \frac{w_{\Delta u} w_{\Delta e}}{4 - 2\alpha} \\ &= \frac{4 - 2\alpha_0}{4 - 2\max(w_e|e|, w_{\Delta e}|\Delta e|)} \frac{1}{4 - 2\alpha_0} w_{\Delta u} w_{\Delta e} \\ &= \gamma \frac{1}{4 - 2\alpha_0} w_{\Delta u} w_{\Delta e} \\ &= \gamma K_{c\alpha_0}^{(F)}, \end{aligned} \tag{16}$$

where $K_{c\alpha_0}$ is the gain of FLC when the system is at its α_0 contour. Clearly, the fuzzy PID controller has the property that its $T_i^{(F)}$ and $T_d^{(F)}$ are fixed and $K_c^{(F)}$ is variable in terms of different e and Δe. Because we will have

$$\gamma(e, \Delta e) = \frac{4 - 2\alpha_0}{4 - 2\max(w_e|e|, w_{\Delta e}|\Delta e|)}, \tag{17}$$

which means $\gamma < 1$, $\gamma = 1$ or $\gamma > 1$ when normalized system states ($w_e e$ and $w_{\Delta e}\Delta e$) are inside, on, or outside the α_0 contour, respectively. From the second equation of (13), replacing K_c by $K_c^{(F)}$, we obtain the closed-loop gain

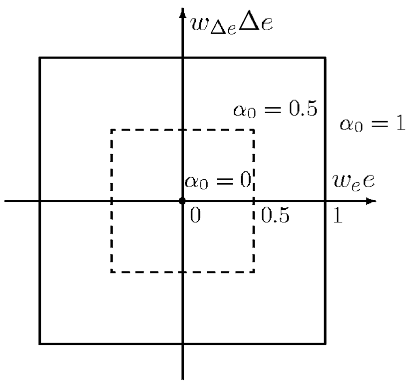

Fig. 3. Equivalent gain/phase margin contours of different α_0.

margin for the fuzzy PID as

$$A_{\mathrm{m}}^{(\mathrm{F})} = \frac{A_{\mathrm{m}\alpha_0}}{\gamma}, \tag{18}$$

where

$$A_{\mathrm{m}\alpha_0} = \frac{\omega_{\mathrm{p}}\tau_1}{K_{\mathrm{c}\alpha_0}^{(\mathrm{F})} K_{\mathrm{p}}}.$$

Eq. (18) shows that the gain margin is always reciprocal to γ. Therefore, from (17) we can conclude that no matter what α_0 is used, the further away from outside the equivalent gain/phase margin contour, the larger is the γ and consequently, the smaller is the gain margin. Similarly, the closer to the steady state, the smaller is the γ and then the larger is the gain margin. In short, $A_{\mathrm{m}}^{(\mathrm{F})}$ increases when $|e|$ or $|\Delta e|$ decreases, which means that the loop gain $K_{\mathrm{c}}^{(\mathrm{F})} K_{\mathrm{p}}$ decreases and the system 'safety factor' increases. Note that this is a desirable property, as this ensures that the system is less sensitive to measurement noise near the steady state and is quick in response when off the steady state.

The property of α_0 can be used to allocate the equivalent gain/phase margin contour. For example, if $\alpha_0 = 0$ the system has the same local stability property as the one controlled by the conventional PID controller around the steady state, and the equivalent gain/phase margin contour shrinks to a single point located at the center of the $e/\Delta e$ plane. In this situation, the controller gain will reach its minimum only when the system is at its steady state. In this study, we set $\alpha_0 = 0$ to ensure that the fuzzy PID controller has the same local stability as the conventional PID controller around the steady state and higher gain property of the steady state.

3. Stability and performance analysis

In the previous section, the properties of gain scheduling and local stability have been discussed in the sense of gain and phase margins. Since the concepts of gain and phase margins are essential for linear control systems, the above discussions are qualitative and approximate ones. In order to explore the quantitative relationship between the fuzzy and conventional PID controllers and evaluate the global stability, we need more strict and more general analysis methods which can be applied to both nonlinear processes and nonlinear controllers. The small-gain theorem is an appropriate tool for this purpose. It should be noted that the new fuzzy PID is tuned based on a simple model of second order with dead time. This implies that less information is available in the stage of controller design. Hence, the stability analysis is imperative, especially when the controlled process is of general nonlinear uncertain classes such as BIBO types.

3.1. BIBO stability condition of the fuzzy PID control system

In this subsection, we will analyze the bounded-input/bounded-output (BIBO) stability of the fuzzy PID control system. By using the small-gain theorem, we will find the generalized sufficient BIBO stability condition of the proposed fuzzy PID control system. Consider a general case where the process under control, which is denoted by $g(\bullet)$, is nonlinear and the reference input is $r(k)$. By using the control law (10), we have

$$\begin{aligned} u_{\mathrm{PID}}^{(\mathrm{F})}(k) &= \Delta u_1^{(\mathrm{F})}(k) + u_2^{(\mathrm{F})}(k) + \sum_{i=0}^{k-1} \Delta u_1^{(\mathrm{F})}(i) \\ &= \Delta u_{\mathrm{PID}}^{(\mathrm{F})}(k) + u_{\mathrm{PID}}^{(\mathrm{F})}(k-1), \end{aligned} \tag{19}$$

where

$$\Delta u_{\mathrm{PID}}^{(\mathrm{F})}(k) = \Delta u_1^{(\mathrm{F})}(k) + u_2^{(\mathrm{F})}(k) - u_2^{(\mathrm{F})}(k-1). \tag{20}$$

By denoting

$$\begin{aligned} &\Delta u_{\mathrm{PID}}^{(\mathrm{F})}(k) = f(e(k)), \\ &y(k) = g(\Delta u_{\mathrm{PID}}^{(\mathrm{F})}(k)), \end{aligned} \tag{21}$$

it is easy to obtain an equivalent closed-loop control system as shown in Fig. 4.

Theorem. *A sufficient condition for the nonlinear fuzzy PID control system to be BIBO stable is that the given nonlinear process has a bounded norm (gain) as $\|g\| < \infty$ and the parameters of the fuzzy* PID *controller, $w_e, w_{\Delta e}, w_u$ and $w_{\Delta u}$, (or $K_{\mathrm{c}}^{(\mathrm{F})}(k), T_{\mathrm{i}}^{(\mathrm{F})}$ and $T_{\mathrm{d}}^{(\mathrm{F})}$ in* (16)), *satisfy*

$$K_{\mathrm{c}}^{(\mathrm{F})}(k)\left(1 + \frac{T_{\mathrm{d}}^{(\mathrm{F})}}{T_{\mathrm{i}}^{(\mathrm{F})}} + \frac{\Delta t}{T_{\mathrm{i}}^{(\mathrm{F})}} + \frac{T_{\mathrm{d}}^{(\mathrm{F})}}{\Delta t}\right) \cdot \|g\| < 1, \tag{22}$$

where $\|g\|$ is the operator norm of the given $g(\bullet)$, or the gain of the given nonlinear system, usually defined as (*Desoer & Vidyasagar*, 1975):

$$\|g\| = \sup_{v_1 \neq v_2, k \geq 0} \frac{|g(v_1(k)) - g(v_2(k))|}{|v_1(k) - v_2(k)|}. \tag{23}$$

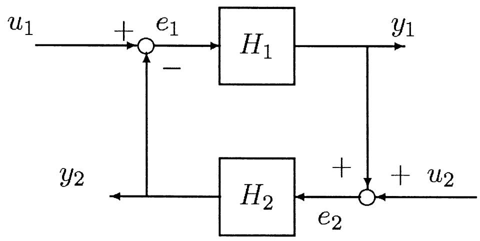

Fig. 4. An equivalent closed-loop fuzzy PID control system.

Proof. The fuzzy PID controller can be considered as a self-tuning adaptive nonlinear PID controller since the gains of this kind of controllers vary in terms of the combination of the inputs $(e, \Delta e)$ of the fuzzy controller. The combination of $(e, \Delta e)$ can be divided into nine regions as shown in Fig. 5. When the inputs of $(e(k), \Delta e(k))$ are in region I, the control law is given by (20), therefore from (6), (7) and (9)

$$\begin{aligned}\Delta u^{(F)}_{PID}(k) &= \Delta u_1^{(F)}(k) + u_2^{(F)}(k) - u_2^{(F)}(k-1)\\ &= K_c^{(F)}(k)\left[\Delta e + \frac{\Delta t}{T_i^{(F)}} e(k)\right]\\ &\quad + K_c^{(F)}(k)\,\frac{T_d^{(F)}}{T_i^{(F)}}\left[e(k) + \frac{T_i^{(F)}}{\Delta t}\Delta e(k)\right]\\ &\quad - K_c^{(F)}(k-1)\,\frac{T_d^{(F)}}{T_i^{(F)}}\left[e(k-1) + \frac{T_i^{(F)}}{\Delta t}\Delta e(k-1)\right]\\ &= K_c^{(F)}(k)\left(1 + \frac{T_d^{(F)}}{T_i^{(F)}} + \frac{\Delta t}{T_i^{(F)}} + \frac{T_d^{(F)}}{\Delta t}\right)e(k)\\ &\quad - K_c^{(F)}(k)\left(1 + \frac{T_d^{(F)}}{\Delta t}\right)e(k-1)\\ &\quad - K_c^{(F)}(k-1)\left(\frac{T_d^{(F)}}{T_i^{(F)}} + \frac{T_d^{(F)}}{\Delta t}\right)e(k-1)\\ &\quad + K_c^{(F)}(k-1)\,\frac{T_d^{(F)}}{\Delta t} e(k-2) \end{aligned} \tag{24}$$

and thus

$$\begin{aligned}\|f(e(k))\| &\le K_c^{(F)}(k)\left|1 + \frac{T_d^{(F)}}{T_i^{(F)}} + \frac{\Delta t}{T_i^{(F)}} + \frac{T_d^{(F)}}{\Delta t}\right|\\ &\qquad \cdot |e(k)| + v_1 e_{\max},\end{aligned} \tag{25}$$

where

$$\begin{aligned} v_1 &= \max(K_c^{(F)})\left(1 + \frac{T_d^{(F)}}{T_i^{(F)}} + 3\frac{T_d^{(F)}}{\Delta t}\right)\\ &\le 2K_{c\alpha_0}^{(F)}\left(1 + \frac{T_d^{(F)}}{T_i^{(F)}} + 3\,\frac{T_d^{(F)}}{\Delta t}\right)\\ e_{\max} &= \max(|e(0)|, |e(1)|, \dots, |e(k-1)|). \end{aligned} \tag{26}$$

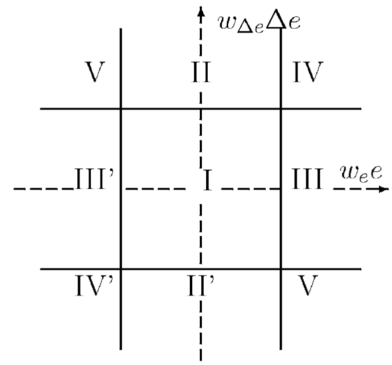

Fig. 5. Different regions of fuzzy PID controller's inputs combinations.

On the other hand,

$$\|g(u^{(F)}_{PID}(k))\| \le \|g\| \cdot |u^{(F)}_{PID}(k)|. \tag{27}$$

Applying the small-gain theorem, we can obtain the sufficient condition for the BIBO stability given by the theorem

$$K_c^{(F)}\left|1 + \frac{T_d^{(F)}}{T_i^{(F)}} + \frac{\Delta t}{T_i^{(F)}} + \frac{T_d^{(F)}}{\Delta \mathrm{t}}\right| \cdot \|g\| < 1.$$

Similarly, when the system state $(e(k), \Delta e(k))$ is in the regions of II and II′, in which the term $w_{\Delta e}\Delta e$ is outside the interval of $[-1,1]$ and becomes a constant due to the saturation. Hence we can obtain the sufficient BIBO stability condition as

$$K_c^{(F)}\left|\frac{T_d^{(F)}}{T_i^{(F)}} + \frac{\Delta t}{T_i^{(F)}}\right| \cdot \|g\| < 1.$$

When $(e(k), \Delta e(k))$ is in the regions of III and III′, in which $w_e e$ is outside the interval of $[-1,1]$, the sufficient BIBO stability condition is found to be

$$K_c^{(F)}\left|1 + \frac{T_d^{(F)}}{\Delta t}\right| \cdot \|g\| < 1.$$

Finally, when $(e(k),\Delta e(k))$ is in the regions of IV, IV′, V and V′, since the control effort is constant, the sufficient BIBO stability condition is that $\|g\|$ is bounded.

By combining all the above conditions together, and noting that $K_c^{(F)} > 0$, $T_i^{(F)} > 0$, $T_d^{(F)} > 0$ and $\Delta t > 0$, the result for the stability of the fuzzy PID control system will be:

$$K_c^{(F)}\left(1 + \frac{T_d^{(F)}}{T_i^{(F)}} + \frac{\Delta t}{T_i^{(F)}} + \frac{T_d^{(F)}}{\Delta t}\right) \cdot \|g\| < 1. \qquad \square$$

Note that in (22), if we eliminate the superscript (F), we will arrive at the sufficient BIBO stability condition for a linear PID-controlled closed-loop system (see Appendix A). Moreover, if the same gain and phase margin specifications are used, the tuning formulae (14) and (15) result in a fuzzy PID controller that has the same gains on its α_0 contour as a conventional PID controller. In other words, the proposed fuzzy PID controller will retain at least the same stability property as its conventional counterpart on and inside its α_0 contour. This result is summarized as follows.

Corollary. *For a nonlinear process controlled stably by a conventional* PID *controller with gain* K_c, *integral time constant* T_i *and derivative time constant* T_d, *if the* PID *controller is replaced by the proposed fuzzy* PID *controller whose control parameters* $w_{\Delta e}$, $w_{\Delta u}$ *and* w_u *are given by the tuning formula* (14) *with* $\alpha = \alpha_0 \in [0,1]$, *the resulting fuzzy* PID *control system will ensure at least the same* (*local*) *stability on and within the* α_0 *contour.*

Remark 5. The significance of the above conclusion is that, one can always replace a conventional PID controller by the proposed nonlinear fuzzy PID controller without losing the stability margin around the equilibrium. In particular, if we take $\alpha_0 = 0$, then in steady state $e(k) = \Delta e(k) = 0$, we have $K_c^{(F)} = K_c$, the conventional PID and the fuzzy PID control systems have exactly the same stability.

Remark 6. From equations (14), (16) and (22), it is easy to derive that the stability condition (22) is independent of the error weighting factor w_e. Therefore, we can use this extra degree of freedom in the design to improve system responses, as discussed in the previous section, while at the same time maintaining the system stability.

3.2. Control efforts between fuzzy and conventional PID controllers

In the previous section we have shown that, by choosing $\alpha_0 = 0$, the fuzzy PID control gain is higher than that of the conventional PID except for the equilibrium in which both are the same. This property ensures that the load disturbance rejection of the fuzzy PID will be at least as good as the conventional one. Here we will explore another novel property of the new fuzzy PID controller: in set-point control the proposed fuzzy PID control system will generate lower control signal profiles compared with the conventional PID controllers while maintaining the higher control gain. This property will effectively reduce the overshoot phenomenon in set-point control. Note that in the PI or PID control, the initial value of the control signal plays an important role because of the integral action. By limiting the initial control effort at low level, the overall control profile will be kept lower. In the remainder of the subsection we will show that the proposed fuzzy PID control system does generate a lower initial control signal compared to the conventional PID control system.

To eliminate the derivative kick in the implementation of PID control a modified derivative term is used as follows (Åström & Hägglund, 1995):

$$D = -K_c T_d \frac{dy}{dt}.$$

Similarly, the input Δe to the fuzzy PID controller is replaced by $-\Delta y$. By transforming the series form of conventional PID controller to the parallel form, and subtracting $u(k-1)$ from $u(k)$ we can obtain

$$\Delta u_{PID}(k) = K_c \frac{T_i + T_d}{T_i}[e(k) - e(k-1)] + \frac{K_c}{T_i} e(k-1)\Delta t - K_c T_d \frac{y(k) - 2y(k-1) + y(k-2)}{\Delta t}. \quad (28)$$

Similarly, for the fuzzy PID controller we have

$$\Delta u_{PID}^{(F)}(k) = \Delta u_1(k) + u_2(k) - u_2(k-1). \quad (29)$$

From (24) we have

$$\Delta u_{PID}^{(F)}(k) = K_c^{(F)}(k)\left[y(k-1) - y(k) + \frac{T_d^{(F)}}{T_i^{(F)}}(e(k) - e(k-1)) \right] + \frac{K_c^{(F)}(k)}{T_i^{(F)}} e(k)\Delta t - T_d^{(F)} \frac{K_c^{(F)}(k)[y(k) - y(k-1)] - K_c^{(F)}(k-1)[y(k-1) - y(k-2)]}{\Delta t}. \quad (30)$$

When a set-point change occurs, we have $k = 0$, $e(0) = r$, $e(-1) = 0$ and $y(0) = y(-1) = y(-2)$. Therefore from (28) and (30), we have

$$\delta u(0) = \Delta u_{PID}^{(F)}(0) - \Delta u_{PID}(0) = \left(K_c^{(F)}(0)\, \frac{T_d^{(F)} + \Delta t}{T_i^{(F)}} - K_c \frac{T_i + T_d}{T_i} \right) r = \left(K_c^{(F)}(0)\, \frac{T_d + \Delta t}{T_i} - K_c \frac{T_i + T_d}{T_i} \right) r. \quad (31)$$

From tuning formula (15) and choosing $\alpha_0 = 0$, we have

$$K_c^{(F)}(0) = \frac{4 - 2\alpha_0}{4 - 2\max(we|e(0)|, w_{\Delta e}|\Delta e(0)|)} K_c = \frac{10}{9} K_c \quad (32)$$

and thus

$$\delta u(0) = \frac{K_c}{9T_i}(T_d + 10\Delta t - 9T_i)r. \quad (33)$$

From (13) we can obtain, when the specified gain and phase margin pair is (3, 45°), Δt is sufficiently small and $L/\tau_1 \geq 0.023$, that: $\delta u(0) < 0$ (noting that $T_d = \tau_2 \leq \tau_1$). Moreover, if $L/\tau_1 \geq 0.1$, we will have: $T_i \in (0.35\tau_1, \tau_1)$, and we will arrive at the result that

$$\delta u(0) \approx -K_c r < 0. \tag{34}$$

Consequently,

$$\Delta u_{PID}^{(F)}(0) < \Delta u_{PID}(0), \tag{35}$$

namely, the initial control effort of fuzzy controller is smaller than that of the conventional PID controller. Moreover, if the original Δe is used in derivative control action of both controllers, the fuzzy PID controller will still give a smaller initial control effort. This is because the saturation of the controller output occurs when there is a set-point change, which leads to the normalized change of error being outside the universe of discourse. This smaller initial control effort of fuzzy controller prevents it from injecting large amounts of energy, which may cause large overshoot, to the system. In the experimental example we will further demonstrate the low control profile property.

4. Experimental evaluation

To compare the fuzzy and conventional PID controllers, an experiment is carried out in which a nonlinear plant is used. The plant employed in the experiment is a coupled-tank system and the purpose of the experiment is to control the fluid level in the second tank (Fig. 6). In process industries, the control of fluid levels in storage tanks is a common and important control problem. By controlling the fluid level in the tank, material balance can be achieved so that inflow and outflow are equal in the steady state.

The basic experimental system consists of two hold-up tanks coupled by an orifice. The input $q(t)$ is supplied by the variable speed pump, which pumps water to the first tank. The orifice between two tanks allows the water to flow into the second tank and then out as an outflow. The basic control problem is to control the water level in the second tank $H_2(t)$ by varying the speed of the pump. The measurement voltage for water level is read in and the control signal for the pump is written out by a computer through A/D and D/A interfaces. The control algorithm is realized by computer programming.

Since the two tanks are coupled by an orifice, the plant is a second-order system. Moreover, the outflow rate is determined by the square root of the fluid level in tank 2 ($H_2(t)$), thus the system is essentially nonlinear. For the design of conventional and fuzzy PID controllers using gain and phase margin specifications, the plant must be linearized, simplified and modeled by a second-order plus dead-time structure. After performing a relay feedback experiment, the ultimate gain and period are obtained as 21.21 and 100, respectively. By conducting another set-point changing experiment, the plant gain can also be obtained as 0.75. Thus a simplified plant model will be

$$G(s) = \frac{0.75e^{-8.067s}}{(1 + 61.45s)^2}$$

and the parameters of the conventional and fuzzy PID controllers are determined based on this model and formulas (13)–(15).

The experiments are carried out during a time period from 0 to 4000 s. The sampling period is 1 s. First, the system is settled at 10 cm. There are set-point changes from 10 to 16 cm, 16 to 11 cm and 11 to 13 cm at time instants of 100, 1100 and 2000, respectively. Moreover, there is a load disturbance at time instant 2800, which is

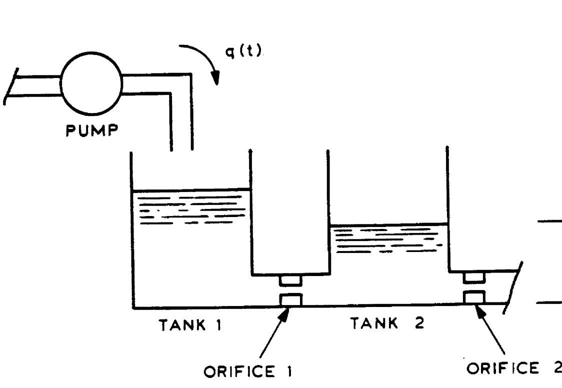

Fig. 6. Coupled tanks system.

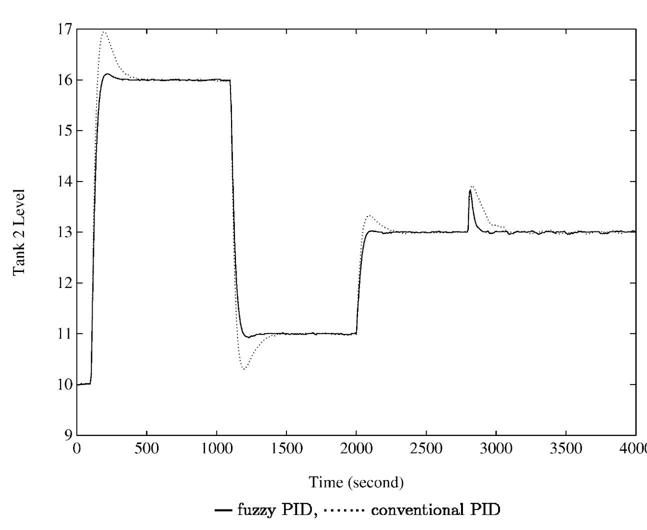

Fig. 7. Tank 2 water level.

a 20 cm^3/s flow introduced by a pump to tank 1 to emulate the change in inflow. The specified gain and phase margins are 3 and 45°, respectively. The size of the orifice is set to 0.396 cm^2. The sampling interval is 1 s and the experiment results are shown in Fig. 7. In the figure, the solid and dotted lines are the water levels of tank 2 controlled by the fuzzy and conventional (Ho et al., 1994) PID controllers, respectively.

After the water level is settled at 13 cm (time instant 2500), the parameters of fuzzy PID controller are re-calculated by increasing χ in (15) from 0.2 to 0.5. From the experiment data, we can find that the closed-loop system performance of fuzzy PID controller is evidently better than that of conventional PID controller. The fuzzy PID controller gives only slight overshoot to set-point changes with almost same response speed as its conventional counterpart. On the other hand, the conventional PID controller gives large overshoots. The magnitude of the system response to load disturbance is relatively smaller and the convergence is faster when using fuzzy PID controller. Experiment results confirm again the advantage of the proposed fuzzy PID controller.

5. Conclusion

In this paper, a new structure of fuzzy PID controller is presented. The parallel combination of fuzzy PI and PD controllers shows its simplicity in determining the control rules and controller parameters. A tuning formula based on gain and phase margins is introduced by which the weighting factors of a fuzzy PID controller can be selected with respect to the second-order plus dead-time plants. The validity of the proposed fuzzy PID controller and gain and phase-margin-based tuning formula is confirmed through theoretical analysis and experiment. Both theoretical and experimental results show that the fuzzy PID controller has the nonlinear properties of (1) higher control gains when the system is away from its steady states; and (2) lower control profile when set-point changes occur. As a result, these nonlinear properties provide the fuzzy PID control system with a superior performance over the conventional PID control system.

Appendix A

A.1. Derivation of BIBO stability condition of linear PID control system

Suppose a nonlinear process N is controlled by a linear PID controller. In the computerized implementation, the PID controller is discretized by using a zero-order holder where s in the transfer function of PID controller is substituted by $(1 - z^{-1})/\Delta t$ and for a linear PID controller in the series form, we have

$$\frac{u(k) - u(k-1)}{\Delta t} = \frac{K_c}{T_i}\left(1 + \frac{T_i}{\Delta t} + \frac{T_d}{\Delta t} + \frac{T_i T_d}{\Delta t^2}\right)e(k) - \frac{K_c}{T_i}\left(\frac{T_i}{\Delta t} + \frac{T_d}{\Delta t} + \frac{2T_i T_d}{\Delta t^2}\right)e(k-1) + \frac{K_c T_d}{\Delta t^2}e(k-2).$$

Referring to Fig. 4, we define

$$\begin{aligned} e_1(k) &= e(k) = r(k) - y(k),\\ e_2(k) &= u(k),\\ u_1(k) &= r(k),\\ u_2(k) &= u(k-1),\\ H_1(e_1(k)) &= \Delta u(k) = u(k) - u(k-1),\\ H_2(e_2(k)) &= N(e_2(k)) = y(k). \end{aligned} \tag{36}$$

Applying the small-gain theorem, we can obtain the following sufficient condition for the BIBO stability for the linear PID controlled system, as

$$K_c\left(1 + \frac{T_d}{T_i} + \frac{\Delta t}{T_i} + \frac{T_d}{\Delta t}\right)\|H_2\| < 1.$$

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Dr. Jian-Xin Xu (J.-X. Xu) received his Bachelor degree from Zhejiang University, China in 1982. He attended the University of Tokyo, Japan, where he received his Master's and Ph.D. degrees in 1986 and 1989, respectively. All his degrees are in Electrical Engineering.

He worked for one year in the Hitachi Research Laboratory, Japan and for more than one year in Ohio State University, U.S.A. as a Visiting Scholar.

In 1991 he joined the National University of Singapore, and is currently an associate professor in the Department of Electrical Engineering. His research interests lie in the fields of learning control, variable structure control, fuzzy logic control, discontinuous signal processing, and applications to motion control and process control problems.

He is a senior member of IEEE.

Prof. Hang Chang Chiteh (C.C. Hang) graduated with a First Class Honours Degree in Electrical Engineering from the University of Singapore in 1970. He received the Ph.D. degree in control engineering from the University of Warwick, England, in 1973. From 1974 to 1977, he worked as a Computer and Systems Technologist in the Shell Eastern Petroleum Company (Singapore) and the Shell International Petroleum Company (The Netherlands). Since 1977, he has been with the National University of Singapore, serving in various positions including being the Vice-Dean of the Faculty of Engineering and Head of the Department of Electrical Engineering. Since October 1994, he has been appointed Deputy Vice-Chancellor. His major areas of research are adaptive and intelligent control in which he has published one book, 190 international journal and conference papers and 4 patents. He was a Visiting Scientist in Yale University in 1983 and in Lund University in 1987 and 1992. From 1992 to 1999, he served as Principal Editor (Adaptive Control) of the Automatica Journal. He was elected a Fellow of IEEE in 1998 and a Fellow of Third World Academy of Sciences in 1999.

Dr. Chen Liu received his Bachelor and Master degrees from Tsinghua University, and Ph.D. degree from National University of Singapore. All his degrees are in Electrical Engineering. His research interests are fuzzy logic control and system tuning.