

# Multiobjective Optimal Location of FACTS Shunt-Series Controllers for Power System Operation Planning

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**Abstract**—This paper develops appropriate models of flexible ac transmission systems (FACTS) shunt-series controllers for multiobjective optimization and also presents a multiobjective optimization methodology to find the optimal location of FACTS shunt-series controllers. The objective functions are the total fuel cost, power losses, and system loadability with and without minimum cost of FACTS installation. The  $\epsilon$ -constraint approach is implemented for the multiobjective mathematical programming (MMP) formulation, including the FACTS shunt-series controllers (i.e., phase-shifting transformer (PST), hybrid flow controller (HFC), and unified power-flow controller (UPFC)). Simulation results are presented for the IEEE 14-bus system. The optimization method is numerically solved using Matlab and general algebraic modeling system (GAMS) software environments. The solution procedure uses nonlinear programming (NLP) and mixed-integer nonlinear programming (MINLP) to solve the optimal location and setting of FACTS incorporated in the optimal power-flow problem considering these objective functions and improving the power system operation. Furthermore, the results demonstrate that the HFC is outperformed by PST and UPFC from the analytical and technical point of views.

**Index Terms**—Flexible ac transmission systems (FACTS), hybrid flow controller (HFC), multiobjective, optimal location, optimal power flow (OPF), phase-shifting transformer (PST), unified power-flow controller (UPFC).

## I. INTRODUCTION

THE MAIN objective of the planning and operation of an electric power system is to satisfy the system load and energy requirement as economically as possible. Therefore, the system planner has to consider a variety of options, including flexible ac transmission systems (FACTS) controllers, and they have to make decisions not only based on technical and cost considerations, but also on return of investment.

FACTS controllers are characterized by their ability to have control algorithms structured to achieve multiple objectives in

a transmission system [1]. They provide an opportunity for enhanced value of transmission in terms of loading capability, reliability, availability, and flexibility of ac transmission.

The objectives of this paper are 1) to provide appropriate models of FACTS shunt-series controllers for multiobjective optimization and 2) to present a multiobjective optimization methodology to find the optimal location of FACTS shunt-series controllers to optimize the total fuel cost, power losses, and system loadability with and without the minimum cost of FACTS installation as objective functions. The shunt-series controllers considered in this paper are the phase-shifting transformer (PST), unified power-flow controller (UPFC), and hybrid flow controller (HFC).

PST is used to control the power through certain paths in a transmission system. The ability of PST to control power flow in a power system has long been recognized [2]–[4], and its models and operational characteristics are well established [5]–[9]. The UPFC, with its concept proposed by Gyugyi in 1991 [10], is one of the most brilliant of FACTS controllers which is capable of providing active and reactive power control independently [5], [6], [10]–[12]. The HFC is a new member of FACTS controllers introduced in [13] and [14], and its modified model has been discussed in [15]. The HFC, which is also called Dynaflow, and developed by ABB [16], [17] includes a conventional PST, a mechanically switched shunt capacitor (MSC), a multimodule thyristor-switched series capacitor (TSSC) and a multimodule thyristor-switched series reactor (TSSR). Since HFC is a mixture of series and shunt compensation, its operation is similar to that of a UPFC and PST [13]. It is expected that in the near future, the application of this device will be favored.

Since the installation of FACTS controllers in a power system is an investment issue, it is necessary for any new installation of FACTS to be very well planned. Among the different assessment tools used for this purpose, optimal power flow (OPF) seems to be the most suitable [18]. The OPF problem aims to achieve an optimal solution of a specific power system objective function by adjusting the power system control variables, while satisfying a set of operational and physical constraints [19]. The optimal location and OPF of FACTS controllers can be formulated as a multiobjective optimization problem. It gives economic and technical benefits and so helps in making a decision for the optimal investment.

Recently, several multiobjective methods as shown have been discussed for solving the multiobjective optimal location of FACTS controllers. The multiobjective evolutionary algorithm (EA) has been applied to optimally locate the UPFC

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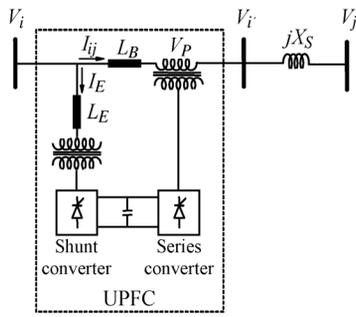


Fig. 3. Basic schematic diagram of UPFC.

Here,  $r$  is the radius of the UPFC operating region and  $\gamma$  is the UPFC phase angle [12].

### III. PROBLEM FORMULATION

Optimal power system operation seeks to optimize the steady-state performance of a power system in terms of one or more objective functions while satisfying several equality and inequality constraints. Generally, the problem can be formulated as follows [18], [20]–[29].

#### A. Objective Functions

In this paper, four objective functions are considered which are the total fuel cost, the power loss, the system loadability, and the cost of FACTS installation. These objective functions are formulated as follows.

- 1) The total fuel cost: The first objective function is to minimize the total fuel cost that can be expressed as

$$F_1 = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (\$/h) \quad (13)$$

where  $P_{Gi}$  is the active power output of the  $i$ th generator,  $NG$  is the total number of dispatched generators, and  $a_i, b_i, c_i$  are the fuel cost coefficients of the  $i$ th generator [29].

- 2) The real power losses: The second objective function is to minimize the real power losses in transmission lines that can be expressed as

$$F_2 = P_{Loss}(x, u) = \sum_{l=1}^{Nl} P_l \quad (14)$$

where  $u$  is a set of control variables,  $x$  is a set of dependent variables,  $P_l$  is the real power losses at line- $l$ , and  $Nl$  is the number of transmission lines.

- 3) The system loadability: The third objective function is to maximize the system loadability that can be described as [22] and [30]

$$F_3 = \rho(x, u) \quad (15)$$

and  $\rho$  can be obtained by assuming the constant power factor at each load in the real and reactive power balance equations as follows:

$$P_{Gi} - \rho P_{Di} = f_{Pi}(x, u) \quad (16)$$

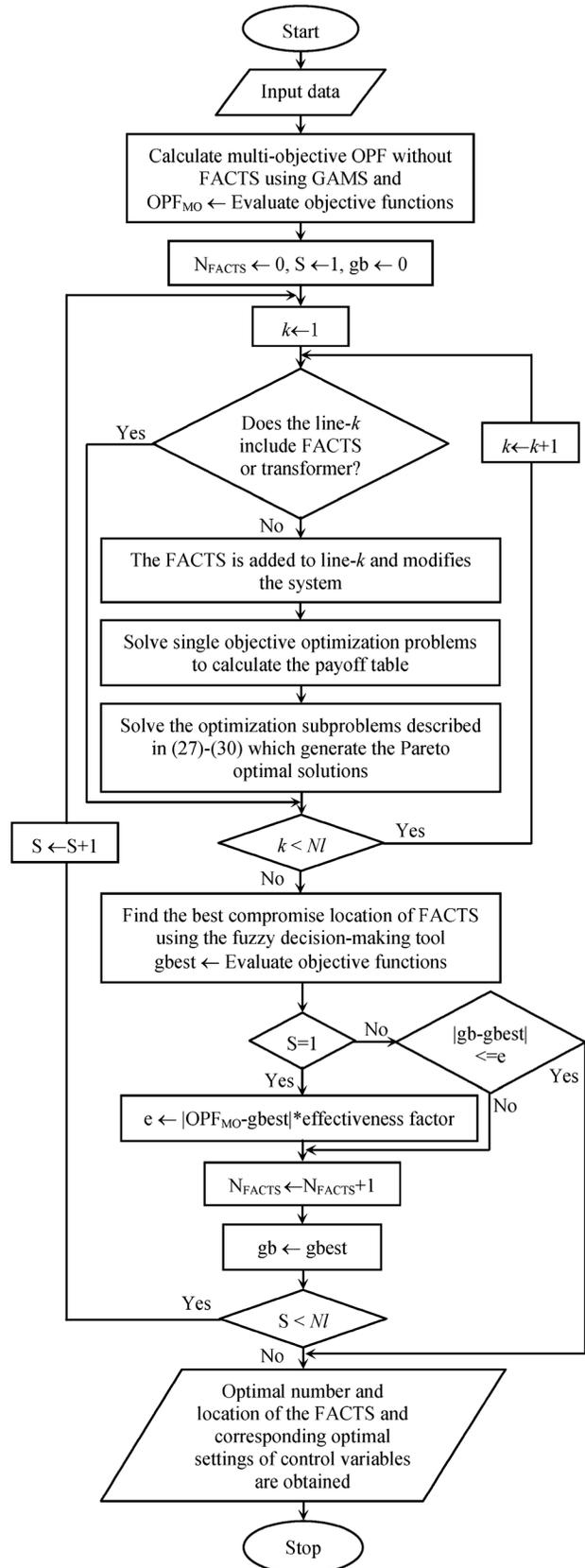


Fig. 4. Flowchart of the proposed algorithm.

$$Q_{Gi} - \rho Q_{Di} = f_{Qi}(x, u) \quad (17)$$

where  $f_{Pi}$  and  $f_{Qi}$  are the real and reactive power-flow equations at bus- $i$  where the FACTS controller parameters

are considered;  $P_{Gi}$  and  $Q_{Gi}$  are the generator real and reactive power at bus- $i$ , respectively; and  $P_{Di}$  and  $Q_{Di}$  are the load real and reactive power at bus- $i$ , respectively.

- 4) The cost of FACTS installation: The fourth objective function is to minimize the cost of FACTS installation that can be mathematically formulated and is given by the following equation:

$$F_4 = \frac{C \times S \times 1000}{8760 \times 5} \quad (\$/h) \quad (18)$$

where  $C$  is the cost of FACTS installation in U.S./kVAr, and  $S$  is the operating range of the FACTS controllers in MVar. Based on the ABB and Siemens database, the cost functions for PST, HFC, and UPFC are developed as [17], [22], [31], and [32]

$$\begin{aligned} C_{\text{PST}} &= 12 \\ C_{\text{HFC}} &= 0.00012S^2 - 0.10764S + 75.288 \\ C_{\text{UPFC}} &= 0.0003S^2 - 0.2691S + 188.22. \end{aligned} \quad (19)$$

In this paper, the five-year planning period is applied to evaluate the cost function.

### B. Constraints

The OPF problem has two sets of constraints, including equality and inequality constraints. These constraints can be expressed as follows [18].

- 1) Equality constraints: The equality constraints of the OPF problem are the power-flow equations corresponding to real and reactive power balance equations, described in (16) and (17), respectively.
- 2) Inequality constraints: The inequality operation constraints in the OPF problem include:

Generation constraints: Generator voltages, real power outputs, and reactive power outputs are restricted by their lower and upper limits as follows:

$$V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max}, \quad i = 1, \dots, NG \quad (20)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad i = 1, \dots, NG \quad (21)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \quad i = 1, \dots, NG. \quad (22)$$

Security constraints: These include the constraints of voltages at load buses and transmission-lines loadings as follows:

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max}, \quad i = 1, \dots, Nd \quad (23)$$

$$S_{Li} \leq S_{Li}^{\max}, \quad i = 1, \dots, Nl \quad (24)$$

where  $Nd$  is the number of load buses, and  $S_{Li}$  is the apparent power flowing through the  $i$ th transmission line.

FACTS constraints: The FACTS shunt-series controller settings are bounded as follows:

$$\text{PST} : \sigma^{\min} \leq \sigma \leq \sigma^{\max}; S_{\text{PST}} \leq S_{\text{PST}}^{\max},$$

$$\text{HFC} : k^{\min} \leq k \leq k^{\max}; 0 \leq K_C \leq K_C^{\max}; 0 \leq K_L \leq K_L^{\max};$$

$$0 \leq K_m \leq K_m^{\max}; S_{\text{HFC}} \leq S_{\text{HFC}}^{\max}, \text{ and}$$

$$\text{UPFC} : r^{\min} \leq r \leq r^{\max}; \gamma^{\min} \leq \gamma \leq \gamma^{\max}; S_{\text{UPFC}} \leq S_{\text{UPFC}}^{\max}$$

where  $S_{\text{PST}}$ ,  $S_{\text{HFC}}$ , and  $S_{\text{UPFC}}$  are the apparent power of PST, HFC, and UPFC controllers in megavolt amperes, respectively.

### C. Problem Statement

In general, aggregating the objectives and constraints, the problem can be mathematically formulated as follows:

$$\begin{aligned} &\text{minimize} && F_1 \\ &\text{minimize} && F_2 \\ &\text{maximize} && F_3 \\ &\text{minimize} && F_4 \\ &\text{subject to} && g(x, u) = 0 \\ &&& h(x, u) \leq 0 \end{aligned} \quad (25)$$

where  $g(x, u)$  and  $h(x, u)$  are the set of equality and inequality constraints, respectively. The multiobjective optimization problem can be converted into a single objective optimization and then solved by using the  $\varepsilon$ -constraint method in the next section.

## IV. MULTIOBJECTIVE MATHEMATICAL PROGRAMMING

In this section, a well-organized technique based on the  $\varepsilon$ -constraint method for solving the MMP problems is proposed. The MMP problems have more than one objective function and there is no single optimal solution that is simultaneously able to optimize all of the objective functions where the decision makers are looking for the “most preferred” solution in contrast to the optimal solution. In MMP, the concept of optimality is replaced by that of Pareto optimality that cannot be improved in one objective function without deteriorating its performance in at least one of the rest. In the current study, the  $\varepsilon$ -constraint method and the fuzzy decision-making tool are described as follows.

### A. $\varepsilon$ -Constraint Method

The  $\varepsilon$ -constraint method optimizes one of the objective functions while the other objective functions are considered as constraints [33]

$$\begin{aligned} &\min F_{1,k}(x) \\ &\text{subject to } F_{2,k}(x) \leq e_{2,k} F_{3,k}(x) \leq e_{3,k} \dots F_{p,k}(x) \leq e_{p,k} \end{aligned} \quad (26)$$

where subscripts  $p$  and  $k$  indicate the number of objective functions and the number of transmission lines which include the FACTS controller, respectively. In order to properly handle this method, the range of every objective function at least for the  $p - 1$  objective functions are required that will be used as constraints. The most common approach is to calculate

these ranges from the payoff table (the table with the results from the individual optimization of the  $p$  objective functions) [33]. After placing the FACTS controller on each line of the power system, the individual optima of the objective functions are calculated to construct the payoff table ( $F_{i,k}^*$  indicates the optimum value of the  $i$ th objective function by placing the FACTS controller on the  $k$ th line.), where the value of other objective functions is computed which are depicted by  $F_1^{i,k}, \dots, F_{i-1}^{i,k}, F_{i+1}^{i,k}, \dots, F_p^{i,k}$ . Consequently, the  $i$ th row of the payoff table contains  $F_{1,k}^i, \dots, F_{i-1,k}^i, F_{i,k}^*, F_{i+1,k}^i, \dots, F_{p,k}^i$ . In this way, all rows of the payoff table are calculated. The range of the  $j$ th objective function is obtained among the minimum and maximum values of the  $j$ th column of the payoff table that is divided into  $q_j$  equal intervals using  $(q_j - 1)$  intermediate equidistant grid points. Thus, we have a total of  $(q_j + 1)$  grid points for the  $j$ th objective function where the total number of optimization subproblems for placing the FACTS controller on each line becomes  $(q_2 + 1) \times (q_3 + 1) \times \dots \times (q_p + 1)$ . The density of the Pareto optimal set representation can be controlled by properly assigning the values to the  $q_i$ . The higher number of grid points leads to the denser representation of the Pareto optimal set but with the cost of higher computation time [34]. In this paper, the number of intervals for the objective functions is selected to be 4. In order to deal with the MMP problem of the power system operation, four objective functions  $F_1, F_2, F_3$ , and  $F_4$  are considered, described in (13)–(15) and (18), respectively. Therefore, the optimization subproblems become the following form:

$$\begin{aligned} & \min F_{1,k}(x) \\ & \text{subject to } F_{2,k}(x) \leq e_{2,i,k} \\ & F_{3,k}(x) \geq e_{3,j,k} \quad F_{4,k}(x) \leq e_{4,l,k} \end{aligned} \quad (27)$$

$$e_{2,i,k} = \max(F_{2,k}) - \left( \frac{\max(F_{2,k}) - \min(F_{2,k})}{q_2} \right) \times i \quad (28)$$

$$i = 0, 1, \dots, q_2$$

$$e_{3,j,k} = \min(F_{3,k}) + \left( \frac{\max(F_{3,k}) - \min(F_{3,k})}{q_3} \right) \times j \quad (29)$$

$$j = 0, 1, \dots, q_3$$

$$e_{4,l,k} = \max(F_{4,k}) - \left( \frac{\max(F_{4,k}) - \min(F_{4,k})}{q_4} \right) \times l \quad (30)$$

$$l = 0, 1, \dots, q_4$$

where  $\max(\cdot)$  and  $\min(\cdot)$  represent the maximum and minimum values of the individual objective function while placing the FACTS controller on the  $k$ th line, respectively. Note that the optimization subproblems should be accompanied by the constraints of the MMP problem described in the previous section.

### B. Fuzzy Decision-Making Tool

After placing the FACTS controller on each transmission line of the power system, the Pareto optimal solutions are obtained by solving the optimization subproblems. Thereafter, the decision maker needs to choose the optimal location of the FACTS controller according to the best compromise among the Pareto

optimal solutions. In this paper, a fuzzy decision-making approach is proposed for the optimal location decision process wherein a linear membership function ( $\mu_i$ ) is defined for each objective function as follows:

$$\mu_{i,k}^n = \begin{cases} 1 & F_{i,k}^n \leq \min(F_i) \\ \frac{\max(F_i) - F_{i,k}^n}{\max(F_i) - \min(F_i)} & \min(F_i) \leq F_{i,k}^n \leq \max(F_i) \\ 0 & F_{i,k}^n \geq \max(F_i) \end{cases} \quad (31)$$

for minimized objective functions and

$$\mu_{i,k}^n = \begin{cases} 0 & F_{i,k}^n \leq \min(F_i) \\ \frac{F_{i,k}^n - \min(F_i)}{\max(F_i) - \min(F_i)} & \min(F_i) \leq F_{i,k}^n \leq \max(F_i) \\ 1 & F_{i,k}^n \geq \max(F_i) \end{cases} \quad (32)$$

for maximized objective functions where  $F_{i,k}^n$  and  $\mu_{i,k}^n$  are the value of the  $i$ th objective function in the  $n$ th Pareto optimal solution of the  $k$ th transmission line which includes the FACTS controller and its membership function, respectively. The membership functions are used to evaluate the optimality degree of the Pareto optimal solutions. The most preferred solution can be selected as follows:

$$\mu_{\text{opt}} = \max_{k=1}^{NI} \left\{ \sup_{n \in k} \frac{\sum_{i=1}^p w_i \cdot \mu_{i,k}^n}{\sum_{n=1}^M \sum_{i=1}^p w_i \cdot \mu_{i,k}^n} \right\} \quad (33)$$

where

$$w_i \geq 0, \quad \sum_{i=1}^p w_i = 1. \quad (34)$$

Here,  $w_i$  is the weight value assigned to the  $i$ th objective function and  $M$  is the number of Pareto optimal solutions in each transmission line which includes the FACTS controller. The weight values  $w_i$  can be selected by the power system dispatcher based on the importance of the economical and technical aspects. Therefore, the optimal location and settings of the FACTS controller based on the adopted weighting factors are obtained by the proposed algorithm as the best Pareto optimal solution.

The flowchart of the proposed algorithm is depicted in Fig. 4. In order to generalize the proposed algorithm, the stop criterion is proposed for a given optimal number of FACTS controllers. The optimal number is controlled by an effectiveness factor. The effectiveness factor is an arbitrary factor between 0 and 1. In this paper, it is equal to 1 and, hence, only one FACTS device is considered.

## V. CASE STUDIES

The effectiveness of the proposed approach is investigated on the IEEE 14-bus test system. The optimal location of PST, HFC, and UPFC and their settings to optimize the objective functions were obtained and discussed. The simulation studies were carried out in Matlab 7.6 and GAMS 23.0 softwares and executed on a 2.66-GHz Pentium-IV PC with 2-GB random-access memory (RAM). The NLP and MINLP optimization problems are modeled in GAMS software and solved using MINOS and DICOPT solvers, respectively [35]. Data on the test system are

TABLE I  
RESULTS OF SINGLE-OBJECTIVE AND MULTIOBJECTIVE OPTIMIZATIONS FOR THE IEEE 14-BUS SYSTEM

Case	Parameters	Without FACTS	PST (Without $F_4$ minimized)	HFC (Without $F_4$ minimized)	HFC (With $F_4$ minimized)	UPFC (With $F_4$ minimized)	
$F_1$	Total Fuel Cost (\$/h)	17278.804	17270.35	17216.64	17226.60	17159.43	
	$\Sigma P_{\text{loss}}$ (MW)	1.7128	1.5987	0.8747	1.0062	1.9297	
	Investment cost (\$/h)	-	7.67	102.53	66.24	105.16	
	FACTS Size (MVA)	-	28.00	65.30	40.81	25.37	
	FACTS Location	-	Line 1-5	Line 1-5	Line 1-5	Line 2-4	
	Execution Time (s)	0.2020	2.8026	3.9950	19.5572	12.6863	
$F_2$	$\Sigma P_{\text{loss}}$ (MW)	1.1280	1.0850	0.7658	0.8030	0.7837	
	Total Fuel Cost (\$/h)	18186.618	18153.08	18397.97	18195.66	18861.42	
	Investment cost (\$/h)	-	6.97	96.51	62.23	90.02	
	FACTS Size (MVA)	-	25.44	61.13	38.20	21.60	
	FACTS Location	-	Line 2-5	Line 2-4	Line 2-4	Line 6-13	
	Execution Time (s)	0.1987	2.7353	4.0611	19.2546	12.8135	
$F_3$	Loadability Index	1.5565	1.5574	1.567	1.565	1.5755	
	Total Fuel Cost (\$/h)	30700.100	30700.100	30700.100	30700.100	30700.100	
	$\Sigma P_{\text{loss}}$ (MW)	6.8621	6.6189	4.2433	4.5605	7.2899	
	Investment cost (\$/h)	-	13.05	146.46	96.03	109.26	
	FACTS Size (MVA)	-	47.64	97.27	60.80	26.39	
	FACTS Location	-	Line 1-5	Line 2-3	Line 2-3	Line 2-3	
Case 1 ( $F_1$ & $F_2$ )	Total Fuel Cost (\$/h)	17533.703	17398.61	17220.06	17221.68	17215.97	
	$\Sigma P_{\text{loss}}$ (MW)	1.300	1.3083	0.8643	0.8643	0.9111	
	$\Sigma Q_{\text{loss}}$ (MVar)	12.6766	12.1315	10.9264	10.8992	11.1041	
	Investment cost (\$/h)	-	6.94	101.48	98.02	245.31	
	FACTS Size (MVA)	-	25.32	64.57	62.17	62.24	
	FACTS Location	-	Line 1-5	Line 1-5	Line 1-5	Line 1-5	
FACTS Settings	-	$\sigma = 3.2108$	$K_C = 0$ $K_m = 1$	$k = 0.123$ $K_L = 0$	$K_C = 0$ $K_m = 1$	$k = 0.117$ $K_L = 0$	
Execution Time (s)	1.1122	17.5423	24.6590	24.2457	17.8447	$r = 0.1387$ $\gamma = 80.012$	
Case 2 ( $F_1$ & $F_3$ )	Total Fuel Cost (\$/h)	25274.444	23894.06	26709.23	26735.04	20685.36	
	Loadability Index	1.3617	1.3068	1.4249	1.4249	1.188	
	$\Sigma P_{\text{loss}}$ (MW)	4.8759	4.0063	3.5712	3.8324	4.1667	
	$\Sigma Q_{\text{loss}}$ (MVar)	24.8826	21.3362	19.6302	20.8114	25.0512	
	Investment cost (\$/h)	-	11.50	129.63	81.46	274.60	
	FACTS Size (MVA)	-	41.97	84.71	50.89	70.44	
	FACTS Location	-	Line 1-5	Line 2-3	Line 2-3	Line 2-3	
	FACTS Settings	-	$\sigma = 5.2457$	$K_C = 0$ $K_m = 1$	$k = 0.150$ $K_L = 0$	$K_C = 0$ $K_m = 0$	$k = 0.096$ $K_L = 0$
	Execution Time (s)	1.1101	15.6165	23.6091	24.1082	16.3960	$r = 0.1593$ $\gamma = 180$
Case 3 ( $F_2$ & $F_3$ )	$\Sigma P_{\text{loss}}$ (MW)	4.7220	4.3912	3.2162	3.0134	3.3919	
	Loadability Index	1.427	1.4184	1.4233	1.424	1.4487	
	Total Fuel Cost (\$/h)	27676.998	27492.35	26720.97	27516.17	27985.16	
	$\Sigma Q_{\text{loss}}$ (MVar)	23.3625	21.8285	18.6395	17.3633	18.5705	
	Investment cost (\$/h)	-	10.06	155.52	85.04	185.13	
	FACTS Size (MVA)	-	36.70	104.20	53.30	45.95	
	FACTS Location	-	Line 1-5	Line 1-5	Line 2-3	Line 2-3	
	FACTS Settings	-	$\sigma = 4.5973$	$K_C = 5$ $K_m = 1$	$k = 0.172$ $K_L = 0$	$K_C = 0$ $K_m = 0$	$k = 0.100$ $K_L = 0$
	Execution Time (s)	1.0941	16.0585	23.7409	23.4578	15.6337	$r = 0.091$ $\gamma = 95.822$
Case 4 ( $F_1$ & $F_2$ & $F_3$ )	Total Fuel Cost (\$/h)	21622.168	21931.66	21878.13	24784.61	23937.21	
	$\Sigma P_{\text{loss}}$ (MW)	3.2874	3.2116	1.9203	3.2781	4.0924	
	Loadability Index	1.209	1.2231	1.225	1.3239	1.311	
	$\Sigma Q_{\text{loss}}$ (MVar)	19.4722	18.6811	14.0144	18.6062	23.0957	
	Investment cost (\$/h)	-	10.60	130.03	71.81	143.99	
	FACTS Size (MVA)	-	38.67	85	44.47	35.21	
	FACTS Location	-	Line 1-5	Line 1-5	Line 4-5	Line 2-5	
	FACTS Settings	-	$\sigma = 4.8321$	$K_C = 0$ $K_m = 1$	$k = 0.165$ $K_L = 0$	$K_C = 7$ $K_m = 1$	$k = 0.005$ $K_L = 0$
	Execution Time (s)	1.2975	19.5889	29.3228	28.1455	19.4638	$r = 0.0673$ $\gamma = 137.823$

TABLE II  
RESULTS OF THE BEST PARETO OPTIMAL SOLUTIONS BASED ON  $F_1$ ,  $F_2$ ,  $F_3$ , AND DECISION-MAKING ASPECTS FOR THE IEEE 14-BUS SYSTEM

FACTS device	Parameters	$F_1$ -based best Pareto optimal solution	$F_2$ -based best Pareto optimal solution	$F_3$ -based best Pareto optimal solution	Decision making-based best Pareto optimal solution
PST	Total Fuel Cost (\$/h)	21931.96	21931.96	24136.20	21934.89
	$\sum P_{\text{loss}}$ (MW)	3.2142	3.2142	3.5174	3.2761
	$\sum Q_{\text{loss}}$ (MVar)	18.7197	18.7197	19.1935	19.0631
	Loadability Index	1.2231	1.2231	1.3068	1.2230
	Investment cost (\$/h)	9.29	9.29	7.94	12.33
	FACTS Size (MVA)	33.91	33.91	28.99	44.99
	FACTS Location	Line 1-5	Line 1-5	Line 1-5	Line 2-4
HFC	Total Fuel Cost (\$/h)	21953.10	22147.85	25115.91	24784.61
	$\sum P_{\text{loss}}$ (MW)	3.0367	2.0123	2.3639	3.2781
	$\sum Q_{\text{loss}}$ (MVar)	19.0758	14.3452	15.4820	18.6062
	Loadability Index	1.2247	1.2376	1.3280	1.3239
	Investment cost (\$/h)	44.51	118.99	72.53	71.81
	FACTS Size (MVA)	26.90	76.97	44.94	44.47
	FACTS Location	Line 6-13	Line 1-5	Line 2-3	Line 4-5
UPFC	Total Fuel Cost (\$/h)	21659.11	21924.20	23947.41	23937.21
	$\sum P_{\text{loss}}$ (MW)	5.6914	2.9869	4.4805	4.0924
	$\sum Q_{\text{loss}}$ (MVar)	31.2551	18.9153	24.5885	23.0957
	Loadability Index	1.2355	1.2239	1.3290	1.3111
	Investment cost (\$/h)	286.82	151.18	187.60	143.99
	FACTS Size (MVA)	73.92	37.07	46.60	35.21
	FACTS Location	Line 1-5	Line 6-13	Line 2-3	Line 2-5

taken from [36]. Characteristics of FACTS controllers are given in the Appendix.

#### A. IEEE 14-Bus Test System

In order to study the effect of the FACTS location and its settings on the indices of power system operation, the performance of PST, HFC, and UPFC are investigated on the IEEE 14-bus system. The results of single objective and multiobjective optimizations are obtained and compared for the objective functions. Based upon (19), the PST cost is low compared to that of the UPFC and HFC where it is not minimized but its value is computed.

The results of the single objective optimization are presented in Table I. In the case of maximum system loadability, UPFC gives the best performance compared to PST and HFC although its power losses are higher than that of HFC. It is noticeable that by maximizing the system loadability without minimizing the cost of UPFC installation, the loadability index of UPFC is equal to 1.598 where its power losses are equal to 8.946. After summing the cost of the FACTS controller into the total fuel cost, HFC shows a satisfactory improvement compared to PST and UPFC in minimum power losses and minimum total fuel cost as single objective functions. It is noted that the UPFC losses are higher than those of the PST and HFC, which is neglected in this study. Furthermore, the system loadability is equal to 1 if it is not maximized.

In order to study the conflict among the objective functions, the multiobjective optimization is performed in four cases. In

case 1, the total fuel cost and the power losses are minimized. The total fuel cost and the system loadability are optimized in case 2. Case 3 is studied to optimize the power losses and the system loadability. The total fuel cost, the power losses, and the system loadability are optimized in case 4. All of the cases are carried out with and without minimum cost of FACTS installation as the objective function. In all cases, the same weight values are assigned to the objective functions. According to the adopted weighting factors, the most preferred compromise solution is selected among the Pareto optimal solutions using the fuzzy decision-making process in each case. The results of multiobjective optimization are shown in Table I. Some interesting observations which can be derived from this table are as follows.

In case 1, the optimal location of FACTS controllers is obtained on lines 1–5. It is observed that the HFC achieves the best performance in comparison with the other controllers after summing the investment cost of the FACTS controller into the total fuel cost. Since increasing the system loadability increases all of the loads and, hence, increases the total fuel cost and the power losses, operating HFC in case 2 is better than that of PST and UPFC as well as reducing active and reactive power losses and total fuel cost in comparison with the other controllers. In addition, the results obtained with HFC and UPFC show nearly the same performance in case 3 although the total fuel cost of HFC is lower than that of the UPFC. In case 4, the HFC gives satisfactory effectiveness compared to that of the UPFC due to higher system loadability of HFC with respect to the UPFC with a reduction in active and reactive power losses compared with the UPFC.

TABLE III  
RESULTS OF MULTIOBJECTIVE OPTIMIZATION WITH CHANGES IN OPERATING CONSTRAINTS FOR THE IEEE 14-BUS SYSTEM

FACTS device	Parameters	Mode 1	Mode 2	Mode 3	Mode 4
PST	Total Fuel Cost (\$/h)	21934.89	21931.62	21930.65	21927.19
	$\Sigma P_{\text{loss}}$ (MW)	3.2761	3.0559	3.3260	3.0874
	$\Sigma Q_{\text{loss}}$ (MVar)	19.0631	17.7468	18.8451	17.4924
	Loadability Index	1.2230	1.2237	1.2203	1.2211
	Investment cost (\$/h)	12.33	11.42	10.33	10.24
	FACTS Size (MVA)	44.99	41.70	37.71	37.38
	FACTS Location	Line 2-4	Line 2-4	Line 2-5	Line 2-5
	FACTS Settings	$\sigma = 4.7389$	$\sigma = 4.0772$	$\sigma = 3.9312$	$\sigma = 3.6142$
HFC	Total Fuel Cost (\$/h)	24784.61	24908.32	23623.22	24793.59
	$\Sigma P_{\text{loss}}$ (MW)	3.2781	3.2499	3.9506	3.0612
	$\Sigma Q_{\text{loss}}$ (MVar)	18.6062	17.9611	20.8648	16.8848
	Loadability Index	1.3239	1.3241	1.2899	1.3213
	Investment cost (\$/h)	71.81	23.56	11.06	73.18
	FACTS Size (MVA)	44.47	13.98	6.5	45.37
	FACTS Location	Line 4-5	Line 3-4	Line 9-10	Line 4-5
	FACTS Settings	$K_C = 7$ $k = 0.005$ $K_m = 1$ $K_L = 0$	$K_C = 0$ $k = -0.021$ $K_m = 0$ $K_L = 0$	$K_C = 7$ $k = 0.003$ $K_m = 0$ $K_L = 0$	$K_C = 7$ $k = 0.005$ $K_m = 1$ $K_L = 0$
UPFC	Total Fuel Cost (\$/h)	23937.21	23728.38	23767.62	23724.95
	$\Sigma P_{\text{loss}}$ (MW)	4.0924	5.0747	4.4315	5.1643
	$\Sigma Q_{\text{loss}}$ (MVar)	23.0957	27.4842	23.6496	25.9942
	Loadability Index	1.3111	1.3221	1.3120	1.3158
	Investment cost (\$/h)	143.99	253.70	209.34	236.56
	FACTS Size (MVA)	35.21	64.57	52.42	59.83
	FACTS Location	Line 2-5	Line 1-5	Line 1-5	Line 1-5
	FACTS Settings	$r = 0.0673$ $\gamma = 137.823$	$r = 0.1396$ $\gamma = 174.977$	$r = 0.1200$ $\gamma = 151.2863$	$r = 0.1286$ $\gamma = 173.98$

In all cases, the execution time of the proposed algorithm is calculated and tabulated in Table I. Since there is no previous literature specifically examining the effect of the FACTS device on all transmission lines, the execution time cannot be compared to that of the other methods. However, as can be inferred from the literature [37], [38], they are less than that of the other methods.

In order to evaluate the performance of the fuzzy decision-making tool, the optimal locations of FACTS devices based on the total fuel cost, power losses, system loadability, and decision-making aspects among the Pareto optimal solutions associated with the simultaneous optimization of all objective functions are presented in Table II. As can be observed in Table II, the fuzzy decision-making tool indicates good performance to choose the most preferred compromise solution among the Pareto optimal solutions compared to the selection criteria based on objective functions.

In addition, the sensitivity to changes in operating constraints to simultaneously optimize all objective functions is considered and compared without any changes to those constraints in four modes. In mode 1, no changes were applied on operating constraints. In mode 2, a variation in the range of voltages from  $0.94 \leq V \leq 1.06$  to  $0.9 \leq V \leq 1.1$  is assumed. A 10% decrease in nominal rating of the transmission lines is taken into account in mode 3. Mode 4 is planned to check the simultaneous

changes of modes 2 and 3. It is noticeable that by increasing the nominal rating of the transmission lines, the results will be similar to that of mode 1 since the boundary condition of inequality (24) is satisfied in mode 1 and, hence, it does not affect the optimization process in comparison with mode 1. The results of multiobjective optimal location of PST, HFC, and UPFC with respect to all modes are shown in Table III.

According to the single objective optimization, the results of mode 2 demonstrate a decrease in the total fuel cost and power losses as well as an increase in the system loadability compared to that of mode 1 while the results of mode 3 are in contrast point to those of mode 2. The assumptions considered in mode 4 show a decrease in the power losses and system loadability in addition to an increase in the total fuel cost compared to that of mode 1. In a similar way, the reaction to changes in operating constraints can be deduced from the results obtained by the multiobjective optimization presented in Table III.

## VI. CONCLUSION

In this paper, a multiobjective mathematical programming for allocation of the FACTS shunt-series controller is developed to involve the objective functions using the  $\varepsilon$ -constraint method for generating the Pareto optimal solutions. The fuzzy decision-making approach is proposed to obtain the best Pareto optimal location and settings of the FACTS controller among the Pareto

optimal solutions. In order to evaluate the effectiveness of the proposed approach, the performance of PST, HFC, and UPFC are investigated on the IEEE 14-bus test system.

The optimal location of PST, HFC, and UPFC and their settings to optimize total fuel cost, power losses, system loadability, and cost of FACTS installation as objective functions, using NLP and MINLP as solution procedures, were obtained and discussed. The various results shown in the tables compare the scenario without FACTS and with PST, HFC, and UPFC for the test system.

According to the adopted weighting factors, the most preferred compromise solution is selected among the Pareto optimal solutions using the fuzzy decision-making process in each case study. The results show the effectiveness of the proposed algorithm to optimally locate the FACTS shunt-series controller in a transmission system. In addition, the results illustrate that HFC can be applied for the best satisfaction of the dispatcher requirement based on the technical and economical aspects.

#### APPENDIX

##### Data of PST, HFC, and UPFC

$$X_E = 0.001 \text{ p.u.}; X_B = 0.007 \text{ p.u.}; S_{base} = 100 \text{ MVA.}$$

$$\text{PST: } -20^\circ \leq \sigma \leq 20^\circ; k = \tan \sigma (\text{quadrature booster}).$$

$$\text{HFC: } -0.26 \leq k \leq 0.26; X_C = 0.0076 \text{ p.u.};$$

$$X_L = 0.0038 \text{ p.u.};$$

$$0 \leq K_C \leq 7; 0 \leq K_L \leq 3; 0 \leq K_m \leq 2;$$

$$Y_{MSC} = 0.25 \text{ p.u..}$$

$$\text{UPFC: } 0 \leq r \leq 1; -\pi \leq \gamma \leq \pi.$$

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