A Study of Chattering Suppression in Sliding Mode-Based Missile Guidance Law

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Abstract—**A sliding mode controller with chattering suppression for missiles guidance law is designed under the presence of target maneuvering. To reduce the chattering in sliding mode control, constitute a continuous function instead of signal function which guarantees the practicability of the control scheme. In the numerical simulation, it is obviously that the proposed guidance law reduces the chattering phenomenon and**

Keywords-guidnace law;sliding mode control; maneuvering target; Missile

I. INTRODUCTION

This Perhaps, the most widely used guidance law for short range homing missiles is the PN, for reasons of its inherent simplicity and ease of implementation[1,2]. It is well known that the PN law seeks to null the LOS rate against no maneuvering targets, by making the missile heading rate proportional to the LOS rate. However, with the advent of highly maneuverable targets, simple PN shows up less favorably. Moreover, the neglected aerodynamic drag affects the missile maneuverability and velocity, resulting in a loss of performance at higher altitudes and in the rase of retreating targets.

To remove some of these shortcomings, a new class of guidance law based on Sliding Mode Control (SMC) theory is proposed in [3]. The main advantage of these guidance laws is that they are less sensitive to unmodeled dynamics. More importantly, they are relatively simple to implement and offer robustness against a wide variety of target maneuvers.

Sliding mode control (SMC) is an efficient control method to stabilize systems with nonlinearity and uncertainty features. Especially, ideal sliding mode control system is insensitive to perturbation of parameters and external disturbances[4-6]. Although the technique has good robustness properties, pure sliding mode control presents drawbacks that include large control requirements and chattering[7,8]. To reduce the chattering and get better trending performance, constitute a continuous function instead of signal function in the SMC design.

The paper is organized as follows. Section 2 introduces an existing mathematical model in literature for the dynamics of air-to-air missile. Section 3 illustrates the design of sliding mode controller including stability analysis. The proposed sliding mode control scheme is validated by simulations in Section 4. Finally, conclusions are presented in Section 5.

II. MATHEMATICAL MODEL OF BTT MISSILE

The planner CLOS guidance problem can be formulated as a tracking problem for a time-varying nonlinear system [4]. Fig.1 depicts the 2-D missile target pursuit diagram. The origin of the inertial frame is located at the ground tracker. The *Z* axis is vertical upward and the X axis is horizontal. The origin of the missile body frame is fixed at the missile centerline. One can drive planar missile-target engagement kinematics without accounting for gravity, the LOS angel and LOS angel rate can be represented as follows[9]:

$$\dot{q} = \frac{1}{R}[V_m \sin\eta_m - V_t \sin\eta_t] \tag{1}$$

$$\dot{R} = -V_m \cos\eta_m + V_t \cos\eta_t \tag{2}$$

$$q = \theta_m + \eta_t \tag{3}$$

Where q is the LOS angle, V_m, V_t are missile velocity and target velocity. The range between the adversaries is *R*. The flight path angles of missile and target are denoted by η_m, η_t. Orientation of missile body reference frame XOZ with respect to inertial frame X_0OY_0 is presented in Fig. 1.

Defining $a_m = V_m\dot{\theta}_m$, $a_t = V_t\dot{\theta}_t$ and taking the derivative of equation (1) with respect to time, and considering (2) (3), we obtain

$$\ddot{q} = \frac{1}{R}[-2\dot{R}\dot{q} - a_m\cos\eta_m + \dot{V}_m\sin\eta_m + a_t\cos\eta_t - \dot{V}_t\sin\eta_t] \tag{4}$$

It is assumed that $x_1 = q, x_2 = \dot{q}$, the equation (4) can be written as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -a\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -b\end{bmatrix}u + \begin{bmatrix}0\\ b\end{bmatrix}d \tag{5}$$

where, $a = 2\dot{R}/R$, $b = 1/R$. $u = a_m\cos\eta_m - \dot{V}_m\sin\eta_m$, $d = a_t\cos\eta_t - \dot{V}_t\sin\eta_t$.

III. SLIDING MODE CONTROLLER DESIGN

The objective of continuous SMC design is to achieve a continuous control input *u* for a given reference command, so that the LOS rate approaches zero asymptotically. Define the expected response with decoupled characteristic as:

$$s = \dot{q} \tag{6}$$

In order to guarantee the system approaching sliding mode surface well, we bring forward an adaptive trending law[10]:

$$\dot s = -\frac{k|\dot R|}{R}s - \frac{Q}{R}\mathrm{sgn}(s) \quad (7)$$

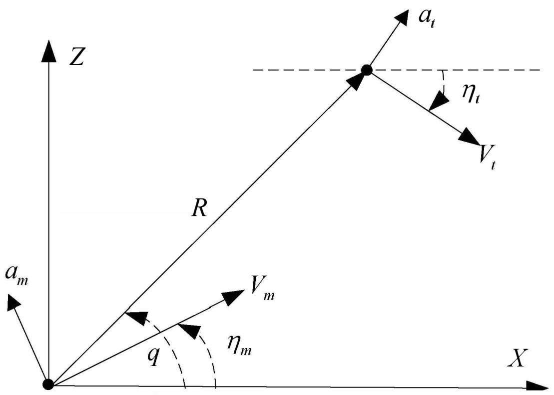


Fig. 1 Planar engagement geometry

Where k and Q are some positive numbers. From Equ. (7), the trending velocity will increase with R, otherwise, the trending velocity will be reduced. By adaptation of the parameter, the system can reach the sliding mode surface at a smaller trending velocity compared to the condition that $\dot s$ is fixed, so that it can reduce chattering of sliding mode surface.

Substituting Eqs. (6), (7) into Eq. (5), the control value of guidance law is

$$u = a_m\cos\eta_m - \dot V_m\sin\eta_m = (k+2)|\dot R|s + Q\mathrm{sgn}(s) + d \quad (8)$$

It is shown in [8] that the additive bias term is an estimate of the target acceleration, when the system is in sliding mode steady state. In other words, the guidance law （8） behaves like the APN guidance law in the neighborhood of $\dot q = 0$. This guidance law can be regarded as a PN guidance law with time-varying navigation gain, due to the presence of sliding mode term and with a switched bias term.

However, the main advantage of this guidance law over APN is that it does not require any explicit target maneuver estimation. Besides the uncertainty regarding the target acceleration, there exist other unmodeled dynamics like missile velocity variations, neglected seeker and track loop dynamics, etc. It is well known that sliding-mode control has the property of strong robustness to uncertainties when the system is controlled on a sliding surface. However, there exists a switching function in control design that always leads to the undesired exciting in the high frequency domain for the control system. So in this paper, to reserve the advantage of robustness of the sliding-mode control and to eliminate the disadvantage of high frequency switching for systems, a robust approximate the sliding-mode control with the integral switching surface is proposed.

In order to resolve this problem, the controller output can be simplified as

$$u = a_m\cos\eta_m - \dot V_m\sin\eta_m = (k+2)|\dot R|s + Q\mathrm{sgn}(s) \quad (9)$$

Take Lyapunov function as

$$V = \frac{1}{2}s^2 \quad (10)$$

Submitting Eq. (9) into Eq. (4) and the derivation of Lyapunov function is

$$\begin{aligned}\dot V = s\dot s &= \frac{1}{R}[-k|\dot R|\dot q^2 - Q\mathrm{sgn}(\dot q)\dot q + \hat a_t\cos\eta_t\dot q - \hat{\dot V}_t\sin\eta_t\dot q]\\ &= \frac{1}{R}[-(k - \frac{\hat a_t}{\dot q|\dot R|}\cos\eta_t + \frac{\hat{\dot V}_t}{\dot q|\dot R|}\sin\eta_t)|\dot R|\dot q^2 - Q\mathrm{sgn}(\dot q)\dot q]\\ &= \frac{1}{R}[-(Q - \frac{\hat a_t\dot q}{\dot q}\cos\eta_t + \frac{\hat{\dot V}_t\dot q}{\dot q}\sin\eta_t)|\dot q| - k\dot q^2|\dot R|] \quad (11)\end{aligned}$$

With the value $\dot V < 0$ selected as above, it can be shown that Eq.(17) is satisfied.

$$\begin{aligned}&Q > |\hat a_t| + |\hat{\dot V}_t|\\ &k > 0\end{aligned} \quad (12)$$

Where $\hat a_t, \hat{\dot V}_t$ are estimated values of perturbation of target. In order to avoid chattering which was resulted from discontinuity denotation function sgn(.) in Eq.(9).

Defining

$$[\mathrm{sgn}\, s]_{ad} = \begin{cases}\mathrm{sgn}(s) & |s| \ge \dfrac{\pi}{2\omega}\\ \omega\dot s\cos\omega s & |s| < \dfrac{\pi}{2\omega}\end{cases} \quad (13)$$

It can be tested, $\omega \to \infty, [\mathrm{sgn}(s)]_{ad} \to \mathrm{sgn}(s)$ .Take Lyapunov function still as

$$V = \frac{1}{2}s^2$$

Then derivative of the Lyapunov function can be derived as

$$\begin{aligned}\dot V \le &\frac{1}{R}[-(Q - \frac{\hat a_t\dot q}{\dot q}\cos\eta_t + \frac{\hat{\dot V}_t\dot q}{\dot q}\sin\eta_t)|\dot q| - k\dot q^2|\dot R|]\\ &-d[\mathrm{sgn}\,\dot q]_{ad}\dot q + d|\dot q| \quad (14)\end{aligned}$$

When $|\dot q| \ge \frac{\pi}{2\omega}$, Q, k satisfy Eq. (12), then $\dot V < 0$.

When

$$|\dot q| < \frac{\pi}{2\omega}, \quad (15)$$

$$\begin{aligned}\dot V \le &\frac{1}{R}[-(Q - \frac{\hat a_t\dot q}{\dot q}\cos\eta_t + \frac{\hat{\dot V}_t\dot q}{\dot q}\sin\eta_t)|\dot q|\\ &-d|\dot q|(1 - \sin\omega\dot q\,\mathrm{sgn}\,\dot q) < 0 \quad (16)\end{aligned}$$

Relations (15) and (16) mean that for $|\dot q| < \frac{\pi}{2\omega}$, $\dot q$ will keep in this area asymptotically. Obviously , according to Lyapunov theorem, the system is asymptotically stable near the equilibrium point. Combining Eq.(12) with Eq.(14) give the guidance law.

IV. SIMULATION RESULTS

In the section, we take the air-to-air missile as an example to simulate. In simulation, we consider the dynamic property of missile and target as ideal two-order property, Simulation conditions are as follows:

1. The missile and target property

Missile velocity is V_m =720 (m/s); Target velocity is V_t = 520 (m/s)

Scenario 1: Target acceleration in pitch channels: When $t < 1\,\text{s}$, $a_t = -5gm/s^2$ when $t > 1\,\text{s}$, $a_t = 5gm/s^2$

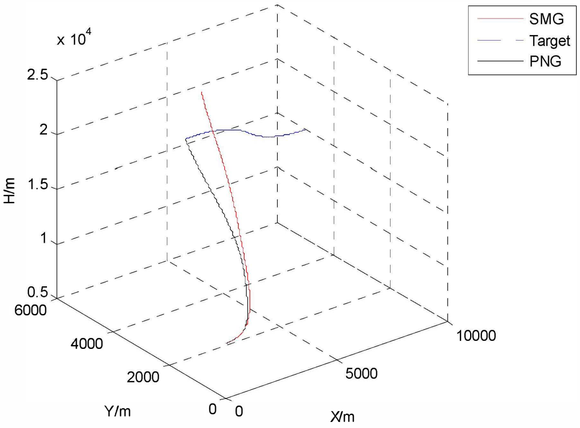
Scenario 2: Target acceleration in pitch channels: When $t < 2\,\text{s}$, $a_t = -10gm/s^2$ when $t > 2\,\text{s}$, $a_t = 10gm/s^2$

2. Initial conditions

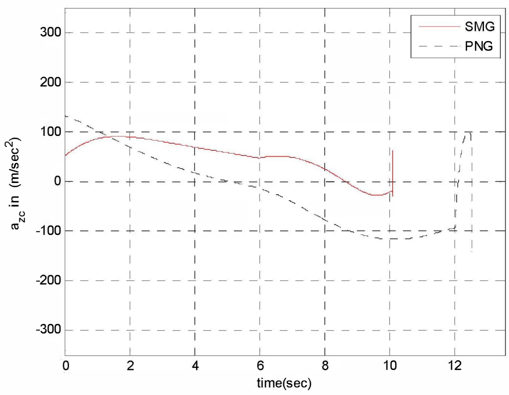
The initial relative range of Missile-to-target was chosen: R (0) = 20000 m; The initial azimuths of LOS to reference coordinate system were chosen 30°; The initial azimuths of Target to LOS were chosen 20°.

3. Select k=1, Q=20 and all simulation steps were chosen as $0.001\,\text{s}$.

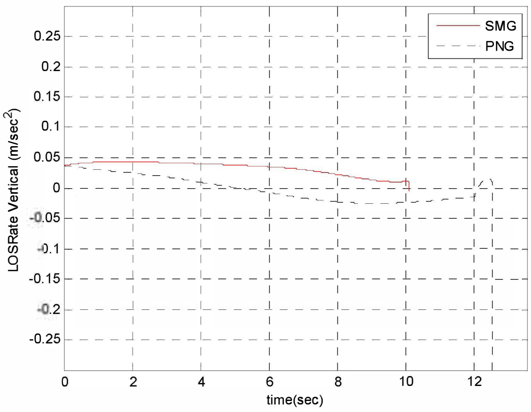
The distributing curve of missile acceleration was shown in Figure.2, which reflect furthest the leading property of guidance laws. Simulation results can been shown that the meeting time of the missile to target is 10.099s in the present of the planner guidance law with angle constraints, and the miss distance 0.72m with SMG(Sliding Mode Guidance), the meeting time is 12.5360s and the miss distance 1.95m with PNG. Based on simulation results, the guidance law with SMG is of obvious effect, which can reduce the larger overload, and decrease the miss distance of the air-to-air missile to highly maneuvering target at the terminal stages.



(a) Trajectory in scenario 2 with SMC guidance law and PNG



(b) History of acceration commands



(c) History of LOS rate

Fig.2 Engagement scenario 2 with SMC guidance law

V. CONCLUSION

This paper has studied the sliding mode controller design for air-to-air missiles based on sliding mode control theory. The performance is investigated in the presence of target maneuvering. Moreover, we constitute a sliding mode function instead of signal function which can reduce the chattering, guarantees the practicability of the control scheme. Simulation shows the well tracking performance of the controller. Chattering is erased and the robustness is intensified.

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