Journal of Process Control 19 (2009) 1036–1042

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

RNGA based control system configuration for multivariable processes

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ARTICLE INFO

Article history:
Received 15 April 2008
Received in revised form 12 January 2009
Accepted 15 January 2009

Keywords:
Multivariable processes
Decentralized control
Interaction measurement
Loop pairing
Relative gain array
Niederlinski index
Relative normalized gain array
Controller independent
Simulation

ABSTRACT

This paper presents a new control-loop configuration criterion for multivariable processes. Both the steady-state and transient information of the process transfer function are investigated. A new interaction measurement, relative normalized gain array, is proposed for evaluating control-loop interactions. Consequently, a new loop pairing criterion based on the relative normalized gain array is proposed for control structure configuration. The main contribution of this work is that it systematically analyzed the process transferring characteristics from both steady-state and transient perspectives and derived a feasible solution for the problem. Several examples, for which the conventional relative gain array based loop pairing criterion gives an inaccurate interaction assessment, are employed to demonstrate the effectiveness of the proposed interaction measure and loop pairing criterion.

1. Introduction

Despite the availability of sophisticated methods for designing multivariable control systems, decentralized control remains dominant in industry applications mainly due to: (1) it requires fewer parameters to tune which are easier to be understood and implemented; and (2) loop failure tolerance of the resulting control system can be assured during the design phase. Therefore, they are more often used in process control applications [1,2]. However, the potential disadvantage of using the limited control structure is the deteriorated closed-loop performance caused by interactions among loops as a result of the existence of non-zero off-diagonal elements in the transfer function matrix [3,4]. Thus, the primary task in the design of decentralized control systems is to determine loop configuration, i.e. pair the manipulated variables and controlled variables to achieve the minimum interactions among control loops so that the resulting multivariable control system mostly resembles its single-input single-output counterparts and the subsequent controller tuning is largely facilitated by SISO design techniques [5].

Since the pioneering work of Bristol [6], the relative gain array (RGA) based techniques for control-loop configuration have found widespread industry applications, including blending, energy conservation, and distillation columns, etc. [7–10]. The RGA based techniques have many important advantages, such as very simple in calculation as it only uses process steady-state gain matrix and scaling independent, etc. [11]. To simultaneously consider the closed-loop properties, the RGA based pairing rules are often used in conjunction with the Niederlinski index (NI) [12] to guarantee the system stability [3,5,11,13–15]. However, it has been pointed out that this RGA and NI based loop paring criterion is a necessary and sufficient condition only for a 2×2 system and it becomes a necessary condition for 3×3 and higher dimensional systems [11,16]. Moreover, using steady-state gain alone may result in incorrect interaction measures and consequently loop pairing decisions, since no dynamic information of the process is taken into consideration.

To overcome the limitations of RGA based loop pairing criterion, several pairing methods have later been proposed by using the dynamic RGA (DRGA) to consider the effects of process dynamics, which employ the transfer function model instead of the steady-state gain matrix to calculate RGA [17–19]. In DRGA, the denominator involved achieving perfect control at all frequencies, while the numerator was simply the open-loop transfer function. Recently, McAvoy et al. proposed a significant DRGA approach [20]. Using the available dynamic process model, a proportional output optimal controller is designed based on the state space approach and the resulting controller gain matrix is used to define a DRGA. Several examples in which the normal RGA gives the inaccurate interaction measure and wrong pairings were studied and in all cases the new DRGA method gives more accurate interaction

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0959-1524/$ - see front matter
doi:10.1016/j.jprocont.2009.01.004

assessment and the best pairings. However, DRGA is often controller dependent [20], which makes it more difficult to calculate and to be understood by practical control engineers. To combine the advantages of both RGA and DRGA, Xiong et al. [21] introduced a relative effective gain array (REGA) based loop pairing criterion by employing the steady-state gain and bandwidth of the process transfer function element. Since the REGA considers both the steady-state and the transient information of the process, it provides a more comprehensive description for loop interactions. Another advantage of REGA is that it is controller independent which is more superior to other existing loop pairing methods. However, since the calculation of REGA depends on the critical frequency point of individual element, different selection criteria for critical frequency points result in different REGAs, subsequently, cause uncertainties in control structure configurations.

In this paper, we propose a new loop pairing criterion based on a new method for interaction measurement. Through investigating both the steady-state and transient information of the process transfer function, the normalized gain is defined to provide a more comprehensive description of each process input to output channel. The relative normalized gain array (RNGA) is then introduced for loop interaction measurements. Consequently, a new loop pairing criterion based on the RNGA is proposed for control structure configuration. The main advantages of this method are: (1) Compared with RGA method, it considers not only the process steady-state information but also transient information; (2) compared with DRGA method, it also provides a comprehensive description of dynamic interaction among individual loops without requiring the specification of the controller type and with much less computation; (3) compared with REGA method, it requires even less calculation but resulting in an unique and optimal loop pairing decision; and (4) it is very simple for field engineers to understand and work out pairing decisions in practical applications. Several examples, for which the RGA based loop pairing criterion gives an inaccurate interaction assessment, are employed to demonstrate the effectiveness of the proposed interaction measure and loop pairing criterion.

2. Preliminaries

Consider an $n \times n$ system with a decentralized feedback control structure as shown in Fig. 1, where, $\mathbf{r} = [r_1 \ \cdots \ r_n]^T$, $\mathbf{u} = [u_1 \ \cdots \ u_n]^T$ and $\mathbf{y} = [y_1 \ \cdots \ y_n]^T$ are vectors of references, inputs and outputs respectively; $\boldsymbol{G}(s) = [g_{ij}(s)]_{n\times n}$ is system's transfer function matrix and $\mathbf{C}(s) = \mathrm{diag}\{c_1(s), \ldots, c_n(s)\}$ is the decentralized controller; $i, j = 1, \ldots, n$ are integer indices.

The loop pairing problem defines the control system structure, i.e., which of the available plant inputs are to be used to control each of the plant outputs. The most popular loop pairing method is the RGA and NI based pairing rules as follows [6,11,12].

The relative gain for variable pairing $y_i - u_j$ is defined as the ratio of two gains representing, first, the process gain in an isolated loop and, second, the apparent process gain in the same loop when all other loops are closed,

$$\lambda_{ij} = \frac{(\partial y_i/\partial u_j)_{u_{l\neq j}\text{constant}}}{(\partial y_i/\partial u_j)_{y_{k\neq i}\text{constant}}} = g_{ij}[\mathbf{G}^{-1}]_{ji},$$

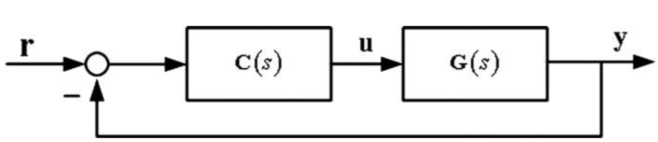

Fig. 1. Block diagram of general decentralized control system.

and RGA, $\boldsymbol{\Lambda}(\mathbf{G})$, in matrix form is defined as,

$$\boldsymbol{\Lambda}(\mathbf{G}) = \{\lambda_{ij} | i, j = 1, 2, \ldots, n\} = \mathbf{G} \otimes \mathbf{G}^{-T},$$

where $\otimes$ is the Hadamard product and G^{-T} is the transpose of the inverse of $\boldsymbol{G}$.

Furthermore, if all n loops are closed, the multi-loop system will be unstable for all possible (any) values of controller parameters (i.e., it will be "structurally monotonic unstable"), if the NI is negative, i.e.

$$NI = \frac{\det[\mathbf{G}(j0)]}{\prod_{i=1}^{n} g_{ii}(j0)} < 0,$$

where $\det[\mathbf{G}(j0)]$ denotes the determinant of matrix $\mathbf{G}(j0)$. The sign of NI, i.e., NI > 0, provides a necessary stability condition and consequently, constitutes a complementary tool to the RGA in variable pairing selection.

The pairing rules based on RGA and NI are that manipulated and controlled variables in a decentralized control system should be paired in such a way: (i) the paired RGA elements are closest to 1.0; (ii) the NI is positive, (iii) all paired RGA elements are positive; and (iv) large RGA elements should be avoided.

One of the main advantages of above pairing rules is that the interaction evaluation depends on only the steady-state gains. This information is easily obtained from simple identification experiments or steady-state design models. A potential weakness of these rules, however, is the same fact that they on only use the steady-state gains which based the assumption of perfect loop control to determine loop pairing. We use the following example to illustrate this point.

Example 1. Consider a 2×2 process [20] with transfer function matrix

$$\mathbf{G}(s) = \begin{pmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{pmatrix}.$$

The steady-state RGA is obtained as

$$\boldsymbol{\Lambda}(\mathbf{G}(j0)) = \begin{pmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{pmatrix}.$$

Obviously, both diagonal and off-diagonal pairings have positive RGA elements, and it is easy to verify that they also have positive NIs (diagonal pairing: NI = 1.2 and off-diagonal pairing: NI = 6.0, respectively). Since the relative gains of diagonal elements are close to unity which indicates a small amount of interaction between control loops, the diagonal pairing is suggested by the RGA and NI based loop pairing rules. However, Mc Avoy et al. used DRGA and optimal decentralized PI controllers for various configurations, and found that the diagonal pairing resulted in a poor closed-loop performance [20]. The main reason for the poor performance of the diagonal pairing is the dynamic properties of the transfer functions. It can be easily seen that the time constants and delays (10 and 4) of the off-diagonal elements are 10 times smaller than those (100 and 40) of the diagonal ones. In such case, pairing the faster loops (even with smaller steady-state gains) take the advantage of the time scale decoupling such that seriousness of the interactions from the slower loop would be reduced.

3. Relative normalized gain array

In the design of a decentralized control system, it is desired that inputs and outputs with dominant transfer functions be paired together for effective control. Generally, two factors in the open-loop transfer functions will affect the loop pairing decision and should be focused upon when considering the effect of interactions:

- Steady-state information: the steady-state gain of the transfer function $g_{ij}(s)$ reflects the effect of manipulated variable u_j on controlled variable y_i when the system is stable.
- Transient information: the transient information of the transfer function $g_{ij}(s)$ is accountable for the sensitivity of the controlled variable y_i to manipulated variable u_j and, consequently, the promptness of a particular output response to an input and the ability to reject the interactions from other loops.

Hence for decentralized control system design, it is desired that the control structure can be configured based on an effective evaluation of control-loop interactions in terms of both steady-state and transient information. For steady-state information, it can be easily extracted from the process steady-state gain matrix. While for the dynamic information, it can be obtained from the process responses to an input such as pulses, steps, ramps or other deterministic signals. Since step inputs are often used in control system identification and synthesis due to its simple physical interpretation and implementation, we will adopt the step response analysis in our development.

There are several criteria to evaluate the characteristics of a transfer function, here, we adopt integrated error (IE) criterion to evaluate the process dynamic properties as:

- Since the process input may cover the whole frequency domain, an evaluation of overall process dynamics is more interested than those of particular frequency points.
- From the fundamentals of feedback control theory [22,23], there must has at least one zero pole (integrator) in the open-loop transfer function of the feedback control system so that the steady-state closed-loop output error is zero. This zero pole contributes to controller output by IE which is directly related to the process dynamics.

Let

$$g_{ij}(s) = g_{ij}(j0) \times \bar{g}_{ij}(s),$$

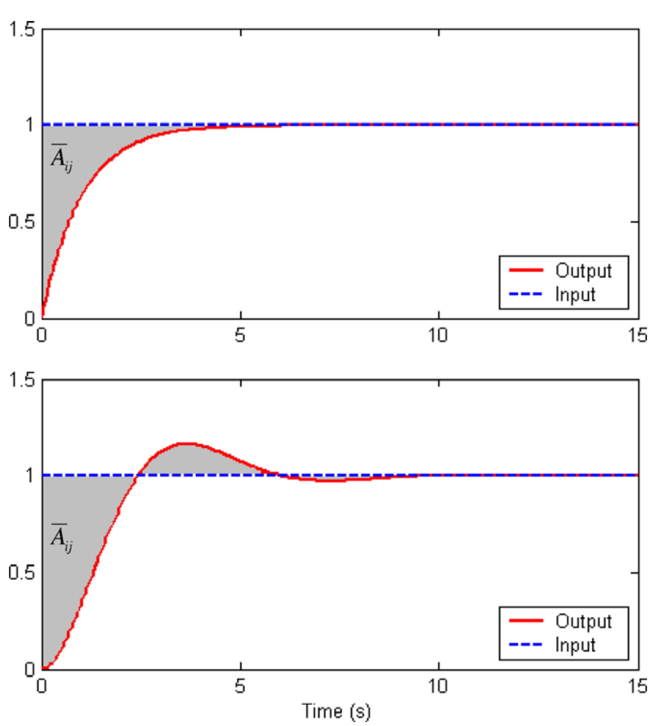

Fig. 2. Typical waveforms of non-oscillatory (top) and oscillatory (bottom) processes with $\bar{A}_{ij}$ indicated by shaded area.

where $g_{ij}(j0)$ and $\bar{g}_{ij}(s)$ with $\bar{g}_{ij}(j0) = 1$ are the steady-state gain and the normalized transfer function of $g_{ij}(s)$ respectively, and assume that the process $\bar{g}_{ij}(s)$ is open-loop stable and its output $\bar{y}_i = \bar{g}_{ij}(s)u_j$ is initially rest at zero, then a unit step disturbance is applied at the process input u_j. Since most industrial processes are either non-oscillatory or even oscillatory but well damped as shown in Fig. 2 ($\bar{A}_{ij}$ indicated by the shade area), the process output $\bar{y}_i$ will go to unity. We thus have

$$\bar{A}_{ij} = \int_0^\infty (\bar{y}_i(\infty) - \bar{y}_i(t))\,dt.$$

As a accumulation of the difference between the expected and the real outputs of process $\bar{g}_{ij}(s), \bar{A}_{ij}$, in fact, is equal to the average residence time $\tau_{ar,ij}$ of $\bar{g}_{ij}(s)$ [24], i.e., $\tau_{ar,ij} = \bar{A}_{ij}$. Apparently, smaller $\tau_{ar,ij}$ indicates that the transfer function has fast response to input disturbance, while larger $\tau_{ar,ij}$ indicates the open-loop process has slower process dynamics. Therefore, the average residence time $\tau_{ar,ij}$ can effectively reflect the process dynamics of $\bar{g}_{ij}(s)$, and accordingly $g_{ij}(s)$.

Thus far, two important parameters for the process $g_{ij}(s)$ are obtained:

- Steady-state gain $g_{ij}(j0)$: the steady-state gain reflects the effect of the manipulated variable u_j to the controlled variable y_i.
- Average residence time $\tau_{ar,ij}$: the average residence time is accountable for the response speed of the controlled variable y_i to manipulated variable u_j.

In order to use above both parameters for interaction measure and loop pairing, we now define the normalized gain (NG) $k_{N,ij}$ for a particular transfer function $g_{ij}(s)$ as

$$k_{N,ij} = \frac{g_{ij}(j0)}{\tau_{ar,ij}}. \tag{1}$$

Eq. (1) indicates that a large value of $k_{N,ij}$ implies that the combination effect of the manipulated variable u_j to the controlled variable y_i and the response speed of the controlled variable y_i to manipulated variable u_j is large. Therefore, the loop pairing with large normalized gain $k_{N,ij}$ should be preferred.

Extend Eq. (1) to all elements of transfer function matrix $\mathbf{G}(s)$, one can obtain the normalized gain matrix $\mathbf{K}_N$ as

$$\mathbf{K}_N = [k_{N,ij}]_{n\times n} = \mathbf{G}(j0) \odot \mathbf{T}_{ar}, \tag{2}$$

where $\mathbf{T}_{ar} = [\tau_{ar,ij}]_{n\times n}$ and $\odot$ indicates element-by-element division.

Since $k_{N,ij}$ indicates the control effectiveness from manipulated variable u_j to controlled variable y_i in terms of steady-state and process dynamics, the bigger the $k_{N,ij}$ value is, the more dominant the loop will be.

Remark 1. Even though more precise higher-order process models can be obtained by either physical model construction (following the mass and energy balance principles) or the classical parameter identification methods, from a practical point of view, the lower order process models are more convenient for control system design. The normalized gains of first order plus delay time (FOPDT) and second order plus delay time (SOPDT) processes are given in appendix.

Similar to the definition of relative gain [6], by replacing the steady-state gain matrix with the normalized gain matrix $\mathbf{K}_N$ of Eq. (2), we define the relative normalized gain (RNG) between output variable y_i and input variable u_j, ϕ_{ij}, as the ratio of two normalized gains:

$$\phi_{ij} = \frac{k_{N,ij}}{\hat{k}_{N,ij}},$$

where $\hat{k}_{N,ij}$ is the effective gain between output variable y_i and input variable u_j when all other loops are closed. When the relative normalized gains are calculated for all the input/output combinations of a multivariable process, it results in an array of the form similar to that of RGA, we call it as relative normalized gain array (RNGA): $\mathbf{\Phi} = [\phi_{ij}]_{n\times n}$, which can be calculated by

$$\mathbf{\Phi} = \mathbf{K}_N \otimes \mathbf{K}_N^{-T}.$$

In analogy to RGA, we here provide some important properties of the RNGA:

(i) The value of ϕ_{ij} is a measure of the effective interaction expected in the ith loop if its output y_i is paired with u_j.
(ii) The elements of the RNGA across any row, or down any column, sum up to 1, i.e.,

$$\sum_{i=1}^{n}\phi_{ij} = \sum_{j=1}^{n}\phi_{ij} = 1.$$

(iii) Let $\hat{k}_{N,ij}$ represent the normalized gain of the ith loop when all the other loops are closed, whereas $k_{N,ij}$ represents the normal, open-loop normalized gain, then:

$$\hat{k}_{N,ij} = \frac{1}{\phi_{ij}} k_{N,ij}.$$

RGA–NI–RNGA based control configuration rules. As RGA and NI tools are based on steady-state information and can provide sufficient conditions for the structurally unstable control configurations, they are adopted here to eliminate those structures with unstable pairing options. Thus, the RGA–NI–RNGA based control configuration rules are developed as: Manipulated and controlled variables in a decentralized control system should be paired in the following way that:

(i) All paired RGA elements are positive.
(ii) NI is positive.
(iii) The paired RNGA elements are closest to 1.0.
(iv) Large RNGA elements should be avoided.

Here, all tools RNGA, RGA and NI offer important insights into the issue of control structure selection. RNGA is used to measure the loop interactions at the whole frequency range, while RGA and NI are used as a sufficient condition to rule out the closed-loop unstable pairings. The significance of development of RNGA are:

(i) RNGA considers not only the process steady-state information but also the transient information in measuring the loop interactions, therefore, it provides more accurate pairing results than that of RGA based pairing criterion.
(ii) RNGA only uses information of open-loop process transfer functions and provides a comprehensive description of dynamic interactions among individual control loops without requiring the specification of controller type, therefore, it is controller type independent and with much less computation load than DRGA method.
(iii) RNGA uses the average residence time to account for the overall process dynamics and is critical frequency point independent, therefore, it requires much less calculation but resulting in a unique and optimal loop pairing decision, which is more efficient than REGA (especially when the process transfer function contains time delay, the critical frequency points for calculating REGA cannot be obtained directly without powerful calculation tools such as MATLAB, etc.).
(iv) RNGA is very simple for field engineers to understand and work out pairing decisions in practical applications.

Remark 2. For system that has m zero poles, the transfer function can be factorized as

$$g_{ij}(s) = \frac{1}{s^m} \times g_{ij}(j0) \times \bar{g}_{ij}(s), \tag{3}$$

where $\bar{g}_{ij}(j0) = 1$. Since these integrators are always removed during controller design [23–25], the normalized gain as well as RNGA can be calculated by using $\bar{g}_{ij}(s)$ in Eq. (3).

4. Case study

4.1. Example 1 continued

According to appendix, the average residence time matrix $\mathbf{T}_{ar}$ is obtained as

$$\mathbf{T}_{ar} = \begin{pmatrix} 140 & 14 \\ 14 & 140 \end{pmatrix}.$$

Above equation indicates that the diagonal pairing has more sluggish response due to larger average residence times. Therefore, even though the diagonal pairing is dominant at steady-state and even very low frequency band $([\omega \in [0 \quad \frac{1}{140}))$, the off-diagonal pairing becomes dominant at middle frequency band $(\omega \in (\frac{1}{140} \quad \frac{1}{14}))$. To consider both steady-state and dynamic information, the normalized gain matrix is obtained as

$$\mathbf{K}_N = \begin{pmatrix} 0.0357 & 0.0714 \\ -0.3571 & 0.0357 \end{pmatrix}.$$

Thus, the RNGA can be calculated as

$$\mathbf{\Phi} = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} = \begin{pmatrix} 0.0476 & 0.9524 \\ 0.9524 & 0.0476 \end{pmatrix}.$$

Apparently, the off-diagonal pairing is the best one with the smallest interactions between control loops, and should be selected.

4.2. Example 2

Consider the two-input two-output process

$$\mathbf{G}(s) = \begin{pmatrix} \frac{5e^{-s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-s}}{100s+1} \end{pmatrix}$$

The RGA, REGA and RNGA are obtained and listed in Table 1.

Table 1 shows that

(i) REGA is critical frequency dependent, which means, with selecting different critical frequencies, $\omega_{c,ij} = \omega_{u,ij}$ or $\omega_{c,ij} = \omega_{b,ij}$, REGA suggests different control structure configurations, however, both RGA and RNGA do not need the critical frequency information, require less computation load especially for those processes with time delays, and result in an unique decision.
(ii) The calculations for both RGA and RNGA are very simple, however, with taking the process dynamics into account, RNGA is more accurate.

To illustrate the validity of above results, decentralized controllers for both diagonal and off-diagonal pairings are designed respectively based on the IMC-PID controller tuning rules [25]. The obtained controller settings are given in Table 2. To evaluate the output control performance, we consider a unit step set-point change ($r_i(t) = 1$) of all control loops one-by-one and the integral

Table 1
The obtained RGA, REGA and RNGA for Example 2.

Tools	RGA	REGA		RNGA
		$\omega_{c,ij}=\omega_{u,ij}$ with $arg[\bar{g}_{ij}(j\omega_{u,ij})]=-\pi$	$\omega_{c,ij}=\omega_{b,ij}$ with $\lvert\bar{g}_{ij}(j\omega_{b,ij})\rvert=\sqrt{2}/2$	
Calculated results	$\begin{pmatrix}0.8333 & 0.1667\\0.1667 & 0.8333\end{pmatrix}$	$\begin{pmatrix}0.9840 & 0.0160\\0.0160 & 0.9840\end{pmatrix}$	$\begin{pmatrix}0.0476 & 0.9524\\0.9524 & 0.0476\end{pmatrix}$	$\begin{pmatrix}0.0876 & 0.9124\\0.9124 & 0.0876\end{pmatrix}$
Conclusions	Diagonal pairing	Diagonal pairing	Off-diagonal pairing	Off-diagonal pairing

square error (ISE) of $e_i(t) = r_i(t) - y_i(t)$ is used to evaluate the control performance

$$\mathrm{ISE} = \int_0^\infty e_i^2(t)\,dt.$$

The simulation results and ISE values are given in Fig. 3. The results show that the off-diagonal pairing gives better overall control system performance.

4.3. Example 3

Consider a 3×3 process with transfer function matrix

$$\mathbf{G}(s) = \begin{pmatrix} \frac{e^{-9s}}{6s^2+17s+1} & \frac{-9e^{-5s}}{s^2+4s+1} & \frac{13e^{-3s}}{3s^2+35s+1} \\ \frac{-5e^{-13s}}{2s^2+19s+1} & \frac{8e^{-2s}}{s^2+33s+1} & \frac{7e^{-5s}}{s^2+3s+1} \\ \frac{-16e^{-3s}}{s^2+5s+1} & \frac{3e^{-7s}}{s^2+14s+1} & \frac{e^{-11s}}{3s^2+25s+1} \end{pmatrix}.$$

The steady-state RGA is obtained as

$$\mathbf{\Lambda}(\mathbf{G}(j0)) = \begin{pmatrix} -0.0054 & 0.3981 & 0.6073 \\ -0.0992 & 0.6912 & 0.4080 \\ 1.1046 & -0.0893 & -0.0153 \end{pmatrix}.$$

From the RGA based loop pairing rules, the off-diagonal pairing with $NI = 1.4537$ is desired for decentralized control configuration. However, with process dynamics considered, RNGA suggests a different decentralized control structure.

According to appendix, both $\mathbf{T}_{ar}$ and $\mathbf{K}_N$ can be obtained easily as

$$\mathbf{T}_{ar} = \begin{pmatrix} 26 & 9 & 38 \\ 32 & 35 & 8 \\ 8 & 21 & 36 \end{pmatrix},$$

$$\mathbf{K}_N = \begin{pmatrix} 0.0385 & -1.0000 & 0.3421 \\ -0.1563 & 0.2286 & 0.8750 \\ -2.0000 & 0.1429 & 0.0278 \end{pmatrix}.$$

Table 2
Decentralized PI controllers for both control configurations of Example 2[a].

Loop	Diagonal pairing		Off-diagonal pairing	
	k_{Pi}	τ_{Ii}	k_{Pi}	τ_{Ii}
1	0.5	100	1.25	10
2	0.5	100	−0.25	10

[a] The PI controller is in form of $c_i = k_{Pi}\left(1 + \frac{1}{\tau_{Ii}s}\right)$.

Table 3
Decentralized PI controllers for both control configurations of Example 3[a].

Control loop	Pairing $y_1-u_3/y_2-u_2/y_3-u_1$			Pairing $y_1-u_1/y_2-u_3/y_3-u_2$		
	k_{Pi}	τ_{Ii}	τ_{Di}	k_{Pi}	τ_{Ii}	τ_{Di}
1	0.0292	35.0	0.0857	−0.0363	4	0.2500
2	0.0142	33.0	0.0303	0.0346	3	0.3333
3	−0.0515	5.0	0.2000	−0.0518	5	0.2000

[a] The PID controller is in form of $c_i = k_{Pi}\left(1 + \frac{1}{\tau_{Ii}s} + \tau_{Di}s\right)$.

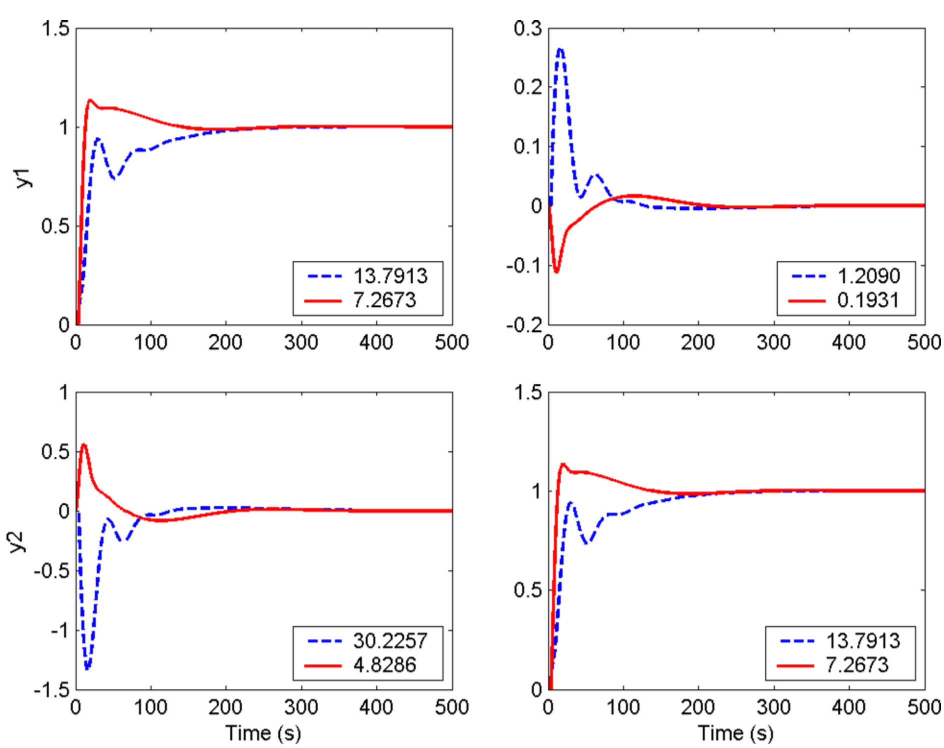

Fig. 3. Simulation results of Example 2 (dotted lines: diagonal pairing, solid lines: off-diagonal pairing).

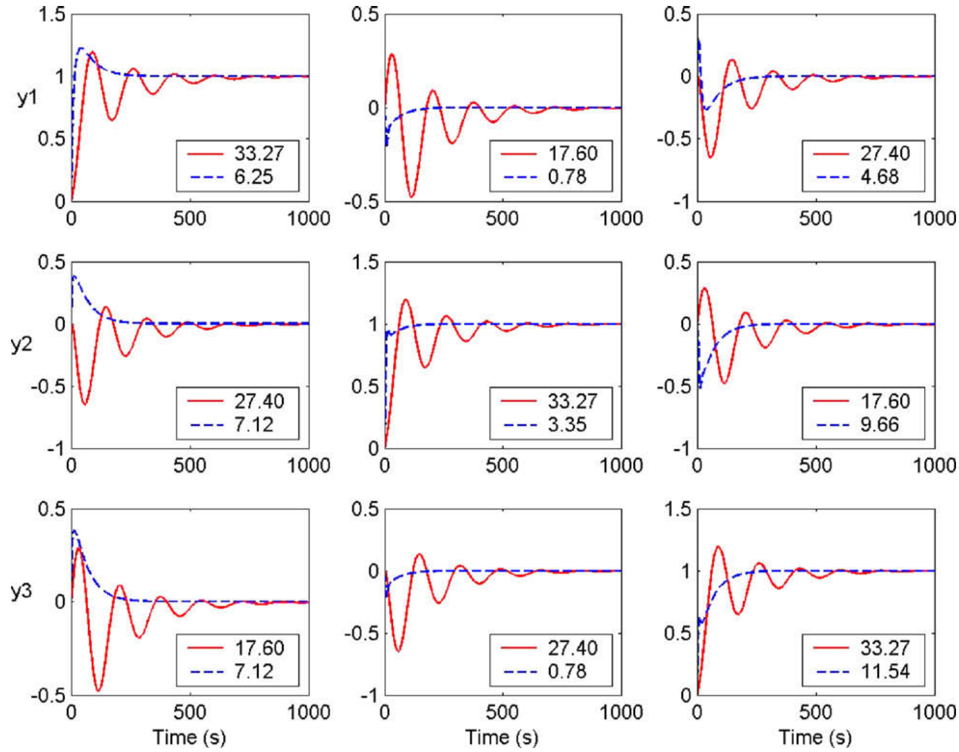

Fig. 4. Simulation results of Example 3 (solid lines: pairing $y_1 - u_3/y_2 - u_2/y_3 - u_1$, dash lines: pairing $y_1 - u_2/y_2 - u_3/y_3 - u_1$).

Then the RNGA is

$$\mathbf{\Phi} = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} = \begin{pmatrix} -0.0024 & 0.9237 & 0.0787 \\ -0.0063 & 0.0829 & 0.9235 \\ 1.0088 & -0.0066 & -0.0022 \end{pmatrix}$$

which indicates that the pairing $y_1 - u_2/y_2 - u_3/y_3 - u_1$ ($NI = 2.3998$) should be preferred for decentralized control. To test whether the suggested pairing is correct or not, decentralized controllers for cases of pairings $y_1 - u_3/y_2 - u_2/y_3 - u_1$ and $y_1 - u_2/y_2 - u_3/y_3 - u_1$ are designed respectively based on the IMC-PID controller tuning rules [25]. The obtained controller settings and simulation results are given in Table 3 and Fig. 4 respectively.

Fig. 4 shows that the overall performance of pairing $y_1 - u_2/y_2 - u_3/y_3 - u_1$ is significantly better than that of pairing $y_1 - u_3/y_2 - u_2/y_3 - u_1$.

Comparatively, however, the RNGA based methodology is much simpler and easier to be implemented.

5. Conclusion

In this paper, a new loop pairing criterion based on a new method for interaction measurement was proposed. Both the steady-state and transient information of the process transfer function are investigated, and the RNGA was introduced for loop interaction measurements. The effectiveness of the method was demonstrated by several examples, for which the RGA based loop pairing criterion gives an inaccurate interaction assessment, while the proposed interaction measure and loop pairing criterion provide accurate results. This method is very easy to be implemented and can be a very useful tool in design of the decentralized and decoupling control systems. The design of the decentralized controller especially for high dimensional processes and the robustness analysis against parametric and structural model errors by using RNGA information are currently under investigation and the results will be reported later.

Appendix A

Normalized gain of FOPDT process

The transfer function for FOPDT process is given as

$$g_{ij}(s) = \frac{k_{ij}}{\tau_{ij}s + 1} e^{-\theta_{ij}s}.$$

The normalized transfer function and its step response in time domain are thus obtained respectively as:

$$\bar{g}_{ij}(s) = \frac{1}{\tau_{ij}s + 1} e^{-\theta_{ij}s},$$

and

$$\bar{y}_i(t) = 1 - e^{-(t-\theta_{ij})/\tau_{ij}}.$$

Subsequently, the average residence time $\tau_{ar,ij}$ can be obtained as

$$\tau_{ar,ij} = \bar{A}_{ij} = \int_0^\infty [\bar{y}_i(\infty) - \bar{y}_i(t)]dt = \int_0^\infty [1 - (1 - e^{-(t-\theta_{ij})/\tau_{ij}})]dt$$
$$= \int_0^\infty e^{-(t-\theta_{ij})/\tau_{ij}} dt = \tau_{ij} + \theta_{ij}.$$

Hence, the normalized gain of $g_{ij}(s)$ is obtained as

$$k_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}} = \frac{k_{ij}}{\tau_{ij} + \theta_{ij}}. \tag{A1}$$

Normalized gain of SOPDT process

The transfer function for SOPDT process is given as

$$g_{ij}(s) = k_{ij} \times \frac{1}{a_{ij}s^2 + b_{ij}s + 1} e^{-\theta_{ij}s}$$
$$= k_{ij} \times \frac{\omega_{n,ij}^2}{s^2 + 2\zeta_{ij}\omega_{n,ij}s + \omega_{n,ij}^2} e^{-\theta_{ij}s}, \tag{A2}$$

where $\omega_{n,ij} = \frac{1}{\sqrt{a_{ij}}}$, and $\zeta_{ij} = \frac{b_{ij}}{2\sqrt{a_{ij}}}$.

Then two cases should be considered:

(i) When $0 < \zeta_{ij} < 1$, the transient function and its step response in time domain are thus obtained respectively as

$$\bar{g}_{ij}(s) = \frac{\omega_{n,ij}^2}{s^2 + 2\zeta_{ij}\omega_{n,ij}s + \omega_{n,ij}^2} e^{-\theta_{ij}s}$$

and

$$\bar{y}_i(t) = \begin{cases} 0 & t < \theta_{ij}, \\ 1 - \frac{e^{-\zeta_{ij}\omega_{n,ij}(t-\theta_{ij})}}{\sqrt{1-\zeta_{ij}^2}} \sin[\omega_{n,ij}\sqrt{1-\zeta_{ij}^2}(t-\theta_{ij}) + \tan^{-1}\frac{\sqrt{1-\zeta_{ij}^2}}{\zeta_{ij}}] & t \geqslant \theta_{ij}. \end{cases}$$

Subsequently, the average residence time $\tau_{ar,ij}$ can be obtained as

$$\begin{aligned} \tau_{ar,ij} = \bar{A}_{ij} &= \int_0^\infty [\bar{y}_i(\infty) - \bar{y}_i(t)]dt \\ &= \int_0^{\theta_{ij}} 1\,dt + \int_{\theta_{ij}}^\infty \frac{e^{-\zeta_{ij}\omega_{n,ij}(t-\theta_{ij})}}{\sqrt{1-\zeta_{ij}^2}} \\ &\quad \times \sin\left[\omega_{n,ij}\sqrt{1-\zeta_{ij}^2}(t-\theta_{ij}) + \tan^{-1}\frac{\sqrt{1-\zeta_{ij}^2}}{\zeta_{ij}}\right]dt \\ &= \frac{2\zeta_{ij}}{\omega_{n,ij}} + \theta_{ij}. \end{aligned}$$

Hence, the normalized gain of $g_{ij}(s)$ is obtained as

$$k_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}} = \frac{k_{ij}}{\frac{2\zeta_{ij}}{\omega_{n,ij}} + \theta_{ij}}.$$

(i) When $1 < \zeta_{ij} < \infty$, the transient function given in Eq. (A2) can be re-written as

$$\bar{g}_{ij}(s) = \frac{1}{(\tau_{1,ij}s+1)(\tau_{2,ij}s+1)} e^{-\theta_{ij}s},$$

with

$$\tau_{1,ij} = \frac{1}{\omega_{n,ij}\left(\zeta_{ij} + \sqrt{\zeta_{ij}^2 - 1}\right)}$$

and

$$\tau_{2,ij} = \frac{1}{\omega_{n,ij}\left(\zeta_{ij} - \sqrt{\zeta_{ij}^2 - 1}\right)}.$$

The step response in time domain is thus obtained as

$$\bar{y}_i(t) = \begin{cases} 0 & t < \theta_{ij}, \\ 1 + \frac{1}{\tau_{2,ij} - \tau_{1,ij}}\left(\tau_{1,ij}e^{-\frac{t-\theta_{ij}}{\tau_{1,ij}}} - \tau_{2,ij}e^{-\frac{t-\theta_{ij}}{\tau_{2,ij}}}\right) & t \geqslant \theta_{ij}. \end{cases}$$

Subsequently, the average residence time $\tau_{ar,ij}$ can be obtained as

$$\begin{aligned} \tau_{ar,ij} = \bar{A}_{ij} &= \int_0^\infty [\bar{y}_i(\infty) - \bar{y}_i(t)]dt \\ &= \int_0^{\theta_{ij}} 1\,dt + \int_{\theta_{ij}}^\infty \frac{-1}{\tau_{2,ij} - \tau_{1,ij}}(\tau_{1,ij}e^{-\frac{t-\theta_{ij}}{\tau_{1,ij}}} - \tau_{2,ij}e^{-\frac{t-\theta_{ij}}{\tau_{2,ij}}})dt \\ &= \tau_{1,ij} + \tau_{2,ij} + \theta_{ij} = \frac{2\zeta_{ij}}{\omega_{n,ij}} + \theta_{ij}. \end{aligned}$$

Hence, the normalized gain of $g_{ij}(s)$ is obtained as

$$k_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}} = \frac{k_{ij}}{\frac{2\zeta_{ij}}{\omega_{n,ij}} + \theta_{ij}}.$$

Combining above both cases and since $b_{ij} = \frac{2\zeta_{ij}}{\omega_{n,ij}}$, the average residence time $\tau_{ar,ij}$ and the normalized gain $k_{N,ij}$ for SOPDT process $g_{ij}(s)$ are

$$\tau_{ar,ij} = b_{ij} + \theta_{ij},$$

$$k_{N,ij} = \frac{k_{ij}}{b_{ij} + \theta_{ij}}.$$

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