



Parallel genetic algorithm in bus route headway optimization

Bin Yu^{a,*}, Zhongzhen Yang^a, Xueshan Sun^a, Baozhen Yao^b, Qingcheng Zeng^a, Erik Jeppesen^c

^a Transportation Management College, Dalian Maritime University, Dalian 116026, PR China

^b School of Civil Engineering & Architecture, Beijing Jiaotong University, Beijing 100044, PR China

^c Centre of Maritime Research, University of Southern Denmark, DK-5230 Odense, Denmark

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ABSTRACT

In this paper, a model for optimizing bus route headway is presented in a given network configuration and demand matrix, which aims to find an acceptable balance between passenger costs and operator costs, namely the maximization of service quality and the minimization of operational costs. An integrated approach is also proposed in the paper to determine the relative weights between passenger costs and operator costs. A parallel genetic algorithm (PGA), in which a coarse-grained strategy and a local search algorithm based on Tabu search are applied to improve the performance of genetic algorithm, is developed to solve the headway optimization model. Data collected in Dalian City, China, is used to verify the feasibility of the model and the algorithm. Results show that the reasonable resource assessment can increase the benefits of transit system.

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1. Introduction

With the increase in concern on the environment pollution and traffic congestion, authorities of most cities in China have formed many strategies on giving priority to the development of urban public transportation system. During transit operation, there are some important tasks including network design, frequency design (headway design), setting timetables, scheduling vehicles to trips, and assignment of drivers [12]. Among these tasks, timetable (dispatching schedule) of bus vehicles is one of the most important aspects in transit operation. The determination of dispatching time of each vehicle is based on the pre-planned time interval between two adjacent vehicles.

In this study, a bus headway (i.e., scheduled dispatching time interval of two successive buses) optimization model is proposed to minimize the total costs of passengers and operators in a given network configuration and demand matrix. At the same time, an integrated approach is proposed to explore the trade-off between passenger costs and operator costs, in which the relative importance and the difference between the two conflicting objectives are considered.

Transit scheduling problems in the real world are often inefficient to be solved by classical optimization techniques because of the large numbers of trips, bus routes and stations [13]. Recently, the heuristics are considered as feasible tools to solve combinato-

rial optimization problems [7]. Genetic algorithm [14], which is a multipurpose optimization tool, has successfully been applied in a wide range of optimization problems [5,10] including transportation fields [3,4,23,25]. For this reason, genetic algorithm (GA) is used in this study to determine bus headways of routes. Since the proposed model is to be applied in a real transit system, a local algorithm based on Tabu search and a coarse grain parallel strategy are introduced into GA to improve the performance of the algorithm.

This paper is organized as following: Section 2 is about the problems of basic notations and formulations; Section 3 contains the solution methods of determining bus route headway; Numerical analysis is carried out in Section 4; and lastly, the conclusions are drawn in Section 5.

2. Optimization model

In this study, the maximized social benefits are defined as minimizing the sum of passenger costs and operator costs [8]. In general, it is reasonable to provide enough capacity for all transit passengers on routes in planning stage. There are, however, the situations in which it is not feasible to provide enough transit capacity to avoid congestion, especially in the real transit system. In this study, the problem of determining bus route headways can be formulated as a nonlinear program subject to the vehicle fleet constraint. In the optimization model, the decision variables are the headway of each direction of bus routes ($h_{l,\Delta}$ denotes the headway in the direction Δ of the bus route l). Firstly, passenger costs (in money) and operator costs (in money) are described separately, and then the two sub-problems are integrated into one single model.

* Corresponding author.

E-mail addresses: ybzhyb@163.com, minlfish@yahoo.com.cn (B. Yu).

2.1. Passenger costs

Passenger costs are defined as the passenger travel time-costs, which include waiting time-costs at stops, riding/dwelling time-costs in vehicles and the boarding/alighting time-costs on/from vehicles.

2.1.1. Waiting time-costs

• Basic notations

Δ — Δ , which is a binary sign, denotes that the first direction or the return direction of route l . Here, $\Delta = 1$ denotes the direction with more vehicles and $\Delta = -1$ denotes the other directions. $T_{l,\Delta}^w$, passenger waiting time-costs in the direction Δ of the bus route l .

C_p^w , coefficients of waiting time-costs.

$u_{l,\Delta,k}$, the number of passengers waiting for buses in the direction Δ of the bus route l at the stop k .

$h_{l,\Delta}$, headway in the direction Δ in the bus route l . It is the decision variable in the model. Since there are several vehicles but not infinity vehicles for a route, it is reasonable that the headway of a route has a lower limit. In addition, over many vehicles will induce a larger demand for parking and storing vehicles. Here, the headway of a route is bigger than 60 s.

$t_{l,\Delta}^e$, expected waiting times for the direction Δ of the bus route l . Since passenger arrivals follow a uniform process, the expected waiting times of a passenger is the half of the headway, i.e., $t_{l,\Delta}^e = h_{l,\Delta}/2$.

$q_{l,\Delta,k}$, alighting proportion of the stop k in the direction Δ of the bus route l . It means that the alighting passengers are divided by the alighting passengers in all stops between current stops and the stops to the destination. For example, there are three stops $k, k1, k2$ (destinations), the alighting passengers are 2, 4, 4, respectively, thus $q_{l,\Delta,k} = 20\%$ ($2/(2+4+4) = 20\%$), $q_{l,\Delta,k1} = 50\%$, $q_{l,\Delta,k2} = 100\%$.

$a_{l,\Delta,k}$, the number of passengers alighting from buses in the direction Δ of the bus route l at the stop k .

$$a_{l,\Delta,k} = v_{l,\Delta,k-1 \rightarrow k} \times q_{l,\Delta,k} \quad (1)$$

V^{\max} , maximum capacity of a standard vehicle.

$n_{l,\Delta,k}$, the number of buses going through the stop k in the direction Δ of the bus route l during the researched period. Assume that the first bus of all routes starts from its origin stop at the same time during the researched period and the times is set to zero. The number of buses going through the stop k can be formulated as follow.

$$n_{l,\Delta,k} = \left\lfloor \frac{H - \bar{t}_{l,\Delta,k-1 \rightarrow k}}{h_{l,\Delta}} \right\rfloor \quad (2)$$

where $\lfloor y \rfloor$ is the floor integer symbol, which returns the largest integer less than or equal to y . For example, if $y = 5.2$, $\lfloor y \rfloor = 5$.

$v_{l,\Delta,k \rightarrow k+1}$, the number of passengers in buses in the direction Δ of the bus route l from the stops k to $k+1$.

$$v_{l,\Delta,k \rightarrow k+1} = v_{l,\Delta,k-1 \rightarrow k} + b_{l,\Delta,k} - a_{l,\Delta,k} \quad (3)$$

$b_{l,\Delta,k}$, the number of passengers boarding on buses in the direction Δ of the bus route l at the stop k .

$$b_{l,\Delta,k} = \begin{cases} u_{l,\Delta,k} & u_{l,\Delta,k} \leq n_{l,\Delta,k} \times (V^{\max} - v_{l,\Delta,k-1 \rightarrow k} + a_{l,\Delta,k}) \\ n_{l,\Delta,k} \times (V^{\max} - v_{l,\Delta,k-1 \rightarrow k} + a_{l,\Delta,k}) & \text{otherwise} \end{cases} \quad (4)$$

$\delta_{l,\Delta,k \rightarrow k+1}$, the number of non-served passengers at the stop k in the direction Δ of the bus route l . Since the provision of capacity on one route cannot always carry all passengers, $\delta_{n,l,k \rightarrow k+1}$ is

introduced, which denotes the number of non-served passengers at the stop k in the direction Δ of the bus route l .

$$\delta_{l,\Delta,k \rightarrow k+1} = u_{l,\Delta,k} - b_{l,\Delta,k} \quad (5)$$

φ , penalty coefficient for additional waiting times of non-served passengers.

• Formulation

The time-costs $T_{l,\Delta}^w$ of passengers waiting for buses in the direction Δ of the bus route l are formulated as follow:

$$T_{l,\Delta}^w = C_p^w \sum_k [u_{l,\Delta,k} \times t_{l,\Delta}^e + \varphi \times \delta_{l,\Delta,k} \times h_{l,\Delta}] \quad (6)$$

If the bus capacity can satisfy the passenger demand at the segment $k \rightarrow k+1$, the non-served passengers is 0, i.e., all the passengers can catch the first bus arriving at stop k in the direction of the route l after their arrival. Otherwise, the non-served passengers, besides the waiting times for the first bus, yet have to wait for the following buses. We assume that passengers are sequentially (on a first come, first served basis). Thus, the additional waiting times of non-served passengers equal to the headway in the direction Δ of the route. Generally, the additional waiting times of passengers are more costly. φ is a constant, which is used to reflect the influence of additional waiting times of passengers. It can increase with the increment of the missed vehicles or it may be a constant integer bigger than 1.

2.1.2. Riding/dwelling time-costs

• Basic notations

C_p^r , coefficients of the riding/dwelling time-costs.

$\omega_{l,\Delta,k \rightarrow k+1}$, comfortable level [11,25] of passengers from the stops k to $k+1$ in the direction Δ of the bus route l . Here, the comfort index is approximated by crowded level, which is used to tune with the weight of bus crowding.

$$\omega_{l,\Delta,k \rightarrow k+1} = \frac{v_{l,\Delta,k \rightarrow k+1}}{n_{l,\Delta,k} \times V} \quad (7)$$

$\bar{t}_{l,\Delta,k \rightarrow k+1}$, average running times from the stops k to $k+1$ in the direction Δ of the bus route l . In most existing researches, average running times in the two directions of a route are set as the same. However, considering inappropriate layout of living area and working area in most large cities in China, tide traffic phenomenon usually occurs in some main roads, especially during the rush hours. Thus the traffic in two directions of the main roads is imbalanced. Therefore, average running times of two directions of one route are computed, respectively.

$t_{l,\Delta,k}^d$, bus dwelling times at the stop k in the direction Δ of the bus route l .

t_0^d , constant times for vehicle acceleration, deceleration and door opening/closing.

H , researched period (e.g. a rush hour).

$\xi_{l,\Delta,k}$, crowded coefficient for boarding and alighting at the stop k in the direction Δ of the bus route l . Zolfaghari et al. [28] pointed out dwelling times of buses at stops are related to the bus load, besides the number of passengers boarding and alighting. This indicated that the boarding and alighting times of passengers would increase in the crowded conditions. Here,

the crowded coefficient is used to reflect the influence of the bus load to the boarding and alighting times of passengers.

V , rated capacity of a standard vehicle.

$$\xi_{l,\Delta,k} = \begin{cases} 1 & v_{l,\Delta,k \rightarrow k+1} \geq (n_{l,\Delta,k} \times V) \\ (v_{l,\Delta,k \rightarrow k+1} / (n_{l,\Delta,k} \times V))^2 & \text{otherwise} \end{cases} \quad (8)$$

\bar{b} , \bar{a} , average boarding and alighting times of a passenger.

$$t_{l,\Delta,k}^d = t_0^d + \max(b_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{b}, a_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{a}) \quad (9)$$

• Formulation

Riding/dwelling time-costs of the passengers $T_{l,\Delta}^r$ in the direction Δ of the bus route l are defined below:

$$T_{l,\Delta}^r = C_p^r \sum_k [(\omega_{l,\Delta,k \rightarrow k+1} \times \bar{t}_{l,\Delta,k \rightarrow k+1} \times v_{l,\Delta,k \rightarrow k+1}) + ((v_{l,\Delta,k \rightarrow k+1} - b_{l,\Delta,k}) \times t_{l,\Delta,k}^d)] \quad (10)$$

Riding times of passengers on the segment $k \rightarrow k+1$ are the products of the bus running times on the segment $k \rightarrow k+1$ ($\bar{t}_{l,\Delta,k \rightarrow k+1}$) and the number of the passengers ($v_{l,\Delta,k \rightarrow k+1}$) that can be carried by the direction Δ of the bus route l .

Although some buses could go through their current stops (e.g. stop k) and not arrive at their following stops (e.g. the stop $k+1$) at the end of the researched period, for simplification, we assume that these buses would have reached their following stops and computed the riding time of the passengers in these buses. Dwelling times of passengers are the waiting times of on-board passengers at the stop.

2.1.3. Boarding/alighting time-costs

• Basic notations

$T_{l,\Delta}^{ba}$, passenger boarding/alighting time-costs in the direction Δ of the bus route l .

C_p^{ba} , coefficients of boarding/alighting time-costs.

• Formulation

Passenger boarding/alighting time-costs $T_{l,\Delta}^{ba}$ in the direction Δ of the bus route l are defined below:

$$T_{l,\Delta}^{ba} = C_p^{ba} \sum_k [b_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{b} + a_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{a}] \quad (11)$$

$$\begin{aligned} T &= w^{\text{passenger}} (T_{l,\Delta}^w + T_{l,\Delta}^r + T_{l,\Delta}^{ba}) + w^{\text{operator}} \times T_{l,\Delta}^o \\ &= w^{\text{passenger}} \left[C_p^w \sum_k (u_{l,\Delta,k} \times t_{l,\Delta,k}^e + \varphi \times \delta_{l,\Delta,k} \times h_{l,\Delta}) + C_p^r \sum_k [(\omega_{l,\Delta,k \rightarrow k+1} \times \bar{t}_{l,\Delta,k \rightarrow k+1} \times v_{l,\Delta,k \rightarrow k+1}) + (v_{l,\Delta,k \rightarrow k+1} - b_{l,\Delta,k}) \times t_{l,\Delta,k}^d] \right. \\ &\quad \left. + C_p^{ba} \sum_k [b_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{b} + a_{l,\Delta,k} \times \xi_{l,\Delta,k} \times \bar{a}] \right] + w^{\text{operator}} \left[C_o^f \left\lceil \frac{H}{h_{l,\Delta}} \right\rceil + C_o^v \sum_k n_{l,\Delta,k} \times \bar{t}_{l,\Delta,k \rightarrow k+1} \right] \end{aligned} \quad (14)$$

2.2. Operator costs

• Basic notations

$r_{l,\Delta,k}$, average arrival rate at the stop k in the direction Δ of the route l . Assume passenger arrivals follow an uniform process during the researched period [6,27]. The average arrival rate equals to the boarding passengers at the stop during the period divided by H .

C_o^f , C_o^v , coefficients of fixed operational costs and variable operational costs, respectively.

$T_{l,\Delta}^o$, operator time-costs in the direction Δ of the bus route l .

• Formulation

Generally, operators and transit agencies all expect to provide the transit service in an economic efficiently way. Operator costs

$T_{l,\Delta}^o$ are consisting by fixed operational costs and variable operational costs.

$$T_{l,\Delta}^o = C_o^f \left\lceil \frac{H}{h_{l,\Delta}} \right\rceil + C_o^v \sum_k n_{l,\Delta,k} \times \bar{t}_{l,\Delta,k \rightarrow k+1} \quad (12)$$

where $\lceil y \rceil$ is the ceiling integer symbol, which returns the smallest integer more than or equal to y . For example, if $y = 5.2$, $\lceil y \rceil = 6$.

The fixed costs $C_o^f \times \lceil H/h_{l,\Delta} \rceil$ are consisting by capital discount costs, maintenance costs, salaries of the drivers, etc. The variable costs mainly concern about fuel consumptions. Like calculating passenger riding time-costs, we also assume that all the buses can arrive at following stops at the end of the researched period.

2.3. Model integration

Bus route headway optimization should find the trade-off between passenger costs and operator costs. Generally, if headways are excessively small (too many buses are dispatched), operators have to suffer excessively operational costs. However, if headways are excessively large, some service criteria may not be met which resulted in unsatisfied passengers who may choose the alternative means of transportation. One of the most widely used methods for solving multi-objective optimization problems [16,17] is the weighted sum method, which can transform the multiple objective optimization into a single objective function by weight factors. For simplification, a convex combination of the two objective functions is used in this study and combined the model of passenger costs (in money) and operator costs (in money) for total costs (T) of passengers and operators can be expressed as:

• Basic notations

g_l , approximate fleet size of the bus route l . Omitting dwelling times of buses at terminal, the needed fleet size of the more one of two directions is viewed as approximate fleet size of the route.

$$g_l = \max \left(\left\lceil \sum_{\Delta} \sum_k \bar{t}_{l,\Delta,k \rightarrow k+1} / h_{l,1} \right\rceil, \left\lceil \sum_{\Delta} \sum_k \bar{t}_{l,\Delta,k \rightarrow k+1} / h_{l,2} \right\rceil \right) \quad (13)$$

G , the total number of bus vehicles.

• Formulation

s.t.

$$h_{l,\Delta} \geq 60 \quad (15)$$

$$\sum_l g_l \leq G \quad (16)$$

$$\omega_{l,\Delta,k \rightarrow k+1} > 0 \quad (17)$$

$$w^{\text{passenger}}, w^{\text{operator}} \geq 0 \quad (18)$$

$$w^{\text{passenger}} + w^{\text{operator}} = 1 \quad (19)$$

where $w^{\text{passenger}}$ and w^{operator} are the factors controlling the weights of passenger costs and operator costs. In practice, the proper selection of the weight factor for each objective is difficult to be determined, because the definition of weights is not precise, nor are the values given by a decision-maker [19]. In fact, $w^{\text{passenger}}$ and w^{operator} can be determined by decision-makers, e.g.

$w^{\text{passenger}} = w^{\text{operator}}$, which reflect the subjective judgment or intuition of decision-makers. However, analysis results are based on the weights can be influenced by the decision-makers due to their lack of knowledge or experience. Therefore, it is an essential and challenging task to develop an objective method to assess the relative weight of the alternatives. An example of bus headway optimization weight factor determination is described in the following section.

3. Parallel genetic algorithm

Genetic algorithm is a search algorithm based on the concepts of natural selection and genetic operations. Many researchers attempted to improve the performance of GA by some methods [22]. Recently, parallel genetic algorithms (PGAs) have become one of the most effective strategies. Actually, PGA basically consists of various GAs, each processing a part of the population or independent populations, with or without communication between them. Therefore, PGA can increase the diversity of population and reduce computation time. Generally, it can be divided into three types [1], namely master-slave type, coarse-grained type and fine-grained type. Here, coarse-grained PGA is used since it costs less and can obtain a near-line acceleration ratio. Moreover, coarse-grained parallelization schemes run several subpopulations in parallel. So it is especially suitable for the cluster system with lower communication bandwidth.

3.1. Encoding

In this research, decision variables of the algorithm are the headways of two directions of each route. Here, an integer encoding scheme is selected to represent bus headways and then a typical chromosome is as follows:

$$\{e_{1,1}, e_{1,2}, e_{2,1}, e_{2,2}, \dots, e_{l,1}, e_{l,2}, \dots, e_{N,1}, e_{N,2}\} \quad (20)$$

The bus departure interval for a route is rarely less than 1 min (for example, the shortest departure interval for Dalian in China is 1 min), and thus we assume that each gene cannot be less than 1 min (i.e., 60 s). The initial population of chromosomes are generated by a probabilistic methodology. Firstly, temporary headways (genes), $e'_{l,\Delta}$ ($60 \leq e'_{l,\Delta} \leq 3600$), are obtained by producing some random numbers. Thus, some temporary chromosomes that consist of the temporary genes are constructed. Headways of routes are generally limited by the fleet size of bus vehicles. The temporary chromosomes need to be checked whether to satisfy the fleet constraint. The approximate fleet size g_l of each temporary chromosome is firstly calculated. Then, a scaled coefficient γ is gained according as the approximate fleet size and the total fleet size. The genes of the initial chromosomes are computed as formula (23).

$$\{e'_{1,1}, e'_{1,2}, e'_{2,1}, e'_{2,2}, \dots, e'_{l,1}, e'_{l,2}, \dots, e'_{N,1}, e'_{N,2}\} \quad (21)$$

$$\gamma = \frac{\sum_l g_l}{G} \quad (22)$$

$$e_{l,\Delta}^0 = \begin{cases} \lceil \gamma \times e'_{l,\Delta} \rceil & \lceil \gamma \times e'_{l,\Delta} \rceil > 60 \\ 60 & \text{otherwise} \end{cases} \quad (23)$$

If a gene $e_{l,1}$ is smaller than 0, the value of the gene is set as zero and the scaled coefficient is re-calculated. For example, if $e_{l,\Delta}^0 < 60$, then set $e_{l,\Delta}^0 = 60$. An initial chromosome is as follows.

$$\{e_{1,1}^{(0)}, e_{1,2}^{(0)}, e_{2,1}^{(0)}, e_{2,2}^{(0)}, \dots, e_{l,1}^{(0)}, e_{l,2}^{(0)}, \dots, e_{N,1}^{(0)}, e_{N,2}^{(0)}\} \quad (24)$$

3.2. Fitness function

GA is an optimal searching method to find the maximum fitness of the individual chromosome, so it is necessary to transform the minimal objective of the problem to a maximum fitness function [10,14]. Here, a constant Q is introduced to transform the fitness function from the total cost function. Generally, the genetic operations may violate total fleet size constraint. There are two approaches to deal with this situation. The first one is to assign a very high penalty cost for such candidate solutions and accordingly reduce their probability of being selected in the forthcoming search. The second approach is to try to fix the resultant violations by adjusting the headways. The advantage of the first approach over the second one is that it is more suitable in according with natural selection and evolution, and it enables GA to investigate further points in the search space. Therefore, the first approach is adopted to deal with the violation situation. Then, the chromosomes are evaluated as follows.

$$F = \frac{Q}{T + \Phi(\tau) \left(\sum_l g_l - G \right)} \quad (25)$$

$$\Phi(\tau) = \begin{cases} \beta_1 \tau^{\beta_2} & \text{if } \sum_l g_l > G \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

where F is the fitness function. Q is a constant. $\Phi(\tau)$ is the penalty coefficient at the generation τ . β_1, β_2 are control coefficients, which can determine the penalty extent for the invalid individuals. They can usually be estimated through simulation.

3.3. Selection operation

The basic part of the selection process is to select from one generation to create the basis of the next generation stochastically. The fittest individuals have a greater chance of survival than weaker ones is required. Here, the *Roulette wheel selection* method is used to select the chromosomes. Besides that, the *Elitism* is also used for the selection. Elitism is a selection method where the best chromosomes in the population are automatically copied into the next generation. That is, if the elitism parameter was set to ξ then the top ξ chromosomes in the population are copied to the next generation.

3.4. Crossover operation

The crossover operator is associated with a crossover rate p_c . An arithmetic crossover [26] is designed. Here, a random multi-stratum crossover method is adopted. For example, for a particular crossover process, at the generation $\tau - 1$, the two selected parent chromosomes E_1^τ and E_2^τ are as Eq. (27). If the random $\delta'_l > 0.5$, that mean two genes will be crossed, otherwise, the genes of parent chromosome would remain to the new chromosome directly. The crossover of two parent chromosomes is as Eq. (28).

$$\begin{aligned} E_1^\tau &= \{e_{1,1}^{(\tau)}, e_{1,2}^{(\tau)}, e_{2,1}^{(\tau)}, e_{2,2}^{(\tau)}, \dots, e_{l,1}^{(\tau)}, e_{l,2}^{(\tau)}, \dots, e_{N,1}^{(\tau)}, e_{N,2}^{(\tau)}\} \\ E_2^\tau &= \{e_{2,1}^{(\tau)}, e_{2,2}^{(\tau)}, e_{2,1}^{(\tau)}, e_{2,2}^{(\tau)}, \dots, e_{2,1}^{(\tau)}, e_{2,2}^{(\tau)}, \dots, e_{N,1}^{(\tau)}, e_{N,2}^{(\tau)}\} \end{aligned} \quad (27)$$

$$\begin{cases} e_{l,\Delta}^{(\tau)} = \left[\delta'_l e_{1,\Delta}^{(\tau-1)} + (1 - \delta'_l) e_{2,\Delta}^{(\tau-1)} \right] \\ e_{l,\Delta}^{(\tau)} = \left[\delta'_l e_{2,\Delta}^{(\tau-1)} + (1 - \delta'_l) e_{1,\Delta}^{(\tau-1)} \right] \\ e_{l,\Delta}^{(\tau)} = e_{1,\Delta}^{(\tau-1)} \\ e_{l,\Delta}^{(\tau)} = e_{2,\Delta}^{(\tau-1)} \end{cases} \quad \begin{matrix} \text{if } \delta'_l > 0.5 \\ \\ \text{otherwise} \end{matrix} \quad (28)$$

where δ'_l, δ''_l are random numbers between (0, 1).

3.5. Mutation operation

Like the crossover operator, the mutation operator is also associated with a mutation rate (p_m) to determine whether the mutation operator is to be applied to the chromosome or not. Since there are two directions in each bus route, both the headways (genes) of two directions of each route need to be mutated in each mutation. If $E^{\tau-1}$ denotes a parent chromosome in the generation $\tau - 1$ and the genes $e_{l,1}^{(\tau-1)}$, $e_{l,2}^{(\tau-1)}$ are selected for the mutation, the result of the mutation of $E^{\tau-1}$ and the mutated chromosome in the generation τ are shown in (29).

$$\begin{cases} e_{l,1}^{(\tau)} = \left[e_{l,1}^{(\tau-1)}(1 + \delta_l''') \right] & \text{if } \delta_l''' > 0.5 \\ e_{l,2}^{(\tau)} = \left[e_{l,2}^{(\tau-1)}(1 - \delta_l''') \right] & \text{otherwise} \end{cases} \quad (29)$$

where δ_l''' , δ_l''' are random numbers between (0, 1).

3.6. Local search algorithm based on TABU search

GA is a suitable method for global optimization problems. To improve the local optimization performance of the GA, a local search algorithm based on TABU search [9] is introduced to find the local optimum in a well-defined local region. Since, the frequent local searches can increase the computation time, the method is implemented with parallel strategy of GA (Section 3.6) in this study.

Tabu search is an iterative procedure that proceeds by transforming one solution into another by making moves. It has successfully been applied in solving the optimization problems in transportation fields [2,15,18,20,21]. The heuristics requires an initial solution and a neighborhood structure. In this study, the initial solution in the local search is to use the current optimal solution in the GA proposed in the previous subsection. Then, the neighbors of the initial solution are examined and the best non-forbidden move is selected. The neighborhood structure in the local search can be described as follows. For example, the chromosomes E^τ denotes the initial solution (Eq. (30)). Firstly, randomly select two genes, e.g. $e_{l,1}^{(\tau)}$, $e_{l,2}^{(\tau)}$ and $e_{l',1}^{(\tau)}$, $e_{l',2}^{(\tau)}$. A neighbor of the initial solution is as Eq. (31).

$$E^\tau = \{e_{1,1}^{(\tau)}, e_{1,2}^{(\tau)}, e_{2,1}^{(\tau)}, e_{2,2}^{(\tau)}, \dots, e_{l,1}^{(\tau)}, e_{l,2}^{(\tau)}, \dots, e_{l',1}^{(\tau)}, e_{l',2}^{(\tau)}, \dots, e_{N,1}^{(\tau)}, e_{N,2}^{(\tau)}\} \quad (30)$$

$$\begin{cases} e_{l,\Delta}^{(\tau)} = \left[\delta_l'''' e_{l,\Delta}^{(\tau)} + (1 - \delta_l''') e_{l',\Delta}^{(\tau)} \right] \\ e_{l',\Delta}^{(\tau)} = \left[\delta_l'''' e_{l',\Delta}^{(\tau)} + (1 - \delta_l''') e_{l,\Delta}^{(\tau)} \right] \end{cases} \quad (31)$$

where δ_l'''' is the random number between (0, 1).

In the local search, a Tabu list is used to prevent generating the degradation solution that has already tested in previous iterations. The size of the Tabu list can influence the search quality, and in our local search the large and fixed Tabu list is used, i.e., the size of the Tabu list is set to 20. The local search algorithm continues until the maximum total number of the iterations or the maximum number of the iterations without improvement of the best solution.

3.7. Coarse-grained strategy

The coarse-grain strategy runs several subpopulations in parallel. The information exchange among these subsets is done at certain the intervals (*epoch*) of iterations. By exchanging the “out-standing chromosomes” between subsets, the search spaces of the subsets are diversified to effectively prevent the premature convergence. Let η and P_η represent the amount of subsets and their scale, respectively, thus the total population $P_{size} = \eta \times P_\eta$.

The common migrating strategy [1,26] is adopted, in which one best individual to migrate, and then to replace the worse individual in a subset with the migrated ones from nearby subset. Here a ring topology is used, which means that subsets x exchange individually with subset $x + 1$ during migration.

3.8. Stopping criterion

When the average of the fitness values of all the individuals is greater than 90% of the fitness values of the best individual or when the algorithm repeats the prepared maximum number of generations, the PGA is considered to have converged and therefore is stopped.

The process of the PGA proposed in this study is described as Fig. 1.

4. Numerical test

The model and the algorithm are tested with the data of Dalian City in China. Dalian's population is about 2 million, the build-up area is about 180 km², and the road network consists of 3200 links and 2300 nodes. There are totally 89 bus lines (Fig. 2) and 3004 bus stops, which extend 1130 km, and with 4130 vehicles in it. Passenger origin-destination (OD) stop matrix is obtained from our former research [24].

4.1. Weight identification

Before optimizing the headways of routes, the weights of passenger costs and operator costs should be determined. It is often difficult for decision-makers to determine the weights because the definition of weights itself is not precise. This paper proposed an integration approach to determine the weights which considers both the relative importance and the difference of operator costs and passenger costs. The sample data consisted of the passenger costs and operator costs of the 89 bus routes ($I = 89$) of transit system in Dalian City. The passenger costs and operator costs between the 89 routes are different due to the different numbers of passengers and vehicles. Thus, the costs of the 89 routes need to be normalized. Firstly, the maximum passenger costs and operator costs among the 89 bus routes were used to scale the sample data. Here, the two attributes were scaled to the range between 0 and 1.

$$\begin{cases} \hat{T}_l^{\text{passenger}} = \frac{T_l^{\text{passenger,max}} - T_l^{\text{passenger}}}{T_l^{\text{passenger,max}} - T_l^{\text{passenger,min}}} \\ \hat{T}_l^{\text{operator}} = \frac{T_l^{\text{operator,max}} - T_l^{\text{operator}}}{T_l^{\text{operator,max}} - T_l^{\text{operator,min}}} \end{cases} \quad (32)$$

$$\begin{cases} T_l^{\text{passenger,max}} = \max(T_l^{\text{passenger}}) \\ T_l^{\text{operator,max}} = \max(T_l^{\text{operator}}) \end{cases} \quad (33)$$

$$\begin{cases} T_l^{\text{passenger,min}} = \min(T_l^{\text{passenger}}) \\ T_l^{\text{operator,min}} = \min(T_l^{\text{operator}}) \end{cases} \quad (34)$$

where $T_l^{\text{passenger}}$ and T_l^{operator} denote the origin passenger costs and operator costs of the existing routes, respectively. $T_l^{\text{passenger,max}}$ and $T_l^{\text{passenger,min}}$ indicate the maximum and minimum passenger costs. Similarly, $T_l^{\text{operator,max}}$ and $T_l^{\text{operator,min}}$ indicate the maximum and minimum operator costs. $\hat{T}_l^{\text{passenger}}$ and $\hat{T}_l^{\text{operator}}$ indicate the scaled passenger costs and operator costs of the existing routes.

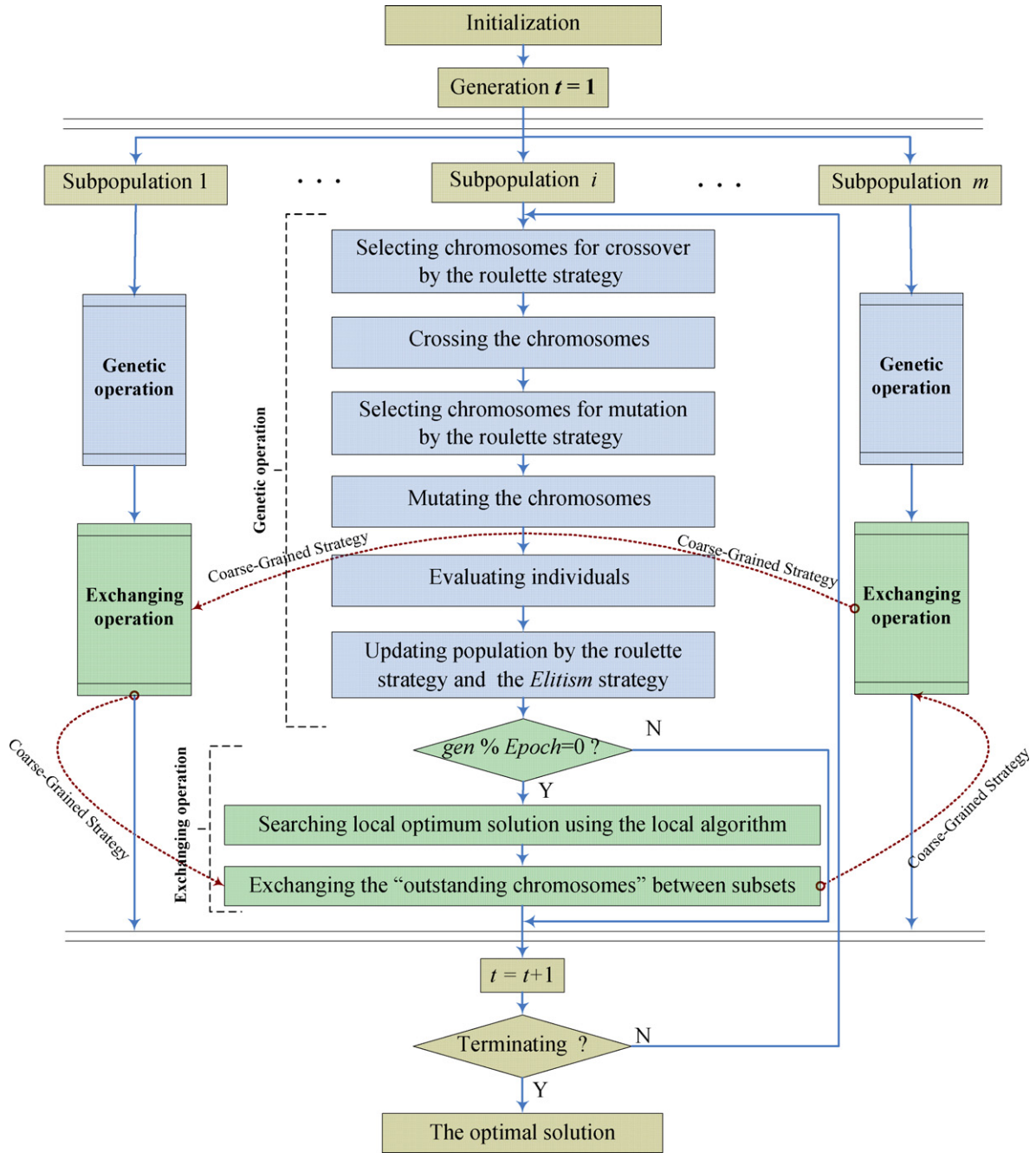


Fig. 1. The flowchart of the PGA.

Then, the passenger weight and operator weight can be computed.

$$\begin{cases} w^{\text{passenger}} = \frac{1}{2}(w^{\text{passenger},G} + w^{\text{passenger},C}) \\ w^{\text{operator}} = \frac{1}{2}(w^{\text{operator},G} + w^{\text{operator},C}) \end{cases} \quad (35)$$

$$\begin{cases} w^{\text{passenger},G} = \frac{\bar{\chi}^{\text{passenger}}}{\bar{\chi}^{\text{passenger}} + \bar{\chi}^{\text{operator}}} \\ w^{\text{operator},G} = 1 - w^{\text{passenger},G} \end{cases} \quad (36)$$

$$\begin{cases} \bar{\chi}^{\text{passenger}} = \frac{1}{L} \sum_{l=1}^L \hat{f}_l^{\text{passenger}} \\ \bar{\chi}^{\text{operator}} = \frac{1}{L} \sum_{l=1}^L \hat{f}_l^{\text{operator}} \end{cases} \quad (37)$$

$$\begin{cases} w^{\text{passenger},C} = \frac{s^{\text{operator}}}{s^{\text{passenger}} + s^{\text{operator}}} \\ w^{\text{operator},C} = 1 - w^{\text{passenger},C} \end{cases} \quad (38)$$

Table 1
Parameters in headway optimization model.

Parameter	H	V	V^{\max}	\bar{b}, \bar{a}	φ	t_0^d	C_p^w	C_p^r	C_p^{ba}	C_o^f	C_o^v
Value	3600s	80	120	3 s	2	3 s	2.7 RMB ^a /h	2.0 RMB/h	1.0 RMB/h	8.75 RMB/vehicle	3 RMB/min

^a RMB (Reminbi).

$$\begin{cases} (s^{\text{passenger}})^2 = \frac{1}{L} \sum_{l=1}^L (\hat{T}_l^{\text{passenger}} - \bar{x}^{\text{passenger}})^2 \\ (s^{\text{operator}})^2 = \frac{1}{L} \sum_{l=1}^L (\hat{T}_l^{\text{operator}} - \bar{x}^{\text{operator}})^2 \end{cases} \quad (39)$$

where $w^{\text{passenger}}$ and w^{operator} denote the weights for passenger costs and operator costs of the routes. L denotes the number of the sample data, here $L=89$. $w^{\text{passenger},G}$ and $w^{\text{operator},G}$ indicate the weights considering the relative importance between multiple objectives (the passenger costs and operator costs), i.e., the average proportion of passenger/operator costs of each route in its total costs. Similarly, $w^{\text{passenger},C}$ and $w^{\text{operator},C}$ indicate the weights considering the difference between multiple objectives (the passenger costs and operator costs), i.e., the difference between passenger/operator costs of routes. $\bar{x}^{\text{passenger}}$ and $\bar{x}^{\text{operator}}$ denote the mean passenger costs and operator costs of the sample data. $s^{\text{passenger}}$ and s^{operator} denote the variance for passenger costs and operator costs of the sample data.

To describe the computation process of the weight identification, we proposed a simple example. There are three routes. Assume that the passenger costs and operator costs of the three routes are (passenger costs = 0.8, operator costs = 0.76), (passenger costs = 0.9, operator costs = 0.8) and (passenger costs = 0.7, operator costs = 0.78), respectively.

Then, the functionality weights and proportionality weights of two indexes are computed, $w^{\text{passenger},G} = 0.49$, $w^{\text{operator},G} = 0.51$, $w^{\text{passenger},C} = 0.55$ and $w^{\text{operator},C} = 0.45$. Thus, the weights of the passenger costs and the operator costs are determined, $w^{\text{passenger}} = 0.52$, $w^{\text{operator}} = 0.48$. From calibration results, there is almost no significant difference between the coefficients of the passenger costs and the operator costs. However, the coefficients are calibrated by practice data of bus system in Dalian City. This indicates that passenger costs and operator costs are similar in the city.



Fig. 2. Bus network in Dalian City.

4.2. Results

The lists of parameters used in the headway optimization model and PGA are shown in **Tables 1 and 2**. The algorithm is implemented in C++, using message passing interface (MPI) library, on 8-computer cluster architecture: windows XP platform environment.

4.2.1. Performance of the proposed algorithm

In order to show the basic behavior of our parallel genetic algorithm, experimental results are given here for various conditions in which there were two sequential GAs with 60 and 80 individuals and eight PGAs: $P_{\text{size}} = 240$ or 320 and $\eta = 4, 6, 8$ and 16 nodes. As the **Table 2** shown, all the parameters are same except P_{size} which change from 240 to 320. **Fig. 3** shows the experimental results.

The sequential GAs (SGA) are the algorithms with $\eta = 1$. Since the proposed model is a complicated problem, the SGA tends to step into premature convergence. It is obvious that the PGAs observe a better quantity than the SGAs. This comportment can be explained principally because when $\eta > 1$, the migration operation between sub-populations can diversify each subset, widen the searching space, and improve the optimization quality. Furthermore, it can be observed that the better performance among PGAs appears at $\eta = 6$ and $\eta = 8$. Compared with computation time of several algorithms, the convergence speeds of the PGAs with $\eta = 8$ and $\eta = 16$ are faster than the ones of other PGAs. Weighting the optimization quality and computation time, we select the PGA with $\eta = 8$ and $P_{\text{size}} = 320$.

To examine the efficacy of the PGA, we continue experimenting 10 times, and **Fig. 4** shows the convergence of the calculation. It can be observed that the fitness increases fast before the 1100th generation, and then it changes smoothly. The best fitness appears at about the 1500th generation. Furthermore, the fitness among ten experiments hardly changes again. This means our algorithm has a good converge and we can conclude that after about 1500 generations of evolution the optimal solutions can be found.

Then, we test the coarse-grained strategy and the local search algorithm. The performances of several algorithms are compared, which include SGA, SGA with the local search algorithm (denoted by SGA-L), GA with coarse-grained strategy (denoted by CGA) and the proposed algorithm (PGA). The several algorithms continue experimenting 10 times and the best solution, the worst solution and the average solution of the 10 results are shown in **Fig. 5**.

We can find that the performances of SGA-L, CGA and PGA are better than the one of SGA. This is just as expected as the more efforts an algorithm expends, the better performance it certainly gains. Compared with SGA-L, CGA generally provides better solution. This can be attributed that the coarse-grain strategy diversifies the population and prevents the algorithm from trapping in local optimization. Furthermore, the introduction of the local search algorithm into CGA (PGA) can adequately search the local region and improve the solutions. This indicates that the incorporation

Table 2
Parameters in PGA.

Parameter	Q	p_c	p_m	ξ	τ^{\max}	β_1	β_2	λ	epoch
Value	10,000	0.6	0.1	1	2000	1	1	3	50

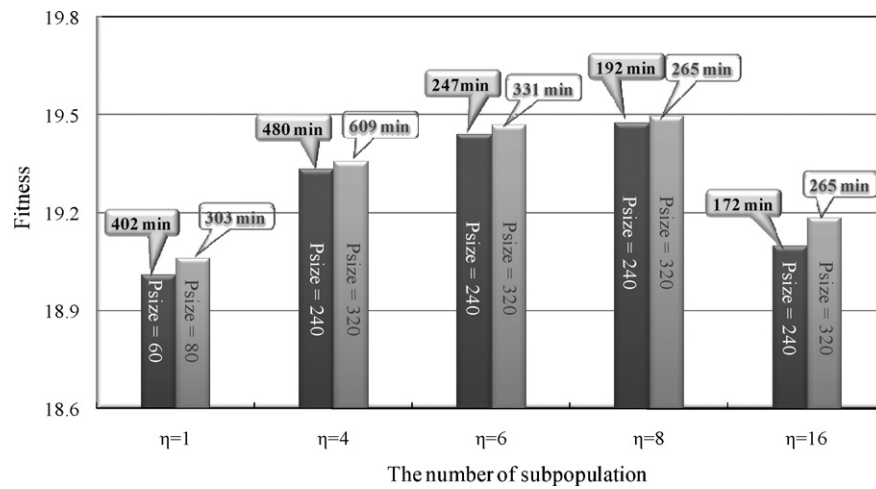


Fig. 3. Experimental result of several parameter combinations.

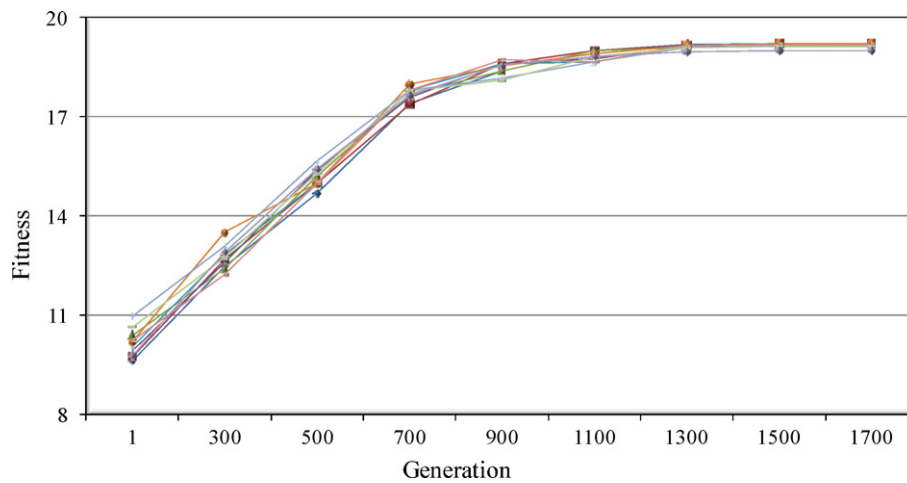


Fig. 4. The result of each calculation.

of the coarse-grained strategy and the local search algorithm can greatly improve the performance of the algorithm.

In order to further examine the performance of the algorithm, here, a multi-objective genetic algorithm (MOGA) is introduced.

There are two objective functions in the MOGA: one is the minimum total passenger cost, and the other is the minimum operation costs. The coding of the MOGA is consistent to the algorithm proposed in this paper.

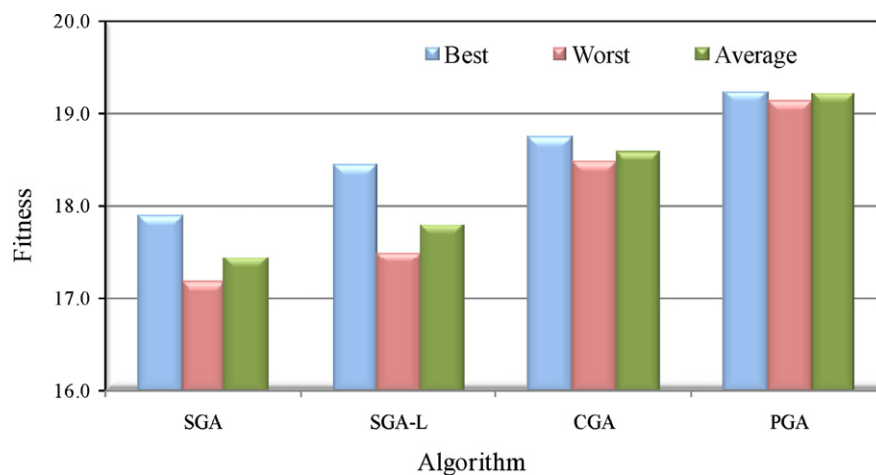


Fig. 5. The comparison of several algorithms.

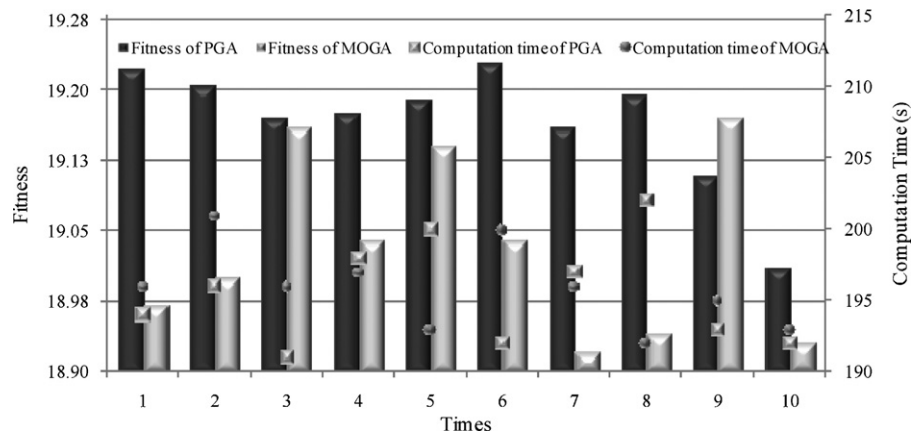


Fig. 6. The computational results of two algorithms.

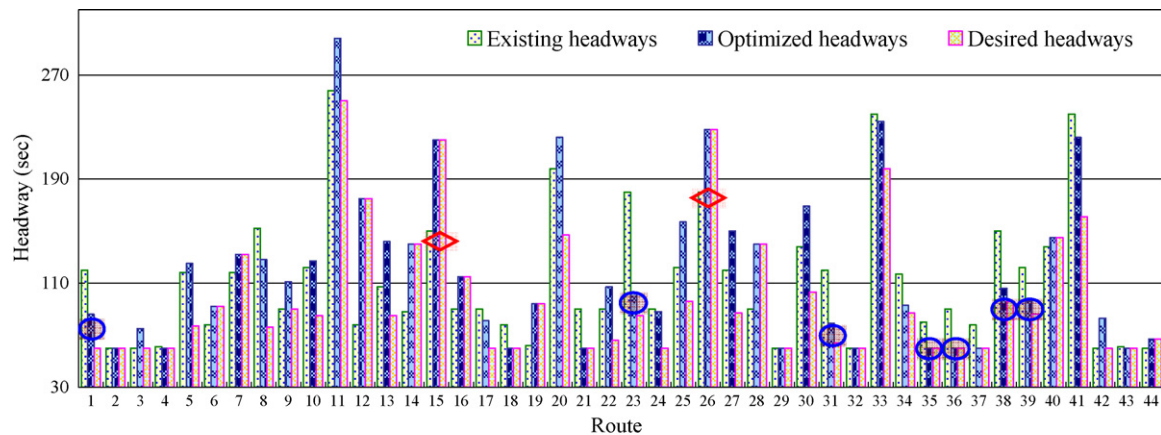
First, each chromosome is sorted according to two objectives, respectively. After sorting each object, the objective function of the overall performance can be got.

$$E_i(X_j) = \begin{cases} (N - R_i(X_j))^2 & R_i(X_j) > 1 \\ kN^2 & R_i(X_j) = 1 \end{cases} \quad (40)$$

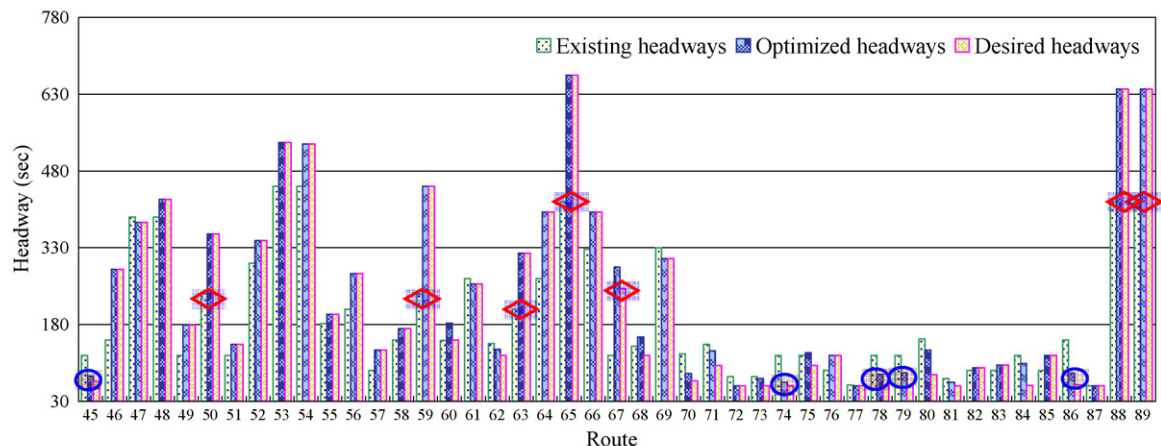
$$E(X_j) = \sum_i E_i(X_j) \quad (41)$$

where n is the number of the object; N is the total number of individuals; X_j is the individual j in population; R_i is the number for sorting all individual quality in the population; $E_i(X_j)$ is the fitness of X_j on the target i ; K is the constant between (1, 2), which is used to increase the fitness when the individual function value performs optimal. Individual choice is adopted by the roulette wheel way. Here, K is set to 1.5. $E(X_j)$ is the final fitness value of the chromosomes j .

For example, there are three chromosomes in a population. Assume that the order of three chromosomes according to two



(a) Route 1-44



(b) Route 45-89

Fig. 7. Comparison of three headways in the direction with more passengers.

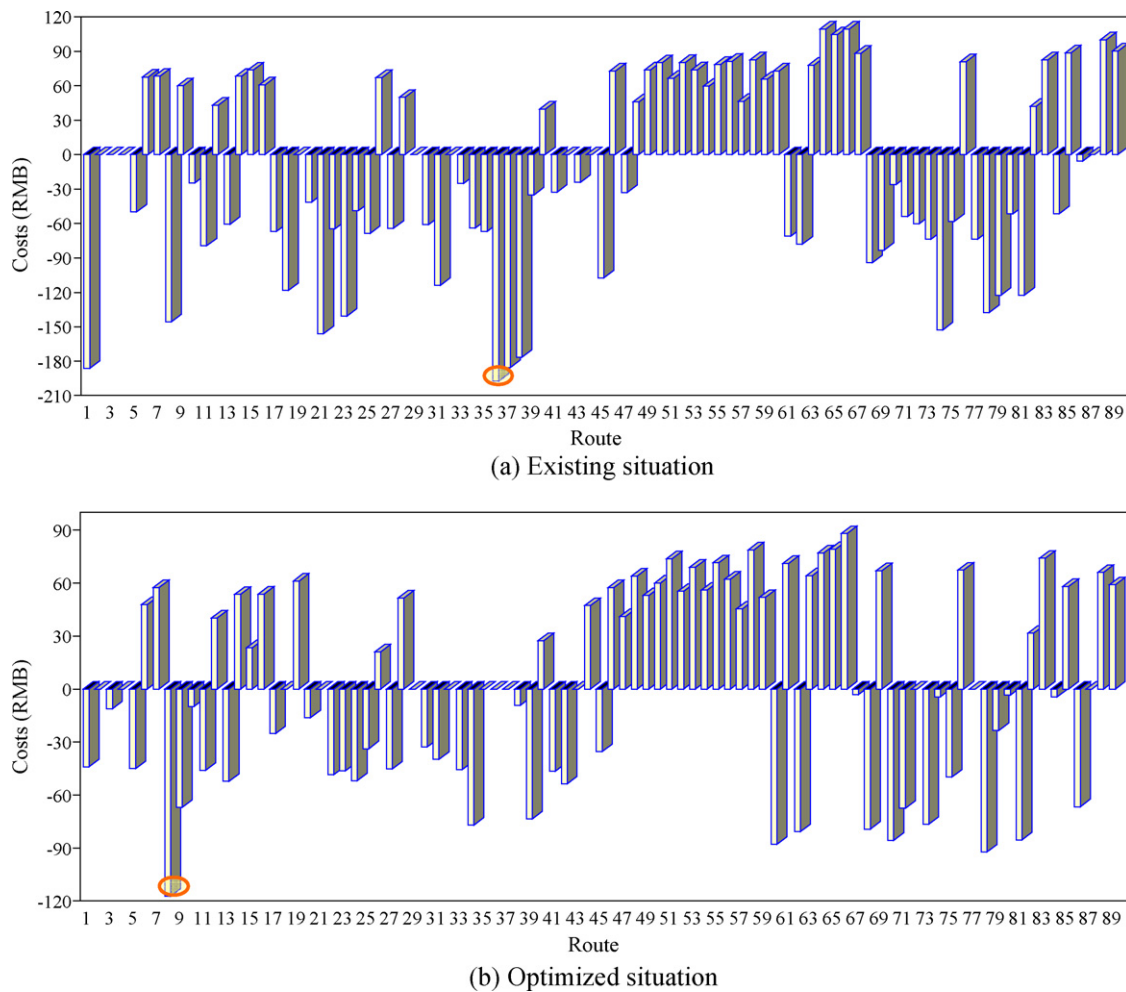


Fig. 8. Decreasing or increasing costs by increasing an operation vehicle.

objectives are (1, 3), (2, 1) and (3, 2), respectively. Then, $N=3$, $R_1(x_1)=1$, $R_2(x_2)=3$, $R_1(x_2)=2$, $R_2(x_2)=1$, $R_1(x_3)=3$ and $R_2(x_3)=2$. Thus, the fitness values of three chromosomes are set to $E(x_1)=13.5$, $E(x_2)=14.5$ and $E(x_3)=1$.

In the MOGA, the same crossover and mutation operations with the proposed algorithm are used. To be fair, the MOGA also uses the parallel strategy. Then, in the same condition, we calculated continuously 10 times two algorithms. Fig. 6 shows the computational results. Obviously, both algorithms have good stability, for example, the difference between the best and worst solutions is less than 5%. In addition, the solution optimized by the MOGA, passenger total cost is lower while there is a lower operator cost in the solution from our algorithm. This is because the MOGA is based on the ranking of chromosomes to select the target chromosome, rather than the objective values. Therefore, when the quality of chromosomes has a larger difference, the MOGA has difficulty in distinguishing chromosomes according to evolution. In addition, from the computational time, one can found the convergence speeds of the two algorithms are similar. On the whole, the optimization qualities of the two algorithms are similar.

4.2.2. Headway optimization model

To validate the proposed model, the optimized headways of routes are compared with the existing headways. The total costs of the current transit system are 573.2 thousand RMB (the operator costs is 289.8 thousand RMB and the passenger costs is 283.4 thousand RMB). Compared with current situation, the operator costs (272.2 thousand RMB) and the passenger costs (243.7 thou-

sand RMB) of the transit system with optimized headways are decreased by about 6% and 14%, respectively. This can attribute to the unreasonable resource allocation in current situation and can also indicate that the integrating resources, the service level of system and the efficiency of resources can be improved.

Furthermore, from optimization results, it is found that the total fleet size of the transit network with optimized headways just equals to the existing one (the maximum fleet size constraint). This implies that the fleet size of the transit system in Dalian City can perhaps not be enough for demands. Therefore, we compute the desired headways of routes using the same data sets as the proposed model through releasing maximum fleet size constraint. Thus, if the costs to purchase vehicles are not being considered, the total costs of the transit system with the desired headways can be greatly decreased by about 18.3%. This can be seen as the one proof for the crowded condition of rush hours in Dalian City. The details of the comparison of three headways in the direction with more passengers of each route are shown in Fig. 7.

It is obvious that the headways of the three situations are different. As a whole, desired headways of most routes are lower or equal to the existing headways or optimized headways. It is also observed that the optimized headways of some routes are much lower than the existing headways of these routes, e.g. route 1, 23, 31, 35, 36, 38, 39, 45, 74, 78, 79 and 86, etc. In fact, these routes are indeed heavily crowded in the rush hours. It is necessary to increase the vehicles to operation to improve service level of these routes. Contrarily, the optimized headways of some routes are higher than the existing headways of these routes, e.g. route 15, 26, 50, 59, 63,

65, 67, 88 and 89, etc. Half of these routes are comprised of an affiliated company, which includes route 60–77. The routes of the affiliated company mainly serve in suburb. From optimized results after resource integration, the comfortable level of passengers in crowded routes can be improved.

From Fig. 7, the existing fleet size is insufficient for the demand in rush hours. However, it is difficult and impossible to purchase enough vehicles to satisfy all routes simultaneously. Therefore, it is necessary to analyze the sensitivity to increase operation vehicles into a route. If the lower limit of headways of routes is set as 60 s, the sensitivity analysis to the routes satisfying the lower limit constrain of headways is shown in Fig. 8. It can be observed that in the transit system with existing headways increasing an operation vehicle into the route 36 can decrease the most costs of among all the routes, while increasing an operation vehicle into the route 8 can gain the most benefit in the transit system with the optimized headways. In addition, we can find that some routes in the existing situation or in the optimized situation bring more costs after increasing an operation vehicle. This indicates that these routes are not crowded. Increasing more operation vehicles to these routes will aggravate the imbalance or break the balance between passenger costs and operator costs. The sensitivity analysis can provide a reference when the transit system or bus companies increase operation vehicles.

5. Conclusions

Headway design is a necessary product for transit system, and it is also true that a transit agency will often evaluate and determine headways of routes. This paper presents a headway optimization model based on a given network configuration and demand matrix. This model synthetically considers the passenger costs and operator costs. Also, an objective approach integrating the functionality and proportionality to weight determination is proposed to find an acceptable balance between the operator costs and the passenger costs. Parallel genetic algorithm is used to solve the headway optimization model and parameters in the algorithm are also tested. Data of transit system in Dalian City, China, is collected to test the model and the algorithm. The existing passenger costs and operator costs of the bus routes of transit system in Dalian City are used to determine the weight between two costs. Results show that PGA is a powerful tool for bus route headway optimization in this paper. Furthermore, results also suggest that resource integration can improve the service level of transit system.

Acknowledgements

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