



Fuzzy logic system based prediction effort: A case study on the effects of tire parameters on contact area and contact pressure



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ARTICLE INFO

Article history:

Received 5 December 2012

Received in revised form 23 April 2013

Accepted 6 October 2013

Available online 14 October 2013

Keywords:

Fuzzy logic system

Contact area

Contact pressure

Soil bin

Prediction model

ABSTRACT

Various methodologies of artificial intelligence have been recently used for estimating performance parameters of soil working machines and off-road vehicles. Due to nonlinear and stochastic features of soil–wheel interactions, application of knowledge-based Mamdani max–min fuzzy expert system for estimation of contact area and contact pressure is described in this paper. Fuzzy logic model was constructed by use of the experience of contact area and contact pressure utilizing data obtained from series of experimentations in soil bin facility and a single-wheel tester. Two paramount tire parameters: wheel load and tire inflation pressure are the input variables for our model, each has five membership functions. As a fundamental aspect of the fuzzy logic based prediction systems, a set of fuzzy if-then rules were used in accordance with fuzzy logic principles. 25 linguistic if-then rules were included to develop a complicated highly intelligent predicting model based on Centroid method at defuzzification stage. The model performance was assessed on the basis of several statistical quality criteria. Mean relative error lower than 10%, satisfactory scattering around unity-slope line (T), and high coefficient of determination, R^2 , were obtained by the fuzzy logic model proposed in this study.

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1. Introduction

1.1. Literature review

Significant upsurge in global population increases food demand which results in mechanized agricultural practices. Heavy tillage practices, multiple passages, and giant wheeled vehicles traversing are of major sources of topsoil and subsoil compaction. Estimation of tire contact patch area contributes to determination of contact pressure and stress-strain propagation that are compaction originators [1]. Soil stress with its lateral production (i.e. soil compaction) is function of contact area [2]. Therefore, estimation of contact area as a major factor influencing the area of soil compaction and further soil–wheel interaction parameters has to be obtained precisely. Although data on soil–tire interface contact area for modern tires and the experimentations carried out are insufficient [3], contact area measurements have been conducted by many researchers for conventional tractors [4,5].

Role of contact area on contact pressure should also be considered as the area of contact in which load is applied to soil that

forms contact pressure. Various investigations in literature have mentioned tire inflation pressure and wheel load as the main influential parameters on formation of contact pressure [2]. Diserens [6] explored a wide range of tires and reported contradictory results for the effect of inflation pressure on the contact area and concluded that inflation pressure had negligible effect on the contact area. Several researchers declared an increment of contact area with respect to decrement of inflation pressure [3]. The effect of wheel load on contact pressure is less clear while the effect of inflation pressure has been noticeably confirmed [2].

1.2. Fuzzy logic method

Self-learning approaches are applied in software computations comprising statistics, machine learning, neural networks and fuzzy logics [7]. Artificial intelligence has featured as promising, instrumental and practical technique of soft computing technologies in science and engineering domains. Fuzzy logic deals with the concept of partial truth theory and provides a methodology to model uncertainty and the human way of thinking, reasoning and perception [8]. Fuzzy logic systems are rule-based or knowledge-based systems first formalized by Zadeh [9]. Since the fuzzy set, a class of objects with a continuum of grades of membership, is descriptive of vague impressions than numerical, variables are therefore better described by linguistic terms. Fuzzy logic sets are characterized by membership functions, also known as characteristic functions that

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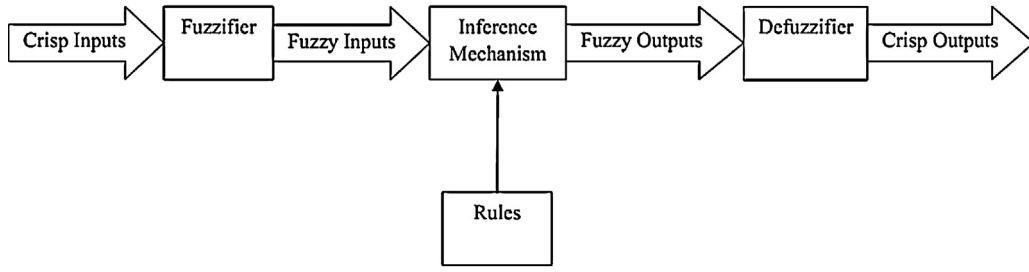


Fig. 1. A general framework of the fuzzy logic system.

assign to each object a degree of membership varying between zero and one. Variety of membership functions are in practice such as S-shaped, Z-shaped, Triangular, and Trapezoidal shaped functions. The triangular membership functions are formed using straight lines. These straight line membership functions have the advantage of simplicity. Because of their smoothness and concise notation, Gaussian membership functions are popular methods for specifying fuzzy sets. These curves have the advantage of being smooth and nonzero at all points [10].

- This spline-based function of x is so named because of its Z-shape. The parameters a and b locate the extremes of the sloped portion of the curve as given by.

$$f(x; a, b) = \begin{cases} 1; & x \leq a \\ 1 - 2\left(\frac{x-a}{b-a}\right)^2; & a \leq x \leq \frac{a+b}{2} \\ 2\left(\frac{x-b}{b-a}\right)^2 & \frac{a+b}{2} \leq x \leq b \\ 0; & x \geq b \end{cases} \quad (1)$$

- Triangular-shaped built-in membership function. The triangular curve is a function of a vector, x , and depends on three scalar parameters a , b , and c , as given by:

$$f(x; a, b, c) = \begin{cases} 0; & x \leq a \\ \left(\frac{x-a}{b-a}\right); & a \leq x \leq b \\ \left(\frac{c-x}{c-b}\right) & b \leq x \leq c \\ 0; & c \leq x \end{cases} \quad (2)$$

Or closely by

$$f(x; a, b, c) = \max \left(\min \left(\frac{x-a}{b-a}, \frac{c-x}{c-b} \right), 0 \right) \quad (3)$$

where the parameters a and c locate the “feet” of the triangle and the parameter b locates the peak.

- The symmetric Gaussian function depends on two parameters σ and c , listed in order in the vector [sig c], as given by:

$$f(x; \sigma, c) = e^{-(x-c)^2/2\sigma^2} \quad (4)$$

A fuzzy logics system (FLS) consists of four main steps as briefly defined: (a) Fuzzifier that converts numerical input values to fuzzy sets, (b) Rules that are the heart of FLS and are described commonly by if-then true rules, (c) Inference to combine rules and map input variables to output variables, and (d) Defuzzifier to reconvert the outputs of inference stage to crisp output values. Fig. 1 shows a general framework of fuzzy logic system.

The advantage of Fuzzy based systems is nonlinear mapping, computational flexibility and accurate estimations that make it applicable to be used as a reliable alternative modeling technique

in engineering [9,11]. To the best knowledge of authors, literature is poor regarding the investigation of the effects of tire inflation pressure and wheel load on contact area and contact pressure predicted by a constructed fuzzy rule-based approach on the basis of Mamdani max–min technique. The readily available data samples obtained from soil bin facility were used for FLS developed model.

2. Research method

2.1. Experimental phase

A capacious soil bin facility was designed and constructed in 2010 in the Faculty of Agriculture, Urmia University, Iran. This soil bin features 23 m length, 2 m width and 1 m depth [12]. This long channel could accommodate a carriage, a single-wheel tester, and tillage tools. A three-phase electromotor of 30 hp was used to move the single-wheel tester, mounted on carriage, by use of chain system. The carriage had the ability to traverse at any desired speed of between 0 and 20 km/h. The system set up is shown in Fig. 2. A load cell of 20 kN capacity was situated vertically between the single-wheel tester and power bolt to measure the value of vertical loading. Data were sent to a digital indicator and then to a data logger device. Transmitted files were recorded with txt files format and subsequently were imported to MATLAB software (version 7.6, 2008, Mathworks Company) for being processed. Summary of treatments being tested is shown in Table 1. In order to determine contact area experimentally, at each treatment, white powder was spread on periphery of soil–tire interface to define contact area. A digital camera was used to take images of contact areas. Image processing method was then used to define contact area.

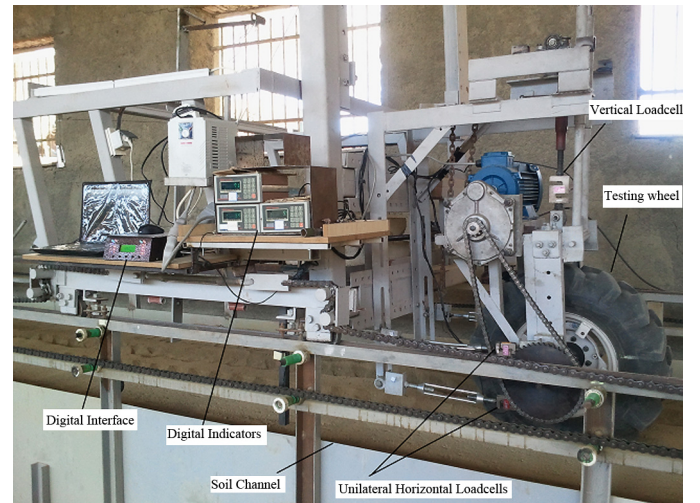


Fig. 2. General depiction of the soil bin system setup for experimental phase of study.

Table 1
Summary of input and output variables ranges.

Input (independent variables)	Parameter	Unit	Levels					
1	Wheel load	kN	0.75	1.75	2.75	3.75	4.75	5.75
2	Inflation pressure	kPa	70	100	140	175		
Output (dependent variables)								
1	Contact area	m ²						
2	Contact pressure	kPa						

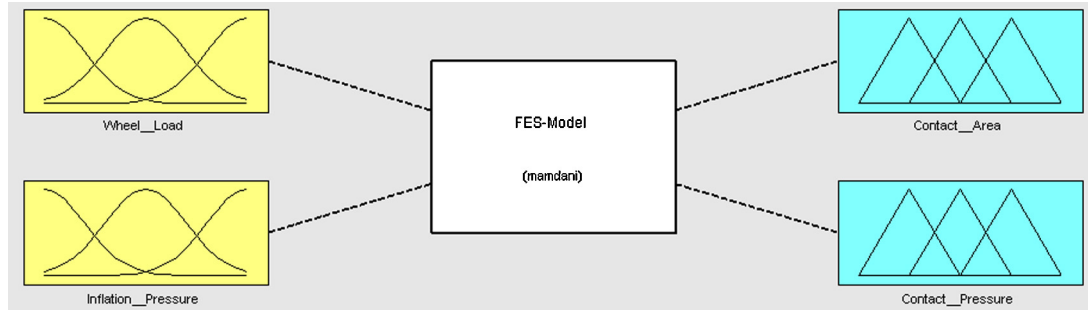


Fig. 3. The structure of fuzzy expert system (Mamdani).

The soil bin was filled with clay-loam soil that exists in most of regions of Urmia, Iran. Leveler, harrow, and roller were of equipment utilized for reverting the soil condition after traversing to the soil condition before traversing. This is fundamental step in experimentations since that the experiments should be carried out in similar conditions. Soil constituents and properties are defined in Table 2.

2.2. Fuzzy logics system

Fuzzy expert system (FES) was used for estimation of the study objectives as affected by input variables of tire inflation pressure and wheel load by implementation of fuzzy logics theory in MATLAB software (version 7.6, 2008, Mathworks Company). The variables are required to be transformed with fuzzification to lexical parameters of very low (VL), low (L), medium (M), high (H) and very high (VH). In order to obtain linear interpolation of outputs, a Mamdani max–min formula was proffered to Sugeno rule (Fig. 3). Furthermore, there are other advantages of Mamdani max–min approach over Sugeno rule such as its higher intuitiveness, general acceptance, and well suitability to human inputs. Aiming to avoid over-fitting or over-learning drawback which reduces forecasting ability, cross validation process was incorporated in the investigation during the selection procedure by splitting the datasets into training and testing portions. The testing set was used to guarantee the model generalization and to avoid over-fitting the model to the training data set. Hence, 75% of datasets were assigned for training the model and 25% remaining were used for testing partition.

Because of the reported accuracy of Z-shaped, triangular and Gaussian memberships in literature [13–15], these functions were used in the investigations to discover the optimum representation. The membership function defines the quality of mapping each point

of the input space to a degree of membership varying between 0 and 1. Primary amounts for membership functions as well as number of them are dependent to research condition. Owing to superior accuracy, the uniformly triangular membership functions were opted for triple input variables as well as the output variable.

2.3. Membership functions and fuzzification of input and output variables

To construct any FLS model, it is initially essential to fuzzify all input and output parameters. For fuzzification of the used parameters, the following quantity values were developed.

$$IP(i_1) = \begin{cases} i_1; & 70 \leq i_1 \leq 175 \\ 0; & \text{otherwise} \end{cases} \quad (5)$$

$$W(i_2) = \begin{cases} i_2; & 0.75 \leq i_2 \leq 5.75 \\ 0; & \text{otherwise} \end{cases} \quad (6)$$

$$\text{Contact area}(o_1) = \begin{cases} o_1 & 0.005 \leq o_1 \leq 0.0423 \\ 0; & \text{otherwise} \end{cases} \quad (7)$$

$$\text{Contact area}(o_2) = \begin{cases} o_2 & 57 \leq o_2 \leq 205 \\ 0; & \text{otherwise} \end{cases} \quad (8)$$

The membership functions obtained from equations above are illustrated in Figs. 4 and 5. Moreover, the lexical expressions and membership functions of the utilized rules of Eq. (7) is given below.

$$\mu_{VL}(i_1) = \begin{cases} 1; & i_1 < 70 \\ \frac{95 - i_1}{25}; & 70 \leq i_1 \leq 95 \\ 0; & i_1 \geq 95 \end{cases} \quad (9)$$

$$\mu_{VL}(i_1) = \left\{ \frac{1}{70} + \frac{0.9}{72.5} + \dots + \frac{0.1}{92.5} + \frac{0}{95} \right\} \quad (10)$$

Table 2
Soil constituents and its measured properties.

Item	Value
Sand (%)	34.3
Silt (%)	22.2
Clay (%)	43.5
Bulk density (kg/m ³)	1560
Frictional angle (°)	32
Cone index (kPa)	437

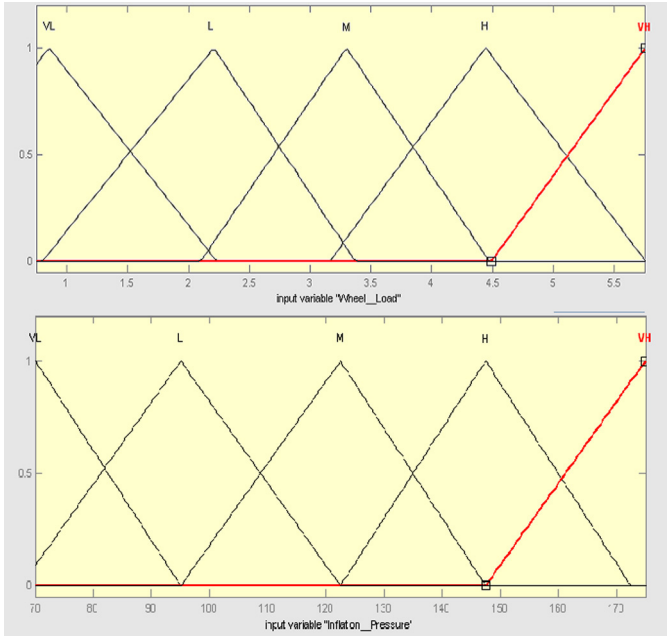


Fig. 4. The prototype membership functions of input variables.

$$\mu_L(i_1) = \begin{cases} \frac{i_1 - 67.5}{27.5}; & 67.5 \leq i_1 < 95 \\ \frac{122.5 - i_1}{27.5}; & 95 \leq i_1 \leq 122.5 \\ 0; & i_1 \geq 122.5 \end{cases} \quad (11)$$

$$\mu_L(i_1) = \left\{ \frac{0}{67.5} + \frac{0.1}{70.25} + \dots + \frac{0.9}{92.25} + \frac{1}{95} + \dots + \frac{0.1}{119.75} + \frac{0}{122.5} \right\} \quad (12)$$

$$\mu_M(i_1) = \begin{cases} \frac{i_1 - 95}{27.5}; & 95 \leq i_1 < 122.5 \\ \frac{147.5 - i_1}{25}; & 122.5 \leq i_1 \leq 147.5 \\ 0; & i_1 \geq 147.5 \end{cases} \quad (13)$$

$$\mu_M(i_1) = \left\{ \frac{0}{95} + \frac{0.1}{97.75} + \dots + \frac{0.9}{119.75} + \frac{1}{122.5} + \frac{0.9}{125} + \dots + \frac{0.1}{145} + \frac{0}{147.5} \right\} \quad (14)$$

$$\mu_H(i_1) = \begin{cases} \frac{i_1 - 122.5}{25}; & 122.5 \leq i_1 < 147.5 \\ \frac{147.5 - i_1}{25}; & 147.5 \leq i_1 \leq 172.5 \\ 0; & i_1 \geq 172.5 \end{cases} \quad (15)$$

$$\mu_H(i_1) = \left\{ \frac{0}{122.5} + \frac{0.1}{125} + \dots + \frac{0.9}{145} + \frac{1}{147.5} + \frac{0.9}{150} + \dots + \frac{0.1}{170} + \frac{0}{172.5} \right\} \quad (16)$$

$$\mu_{VH}(i_1) = \begin{cases} 0; & i_1 < 147.5 \\ \frac{i_1 - 147.5}{27.5}; & 147.5 \leq i_1 \leq 175 \\ 1; & i_1 > 175 \end{cases} \quad (17)$$

$$\mu_{VH}(i_1) = \left\{ \frac{0}{147.5} + \frac{0.1}{150.25} + \dots + \frac{0.9}{172.25} + \frac{1}{175} \right\} \quad (18)$$

After completion of knowledge-based rules applied to the fuzzy inputs, the fuzzy value obtained should be converted to a crisp value that is known as defuzzification process. In this stage, truth degrees of the rules are determined for the each rule by use of the min and then by taking max between working rules. Consequently the output variable is computable.

2.4. If-Then true rule

The fuzzy sets are essential to perform the fuzzy model based on a rule using an implication function. This implication function is, however, known as If-then true rule or called linguistic rule. The rules determine the input and output membership functions that will be used in inference procedure. For implementation of analysis, a total of 25 rules were generated as following.

1. If (Wheel Load is VL) and (IP is VL) then (Contact area is L) and (Contact pressure is VL)
2. If (Wheel Load is VL) and (IP is L) then (Contact area is VL) and (Contact pressure is VL)
3. If (Wheel Load is VL) and (IP is M) then (Contact area is VL) and (Contact pressure is VL)
4. If (Wheel Load is VL) and (IP is H) then (Contact area is VL) and (Contact pressure is M)
5. If (Wheel Load is VL) and (IP is VH) then (Contact area is VL) and (Contact pressure is M)
6. If (Wheel Load is L) and (IP is VL) then (Contact area is M) and (Contact pressure is VL)
7. If (Wheel Load is L) and (IP is L) then (Contact area is M) and (Contact pressure is L)
8. If (Wheel Load is L) and (IP is M) then (Contact area is L) and (Contact pressure is L)
9. If (Wheel Load is L) and (IP is H) then (Contact area is L) and (Contact pressure is H)
10. If (Wheel Load is L) and (IP is VH) then (Contact area is L) and (Contact pressure is H)
11. If (Wheel Load is M) and (IP is VL) then (Contact area is H) and (Contact pressure is L)
12. If (Wheel Load is M) and (IP is L) then (Contact area is H) and (Contact pressure is M)

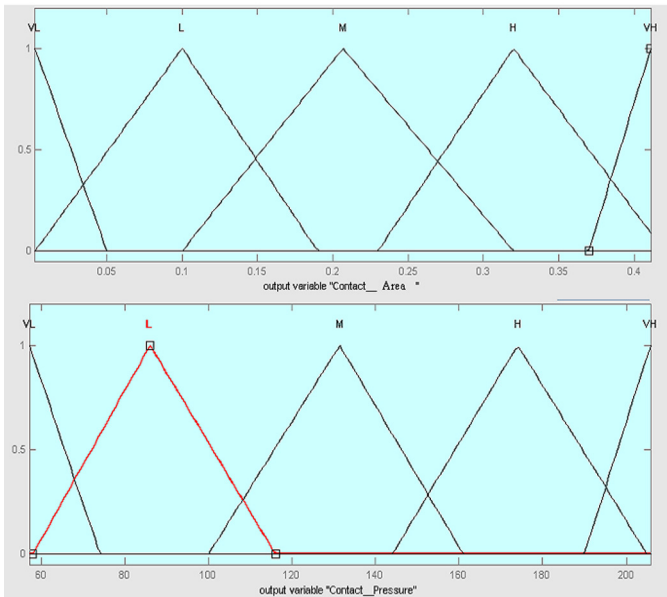


Fig. 5. The prototype membership function of the output variables.

13. If (Wheel Load is M) and (IP is M) then (Contact area is M) and (Contact pressure is L)
14. If (Wheel Load is M) and (IP is H) then (Contact area is M) and (Contact pressure is H)
15. If (Wheel Load is M) and (IP is VH) then (Contact area is L) and (Contact pressure is VH)
16. If (Wheel Load is H) and (IP is VL) then (Contact area is VH) and (Contact pressure is L)
17. If (Wheel Load is H) and (IP is L) then (Contact area is H) and (Contact pressure is M)
18. If (Wheel Load is H) and (IP is M) then (Contact area is H) and (Contact pressure is L)
19. If (Wheel Load is H) and (IP is H) then (Contact area is H) and (Contact pressure is VH)
20. If (Wheel Load is H) and (IP is VH) then (Contact area is M) and (Contact pressure is VH)
21. If (Wheel Load is VH) and (IP is VL) then (Contact area is VH) and (Contact pressure is M)
22. If (Wheel Load is VH) and (IP is L) then (Contact area is VH) and (Contact pressure is H)
23. If (Wheel Load is VH) and (IP is M) then (Contact area is H) and (Contact pressure is M)
24. If (Wheel Load is VH) and (IP is H) then (Contact area is H) and (Contact pressure is VH)
25. If (Wheel Load is VH) and (IP is VH) then (Contact area is H) and (Contact pressure is VH)

2.5. Defuzzification of input and output variables

Subsequent to that the fuzzification process completed, the process called defuzzification is required to convert fuzzy values to output crisp values. In defuzzification, the centeroid (center of area) method was utilized as (Eq. (21)) to obtain crisp output value.

$$Z^* = \frac{\int Z \cdot \mu_C(Z) \cdot dZ}{\int \mu_C(Z) \cdot dZ} \quad (19)$$

where Z^* is the crisp output, Z is the fuzzified value and μ_C is the membership degree of fuzzified value, respectively.

2.6. FES model performance

In order to evaluate the performance of developed Fuzzy expert system (FES) model, the mathematical and statistical approaches were applied on the basis of below criteria.

$$\text{Error} = \frac{1}{N} \sum_{i=1}^N \left| \frac{Y_a - Y_p}{Y_a} \right| \times 100\% \quad (20)$$

$$T = 1 - \frac{\sum_{i=1}^N (Y_{i,a} - Y_{i,p})^2}{\sum_{i=1}^N (Y_{i,a} - \bar{Y})^2} \quad (21)$$

$$R^2 = \frac{\left(\sum_{i=1}^N (Y_{a-Y_a}^-) (Y_{p-Y_a}^-) \right)^2}{\sum_{i=1}^N (Y_{a-Y_a}^-) \cdot \sum_{i=1}^N (Y_{p-Y_a}^-)^2} \quad (22)$$

where $Y_{i,a}$ and $Y_{i,p}$ are i th output variables that obtained by measurement and predicted by FES model, respectively, \bar{Y} is the average over N samples. T value computes the scattering around the line (1:1), a T value close to 1 is prevailed.

3. Results and discussion

Results showed that increase of wheel load results in increment of contact area. The highest contact area for each of inflation pressures was measured for the highest wheel load. This can be

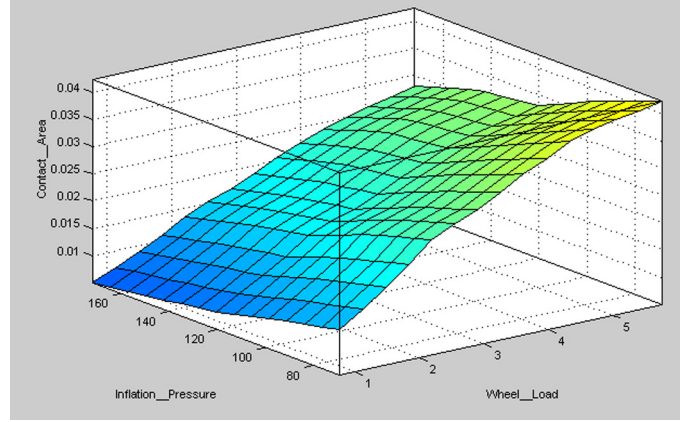


Fig. 6. Surface viewer of fuzzy system considering inflation pressure and wheel load as input variables for contact area prediction.

attributable to increased wheel deformation under rising wheel loads. Higher wheel loads tend to press the wheel into the soil. Soil initiates to deform until resists against applied vertical load. The higher amount of vertical load acting on wheel, the bigger resistance force would be entered to tire in opposite direction. This phenomenon produces wheel deformation that affects contact area.

For a constant inflation pressure, increase of wheel load increases the mean ground contact pressure. This process is confirmed by Schjønning et al., [3]. Similarly, contact pressure increases by augmentation of inflation pressure.

Surface reviewer for fuzzy system for depiction of interrelations of input variables to effect contact area and contact pressure are illustrated in Figs. 6 and 7, respectively. Considering Figs. 6 and 7, they are in confirmation with physical interpretations presented above for the effects of input variables on output variables.

The results of developed FES model for prediction of contact area and contact pressure were compared with the experimental results. Fig. 8 during the number of 18 cases (treatments) depicts the variations of contact area for the experimental effort and the FES model and also mapping between the input and output. Fig. 9 during the number of 18 cases (treatments) depicts the variations of contact pressure for the experimental effort and the FES model and also mapping between the input and output. A satisfactory accordance between experimental data and FES predictive model is perceivable in these figures.

For completion of model applicability, the FES developed models were statistically analyzed (Tables 3–5). The

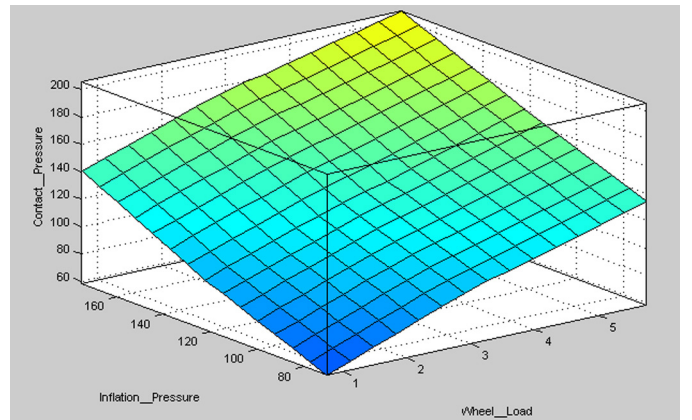


Fig. 7. Surface viewer of fuzzy system considering inflation pressure and wheel load as input variables for contact pressure prediction.

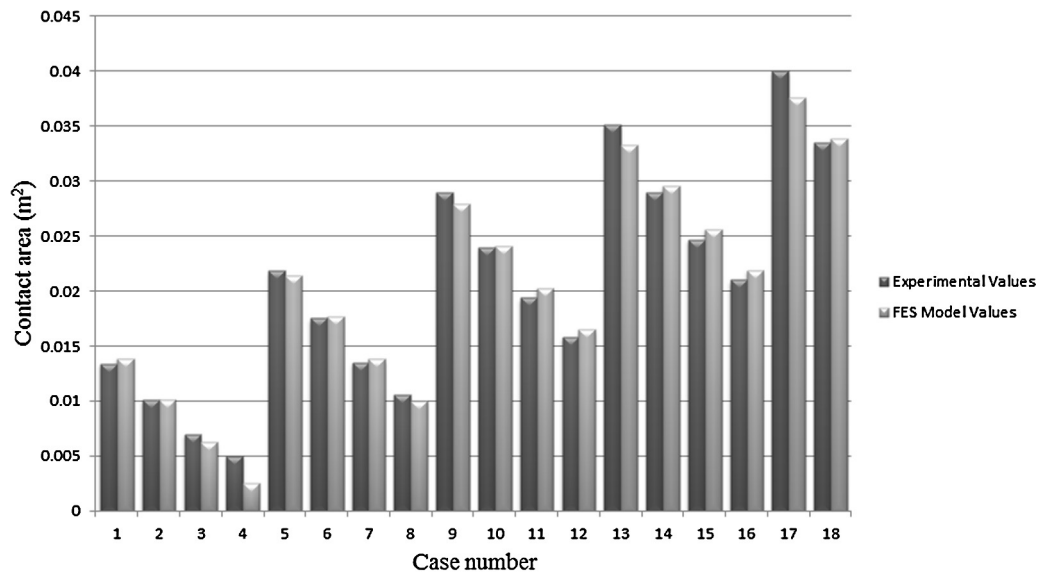


Fig. 8. Comparison of actual experimental data and FES model over 18 treatments for contact area in training partition.

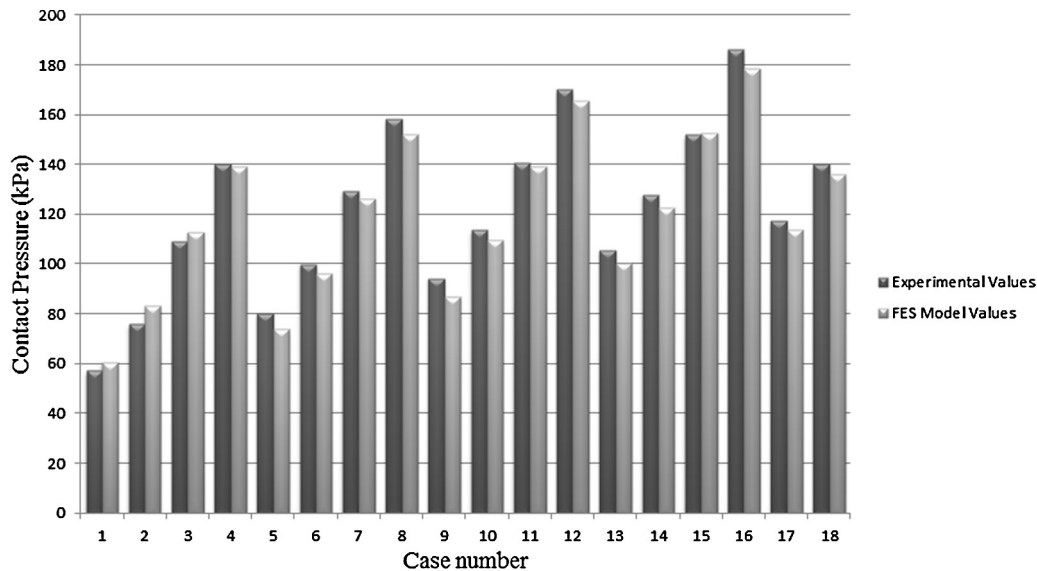


Fig. 9. Comparison of actual experimental data and FES model over 18 treatments for contact pressure in training partition.

Table 3

Statistical specifications of the developed FES models by Z-shape membership function.

Model	Error	R^2	T value
FES contact area model	0.000065848	0.9867	0.981
FES contact pressure model	0.229537	0.9782	0.977

Table 4

Statistical specifications of the developed FES models by Triangular membership function.

Model	Error	R^2	T value
FES contact area model	0.000033315	0.9942	0.992
FES contact pressure model	0.164041	0.9891	0.986

obtained results revealed that triangular membership outperformed Z-shaped and Gaussian membership functions on the basis of lower mean relative error, closer scattering around unity slope-line and better coefficient of determination that

Table 5

Statistical specifications of the developed FES models by Guassian membership function.

Model	Error	R^2	T value
FES contact area model	0.000055938	0.9857	0.977
FES contact pressure model	0.198923	0.9724	0.965

validates the applicability and usefulness of the proposed model.

4. Concluding remarks

For agricultural engineering, machine design, and energy management precise estimations of contact area and contact pressure are inevitable. This paper approves fuzzy logic system's accurate predictions, since Fuzzy logic system includes the privilege of simplicity (lexical interpretations for inputs and outputs), applicability

for nonlinear interactions and robustness over the other conventional methods. We utilized 25 If-Then true rules with Mamdani max–min inference supposition with Centroid defuzzification formula to predict the contact area and contact pressure considering the experimental data obtained in soil bin facility in clay-loam soil condition. The findings divulged the promising application of FES in terms of various statistical criteria. This model can be used for further relevant studies as a reference, however can be improved for more generalization.

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