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Simultaneous allocation of capacitors and voltage regulators at distribution networks using Genetic Algorithms and Optimal Power Flow

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ABSTRACT

The high reactive power level demanded by the distribution systems, the loads growth and consequent increase of system losses introduce variations at the buses voltage magnitudes, which compromise the quality of the supplied electric energy. To assure high quality, some devices such as voltage regulators – VRs and capacitors banks – CBs, are installed to allow effective control of voltage magnitude, reactive power and power factor. The present work proposes a methodology to allocate simultaneously these devices using both Genetic Algorithms – GAs and Optimal Power Flow – OPF. The strategy proposed involves the adoption of GA for the allocation of CBs with specification for the type of bank (fixed or automatic) and the reactive power (kvar), as well as the allocation of VRs with adjustment of their secondary voltage. The OPF is responsible for the solution of the power balance equations, tap adjustments of the VRs that assure the voltage level at their exits according to the voltage level specified by the GA for the diverse load curve and for the attainment of the nominal current of the VRs allocated.

1. Introduction

It is important for the distribution companies optimize their operation by diminishing the losses, obeying the demanded standards and following the necessary regulations. To assure adequate levels of voltage and losses at various points of the distribution network, the use of some devices that accomplish a voltage effective control, a reactive power control and a power factor control is essential. The usual equipments to carry through these controls are the voltage regulators (VRs), the transformers with changing taps (located at the substations) and parallel capacitors (capacitors banks – CBs). Some actions such as load transference from a more loaded feeder to a less loaded one, cross-section exchange, construction of new feeders, primary voltage change and construction of a new substation (SU) are necessary as well.

The supplied benefits of inserting regulating devices depend on how they are inserted into the system, that is, it depends on the localization, capacity and adjustments. These choices are complex because the distribution systems reach large areas. So, in order to help the allocation of these devices that make the voltage and the reactive compensation, this work intends to present a methodology established in terms of Genetic Algorithms (GA) and Optimal Power Flow (OPF) that allocates CBs and VRs simultaneously, minimizing, among other subjects, system losses and voltage deviations.

In terms of only CBs allocation, the first few methods [1–4] have been analytical, searching for economy of energy, reduction of losses and cost of banks' installation. However, they presented high computational efforts that gave birth to new approaches based on the application of heuristic techniques and relaxed versions of the problem. Searching for results that showed the reality of the problem in a more appropriate manner [5,6] considered the conductors of different sections and loads not uniformly distributed.

At the beginning of the nineties, the problem of CBs allocation was studied through the use of diverse techniques such as: Simulated Annealing [7,8], GA [9], Fuzzy Dynamic Programming [10], Fuzzy Systems [11–13] and other directed approaches through electric calculations and numerical methods [14,15], GA and Simulated Annealing [16], GA considering harmonic voltage distortion [17], GA considering an approach for complete distribution systems [18–20], Taboo Search [21] and other hybrid mathematical models [22,23] and heuristic algorithm [24].

In case of VR allocation, there are a few studies. For example, in [25], the optimal VR allocation for radial distribution systems was based on the minimization of an objective function (*OF*) that evaluates the cost of investment and maintenance of these equipments as well as the cost of losses of the network under analysis. In [26], a multi-objective function, focused on the losses and voltage drops, was taken into account using a micro-GA to find the solution.

In terms of simultaneous CBs and VRs allocation, there are only the studies [27,28]. These papers investigated the problem of con-

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temporaneously choosing optimal locations and sizes for both shunt capacitors and series voltage regulators in three-phase unbalanced distribution systems. The sizing and placement procedure not only minimized the power losses along distribution feeders but also made sure that both capacitors and series regulators had the minimum possible impact on the harmonic distortion of bus voltages in the system.

As this field is not enough considered, questions yet come up: which of these devices is the most adequate among so many other alternatives and objectives, besides the minimum cost of equipment and losses?

This study aims to answer this question, presenting a methodology to allocate CBs and VRs simultaneously, considering other objective functions related to the voltage profile. It uses, among the cited techniques, the GA, because of their easy applicability in combination problems. Differently of other studies, to evaluate the individuals supplied by the GA, the proposed methodology uses an OPF instead of the traditional load flows, previously accepted for radial distribution [29]. The OPF, solved by the Non-Linear Primal–Dual Interior Point Method [30], was chosen because it makes possible the optimization of tap regulators that assure the voltage level maintenance specified by the GA for different load platforms.

The exclusive allocation of VRs works mainly with voltage profile improvement and the exclusive allocation of CBs works mainly with supplement of reactive power. The simultaneous allocation of them presents a better combination of voltage profile and losses. So, this paper proposes a multi-objective problem that considers the costs of the equipment and losses (as majority of the works does) and considers the minimization of voltage limits and voltage drop violations along the feeder.

The study is organized as follows. First of all, some basic characteristics of voltage regulation, GA and mathematical formulation are shown. Then, some considerations about the OPF and the structure of algorithm are set. Finally, the results and conclusions are displayed for a 70-bus system [31].

2. Voltage regulation

To realize the allocation, initially, it is necessary to describe the limits of voltage as well as the equipments used for voltage control, focusing on the most relevant aspects.

For example, according to some countries, the magnitudes of voltage in steady state must be placed between 93% and 105% of nominal voltage of the system. There might be yet another cost for the distribution concessionaire, when there are voltage drops over 4% [32], beyond the costs with active power losses.

The imposition of these limits induces the concessionaires to raise the quality of service to their customers. But, to fulfill all these regulations, a detailed study of voltage correction alternatives must be carried out for it to be effective and inexpensive.

Among all the known possible alternatives of voltage control, two will be deeply analyzed in this study: the installation of CBs and VRs.

The CBs are used to make reactive compensation, minimize active losses and improve the voltage profile within acceptable limits. The amount of supplied compensation is related to the localization of capacitors at the distribution system, size, amount and the kind of capacitors to be installed [9].

The CBs available are either fixed or automatic, and their values are specified in terms of reactive power (kvar). The fixed banks are in operation permanently. The automatic banks can turn on and off depending on the load condition and adequate controls.

The VRs are equipments destined to maintain the voltage level at rural and urban distribution networks when they are submitted to voltage variations outside the specified limits.

A VR is basically an autotransformer that has taps and a control circuit responsible for commuting these taps whenever the voltage at the exit of the regulator violates the predetermined limits. Basically, there are three types of VRs: Autobooster, Line-Drop Compensation (LDC) and 32 steps.

This study used VRs of 32 steps because this regulation system is preferred by the concessionaires [33]. This kind of VR allows a constant and daily specified voltage at a pre-determined point of system. Each variation of the taps corresponds to 0.625%, for the 32 steps of voltage variation.

Some considerations about CBs and VRs allocation are still listed below:

(i) the methodology, described in the next section, determines the buses of the distribution network where the CBs must be installed, specifying the size (kvar) and the type (fixed or automatic);
(ii) the fixed CBs are used to make reactive compensation for all load levels; the automatic ones are used for middle and heavy load;
(iii) the VRs (32 steps, Type B) allow total voltage regulation of ±10%;
(iv) the methodology determines the lines of the network where the regulators must be installed, specifying the voltage levels to be adjusted at their exits.

3. Allocation's methodology

Next, the BC and VR allocation's methodology is formulated using GA.

The GA are evolutionary programs inspired by the Theory of Natural Selection. They act on a population of individuals based on the fact that the individuals with good genetic characteristics have greater survival possibilities and greater possibilities to produce more suitable individuals, while the less suitable individuals tend to disappear.

The GA are based, initially, on the generation of a population formed by a set of individuals that can be seen as possible solutions to the problem. During the evolutionary process, this population is evaluated. Fitness is calculated for each individual, reflecting its ability to adapt. A percentage of the most adapted individuals is retained, and others discarded. The members kept for the election can suffer modifications in their characteristics through recombination and mutations, generating descendents for the next generation that maintain the characteristics of the previous generation and make possible the variability of individuals in the population.

The genetic operators used are the selection via roulette with: mutation with tax of 10%, dispersed crossing with tax of 70% and elitism of two individuals. The creation of the initial population was random and the stopping criterion was a maximum number of generations equal to 10,000. The population includes 10 individuals.

In this study, the individual is a binary sequence that contains the localization, type and size of the CBs, and the localization and exit voltage of the VRs.

Fig. 1 presents this codification. It is formed by two parts. The first one indicates the line to receive a VR and the exit voltage of it. The second part indicates the bus to receive a CB, the type (fixed or automatic) and the size of it (kvar).

The fitness of each individual is obtained by the value *OF* that will be described in Eq. (1).

The *OF* prioritizes the reduction of electric losses, voltage profile improvement and cost of the devices.

If there are no economic limits of investment, as many CBs and VRs as are necessary in order to get satisfactory voltage profile may be placed. However, many companies have budget restrictions. So,

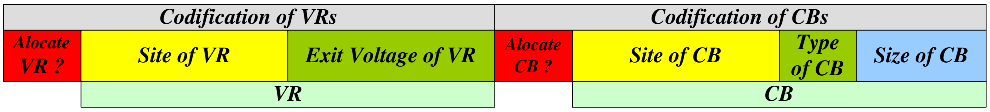

Fig. 1. Codification of an individual.

Table 1
Number of hours for each load level and days/year.

Load level (i)	Number of hours (H_{ij})		
	Working day	Saturday	Sunday/holiday
Heavy	6	6	4
Middle	12	10	12
Light	6	8	8
No. of days (D_{ij})	249	52	64

this study limits the number of CBs (n_max_CB – maximum number of CBs) and VRs (n_max_VR – maximum number of VRs) that can be placed.

The objective function that contemplates all the criteria previously cited is:

$$\min[f_1, f_2, f_3, f_4, f_5] \tag{1}$$

where f_1 is the cost of the active losses; f_2 the cost of voltage violation; f_3 the cost of voltage drop violation; f_4 the cost of CBs; f_5 is the cost of VRs.

The evaluation of cost is made for different load levels. Thus, nine load situations are considered through heavy, middle and light load levels (i) that occur for each one of the three kinds of days (j), namely, working day, Saturday and Sunday/holiday. The number of hours per day for each load condition (H_{ij}) and number of days per year (D_j) are presented in Table 1.

Table 2 complements Table 1, specifying the load levels through the percentage of total load adopted for each level.

Next, all the criteria will be described.

3.1. Cost of active power loss

The losses, $Loss_{ij}$, obtained before and after the installation of the devices, are calculated for each load condition, i, and day of the week, j. Considering the number of hours that each load condition uses during a day, and the number of days of each week during 1 year, the value of total energy consumed by the losses during one whole year can easily be obtained. This value of energy is multiplied by the *Tariff* (\$/MW h), thus obtaining the cost of losses acquired during the period of 1 year:

$$f_1 = Tariff \cdot \sum_{i=1}^{3}\sum_{j=1}^{3} Loss_{ij} \cdot H_{ij} \cdot D_j \tag{2}$$

3.2. Cost of voltage limit violation

A distribution system has to pay attention to pre-established voltage levels for the steady state operation, because it can be fined if there are violations. The optimizing problem must also consider the voltage magnitude limits by incorporating them into the *OF* and taxing them with a fixed cost (*Cost_Violation*).

Hence, after simulating the OPF, we look for buses whose magnitudes are beyond the established regulations. The limit violations are obtained as follows:

$$\begin{aligned}
&\text{If } V^{\min} \leqslant |\dot{V}_k| \leqslant V^{\max} \rightarrow Violation_k = 0\\
&\text{If } |\dot{V}_k| < V^{\min} \rightarrow Violation_k = V^{\min} - |\dot{V}_k| \quad k = 1, \ldots, nb\\
&\text{If } |\dot{V}_k| > V^{\max} \rightarrow Violationk_k = |\dot{V}_k| - V^{\max}
\end{aligned}$$

Table 2
Percentage of total load for each load level.

Load level (i)	Percentage of total load [24] (%)		
	Working day	Saturday	Sunday/holiday
Heavy	130	120	110
Middle	80	70	60
Light	50	40	40

where $\dot{V}_k$ is the complex voltage at bus k; $V^{\min}$ the minimum voltage magnitude; $V^{\max}$ the maximum voltage magnitude; $Violation_k$ the voltage violation at bus k; nb is the total number of buses.

The voltage violations ($Violation_k$) of all buses for each load condition are added and then multiplied by the *Cost_Violation*, as described by the following equation:

$$f_2 = Cost\_Violation \cdot \sum_{i=1}^{9} Voltage\_violation_i \tag{3}$$

where $Voltage\_violation_i$: sum of voltage violations ($Violation_k$) for every load condition i.

3.3. Cost of voltage drop violation

There is an additional cost when voltage drops over 4% occur between the initial bus of the distribution network and any other bus of the feeder [29]. Thus, when the voltage differences exceed 4%, an additional cost needs to be applied to the *OF*:

$$f_3 = 1.144 \cdot \sum_{i=1}^{3}\sum_{j=1}^{3}\left\{H_{ij} \cdot D_j \cdot \sum_{b \in \Phi}\left[(QT_{bij} - 4)^{1.45} \cdot C_{ij}\right]\right\} \tag{4}$$

where Φ is the set of buses with voltage drop over 4%; b the bus with voltage drop over 4%; QT_{bij} the voltage drop at bus b for the load level i and day j; C_{ij} is the load relating to the load level i and day j.

3.4. Cost of CB

The costs related to installation of fixed and automatic CBs are:

$$f_4 = K \cdot \sum_{bbc \in \Omega} (\chi_{bbc}^{fix} \cdot Co_{bbc}^{fix}) + (\chi_{bbc}^{aut} \cdot Co_{bbc}^{aut}) \tag{5}$$

where bbc is the bus with CB installed; Ω the set of buses with CBs installed; $\chi_{bbc}^{fix} = 1$, if the capacitor is fixed and installed at bus bbc and equal 0, contrary case; $\chi_{bbc}^{aut} = 1$, if the capacitor is automatic and installed at bus bbc and equal 0, contrary case; Co_{bbc}^{fix} the installation cost of fixed CB; Co_{bbc}^{aut} is the installation cost of automatic CB.

To equalize the units of the objective function (\$/year), the installations costs of CBs are annualized, using the annualizing coefficient for the installation cost of the devices, K:

$$K = \frac{r(1+r)^l}{(1+r)^l - 1} \tag{6}$$

where r is the annual interest rate; l is the period of lifetime.

3.5. Cost of VR

The sizes of the VRs are not given directly through the codification process of the GA, but are provided indirectly through the decodification of the installation place.

After the establishment of VR localization, the highest value of current that circulates through it can be calculated by OPF and its cost can be evaluated, considering yet an overload of 15%.

So, the costs concerning to the VRs installation are:

$$f_5 = K \cdot \sum_{lrt \in \Psi} Cost_{lrt} \tag{7}$$

where lrt is the line with VRs installed; Ψ the set of lines with VRs installed; $Cost_{lrt}$ the cost of VR installed at line lrt; K is the annualizing coefficient for the installation cost of the VRs.

3.6. Global Criterion Method

Genetic Algorithms use a single objective function and through elitist techniques calculate a set of optimal solutions of which some are selected as Pareto Solutions. Therefore, the Global Criterion Method (GCM) [34] was used to convert the multi-objective function (1) into a single objective function, expressed as:

$$\min \sum_{i=1}^{n} \left(\frac{f_i^* - f_i}{f_i^*} \right)^k \tag{8}$$

where f_i^* is an optimal value, n is equal to the number of objective functions and k is a project value, normally defined as 1 or 2.

Criteria with different values can dominate the final evaluation and harm criteria with lesser magnitudes. For example, the cost of loss has a greater value than the other costs. This fact hinders actions to make better voltage profile and to minimize the cost of the devices.

To solve it, generally, the GCM is adapted, normalizing each criterion through knowledge of the optimal value and the worst case ($f_i^{\max}$):

$$\min \sum_{i=1}^{n} \left(\frac{f_i - f_i^*}{f_i^{\max} - f_i^*} \right) \tag{9}$$

where f_i^* is an optimal value, n is equal to the number of objective functions and $f_i^{\max}$ is the worst case of the objective function f_i.

For simplicity, the ideal values for each criterion are considered equal to zero and the optimization problem is formulated as:

$$\min OF = \frac{f_1}{fmax_1} + \frac{f_2}{fmax_2} + \frac{f_3}{fmax_3} + \frac{f_4}{fmax_4} + \frac{f_5}{fmax_5} \tag{10}$$

where $fmax_1$ is the total loss before the devices connection; $fmax_2$ the subvoltage of 3% at all the buses; $fmax_3$ the voltage drop of 10% throughout the feeder; $fmax_4$ the cost of the most expensive CBs; $fmax_5$ is the cost of the most expensive VRs.

After this normalization, each criterion assumes values between 0 and 1. Or either, this strategy equalizes the magnitude of the functions, becoming all the criteria with the same importance.

If the GCM is formulated as Eq. (8), it would be necessary adequate choices of weights to equalize the criteria or emphasize some ones as plannerś decision. But, the intention of the work is not benefit any one of the functions, and exactly treat all of them equally.

The problem (1), solved by AG, is evaluated for nine load situations that occur for three load levels and three kinds of day. These evaluations are made by the solution of an OPF, formulated as follows.

4. Optimal Power Flow (OPF)

The GA techniques to allocate VRs and CBs demand an evaluation of each individual. To perform this evaluation, it is necessary to obtain the new state of the network with installation of selected equipments in order to verify the losses and the voltage profile. So, it is necessary to solve a load flow problem.

Some efficient methods to find a solution for the load flow problem in radial distribution networks are available in literature. These methods are divided into two main categories: the method of Power Sum, known as “Front and Backward Sweepings” and the methods based on “Implicit Nodal Impedance” [29].

However, in this study, there is the need to adjust the taps of the VRs when they are allocated in order to keep the outside voltage level established by the GA. These adjustments cannot be made by traditional power flows for distribution networks. So, a simplified OPF was shaped and resolved by Interior Points Method.

The OPF determines the state of an electric network, optimizing an objective function and satisfying a set of physical and operational restrictions.

In a conventional OPF, the maximum and minimum limits of voltage magnitudes are considered in all buses, which take up considerable computation time. As the OPF must be resolved many times, velocity and convergence must be guaranteed. Therefore, the maximum and minimum restriction limits of voltage magnitudes are not considered in the process. In order to skirt the disrespect of the voltage limits, it is used an objective function that tries to make the voltage magnitudes at all the buses close to the rated voltage (1 pu). Thus, the objective function formulated is the deviation of the vector of voltage magnitudes from the nominal value, specified as 1 pu.

$$DV = \sum_{i=1}^{nb} (|\dot{V}_i| - 1)^2 \tag{11}$$

In case of equality restrictions, the active and reactive power balance equations as well as the voltage imposition at the exit buses of the allocated regulators are taken into account. These are specified by individual decoding of the GA, beyond the imposition of different voltage magnitudes at the substation bus (SU) for different load levels, as shown in Table 3.

Finally, the restrictions of inequality of the taps are considered when the VRs are installed.

The complete formulation of the OPF is:

$$\min \quad DV \tag{12}$$

Subject to

$$\mathbf{Pg} - \mathbf{Pd} = real[diag(\dot{\mathbf{V}}) \cdot (\dot{\mathbf{Y}} \cdot \dot{\mathbf{V}})^*] \tag{13}$$

$$\mathbf{Qg} - \mathbf{Qd} = imag[diag(\dot{\mathbf{V}}) \cdot (\dot{\mathbf{Y}} \cdot \dot{\mathbf{V}})^*] \tag{14}$$

$$|\dot{V}_1| = V_{saida} \tag{15}$$

$$|\dot{V}_{reg}| = \dot{V}_{reg}^{AG} \quad reg = 1, \ldots, nreg \tag{16}$$

$$\mathbf{Pg}^{\min} \leqslant \mathbf{Pg} \leqslant \mathbf{Pg}^{\max} \tag{17}$$

Table 3
Voltage at the substation for each load level.

Load level (i)	V_{SU} (pu)
Heavy	0.9928
Middle	0.9783
Light	0.9565

$$\mathbf{Qg}^{\min} \leqslant \mathbf{Qg} \leqslant \mathbf{Qg}^{\max} \quad (18)$$

$$\mathbf{a}^{\min} \leqslant \mathbf{a} \leqslant \mathbf{a}^{\max} \quad (19)$$

where **Pg**, **Qg** is the active and reactive generations; **Pd**, **Qd** the active and reactive loads; $\dot{\mathbf{V}}$ the complex voltages; $\dot{\mathbf{Y}}$ the admittance matrix; $\mathbf{Pg}^{\min}$, $\mathbf{Pg}^{\max}$ the minimum and maximum active generation limits; $\mathbf{Qg}^{\min}$, $\mathbf{Qg}^{\max}$ the minimum and maximum reactive generation limits; $|\dot{V}_1|$ the voltage magnitude at substation; $V_{saída}$ the specified voltage magnitude at substatiońs bus; $|\dot{V}_{reg}|$ the voltage magnitude at the exit of voltage regulator, *reg*; $\dot{V}^{AG}_{reg}$ the voltage magnitude at the exit of voltage regulator, *reg*, specified by AG; *nreg* the number of voltage regulators; **a** the relation of voltage magnitudes of voltage regulators; $\mathbf{a}^{\min}, \mathbf{a}^{\max}$ is the minimum and maximum tap setting limits.

For the described model, the Eqs. (13) and (14) represent power balance equations; the Eq. (15) represents the specified bus at the substation, the Eq. (16) represents the specified bus at the exit of voltage regulation obtained by the optimization of problem (10), Eqs. (17) and (18) represent the generation limits and Eq. (19) represents the tap limits of voltage regulators.

This optimization problem (12) and (13) is solved by the primal–dual version of the Interior Points Method.

After the solution of this problem, all the complex voltages are known, which allows the calculations of the currents circulating through the voltage regulators, for each of the nine conditions simulated. The values of the biggest currents are used to obtain the commercial nominal current of each VR and consequently to obtain its equivalent cost.

5. Structure of algorithm

The Genetic Algorithms require the codification of individuals to acquire the solution of the problem. In this study, the individuals are binary types and they indicate:

(i) the buses where the connections of the CBs will be made, specifying the size (kvar) and type (fix or automatic) and
(ii) the lines where the connections of the VRs will be made, specifying the exit voltage for the allocated VRs.

The amount of necessary bits depends on the size of the system to be simulated.

The considered algorithm follows the steps:

1. Simulate the OPF for each load level without the connection of any CB or VR and calculate the operational conditions and total cost *OF*.
2. Create initial population in accordance with the number of buses.
3. Simulate the OPF for each individual and load condition, which adjusts the taps of the VRs in accordance with the exit voltage specified for them, calculates the losses, voltages and voltage drop violations, and the current circulating through the lines with VRs. With these results, calculate total cost, *OF*, and memorize the one with the best performance.
4. If the stop conditions are satisfied (maximum number of iterations), stop, otherwise apply the genetic operators and go to step 3.

6. Results

Fig. 2 presents the 70-bus system [31] used to test the methodology. Bus 1 represents the substation SU.

The value of *Tariff* considered is 98.79 $/kW h and the *Cost_Violation* is 57.00 $/V h [35,36]. The lifetime considered is 20 years and annual rate is 10%. Table 4 presents the prices of automatic and fixed capacitors.

Table 5 presents the prices of VRs for each nominal current.

Table 6 gives the violation of the minimum limit of voltage ($V^{\min} = 0.9$ pu and $V^{\max} = 1.05$) and voltage drops (>4%), without the allocation of voltage regulating devices.

Table 7 presents the costs of the five functions that compose *OF* without the allocation of voltage regulating devices, for a period of 1 year.

The results with the allocations are presented considering: (a) exclusive allocation of CBs, (b) exclusive allocation of VRs and (c) simultaneous allocation of CBs and VRs. They are presented to make comparisons and emphasize the advantage of the proposed methodology.

6.1. Exclusive CBs allocation

Table 8 presents the solution found through the GAs for exclusive CBs allocation. The maximum number of CBs considered is three capacitors.

Fig. 3 demonstrates that there is substantial improvement in voltage around the bus 60 due to the installation of CBs at the buses 62 and 65.

This solution does not eliminate all the existing problems related to the maximum limit of voltage drop violation, as Fig. 4

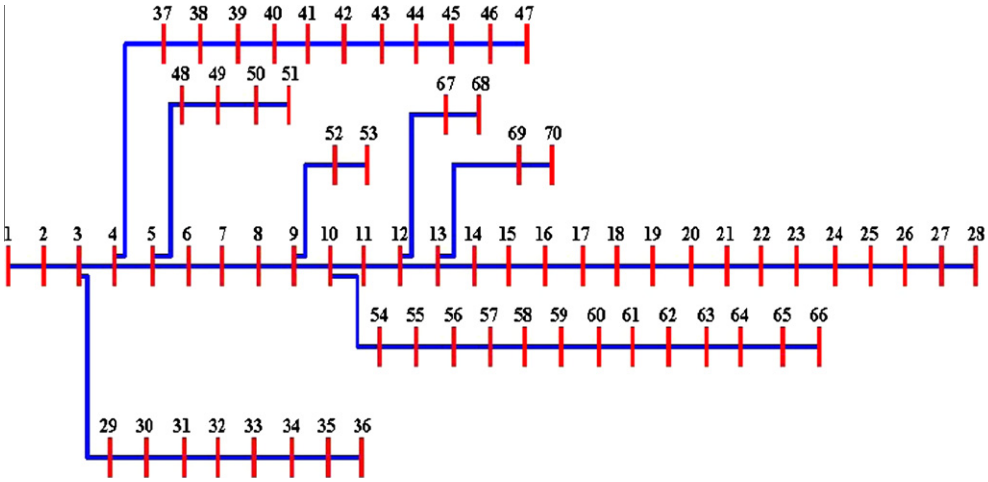

Fig. 2. Distribution network of 70-buses.

Table 4
Prices of automatic and fixed CBs.

(kvar)	Price ($)	
	Fixed	Automatic
150	2750	18,000
300	3000	18,500
600	3750	20,000
900	4250	21,000
1200	4750	22,000
1500	5250	23,000

Table 5
Nominal current and prices of VR (13.8 kV).

Current (A)	Price ($)	Current (A)	Price ($)
50	37,600	250	58,100
100	38,000	300	64,700
150	44,800	350	70,300
200	50,600	400	77,900

Table 6
Problems of violation and drop voltage without allocation.

Day	Load	$V_{bus} < 0.93$ pu (buses)	$\Delta V > 4\%$ (buses)
Working day	Heavy	58–66	15–28 and 57–66
	Middle	60–66	58–66
	Light	59–66	–
Saturday	Heavy	59–66	16–28 and 58–66
	Middle	62–66	59–66
	Light	61–66	–
Sunday holiday	Heavy	59–66	22–28 and 58–66
	Middle	–	62–66
	Light	61–66	–

Table 7
Each *OF* value without devices allocation.

Cost (U\$ 10^6)	Without allocation
Losses	1180
Drop voltage	68,500
Viol. voltage	610
CBs	–
VRs	–

Table 8
Exclusive CBs allocation 70-bus system.

Bus with CB	Type	Power (kvar)	Cost (U\$)
7	Fixed	600	3750
62	Fixed	1500	5250
65	Auto	1500	23,000

shows for working days and heavy load. The problem still persists between buses 57 and 66.

The disadvantages of this kind of allocation are that 29% of voltage drop violation is kept and 17% of losses increased. If more than three capacitors are allocated the voltage drop can diminish but the losses can increase more.

6.2. Exclusive VRs allocation

Table 9 presents the solution found through the GAs for exclusive VRs allocation for the 70-bus system and n_max_VR equal to three.

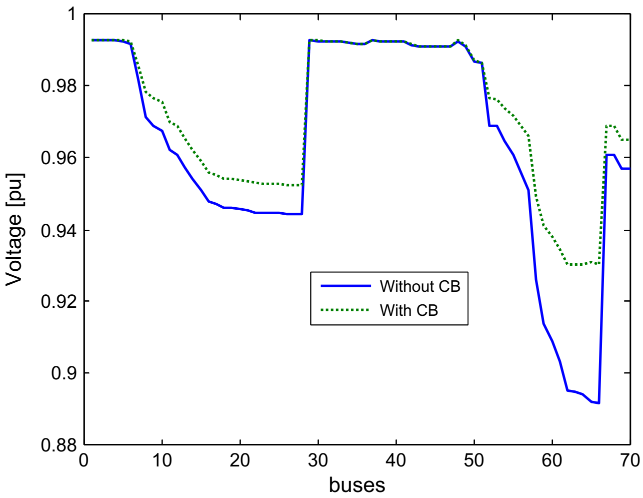

Fig. 3. Voltage profile with and without CBs – working day – heavy load.

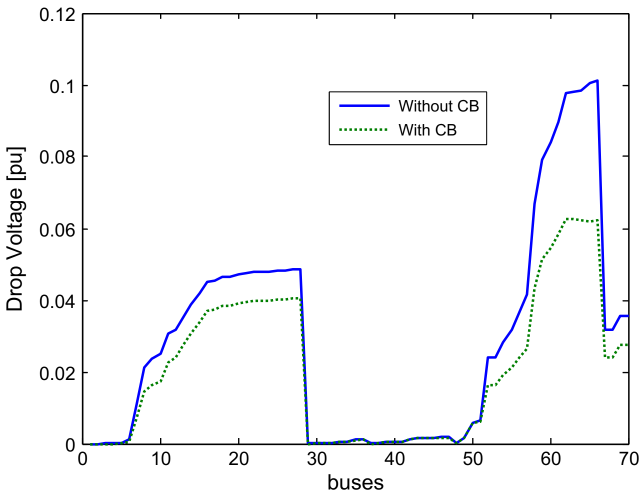

Fig. 4. Voltage drop with and without CBs – working day – heavy load.

Table 9
Exclusive VRs allocation.

Line with VR	Bus after VR	Voltage ajust. (pu)	Curr. (A)	Power (kV A)	Cost (U\$)
7	8	1.04	200	276	51600.00
57	58	1.05	150	207	44800.00

This solution eliminates all the existing problems related to the minimum limit of voltage violation for all kinds of days and load levels, as shown in the voltage profile of Fig. 5 for working days and heavy load.

In terms of voltage drop, the existing problems for all load levels and kinds of days, with drops over 4%, were also eliminated, mainly between buses 15 and 28 and between 54 and 68. Fig. 6 illustrates the voltage drops for working days and heavy load level.

It must be emphasized that the total elimination of voltage drop problems for a system with VRs is due to the fact that the buses after VR allocation are also considered as a reference beyond the substation bus.

Thus, for the calculation of voltage drop greater than 4% after the VRs, the differences in voltage are calculated taking as reference the first bus after the VR and, before this VR, taking as refer-

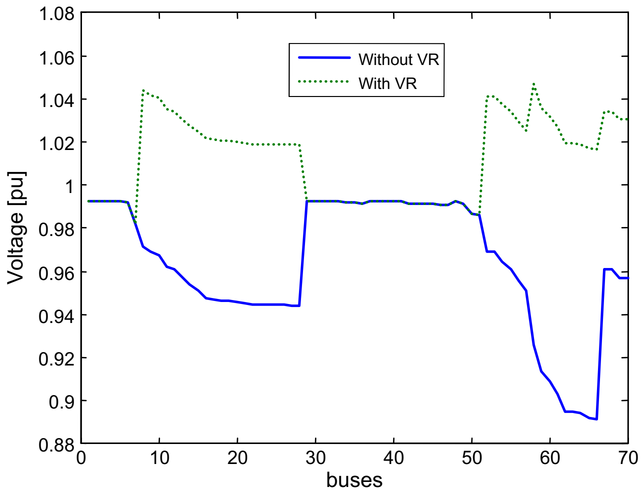

Fig. 5. Voltage profile with and without VRs – working day – heavy load.

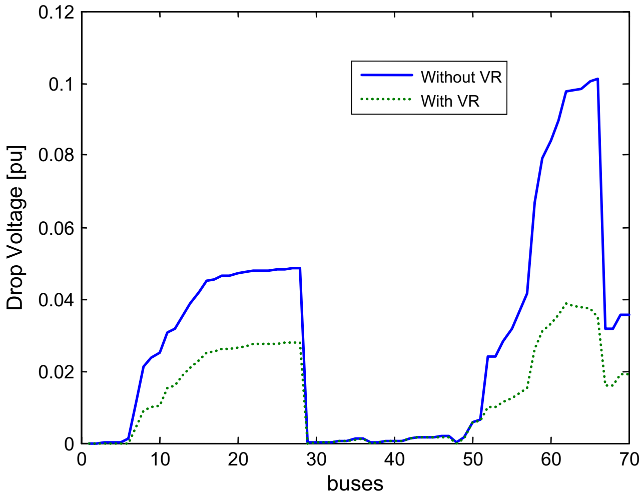

Fig. 6. Drop voltage with and without VR – working day – heavy load.

ence the first bus after the next VR when case more than one VR is placed at the same stretch. The same may also be calculated by taking the substation's bus as reference when any other VR is installed.

6.3. CBs and VRs simultaneous allocation

Tables 10 and 11 present the solution found by the proposed methodology that simultaneously allocates CBs and VRs for the 70-bus system. The maximum number of CB and VR devices that can be placed is three ($n\_\mathrm{max}\_\mathrm{CB} = n\_\mathrm{max}\_\mathrm{VR} = 3$).

This solution, that considers CBs and VRs simultaneously, eliminates all the minimum limit violation problems for all types of days and load levels. The voltage profile, shown in Fig. 7, illustrates the voltages for working days with heavy load level.

In terms of voltage drop, this solution also eliminates all the existing problems of voltage drop over 4%.

Table 12 presents the costs of the five functions that compose the *OF* as well as the simple sum of them (TC), for a period of 1 year.

Comparing the values of Table 12, it can be observed a reduction of the costs related to the active power losses around 120% and 156%, per year, respectively, for the solutions with exclusive VRs allocation and simultaneous CBs and VRs allocation.

Table 10
CB solution for CBs and VRs simultaneous allocation.

Bus with CB	Type	Power (kvar)	Cost (U$)
25	Fixed	150	2750
62	Fixed	900	4250
69	Fixed	150	2750

Table 11
VR solution for CBs and VRs simultaneous allocation.

Line with VR	Bus after VR	Voltage ajust. (pu)	Current (A)	Power (kV A)	Cost (U$)
14	15	1.02	50	69	37,600
57	58	1.03	150	207	44,800

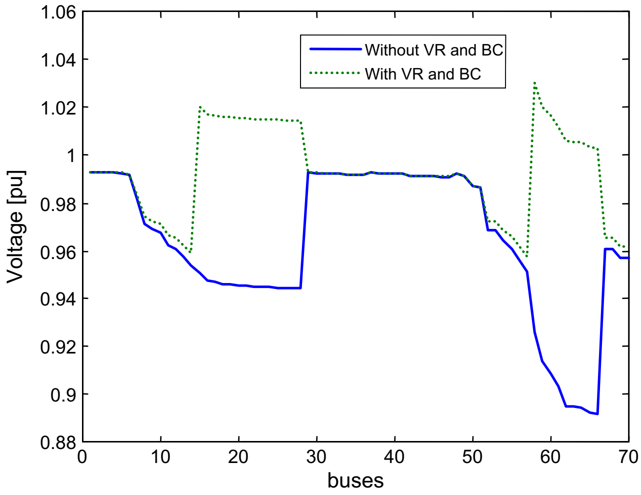

Fig. 7. Voltage profile with and without CBs and VRs – working day – heavy load.

Table 12
Costs and *OF* value with CBs and VRs allocation.

Cost (U$ 10^6)	Without CB and VR	CBs	VRs	CBs and VRs
Losses	1180	1380	990.50	754.35
Drop Voltage	68,500	19,796	0	0
Viol. of voltage	610	0	0	0
CBs	–	0.032	–	0.00975
VRs	–	–	0.0964	0.0824
TC	70,290	21,176	990.5964	754.442

In terms of voltage profile, only three CBs are not sufficient to resolve the drops in voltage along the feeder, two VRs make it better, but three CBs and two VRs decrease the losses substantially and yet improve the voltage profile of all the buses for all types of days and load levels.

In order to reduce these costs, an investment to obtain the required equipment becomes necessary. Then, for the solution with exclusive CBs allocation, U$ 32,000 is required; for the solution with exclusive VRs allocation, an amount of U$ 96,400 is required and finally, for the solution of simultaneous CBs and VRs allocation, a sum of U$ 92,150 is needed, which makes possible an economy of U$ 426 $\times 10^6$ due to the reduction of losses.

The simultaneous allocation of CBs and VRs presents a better combination of results than that obtained through the exclusive allocation because it conciliates the effect of the VRs (improvement of the voltage profile) and the CBs (supplement of reactive power).

7. Conclusions

It is common to allocate only capacitors to resolve problems of losses and low power factor, and to allocate only voltage regulators to resolve problems of voltage profile. This paper proposed a methodology that allocated both at the same time, analyzing the effects of each one and confirming the advantage of a unique proposal at the same time, as already point out in [27,28].

This work proposes a multi-objective problem that considers the costs of the equipment and losses (as the majority of the works does) and differently, considers the violations of voltage limits and of voltage drops along the feeder, conciliating them at the same strategy (cost of the devices, voltage profile and losses).

The BC and VR allocatiońs methodology is formulated using GA. Each individual is evaluated using an OPF instead of the traditional load flows, previously accepted for radial distribution. The OPF was chosen because it makes possible the optimization of tap regulators that assure the voltage level maintenance specified by the GA for different load platforms and to obtain the nominal current of the VRs allocated.

Analyzing the results, the best solution in terms of total cost is that obtained by simultaneous CBs and VRs allocation. This solution is the best (with less investment than required by exclusive VR allocation) and more benefits achieved by the reduction of losses.

This methodology that allocates capacitor and voltage regulators at the same time can be a seed to incorporate latter some observations as: changes in distribution system topology during contingency conditions; effect of switching capacitors and fault current.

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