

# Nonlinear Control and Disturbance Decoupling of HVAC Systems Using Feedback Linearization and Backstepping With Load Estimation

E. Semsar-Kazerooni, *Student Member, IEEE*, M. J. Yazdanpanah, *Member, IEEE*, and Caro Lucas, *Senior Member, IEEE*

**Abstract**—A new control methodology is introduced for nonlinear and MIMO heating, ventilating and air conditioning (HVAC) systems. A full information feedback of states and disturbances is used for disturbance decoupling and model linearization purposes. Then a backstepping controller is applied to the system linearized model. It is shown that in this way, heat and moisture loads can be compensated completely if considered as measurable disturbances. For non-measurable disturbances, a stable observer is designed for estimation purpose. Simulation results show that the closed-loop system has good and fast tracking, offset-free and smooth response with high disturbance decoupling and optimal energy consuming properties in presence of time-varying loads.

**Index Terms**—Estimation, nonlinear systems, observer, tracking.

## NOMENCLATURE

$h_w, h_{fg}$	Enthalpy of liquid water and water vapor.
$W_o, W_s$	Humidity ratio of outdoor and supply air.
$W_3$	Humidity ratio of thermal space.
$V_{he}, V_s$	Volume of heat exchanger and thermal space.
$M_o, Q_o$	Moisture and heat loads.
$T_2, T_3$	Temperature of supply air and thermal space.
$f$	Volumetric flow rate of recirculated air.
$\rho, C_p$	Air mass density, specific heat of air.
$T_o$	Temperature of outdoor air.
$gpm$	Flow rate of chilled water.

## I. INTRODUCTION

HEATING, ventilating and air conditioning (HVAC) systems are among the most challenging plants in the field of process control. The energy consumed by HVAC equipments constitutes 50% of the total world energy consumption [1].

Manuscript received August 4, 2005; revised June 4, 2007. Manuscript received in final form October 24, 2007. First published March 31, 2008; last published July 30, 2008 (projected). Recommended by Associate Editor S. Palanki.

E. Semsar-Kazerooni was with the Electrical and Computer Engineering Faculty, College of Engineering, University of Tehran, Tehran, Iran. She is now with the Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, H3G 1M8, Canada (e-mail: e\_semsar@ece.concordia.ca).

M. J. Yazdanpanah and C. Lucas are with the Center of Excellence for Control and Intelligent Processing, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran, Iran (e-mail: yazdan@ut.ac.ir; lucas@ut.ac.ir).

Digital Object Identifier 10.1109/TCST.2007.916344

HVAC systems include all air-conditioning systems used for cooling or heating the buildings. The purpose of their design is to have a controlled temperature and percentage of humidity in a closed space. In some cases, the problem of HVAC systems control is inevitable from two aspects. The first point is to achieve control objectives with optimal consumption of energy. A mere 1% improvement in energy efficiency of these systems translates into annual savings of millions of dollars at the national level [2]. Moreover, in cases such as a Metro system which is underground or in the towers with hundred stages, it is necessary to have a complete air conditioning system, which keeps the moisture, temperature and perhaps pressure in acceptable ranges.

A wide range of research has been conducted to achieve one or all of these purposes. PID controllers have been used for temperature control for many years [3]. In [4], Direct Digital Control (DDC) is used for the same purpose. The authors in [5] introduced a tuning method for a digital controller based on relay feedback and pole placement strategies. Intelligent methods have been used in [2], [6] for control and identification purposes.

Zaheeruddin introduced optimal and suboptimal controllers for these systems in [7] and [8] and a decentralized controller for both single and multi-zone plants in [9]–[11]. The control goals in these references are mainly set point regulation for temperature and rejection of disturbances. In [12], a robust controller is implemented to a nonlinear HVAC system. For the purposes of control and minimization of actuator repositioning by different methods, [13]–[15] can be referred. In [16], authors introduced a nonlinear optimal strategy and in [17] a nonlinear non-interacting controller is used for purpose of temperature and relative humidity control. There, a feedback linearization method is used for design of a non-interactive control. In [18], the improvements in HVAC system performance are investigated through application of multiple-input, multiple-output (MIMO) robust controllers. The method is tested on an experimental test-bed, as well.

In [19], feedback linearization and backstepping methods are used for the purposes of disturbance decoupling and regulation. The difference between that work and the present one is that in the former, the full state and disturbance knowledge is available whereas in the latter, the disturbances are unavailable and so, a stable observer is designed for disturbance estimation. Also, in the hybrid control domain [20] can be referred in which discrete event system (DES) and continuous variable dynamic system

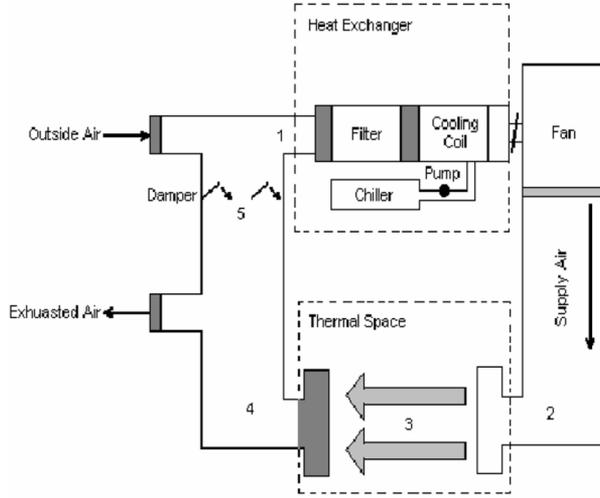


Fig. 1. A single-zone VAV HVAC system.

theories are integrated in order to control the temperature in HVAC systems.

Although many strategies have been used to improve the performance of these systems, there are still many unsolved problems. An example is the problem of load compensation. It is important to keep the air condition in an acceptable mode in the presence of load disturbances such as heat and moisture loads. This should be done without extra energy consumption. In the present paper a new approach for control and disturbance decoupling in HVAC systems is introduced using feedback linearization and backstepping methods. This method is applied to a nonlinear model of the system and the simulation results show its good performance in achieving the pre-specified goals. The organization of the rest of the paper is as follows. In Section II, the problem is defined formally. In Section III the feedback linearization method is described. In Section IV, an observer is designed to estimate the load vector. Backstepping method is explained in Section V. Finally, simulation results and conclusions are given in Sections VI and VII.

## II. PROBLEM STATEMENT

A single-zone Variable Air Volume (VAV) HVAC system is shown in Fig. 1. Its main parts are a circulating air fan, a thermal space, a chiller, dampers, and mixing air components. The detailed operation of the system is given in [1] and its analytical model is presented in the following subsection.

### A. HVAC Model

Several models have been considered for HVAC systems in different references. In [13] and [14], a linear first order model with a time delay is considered. In [21], a nonlinear SISO model is used for simulations. In some other references, a bilinear model is considered for describing the temperature(s) dynamics [16], humidity, or both of them [1].

In this paper, the latter is considered which presents both temperature and humidity dynamics. This seems to present a more realistic model of a single-zone HVAC system. It is obvious that humidity and temperature factors are important in

providing a good conditioned air. Unlike the work in [1] which defines the thermal space temperature and humidity as the outputs, the outputs here are defined as thermal space and supply air temperatures similar to the model in [16]. Moreover, to be more practical, here the actuators dynamics are considered in the system model, as well. The mathematical model of the system without actuators' dynamics is given in (1) and (2), where  $x = [x_1 \ x_2 \ x_3]^T$  is the state vector,  $z = [z_1 \ z_2]^T$  and  $y$  are the input and output vectors and  $\omega = [\omega_1 \ \omega_2]^T$  is a slowly time varying disturbance which consists of heat and humidity loads [1] and is either measurable or observable to be estimated:

$$\dot{x} = f_1 x z_1 + g_1 z + p_1 \omega, \quad y = [x_1 \ x_3]^T \quad (1)$$

$$f_1 = \begin{bmatrix} -\alpha_1 60 & \alpha_2 60 & \alpha_1 60 \\ 0 & -\alpha_1 60 & 0 \\ \beta_1 45 & -\beta_3 45 & -\beta_1 60 \end{bmatrix},$$

$$p_1 = \begin{bmatrix} \alpha_3 & -\alpha_3 h_{fg} \\ 0 & \alpha_4 \\ 0 & 0 \end{bmatrix},$$

$$g_1 = \begin{bmatrix} -\alpha_2 60 W_s & 0 \\ \alpha_1 60 W_s & 0 \\ -\beta_3 60 (.25 W_o - W_s) + \beta_1 15 T_o & 6000 \beta_2 \end{bmatrix}. \quad (2)$$

The states and model parameters are defined as follows:

$$\begin{aligned} z_1 &= f, \quad z_2 = gpm, \quad x_1 = T_3, \\ x_2 &= W_3, \quad x_3 = T_2, \quad \omega_1 = Q_0, \\ \omega_2 &= M_0, \quad \beta_1 = 1/V_{he}, \\ \beta_2 &= 1/\rho C_p V_{he}, \quad \beta_3 = h_w/C_p V_{he}, \\ \alpha_1 &= 1/V_s, \quad \alpha_2 = h_{fg}/C_p V_s, \\ \alpha_3 &= 1/\rho C_p V_s, \quad \alpha_4 = 1/\rho V_s \end{aligned}$$

and the numerical values are listed in Table I. In [1], the required assumptions that are made in deriving this model are given.

The control signal in this model is implemented to liquid valves. The valve dynamical model can be considered as follows in which  $\psi(s)$  is the valve inherent characteristic,  $\tau_v$  is the time constant of the valve and  $z(s)$  is the flow rate of the liquid which enters the valve [21]:

$$z(s) = \psi(s)/(1 + \tau_v s) \quad (3)$$

According to the discussion in [21], by considering the characteristic of a linear valve as  $\psi(s) = ku(s)$ , where  $k$  is a constant gain, the valve transfer function can be written as

$$G(s) = z(s)/u(s) = k/(1 + \tau_v s) \quad (4)$$

where  $u(s)$  and  $z(s)$  are the control and input signals applied to the actuator and plant, respectively. Therefore, if a new state vector is defined as  $X = [x^T \ z^T]^T$ , the augmented model of the system and actuators can be written as:

$$\dot{X} = F(X) + G(X)u + P(X)\omega, \quad y = [x_1 \ x_3]^T \quad (5)$$

TABLE I  
NUMERICAL VALUES FOR SYSTEM PARAMETERS

Parameter	Value
$W_o$	0.0018 g / g(lb/lb)
$W_s$	0.007 g / g(lb/lb)
$M_o$	21 g / c(166.06 lb / hr)
$V_{he}$	1.72 m <sup>3</sup> (60.75 ft <sup>3</sup> )
$C_p$	1.005 Kj / Kg.°C(0.24 Btu / lb.°F)
$\tau_v$	29s(0.008 hr)
$k$	5
$V_s$	1655.5m <sup>3</sup> (58464 ft <sup>3</sup> )
$Q_o$	84960W(289897Btu / hr)
$T_o$	29.5°C(85°F)
$\rho$	1.19 kg / m <sup>3</sup> (0.074 lb / ft <sup>3</sup> )
$h_{fg}$	2507.83Kj / Kg(1078.25 Btu / lb)
$h_w$	790.78Kj / Kg(340 Btu / lb)
$T_{3ref}(x_1)$	22.8°C(73°F)
$T_{2ref}(x_3)$	12.8°C(55°F)

where

$$\begin{aligned}
 F(X) &= [\gamma_1 z_1 \quad \gamma_2 z_1 \quad \gamma_3 z_1 + \gamma_4 z_2 \quad -z_1/\tau_v \quad -z_2/\tau_v]^T, \\
 G(X) &= \begin{bmatrix} 0_{2 \times 3} & \vdots & \begin{bmatrix} k/\tau_v & 0 \\ 0 & k/\tau_v \end{bmatrix} \end{bmatrix}^T, \\
 P(X) &= \begin{bmatrix} \begin{bmatrix} \alpha_3 & 0 \\ -\alpha_3 h_{fg} & \alpha_4 \end{bmatrix} & \vdots & 0_{2 \times 3} \end{bmatrix}^T, \\
 \gamma_1 &= \alpha_1 60(x_3 - x_1) - \alpha_2 60(W_s - x_2), \\
 \gamma_2 &= \alpha_1 60(W_s - x_2), \\
 \gamma_3 &= \beta_1 60(-x_3 + x_1) - \beta_3 60(.25W_o + .75x_2 - W_s) \\
 &\quad + \beta_1 15(T_o - x_1), \\
 \gamma_4 &= -6000\beta_2.
 \end{aligned}$$

The above HVAC model is a time-delayed and MIMO one in which one of the I/O channels has a right half-plane zero, which means that it is a non-minimum-phase system.

### III. LINEARIZATION VIA FEEDBACK

#### A. Relative Degree

Relative degree is the number of times one should differentiate the output in order to have the inputs be appeared. The rel-

ative degree for a MIMO model can be defined as follows [22]. Consider the following system:

$$\begin{cases} \dot{X} = F(X) + \sum_{j=1}^m g_j(X)u_j + \sum_{j=1}^m p_j(X)\omega_j = \\ F(X) + G(X)u + P(X)\omega, \quad X \in R^n, u_j, \omega_j \in R, \\ y_i = h_i(X) \in R, \quad i, j = 1, \dots, m \end{cases} \quad (6)$$

It has a vector input relative degree  $r = \{r_1, \dots, r_m\}$  around  $X_0$ , if the following conditions are met:

1)

$$L_{g_j} L_F^k h_i = 0, \quad 1 \leq j \leq m, \quad 1 \leq i \leq m, \quad k < r_i - 1 \quad (7-a)$$

where  $L_F^k h$  is Lie derivative and is the  $k$ th derivative of  $h(X)$  with respect to  $X$  along  $F$  [22].

2) The following matrix is nonsingular in a neighborhood of operating point,  $X_0$ :

$$A(X) = \begin{bmatrix} L_{g_1} L_F^{r_1-1} h_1(X) & \dots & L_{g_m} L_F^{r_1-1} h_1(X) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_F^{r_m-1} h_m(X) & \dots & L_{g_m} L_F^{r_m-1} h_m(X) \end{bmatrix}. \quad (7-b)$$

#### B. Relative Degree in HVAC Systems Model

Considering the model described in (5), and after some calculations, it can be shown that for this model of HVAC systems we have

$$A(X) = \begin{bmatrix} \gamma_1 k_1/\tau_1 & 0 \\ \gamma_3 k_1/\tau_1 & \gamma_4 k_2/\tau_2 \end{bmatrix}, \quad r = \{2, 2\}. \quad (8)$$

To ensure that the transformation is invertible, this matrix should be nonsingular in a neighborhood and so the two terms  $\gamma_1 k_1/\tau_1$  and  $\gamma_4 k_2/\tau_2$  should be nonzero. The latter is always a nonzero parameter whereas the former is a function of system states and in general cannot be guaranteed to be nonzero. However, for a local linearization, which is the case here, we may always select the operating point so that this term remains nonzero. In other words, if the two terms  $\alpha_1(x_3 - x_1)$  and  $\alpha_2(W_s - x_2)$  are scaled so that they never become of the same order around the operating point, then  $\gamma_1 k_1/\tau_1$  would be nonzero and non-singularity of  $A(X)$  can be guaranteed locally around the operating point  $X_0$ . The operating point selected in the simulations satisfies this condition.

#### C. Exact Linearization via Feedback: MIMO Case

As referred in [22], linearization via feedback is applicable to MIMO systems with or without presence of disturbances in their model. Since the HVAC model considered here has disturbance in its representation, in this paper we focus on the linearization of models with disturbances. It is mentioned in [22] that there exist two different types of feedback for the system described in (6), which transform this model into a linear and controllable one in normal form. Moreover, this feedback can render the outputs independent of the disturbances. The point to be considered

is that, disturbances have to be available or observable to be estimated. Here, a lemma which is mentioned in [22] without any proof is used and a brief proof is given.

*Lemma 1* [22]:

1) There exists a feedback of the form

$$u = \alpha(X) + \beta(X)v \quad (9)$$

which renders the outputs of the system in (6) independent of the disturbance  $\omega$  iff

$$\begin{aligned} L_P L_F^k h_i(X) &= 0 \\ \text{For all } 0 \leq k \leq r_i - 1, \quad 1 \leq i \leq m. \end{aligned} \quad (10)$$

2) There exists a feedback of the form

$$u = \alpha(X) + \beta(X)v + \gamma(X)\omega \quad (11)$$

which renders the outputs of the system in (6) independent of the disturbance  $\omega$  iff

$$\begin{aligned} L_P L_F^k h_i(X) &= 0 \\ \text{For all } 0 \leq k \leq r_i - 2, \quad 1 \leq i \leq m. \end{aligned} \quad (12)$$

*Proof:* The proof is provided in the Appendix.

Although this lemma can be used in some cases, there are many other models, which cannot satisfy the conditions described in this lemma. In this paper, the special conditions, besides those in Lemma 1, are introduced in the following new lemma to linearize the nonlinear model and decouple the output from the disturbances for a wide range of systems to which conditions of Lemma 1 cannot be applied. The advantage of this lemma is that the term  $L_P L_F^{r_i-2} h_i(X)$  does not have to be zero but just independent of states.

*Lemma 2:* There exists a feedback of the form (11), which renders the outputs of system (6) independent of the disturbance vector  $\omega$ , if  $\omega$  is slowly time varying compared to system time constant and the following statements hold

$$\text{i) } L_P L_F^k h_i(X) = 0 \quad \text{for } 0 \leq k < r_i - 2, \quad 1 \leq i \leq m \quad (13\text{-a})$$

$$\text{ii) } L_P L_F^{r_i-2} h_i(X) \text{ is independent of the states.} \quad (13\text{-b})$$

*Proof:* The proof is provided in the Appendix.

*Remark 1:* If matrix  $A(X)$  is nonsingular (at least locally), then the coordinate transformation (38) (see the Appendix) is invertible and hence the corresponding mapping is a diffeomorphism.

*Remark 2:* It is worth mentioning that although the transformed states  $\xi^i$  (see the Appendix) are functions of the disturbance  $\omega$ , this does not affect the problem of disturbance decoupling. The reason is that the model is transformed into normal form regardless of the definition of  $\xi^i$ . Therefore, since  $\omega$  already appears explicitly in (39) (see the Appendix), even if  $\xi^i$  is defined as a function of  $\omega$ , this does not introduce any new dependency on  $\omega$ . Moreover, the output which is the first state in the new coordinates, is not a function of  $\omega$ . As a result, the

disturbance decoupling issue is not violated by adopting the proposed coordinate transformation.

#### D. Linearization via Feedback in HVAC Systems Model

For the present model of HVAC systems, the relative degree is  $r = \{2, 2\}$  and the following relations are held:

$$\begin{aligned} L_P h_1(X) &\neq 0, \text{ For } (0 \leq k \leq 2 - 2 = 0 \text{ or } k = 0), \\ L_P h_2(X) &= 0, \text{ For } (0 \leq k \leq 2 - 2 = 0 \text{ or } k = 0). \end{aligned}$$

Hence, none of the two parts of Lemma 1 can be applied to this model. Now, let us verify the conditions in Lemma 2. Condition (13-a) is satisfied readily. Now consider (13-b)

$$L_P L_F^{r_i-2} h_i(X) = \begin{cases} L_P L_F^0 h_i(X) = [\alpha_3 & -\alpha_3 h_{fg}]^T, & i = 1 \\ [0 & 0]^T, & i = 2 \end{cases}$$

which are independent of the state variables. Hence, Lemma 2 is applicable to this model of HVAC systems. Now a coordinate transformation of the form

$$\xi = \begin{bmatrix} x_1 \\ \gamma_1 z_1 + \alpha_3(\omega_1 - h_{fg}\omega_2) \\ x_3 \\ \gamma_3 z_1 + \gamma_4 z_2 \end{bmatrix} \quad (14)$$

can transform the nonlinear model of the system into the linear form of

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = b_1 + \sum_{j=1}^2 a_{1j} u_j + \sum_{j=1}^2 c_{1j} \omega_j = v_1 \\ \dot{\xi}_3 = \xi_4 \\ \dot{\xi}_4 = b_2 + \sum_{j=1}^2 a_{2j} u_j + \sum_{j=1}^2 c_{2j} \omega_j = v_2 \\ \dot{\eta} = q(\xi, \eta) + S(\xi, \eta)u + R(\xi, \eta)\omega \end{cases} \quad (15)$$

where  $u = A^{-1}(v - b - c\omega) = \alpha(X) + \beta(X)v + \gamma(X)\omega$ , in which  $A(X)$  in (8), is locally nonsingular

$$\begin{aligned} b(X) &= [L_F^2 h_1(X) \quad L_F^2 h_2(X)]^T, \text{ and} \\ c(X) &= \begin{bmatrix} -\alpha_3 \alpha_1 z_1 60 & 60 z_1 (-\alpha_3 \alpha_1 h_{fg} + \alpha_2 \alpha_4) \\ \alpha_3 \beta_1 z_1 45 & 45 z_1 (-\alpha_3 \beta_1 h_{fg} - \beta_3 \alpha_4) \end{bmatrix}. \end{aligned}$$

$\eta = \xi_5$  should be defined in such a way to satisfy the condition  $L_{g_j} \xi_5 = 0$ ,  $j = 1, 2$ , and the mapping  $\phi(X) = \text{col}[\xi_1, \xi_2, \xi_3, \xi_4, \xi_5]$  has a nonsingular Jacobian matrix so that it would be a diffeomorphism.

It can be shown that  $\eta = \xi_5 = x_2$  satisfies these conditions. Moreover, in the case of HVAC systems model, the internal dynamics  $\eta$  is stable and  $A(X)$  is nonsingular (locally), hence the transformation is a local diffeomorphism. Therefore, the system may be transformed into normal form with a stable zero dynamics. Hence, design of a controller for the linear part of the model can be done using common control methods to track a specific reference or regulate the outputs to desired set points. In Section V, the backstepping method is used for this purpose.

#### IV. LOAD ESTIMATION

As elaborated in the previous section, a nonlinear system as in (6), which satisfies special conditions, can be transformed into the linear form (40) using the nonlinear feedback as in (39) (see



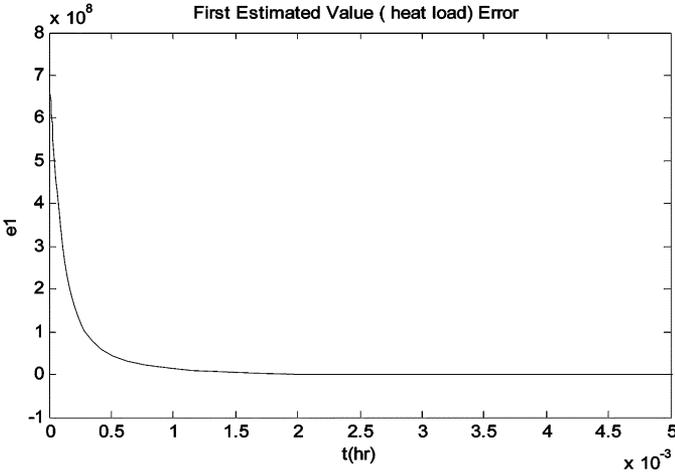


Fig. 2. Estimation error of the first parameter (zoomed in).

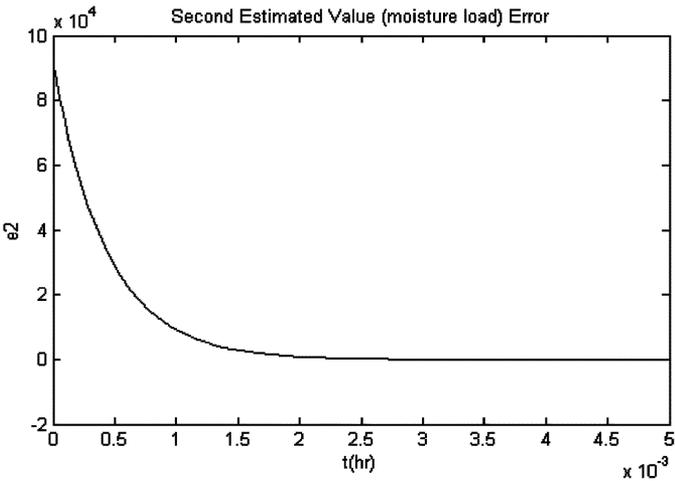


Fig. 3. Estimation error of the second parameter (zoomed in).

### A. Backstepping Control Design for Systems in Normal Form

It can be shown that a system in normal form is a special case of the system in (26) with the following definitions:

$$\begin{aligned} f(\eta^i) &= \begin{bmatrix} 0_{(r_i-2) \times 1} & I_{(r_i-2) \times (r_i-2)} \\ 0 & 0_{(r_i-2) \times 1} \end{bmatrix} \eta^i, \\ g(\eta^i) &= \begin{bmatrix} 0_{(r_i-2) \times 1} \\ 1 \end{bmatrix}, \\ \eta^i &= [\xi_1^i \cdots \xi_{r_i-1}^i]^T, \zeta^i = \xi_{r_i}^i, \\ u_i &= v_i, z = \zeta^i - \varphi(\eta^i). \end{aligned} \quad (29)$$

In this case, candidates for  $V$  and  $\varphi$  can be defined as

$$\begin{aligned} V &= 1/2(\eta^i)^T P \eta^i, \\ \varphi(\eta^i) &= [-a_0 \quad -a_1 \quad \cdots \quad -a_{r_i-2}] \eta^i \end{aligned} \quad (30)$$

where  $a_i$ 's should be selected to guarantee stability of dynamics of  $\eta^i$ , i.e., the matrix  $Q$  defined below should be Hurwitz:

$$\begin{aligned} \dot{\eta}^i &= \begin{bmatrix} 0_{(r_i-2) \times 1} & I_{(r_i-2) \times (r_i-2)} \\ 0 & 0_{(r_i-2) \times 1} \end{bmatrix} \eta^i \\ &+ \begin{bmatrix} 0_{(r_i-2) \times 1} \\ 1 \end{bmatrix} [-a_0 \quad -a_1 \quad \cdots \quad -a_{r_i-2}] \eta^i \\ &= \begin{bmatrix} 0_{(r_i-2) \times 1} & I_{(r_i-2) \times (r_i-2)} \\ -a_0 & -a_1 \quad \cdots \quad -a_{r_i-2} \end{bmatrix} \eta^i \\ &= Q \eta^i. \end{aligned}$$

There can be always a solution for  $a_i$ 's since the pair  $(f(\eta^i), g(\eta^i))$  is both controllable and stabilizable. Hence, the proper selection of  $\varphi$  as in (30) makes the first part of the dynamics in (26) stable. Moreover, we have

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial \eta^i} \dot{\eta}^i \\ &= (\eta^i)^T (PQ + Q^T P) \eta^i \leq - \underbrace{((\eta^i)^T R \eta^i)}_{P.D.} \\ &= -W(\eta) \end{aligned}$$

where the matrix  $P$  satisfies the Lyapunov inequality  $PQ + Q^T P \leq -R$ .

1) *Structure of Backstepping Controller for Systems in Normal Form:* The controller can be derived as follows using (28), and defining

$$\begin{aligned} a &= [a_0 \quad a_1 \quad \cdots \quad a_{r_i-2}]^T \\ \bar{f} &= \begin{bmatrix} 0_{(r_i-2) \times 1} & I_{(r_i-2) \times (r_i-2)} \\ 0 & 0_{(r_i-2) \times 1} \end{bmatrix} : \end{aligned}$$

$$\begin{aligned} v_i &= u_i = \frac{\partial \varphi}{\partial \eta^i} [f(\eta^i) + g(\eta^i) \zeta^i] \\ &\quad - \frac{\partial V}{\partial \eta^i} g(\eta^i) - k_i [\zeta^i - \varphi(\eta^i)] \\ &= -a^T (f(\eta^i) + g(\eta^i) \zeta^i) - (\eta^i)^T P g(\eta^i) - k_i \zeta^i - k_i a^T \eta^i \\ &= -(a^T \bar{f} + g^T P + k_i a^T) \eta^i - (a^T g + k_i) \zeta^i \\ &= -\{[k_i a_0 \quad a_0 + k_i a_1 \quad \cdots \quad a_{r_i-4} + k_i a_{r_i-3} \quad a_{r_i-3} + k_i a_{r_i-2}] \\ &\quad + P_{r_i-1}\} \eta^i \\ &\quad - (a_{r_i-2} + k_i) \zeta^i, \quad i = 1, \dots, m \end{aligned} \quad (31)$$

where  $P_{r_i-1}$  is the last row of matrix  $P$ . This is a state feedback in which two sets of parameters,  $k_i$  and  $a_i$  as well as matrix  $P$  should be designed to guarantee the stability and performance of the closed-loop system.

### B. Applying Backstepping Method to HVAC System Model

As mentioned previously, the HVAC model used in this paper has a vector relative degree for the input as  $r = \{2, 2\}$ . First, it should be noted that the control goal here is to regulate the output around a point  $y_d^i$  not necessarily the origin. So, at the first stage, the model should be transformed to have regulation at the origin of the new model

$$(y^i \rightarrow y_d^i) \Rightarrow (\xi_1^i \rightarrow \xi_{1d}^i) \Rightarrow (\tilde{\xi}_1^i = \xi_1^i - \xi_{1d}^i \rightarrow 0). \quad (32)$$

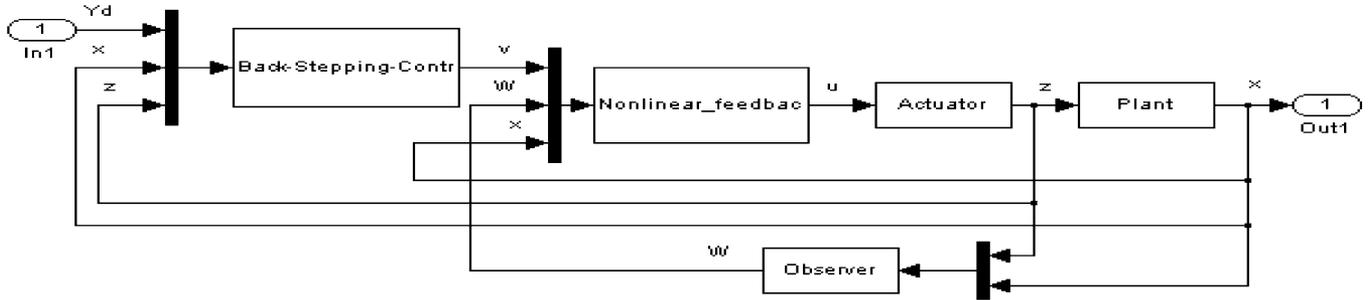


Fig. 4. Diagram of the closed-loop system.

Then we have

$$\begin{aligned}
 r_i = 2 &\Rightarrow (\eta^i = \tilde{\xi}_1^i, \quad \zeta^i = \xi_2^i, \quad u_i = v_i) \\
 &\Rightarrow \begin{cases} \dot{\eta}^i = \dot{\xi}_1^i = \dot{\zeta}^i = \zeta^i \Rightarrow f(\eta^i) = 0, \\ \zeta^i = u_i \end{cases} \\
 g(\eta^i) &= 1
 \end{aligned}$$

and

$$\begin{aligned}
 V &= .5(\eta^i)^T \eta^i, \\
 \varphi(\eta^i) &= -\eta^i, \quad V_a = V + .5z^2, \\
 z &= \zeta^i + \eta^i
 \end{aligned} \tag{33}$$

and hence by applying the coordinate shift around the origin according to (32) the backstepping control law  $u(\zeta^i(X))$  can be obtained from the following relation:

$$u_i = v_i = -(1 + k_i)(\xi_1^i + \xi_2^i - \xi_{1d}^i), \quad i = 1, 2. \tag{34}$$

### C. Control Algorithm

To conclude the control strategy, the following algorithm is presented.

- 1) Use the control law in (39) (or (16)) (see the Appendix) and the transformation in (38) (see the Appendix) to transform the nonlinear model into a linear one.
- 2) Calculate  $v_i$  according to (31) by substituting the corresponding values of  $\zeta^i$  and  $\eta^i$  from (29) [and (38)].
- 3) Having  $v_i, \omega$  (or alternatively  $\hat{\omega}$ ), and  $X$ , the control law  $u$  can be obtained from (39) [or (16)].
- 4) Apply the control  $u$  directly to the system model in (6).

The block diagram of the closed loop system can be seen in Fig. 4.

### D. Stability Analysis

For the case that disturbances are measurable, the closed loop stability is guaranteed (see [19] for detailed discussions). However, if the disturbances are not measurable and they are estimated through an estimator, the dynamics of estimator should be considered in stability analysis. Fortunately, due to the special properties of HVAC systems, we may still use the same analysis as the case when disturbances are measurable. This is due to the fact that dynamics of an HVAC system is a relatively slow dynamics, i.e., it has a time constant of the order of (couple of) hours (e.g., Fig. 5), and the disturbances considered here are either constants or slowly time varying. Hence, if the gain of the observer is selected properly (as in (23)), the stable estimator

can converge fast enough with a time constant of the order of seconds (e.g., Fig. 2). This implies that the integrated system consisting of the plant, controller and observer has multiple time scales and therefore the estimated value of the disturbance has an almost static behavior compared to the plant dynamical behavior using singular perturbation theory [24].

Moreover, the backstepping controller makes the system in normal form (15), stable [23]. Also, by proper selection of operating point, the coordinate transformation (14) would be locally invertible and by appropriate selection of  $\eta$ , the zero dynamics would be stable, therefore the overall linearized HVAC system dynamics with the backstepping controller will be locally stable, even if the disturbance is estimated by an observer.

## VI. SIMULATION RESULTS

Simulation results presented here are achieved by applying the proposed method to a nonlinear, and MIMO model of HVAC systems. In order to show the disturbance decoupling property of the proposed method, simulations are done in the presence of a time-varying load (disturbance). In Figs. 5 and 6, the first and second outputs are seen and in Figs. 7 and 8, the first and second control signals are presented for different values of parameters  $k_1, k_2$ . The set points for the outputs are 22.7 °C (73 °F) and 13.4 °C (56 °F) for the thermal space and supply air temperatures, respectively. The initial state vector is  $X_0 = [21.7^\circ\text{C} (71^\circ\text{F}) \ 0.0092 \text{ lb/lb} \ 12.8^\circ\text{C} (55^\circ\text{F})]^T$  and the parameter values are as in Table I.

The assumed load changes happened at  $t = 25200 \text{ s}$  (7 hr), are from 84960 W (289897 Btu/hr) to 105505 W (360000 Btu/hr) and 21 g/s (166.06 lb/hr) to 25 g/s (197 lb/hr) for the first and second disturbances, respectively (almost 25%). It can be seen that responses are adapted to the changes very fast with no offsets. These results show the capability of the proposed controller in overcoming the effect of disturbances (load changes) and exact regulation properties. These can be compared with the results in [1], for less percentage of the load changes (about 15%). In the simulations brought there, both output responses have some offsets (about 1.7 °C (3 °F)) after load changes happen (see [1] for the simulation results and further descriptions).

Another issue here is the effect of gain  $k_i$  on the responses which is shown for three different values of this parameter. As it becomes larger, the robustness and the speed of the responses would improve whereas the values of overshoots and undershoots in the outputs decrease significantly. However, these are at the price of sharpness of the control signal at the point where



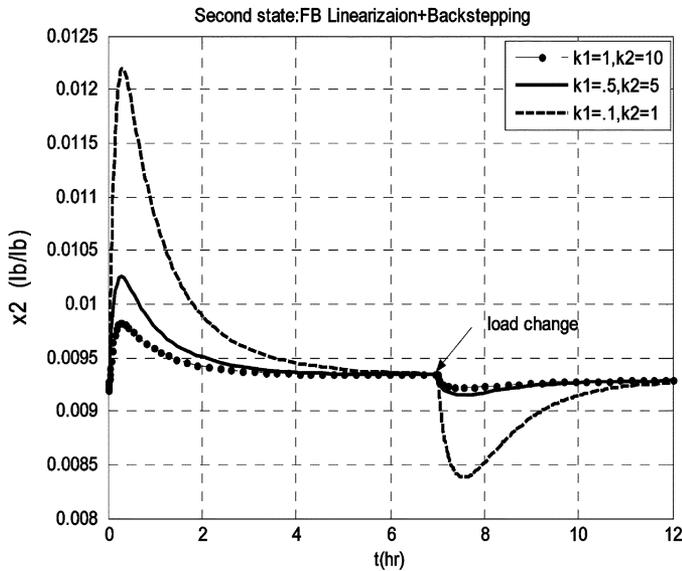


Fig. 9. The second state for different values of  $k_1$ ,  $k_2$ .

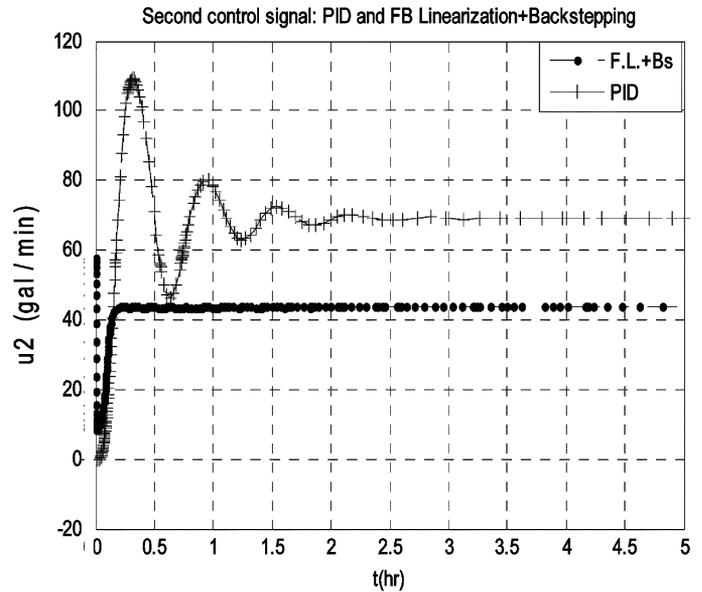


Fig. 11. The second control signal for PID and F.L.+Bs.

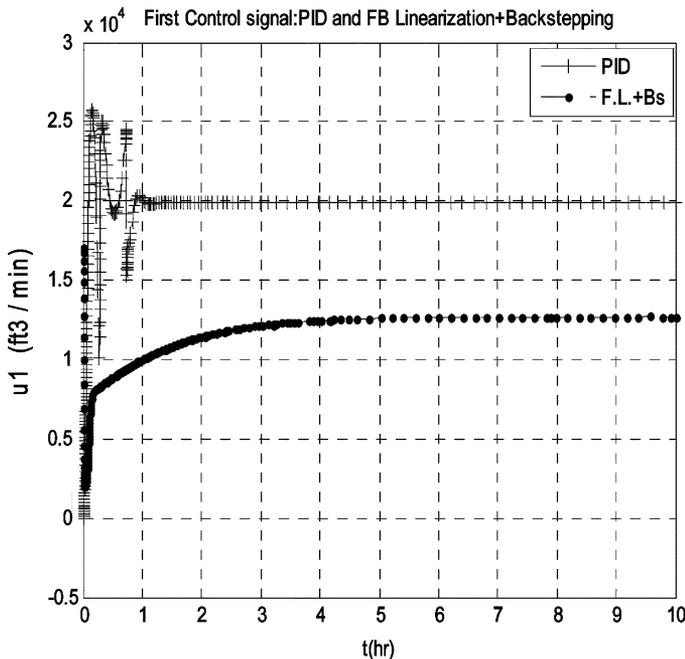


Fig. 10. The first control signal for PID and F.L.+Bs.

less oscillation means less actuator repositioning and the latter in turn, results in longer life of the actuators. This is an important advantage of this method from economical and industrial viewpoints. This property results in an optimal energy consumption, which can be measured via the following index:

$$E = \int_0^t (u - u_{eq})^2 dt \quad (35)$$

where  $u_{eq}$  is the control signal value in the steady state. This index measures the value of the control signal oscillations. The values of  $E$ , shown in Fig. 15, are 1293 and 97 for PID and the proposed method, respectively. There is a great decrease of about 90% in this index in comparison with PID by using

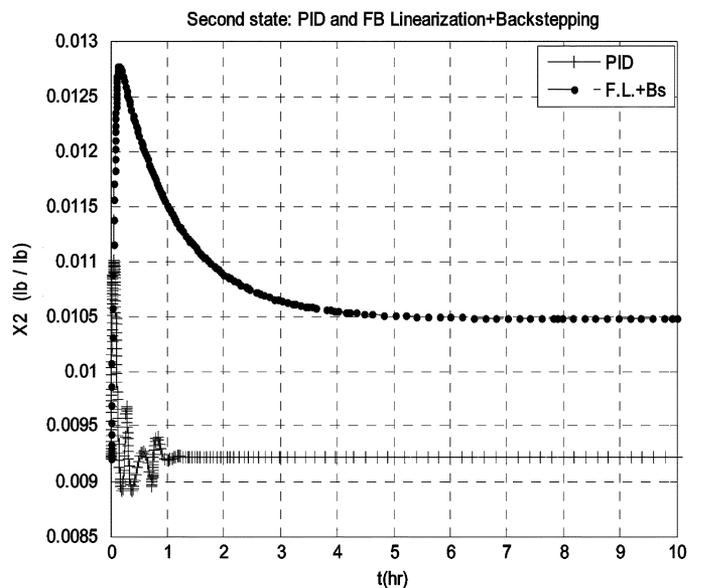


Fig. 12. The second state response for PID and F.L.+Bs.

the new introduced method. The results of comparing the proposed approach with other techniques of adjusting PID parameters follow the same tendency. Consequently, the ability of the proposed technique is verified through this comparison, as well.

## VII. CONCLUSION

In this paper, an algorithm was introduced for disturbance decoupling and tracking in nonlinear MIMO systems. In this method, first the nonlinear model was transformed into a linear one using a nonlinear feedback of states and disturbances. As the conventional feedback linearization methods were not applicable to the present model of HVAC systems, a new definition for the linearizing feedback of states and disturbances was introduced. Also, whenever the disturbances were not available,

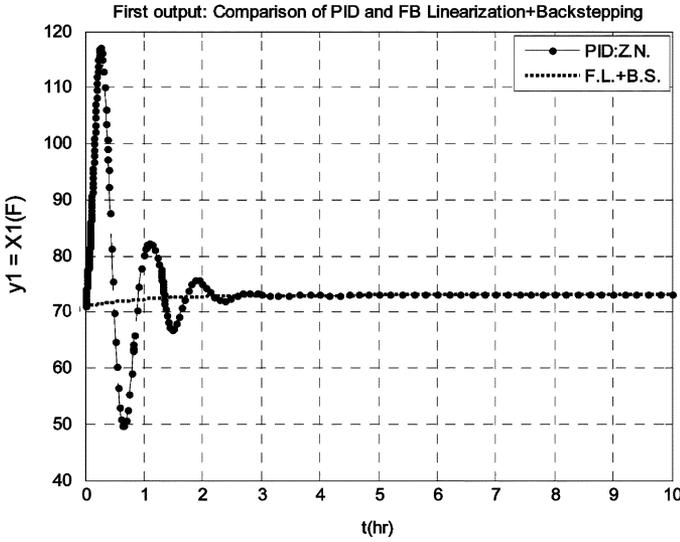


Fig. 13. The first output for PID and F.L.+Bs.

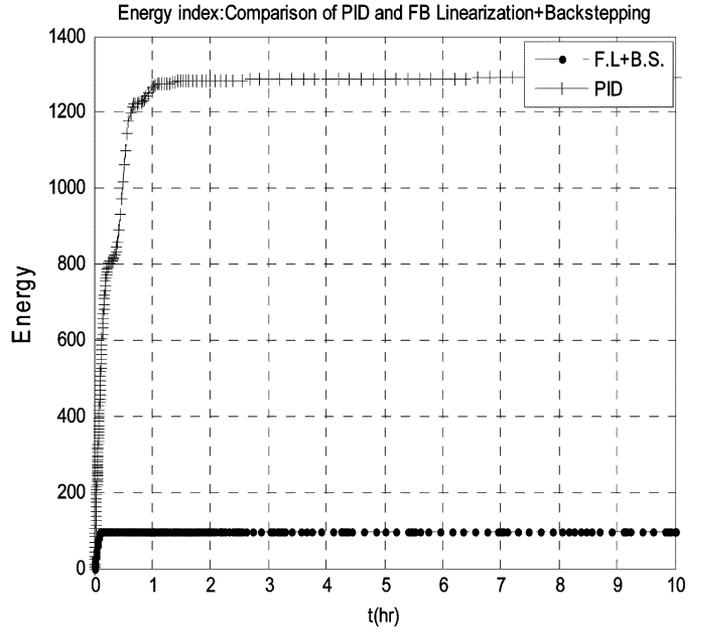


Fig. 15. Energy index comparison for PID and F.L.+Bs.

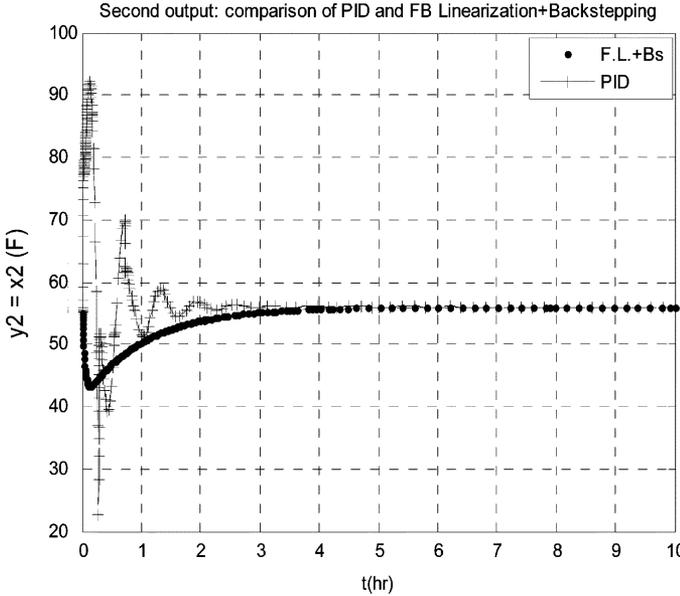


Fig. 14. The second output for PID and F.L.+Bs.

a stable observer was designed to estimate them. The nonlinear feedback not only linearized the system but also decoupled the output from disturbances.

Finally, a state feedback was designed using backstepping method for the system in the normal form. Applying the method to a MIMO, nonlinear model of HVAC systems, showed the effectiveness of the algorithm in reaching at pre-specified design goals such as output regulation and disturbance decoupling in presence of slowly time-varying loads. Moreover, offset and oscillations, which were measured by introducing an energy index, were removed compared to other common control strategies, such as PID controllers. This means less actuator repositioning which in turn results in longer life of actuators and so suggests a more beneficial method from an industrial viewpoint. Although only pulse-like disturbances are checked, the algorithm has the

capability of compensating other type of slowly time varying disturbances which can be a future extension of this work.

#### APPENDIX

*Proof of Lemma 1:* By differentiation of the output in (6) continuously and using Lie derivative notation, we can show that

$$\begin{aligned} \dot{y}_i &= \frac{\partial h_i}{\partial X} \dot{X} \\ &= L_F h_i(X) + (L_G h_i(X))u + (L_P h_i(X))\omega, \\ i &= 1, \dots, m. \end{aligned}$$

However, according to the definition of the input relative degree and condition (10), the coefficients of  $u$  and  $\omega$  in the above relation would be zero till the  $r_i$ th differentiation step. Hence, it is seen that  $y_i^{[j]} = L_F^j h_i(X)$ ,  $j = 1, \dots, r_i - 1$  and  $y_i^{[r_i]} = L_F^{r_i} h_i(X) + (L_G L_F^{r_i-1} h_i(X))u = v_i$ ,  $i = 1, \dots, m$ . System in (6) can be written in normal form using coordinate transformation  $\xi^i = [y_i \quad L_F h_i(X) \quad \dots \quad L_F^{r_i-1} h_i(X)]$  as follows:

$$\begin{aligned} \dot{\xi}_1^i &= \xi_2^i \\ \dot{\xi}_2^i &= \xi_3^i \\ &\vdots \\ \dot{\xi}_{r_i-1}^i &= \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i &= L_F^{r_i} h_i(X) + (L_G L_F^{r_i-1} h_i(X))u \\ &= b_i(X) + a_i(X)u = v_i \\ \dot{\eta} &= q(\xi, \eta) + S(\xi, \eta)u + R(\xi, \eta)\omega \quad (36-a) \\ y_i &= \xi_1^i, \quad i = 1, \dots, m \quad (36-b) \end{aligned}$$

in which  $\eta$  is the dynamics of nonlinear part of the model or the internal dynamics. In this form, it is obvious that  $\omega$

does not appear in the first  $r_i$  equations. Hence, the outputs  $\xi_1^i$ ,  $i = 1, \dots, m$ , are decoupled from disturbances completely. The second part of the lemma can be proved similarly.  $\square$

*Proof of Lemma 2:* As in Lemma 1, by differentiating the output relation continuously and using (13), we will see that

$$\begin{aligned} \dot{y}_i &= L_F h_i(X) \\ &\vdots \\ y_i^{[r_i-1]} &= L_F^{r_i-1} h_i(X) + (L_P L_F^{r_i-2} h_i(X)) \omega \\ y_i^{[r_i]} &= L_F^{r_i} h_i(X) + (L_G L_F^{r_i-1} h_i(X)) u + (L_P L_F^{r_i-1} h_i(X)) \omega. \end{aligned} \quad (37)$$

Now introduce the following new coordinate transformation  $\xi^i$ , different from the one used in [22]:

$$\begin{aligned} \xi_1^i &= h_i(X) \\ \xi_2^i &= L_F h_i(X) \\ &\vdots \\ \xi_{r_i-1}^i &= L_F^{r_i-2} h_i(X) \\ \xi_{r_i}^i &= L_F^{r_i-1} h_i(X) + (L_P L_F^{r_i-2} h_i(X)) \omega. \end{aligned} \quad (38)$$

Needless to say, since  $\omega$  is measurable (or observable to be estimated), the above transformation is well defined. Using (13-a), the definition of input relative degree and properties of Lie derivative, we can see that

$$\begin{aligned} \dot{\xi}_1^i &= L_F h_i(X) = \xi_2^i \\ &\vdots \\ \dot{\xi}_{r_i-1}^i &= \frac{\partial L_F^{r_i-2} h_i(X)}{\partial X} \dot{X} \\ &= L_F^{r_i-1} h_i(X) + L_P L_F^{r_i-2} h_i(X) \omega = \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i &= \frac{\partial}{\partial X} (L_F^{r_i-1} h_i(X) + (L_P L_F^{r_i-2} h_i(X)) \omega) \dot{X} \\ &= L_F^{r_i} h_i(X) + (L_G L_F^{r_i-1} h_i(X)) u \\ &\quad + (L_P L_F^{r_i-1} h_i(X)) \omega \\ &\quad + \frac{\partial}{\partial X} (L_P L_F^{r_i-2} h_i(X)) \omega \dot{X}. \end{aligned}$$

Now, according to (13-b)

$$\frac{\partial}{\partial X} (L_P L_F^{r_i-2} h_i(X)) = 0, \quad i = 1, \dots, m$$

and hence if  $A(X)$ , defined in (7-b), is nonsingular, then

$$\begin{aligned} \dot{\xi}_{r_i}^i &= L_F^{r_i} h_i(X) + (L_G L_F^{r_i-1} h_i(X)) u \\ &\quad + (L_P L_F^{r_i-1} h_i(X)) \omega = v_i \\ \Rightarrow u &= \alpha(X) + \beta(X)v + \gamma(X)\omega \end{aligned} \quad (39-a)$$

$$\begin{aligned} \alpha(X) &= -A^{-1}(X)b(X), \quad \beta(X) = A^{-1}(X), \\ \gamma(X) &= -A^{-1}(X)c(X), \\ c(X) &= \begin{bmatrix} L_P L_F^{r_1-1} h_1(X) \\ \vdots \\ L_P L_F^{r_m-1} h_m(X) \end{bmatrix}, \\ b(X) &= \begin{bmatrix} L_F^{r_1} h_1(X) \\ \vdots \\ L_F^{r_m} h_m(X) \end{bmatrix}. \end{aligned} \quad (39-b)$$

Hence, system (6) may be transformed into the normal form, i.e.,

$$\begin{cases} \dot{\xi}_1^i = \xi_2^i \\ \dot{\xi}_2^i = \xi_3^i \\ \vdots \\ \dot{\xi}_{r_i-1}^i = \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i = v_i, \quad i = 1, \dots, m \\ \dot{\eta} = q(\xi, \eta) + S(\xi, \eta)u + R(\xi, \eta)\omega \\ y_i = \xi_1^i, \quad i = 1, \dots, m. \end{cases} \quad (40-a)$$

$$y_i = \xi_1^i, \quad i = 1, \dots, m. \quad (40-b)$$

It is clear that applying a nonlinear feedback of the states and disturbances in the form of (39) can linearize the nonlinear model of the system into (40). Hence, the outputs,  $\xi_1^i$ ,  $i = 1, \dots, m$ , in the new model, can be decoupled from disturbances, and the proof is complete.  $\square$

## REFERENCES

- [1] B. Arguello-Serrano and M. Velez-Reyes, "Nonlinear control of a heating, ventilating and air conditioning system with thermal load estimation," *IEEE Trans. Contr. Syst. Technol.*, vol. 7, no. 1, pp. 56–63, Jan. 1999.
- [2] J. Teeter and M. Y. Chow, "Application of functional link neural network to HVAC thermal dynamic system identification," *IEEE Trans. Ind. Electron.*, vol. 45, no. 1, pp. 170–176, Feb. 1999.
- [3] G. Geng and G. M. Geary, "On performance and tuning of PID controllers in HVAC systems," in *Proc. IEEE Conf. Contr. Applicat.*, 1993, vol. 2, pp. 819–824.
- [4] T. B. Hartman, *Direct Digital Controls for HVAC Systems*. New York: McGraw-Hill, 1993.
- [5] K. J. Astrom, T. Hagglund, and A. Wallenborg, "Automatic tuning of digital controllers with applications to HVAC plants," *Automatica*, vol. 29, no. 5, pp. 1333–1343, 1993.
- [6] W. Jian and C. Wenjian, "Development of an adaptive neuro-fuzzy method for supply air pressure control in HVAC system," in *Proc. IEEE Conf. Syst., Man, Cybern., 2000*, Oct. 2000, vol. 5, pp. 3806–3809.
- [7] M. Zaheeruddin, "Optimal control of a single zone environmental space," *Building and Environment*, pp. 93–103, 1992.
- [8] M. Zaheeruddin and R. V. Patel, "The design and simulation of a sub-optimal controller for space heating," *ASHRAE Trans.*, vol. 98, pp. 554–564, 1992.
- [9] M. Zaheer-uddin, R. V. Patel, and S. A. K. Al-Assadi, "Decentralized control of a variable air volume system," in *Proc. Amer. Contr. Conf.*, 1994, vol. 3, pp. 3050–3054.
- [10] M. Zaheer-uddin, S. A. K. Al-Assadi, and R. V. Patel, "Decentralized preview control for multiple disturbance rejection in HVAC systems," *Contr. Eng. Practice*, vol. 2, no. 6, pp. 989–1000, 1994.
- [11] M. Zaheer-Uddin, R. V. Patel, and S. A. K. Al-Assadi, "Design of decentralized robust controllers for multizone space heating systems," *IEEE Trans. Contr. Syst. Technol.*, vol. 1, no. 4, pp. 246–261, Dec. 1993.
- [12] C. P. Underwood, "Robust control of HVAC plant II: Controller design," *CIBSE J.*, vol. 21, no. 1, pp. 63–72, 2000.
- [13] M. J. Yazdanpanah, E. Semsar, and C. Lucas, "Minimization of actuator repositioning in delayed processes using flexible neural networks," in *Proc. IEEE Conf. Contr. Applicat.*, 2003, vol. 1, pp. 331–335.
- [14] M. R. Omid and M. J. Yazdanpanah, "Minimization of actuator repositioning using internal model control," in *Proc. 4th Portuguese Autom. Contr. Conf.*, Portugal, Oct. 4–6, 2000.
- [15] M. J. Yazdanpanah, E. Semsar, and C. Lucas, "Minimization of actuator repositioning using neural networks with application in nonlinear HVAC systems," presented at the Eur. Contr. Conf. (ECC'03), Cambridge, U.K., Sep. 2003.
- [16] T. Tigrek, S. Dasgupta, and T. F. Smith, "Nonlinear optimal control of HVAC systems," presented at the 15th IFAC World Congr., Barcelona, Spain, Jul. 2002.
- [17] C. Rentel-Gomez and M. Velez-Reyes, "Decoupled control of temperature and relative humidity using variable-air-volume HVAC system and non-interacting control," in *Proc. IEEE Int. Conf. Contr. Applicat.*, 2001, pp. 1147–1151.

- [18] M. L. Anderson, P. M. Young, D. C. Hittle, C. W. Anderson, J. Tu, and D. Hodgson, "MIMO robust control for heating, ventilating and air conditioning (HVAC) systems," in *Proc. IEEE Conf. Decision Contr.*, 2002, vol. 1, pp. 167–172.
- [19] E. Semsar, M. J. Yazdanpanah, and C. Lucas, "Nonlinear control and disturbance decoupling of an HVAC system via feedback linearization and Backstepping," in *Proc. IEEE Conf. Contr. Applicat.*, 2003, vol. 1, pp. 646–650.
- [20] C. Huang, Y. M. Chan, and L. C. Fu, "Supervised nonlinear control of hybrid system with application to HVAC system," in *Proc. IEEE Int. Conf. Contr. Applicat.*, 2004, vol. 1, pp. 673–678.
- [21] C. P. Underwood, *HVAC Control Systems: Modeling, Analysis and Design*. New York: E&FN SPON, 1999.
- [22] A. Isidori, *Nonlinear Control Systems*. New York: Springer-Verlag, 1995.
- [23] H. K. Khalil, *Nonlinear Systems*. Englewood Cliffs, NJ: Prentice Hall, 1996.
- [24] P. V. Kokotovic, *Singular Perturbation Methods in Control: Analysis and Design*. New York: Academic, 1986.



**E. Semsar-Kazerooni** (S'05) was born in Shiraz, Iran. She received the B.Sc. degree from Shiraz University, Shiraz, Iran, in 2000, and the M.Sc. degree from University of Tehran, Tehran, Iran, in 2003, both in electrical engineering (with distinction). She is currently working towards the Ph.D. degree in electrical engineering at Concordia University, Montreal, Canada.

She was working in electrical and system engineering industries in 2000–2004. Her research interests include nonlinear systems analysis, estimation,

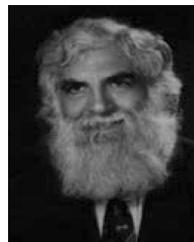
optimal control, process control, neural networks, robotics, unmanned vehicles, team cooperation, consensus achievement, and sensor networks.

Mrs. Semsar-Kazerooni was the recipient of several student awards at Concordia University, Montreal, Canada.



**M. J. Yazdanpanah** (M'05) received the B.Sc., M.Sc., and Ph.D. degrees, all in electrical engineering, from Isfahan University of Technology, Isfahan, Iran, in 1986, Tehran University, Tehran, Iran, in 1988, and Concordia University, Montreal, Quebec, Canada, in 1997, respectively. His Ph.D. dissertation entitled "Control of flexible-link manipulators using nonlinear  $H_\infty$  techniques" was ranked outstanding.

From 1986 to 1992, he worked with the Engineering Research Center, Tehran, Iran, as an R&D engineer, culminating as the chairman of the System Design Division. In 1998, he joined the School of Electrical and Computer Engineering, University of Tehran, Tehran, Iran, where he is now an Associate Professor, serving as instructor of various courses and also supervisor of M.Sc. and Ph.D. students. His research interests are in the areas of analysis and design of nonlinear/robust/optimal/adaptive control systems, robotics, control on networks, and theoretical and practical aspects of neural networks.



**Caro Lucas** (SM'88) received the M.S. degree from the University of Tehran, Tehran, Iran, in 1973, and the Ph.D. degree from the University of California, Berkeley, in 1976.

He is a Professor at the Center of Excellence for Control and Intelligent Processing, Faculty of Electrical and Computer Engineering, University of Tehran. He has served as the Founding Director of Intelligent Systems Research Faculty, IPM (1993–1997), Center of Excellence for Control and Intelligent Processing, UT (2003–2006) and Chairman of the ECE Department at the UT (1986–1988). He was also a Visiting Professor at the American University of Armenia (summer, 2007) Visiting Associate Professor at the University of Toronto, Toronto, Canada (summer, 1989–1990), University of California, Berkeley (1988–1989), an Assistant Professor at Garyounis University (1984–1985), University of California, Los Angeles (1975–1976). His research interests include biological computing, computational intelligence, uncertain systems, intelligent control, neural networks, emotional learning, multiagent systems, smart environments, data mining, business intelligence, financial modeling, and knowledge management.

Dr. Lucas has served as Managing Editor of the *Memories of the Engineering Faculty*, University of Tehran (1979–1991), Reviewer of *Mathematical Reviews* (since 1987), Associate Editor of the *Journal of Intelligent and Fuzzy Systems* (1992–1999), and Chairman of the IEEE, Iran Section (1990–1992). He has also served as the Chairman of several international conferences and member of various editorial boards, research and curricular committees, executive boards, and advisory positions, and has been the recipient of several Iranian and international scientific awards and research grants.