# Neural-Network-Based Decentralized Adaptive Output-Feedback Control for Large-Scale Stochastic Nonlinear Systems

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***Abstract*—This paper focuses on the problem of neural-network-based decentralized adaptive output-feedback control for a class of nonlinear strict-feedback large-scale stochastic systems. The dynamic surface control technique is used to avoid the explosion of computational complexity in the backstepping design process. A novel direct adaptive neural network approximation method is proposed to approximate the unknown and desired control input signals instead of the unknown nonlinear functions. It is shown that the designed controller can guarantee all the signals in the closed-loop system to be semiglobally uniformly ultimately bounded in a mean square. Simulation results are provided to demonstrate the effectiveness of the developed control design approach.**

***Index Terms*—Adaptive control, backstepping, decentralized control, dynamic surface control, neural network (NN), stochastic nonlinear systems.**

## I. Introduction

IN GENERAL, a large-scale system is often considered as a set of interconnected dynamical systems, which comprise some lower order subsystems [1]–[5]. The applications of large-scale systems have been found in many practical systems, such as power systems, computer network systems, and economic systems [6], [7]. The decentralized adaptive technique is often used to handle the control design problems of large-scale systems in that the knowledge of plant parameters and interactions among subsystems is often unknown and that the decentralized design method depends only on local measurements [8].

In the past two decades, the decentralized control for large-scale systems has received considerable attention (see, for example, [9] and [10] and the reference therein). In [10], the problem of decentralized dynamic surface control of large-scale interconnected systems is investigated. However, most of the results focused on deterministic large-scale systems. Recently, a growing attention has been paid on stochastic systems due to the fact that stochastic disturbance exists in many practical systems and that it is often the source of instability (see, for example, [11]–[25]). More recently, decentralized control design method has been applied to large-scale stochastic nonlinear systems [26], [27], which is crucial for control theory, as well as the synthesis of practical control systems.

On the other hand, the method of approximation-based adaptive fuzzy logic control or neural network (NN) control is useful to approximate the unknown nonlinear functions in the systems, such as the method that has been applied to single-input–single-output nonlinear systems in [28]–[36], multiple-input–multiple-output nonlinear systems in [37]–[40], and large-scale nonlinear systems in [8] and [41]–[44]. In most of these researches, adaptive fuzzy or neural controllers are constructed recursively in the framework of the backstepping approach. However, there exists the open problem of the explosion of complexity in the backstepping design procedure. In order to avoid this problem, the dynamic surface control technique was first introduced in [44] for a class of strict-feedback nonlinear systems with unknown functions. Then, this approach was applied to solve a class of nonlinear systems with periodic disturbances in [45] and a class of interconnected nonlinear systems in [46]. The advantage of dynamic surface control is to avoid repeatedly differentiating the virtual control variables by introducing a first-order filter in each step of backstepping design procedure, which greatly simplifies the traditional backstepping control algorithm. However, there are few results available on neural-network-based decentralized adaptive output-feedback control for large-scale stochastic nonlinear systems. Recently, in [47], the authors investigated the problem of adaptive NN output-feedback decentralized stabilization for a class of large-scale stochastic nonlinear strict-feedback systems. However, the design parameters will increase when the order of the system increases, which motivates our research.

In this paper, the problem of adaptive decentralized NN control is investigated for large-scale stochastic nonlinear system.

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It is assumed that only the output of the system is measurable, and there exist unknown nonlinear functions in the systems; therefore an observer-based adaptive NN controller is designed via the backstepping approach. The proposed control method is independent of the prior knowledge of the basis functions of the neural approximators. The main contributions of this paper can be summarized as follows: 1) The approximation-based direct adaptive NN method can solve the decentralized control design problem of the large-scale stochastic nonlinear systems, in which only one adaptation parameter is required, and therefore, the computation burden is greatly reduced; 2) when the state variable information is unknown, decentralized output-feedback control method is proposed to investigate the control design issue for large-scale stochastic nonlinear systems; and 3) in the control design process, the dynamic surface control approach can be utilized to simplify the problem of the explosion of complexity in large-scale nonlinear stochastic systems, which avoids repeated differentiation of the virtual controller by introducing a first-order filter in each step of the backstepping design procedure. It is shown that the proposed control method can guarantee all the signals in the closed-loop system to be semiglobally uniformly ultimately bounded. Finally, a simulation result illustrates the effectiveness of the proposed method. The rest of this paper is organized as follows. The problem to be addressed is formulated in Section II, and controller design is presented in Section III. A design example is provided in Section IV to show the effectiveness of the developed results, and we conclude this paper in Section V.

## II. Problem Formulation and Preliminaries

In this section, the system descriptions are formulated in Section II-A, and the definition of stochastic stability is presented in Section II-B. In Section II-C, the NN is used to approximate the unknown nonlinear functions.

### A. System Descriptions

Consider the following nonlinear stochastic system

$$\begin{aligned} dx_{i,1}(t) &= (x_{i,2}(t) + f_{i,1}(\bar{x}_{i,1}(t)) + h_{i,1}(y))\, dt \\ &\quad + g_{i,1}(y_i)^T dw_i \\ &\vdots \\ dx_{i,n_i-1}(t) &= (x_{i,n_i}(t) + f_{i,n_i-1}(\bar{x}_{i,n_i-1}(t)) + h_{i,n_i-1}(y))\, dt \\ &\quad + g_{i,n_i-1}(y_i)^T dw_i, \\ dx_{i,n_i}(t) &= (u_i + f_{i,n_i}(\bar{x}_{i,n_i}(t)) + h_{i,n_i}(y))\, dt \\ &\quad + g_{i,n_i}(y_i)^T dw_i, \\ y_i(t) &= x_{i,1}(t) \end{aligned} \tag{1}$$

where $\bar{x}_{i,j}(t) = [x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,j}(t)]^T \in R^j$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n_i - 1$, and $x_i(t) = [x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,n_i}(t)]^T \in R^{n_i}$ denote the state vectors of the system, and $u_i \in R$ and $y_i \in R$ represent the input and output of the system, respectively. $f_{i,j}(.)$ stands for the unknown smooth system function with $f_{i,j}(0) = 0$. $h_{i,j}(y)$ is the interconnection between the $i$th subsystem and other subsystems. $g_{i,j}(.)$ is the unknown vector-valued smooth functions with $g_{i,j}(0) = 0$. $w_i$ is an independent $r$-dimensional standard Wiener process.

*Lemma 1:* (Young's Inequality). For $\forall(x, y) \in R^2$, the following inequality holds:

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q$$

where $\varepsilon > 0$, $p > 1$, $q > 1$, and $(p-1)(q-1) = 1$.

*Assumption 1:* For $1 \leq i \leq N$ and $1 \leq j \leq n_i$, there exists positive unknown constant $p_{i,j}$ such that

$$|h_{i,j}(y)| \leq p_{i,j} \sum_{l=1}^{N} \varphi_{ijl}(|y_l|)$$

### B. Stochastic Stability

Consider the following stochastic system

$$dx(t) = f(x(t))\, dt + g(x(t))\, dw \tag{2}$$

where $x \in R^n$ is the system state, $w$ is an $r$-dimensional standard Wiener process, and $f : R^n \to R^n$ and $g : R^n \to R^n$ are locally Lipschitz functions and satisfy $f(0) = g(0) = 0$.

*Definition 1:* For any given $V(x) \in C^2$, which is associated with the stochastic system (2), the infinitesimal generator $\mathcal{L}$ is defined as follows:

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2}\mathrm{Tr}\left\{ g(x)^T \frac{\partial^2 V}{\partial x^2} g(x) \right\}$$

where $\mathrm{Tr}(A)$ is the trace of a matrix $A$.

*Definition 2:* The trajectory $x(t)$ of system (2) is said to be semiglobally uniformly ultimately bounded in $p$th moment if, for some compact set $\Omega \in R^n$ and any initial state $x_0 = x(t_0)$, there exist constant $\varepsilon > 0$ and time constant $T = T(\varepsilon, x_0)$, such that $E[|x(t)|^p] < \varepsilon$ for all $t > t_0 + T$. In particular, when $p = 2$, it is usually called semiglobally uniformly ultimately bounded in mean square.

*Lemma 2:* Consider the stochastic system (2). If there exist functions $V(x) \in C^2$, $\bar{\alpha}_1$, and $\bar{\alpha}_2 \in K_\infty$, and constants $a_0 > 0$ and $b_0 > 0$, such that

$$\begin{aligned} \bar{\alpha}_1(x) \leq &V(x) \leq \bar{\alpha}_2(x) \\ \mathcal{L}V(x) \leq &- a_0 V(x) + b_0 \end{aligned}$$

then, there is a unique solution of system (2) for each $x_0 \in R^n$, and it satisfies

$$E[V(x)] \leq V(x_0)e^{-a_0 t} + \frac{a_0}{b_0} \qquad \forall t > t_0.$$

### C. Approximation-Based NN

In this paper, approximation-based NN will be used to approximate the unknown smooth nonlinear functions. For any continuous unknown smooth nonlinear function $f(Z)$ over a compact set $\Omega_Z \subset R^q$, there exists NN $W^{*T}S(Z)$, such that for a desired level of accuracy $\varepsilon$

$$f(Z) = W^{*T}S(Z) + \delta(Z), \qquad |\delta(Z)| \leq \varepsilon \tag{3}$$

where $W^*$ is the ideal constant weight vector and defined by

$$W^* = \arg \min_{W \in R^N} \left\{ \sup_{Z \in \Omega_Z} \left| f(Z) - W^T S(Z) \right| \right\}$$

$\delta(Z)$ is the approximation error, $W = [w_1, \ldots, w_N]^T$ is the weight vector, and $S(Z) = [s_1(Z), \ldots, s_N(Z)]^T$ is the basis function vector with $N$ being the number of the NN nodes and $N > 1$. Radial basis function $s_i(Z) = \exp[-(Z - \mu_i)^T(Z - \mu_i)/\eta_i^2]$, $i = 1, 2, \ldots, N$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots \mu_{in}]^T$ is the center of the receptive field and $\eta_i$ is the width of the Gaussian function.

*Remark 1:* It is important to note that, although the NNs are universal approximators, it can only be guaranteed within some compact set in the controller design process. Therefore, the stability condition obtained in this paper is semiglobal.

## III. Control Design

In this section, an observer-based controller will be designed to guarantee all the signals in the closed-loop system to be semiglobally uniformly ultimately bounded. To estimate the unmeasured states, we propose the following observer:

$$\dot{\hat{x}}_{i,j} = \hat{x}_{i,j+1} + l_{i,j}(y_i - \hat{x}_{i,1}) \tag{4}$$

where $\hat{x}_{i,n_i+1} = u_i$. Let $\tilde{x}_i = x_i - \hat{x}_i(\hat{x}_i = [\hat{x}_{i,1}, \ldots, \hat{x}_{i,n_i}])$ be the observer error, which satisfies the following equation:

$$d\tilde{x}_i(t) = \left(A_i\tilde{x}_i(t) + f_i\left(\bar{x}(t)\right) + h_i(y)\right)dt + g_i\left(y_i(t)\right)^T dw_i$$

where

$$A_i = \begin{bmatrix} -l_{i,1} & & \\ \vdots & I_{n_i-1} & \\ -l_{i,n_i} & 0\ldots & 0 \end{bmatrix}$$

$$f_i\left(\bar{x}(t)\right) = \left[\, f_{i,1}\left(x_{i,1}(t)\right) \quad \cdots \quad f_{i,n_i}\left(x_{i,n_i}(t)\right) \right]^T$$
$$h_i(y) = \left[\, h_{i,1}(y) \quad \cdots \quad h_{i,n_i}(y) \right]^T$$
$$g_i(y_i) = \left[\, g_{i,1}\left(y(t)\right) \quad \cdots \quad g_{i,n_i}\left(y(t)\right) \right]^T$$

and $l_{i,j}$ is to be designed such that $A_i$ is a strict Hurwitz matrix; therefore, there exists matrix $P_i > 0$ satisfying

$$A_i^T P_i + P_i A_i = -I. \tag{5}$$

Then, the entire system can be expressed as

$$\begin{aligned} d\tilde{x}_i(t) &= \left(A_i\tilde{x}_i(t) + f_i\left(\bar{x}(t)\right) + h_i(y)\right)dt \\ &\quad + g_i\left(y_i(t)\right)^T dw_i, \\ dy_i(t) &= \left(\hat{x}_{i,2}(t) + \tilde{x}_{i,2}(t) + f_{i,1}\left(y_i(t)\right) + h_{i,1}(y)\right)dt \\ &\quad + g_{i,1}(y_i)^T dw_i, \\ d\hat{x}_{i,2}(t) &= \left(\hat{x}_{i,3}(t) + l_{i,2}\tilde{x}_{i,1}(t)\right)dt, \\ &\vdots \\ d\hat{x}_{i,n_i}(t) &= \left(u_i + l_{i,n_i}\tilde{x}_{i,1}(t)\right)dt \end{aligned}$$

where the variables $y_i$ and $\hat{x}_{i,j}$ are available for control design. Now, we introduce a change of coordinates as follows:

$$\begin{aligned} z_{i,1} &= y_i \\ z_{i,j} &= \hat{x}_{i,j} - \alpha_{i,j,f} \end{aligned}$$

where $\alpha_{i,j,f}$ is the output of the first-order filter with $\alpha_{i,j-1}$ as the input. According to Ito's differentiation rule, one can derive

$$\begin{aligned} dz_{i,1} &= \left(\hat{x}_{i,2}(t) + \tilde{x}_{i,2}(t) + f_{i,1}(y_i) + h_{i,1}(y)\right)dt \\ &\quad + g_{i,1}(y_i)^T dw_i \\ dz_{i,j} &= \left(\hat{x}_{i,j+1}(t) + l_{i,j}\tilde{x}_{i,1}(t) - \dot{\alpha}_{i,j,f}\right)dt \\ &\quad \times I = 1, \ldots, N, \quad j = 2, \ldots . n_i. \end{aligned}$$

*Remark 2:* Note that since $g_i(y_i)$ is a smooth function and $g_i(0) = 0$, $g_i(y_i)$ can be expressed as

$$g_i(y_i) = y_i g_i(y_i) = \left[y_i\bar{g}_{i,1}(y_i), \ldots, y_i\bar{g}_{i,n_i}(y_i)\right].$$

In order to avoid the explosion problem of complexity, dynamic surface control approach will be introduced in the backstepping design procedure. In each step, a virtual control function $\hat{\alpha}_i$ should be developed using an appropriate Lyapunov function $V_i$, and the real control law $u_i$ will be designed at last. To begin with the backstepping design procedure, let us define constant

$$\theta_i = \max\left\{N_{i,j}\left\|W_{i,j}^*\right\|^2 : j = 0, 1, 2, \ldots, n_i\right\}.$$

Let $\hat{\theta}_i$ be the estimate of $\theta_i$. The feasible virtual control signal is designed as

$$\alpha_{i,j}(X_{i,j}) = -\frac{1}{2a_{i,j}^2} z_{i,j}^3 \hat{\theta}_i, \quad j = 1, \ldots, n_i - 1 \tag{6}$$

where $X_{i,1} = x_{i,1}$ and $X_{i,j} = (\tilde{x}_{i,1}, \hat{x}_{i,j}, \alpha_{i,j,f}, \dot{\alpha}_{i,j,f})^T$, $j = 2, \ldots, n_i - 1$.

*Theorem 1:* Consider the large-scale stochastic nonlinear system in (1) with observer (4). If a control law is chosen as

$$u_i = -\frac{1}{2a_{i,n_i}^2} z_{i,n_i}^3 \hat{\theta}_i$$

with the intermediate virtual control signals $\alpha_{i,j}$ described as (6) and the adaptive law defined as

$$\dot{\hat{\theta}}_i = \sum_{j=1}^{n_i} \frac{r_i}{2a_{i,j}^2} z_{i,j}^6 - k_{i,0}\hat{\theta}_i \tag{7}$$

where positive constants $a_{i,j}(i = 1, \dots, N, j = 1, \dots, n_i)$, $r_i$, and $k_{i,0}$ are designed parameters, then the closed-loop system can be guaranteed to be semiglobally stable in mean square.

*Proof:* Step $i$, 1: Consider the Lyapunov functional candidate as follows:

$$V_{i,1} = \frac{a_i}{2}\left(\tilde{x}_i^T P_i \tilde{x}_i\right)^2 + \frac{1}{4}z_{i,1}^4 + \frac{1}{2r_i}\tilde{\theta}_i^2$$

where $a_i > 0$ and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. By Definition 1, we have

$$\begin{aligned}\mathcal{L}V_{i,1} = & -a_i\tilde{x}_i^T P_i \tilde{x}_i \|\tilde{x}_i\|^2 + a_i\tilde{x}_i^T P_i\tilde{x}_i\left(2\tilde{x}_i^T P_I(f_i + h_i)\right)\\ &+ \frac{3}{2}z_{i,1}^2 g_{i,1}(z_{i,1})^T g_{i,1}(z_{i,1})\\ &+ 2a_i Tr\Big\{g_i(z_{i,1})\left(2P_i\tilde{x}_i\tilde{x}_i^T P_i\right.\\ &\qquad \left. + \tilde{x}_i^T P_i\tilde{x}_i P_i)g_i(z_{i,1})^T\Big\} - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.\\ &+ z_{i,1}^3\left(\hat{x}_{i,2} + \tilde{x}_{i,2} + f_{i,1}(z_{i,1}) + h_{i,1}(y)\right).\end{aligned} \tag{8}$$

As $f_i \triangleq (f_{i,1}(\bar{x}), \dots, f_{i,n}(\bar{x}))^T$, and $f_{i,j}(\bar{x})$, where $1 \le i \le N$ and $1 \le j \le n_i$, is an unknown function, by Lemma 1, for any given $\varepsilon_{ij0} > 0$, there exists NN $W_{ij0}^{*T} S_{i,0}(X_0)$ such that

$$\begin{aligned}f_{i,j}(X_0) &= W_{ij0}^{*T} S_{i,0}(X_0) + \delta_{ij0}(X_0),\\ |\delta_{ij0}(X_0)| &\le \varepsilon_{ij0}\end{aligned}$$

where $X_0 = \bar{x}$. Therefore

$$\begin{aligned}f_i(X_0) &= W_{i,0}^{*T} S_{i,0}(X_0) + \delta_{i,0}(X_0)\\ \|\delta_{i,0}(X_0)\| &\le \varepsilon_{i,0}.\end{aligned}$$

As $S_{i,0}^T S_{i,0} \le N_{i,0}$ is used and $N_{i,0}$ is the dimension of $S_{i,0}$, and according to the definition of $\theta$, we know $\|W_{i,0}^*\|^4 S_{i,0}^4 \le \theta^2$. Therefore, the following inequality holds:

$$\begin{aligned}2a_i\left(\tilde{x}_i^T\right)^3\|P_i\|^2 f_i &= 2a_i\tilde{x}_i^3\|P_i\|^2\left(W_{i,0}^{*T} S_{i,0}(X_0) + \delta_{i,0}(X_0)\right)\\ &\le \frac{3a_i}{2}\|\tilde{x}_i\|^4 + \frac{a_i}{2}\|P_i\|^8\left\|W_{i,0}^*\right\|^4 S_{i,0}^4\\ &\quad + \frac{3a_i}{2}\|\tilde{x}_i\|^4 + \frac{a_i}{2}\|P_i\|^8\delta_{i,0}^4\\ &= 3a_i\|\tilde{x}_i\|^4 + \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4.\end{aligned} \tag{9}$$

According to Assumption 1, the following inequality holds:

$$\begin{aligned}2a_i\left(\tilde{x}_i^T\right)^3\|P_i\|^2 h_i &\le 2a_i\|\tilde{x}_i\|^3\|P_i\|^2\|h_i\|\\ &\le 2a_i\|\tilde{x}_i\|^3\|P_i\|^2\sum_{j=1}^{n_i}\sum_{l=1}^{N} p_{i,j}\varphi_{ijl}(|y_l|)\\ &\le \frac{a_i}{2}\sum_{j=1}^{n_i}\sum_{l=1}^{N}\varphi_{ijl}^4(|y_l|)\\ &\quad + \frac{3a_i n_i N}{2}p_{i,j}^{\frac{4}{3}}\|P_i\|^{\frac{8}{3}}\|\tilde{x}_i\|^4.\end{aligned} \tag{10}$$

As $\varphi_{ijl}$ is a smooth function, there exists a smooth nonnegative function $\eta_{ijl}^{(1)}(y_l)$ as follows:

$$\begin{aligned}\frac{a_i}{2}\sum_{j=1}^{n_i}\sum_{l=1}^{N}\varphi_{ijl}^4(|y_l|) &\le \sum_{j=1}^{n_i}\sum\_l = 1^N\eta_{ijl}^{(1)}(y_l)y_l^4\\ &\quad + \sum_{j=1}^{n_i}\sum_{l=1}^{N}4a_i\varphi_{ijl}^4(0)\end{aligned} \tag{11}$$

such that From Lemma 1, the terms in (8) lead to

$$\begin{aligned}&2a_i Tr\left\{g_i(z_{i,1})\left(2P_i\tilde{x}_i\tilde{x}_i^T P_i + \tilde{x}_i^T P_i\tilde{x}P_i\right)g_i(z_{i,1})^T\right\}\\ &\le 2a_i n_i\left\|g_i(z_{i,1})\left(2P_i\tilde{x}_i\tilde{x}_i^T P_i + \tilde{x}_i^T P_i\tilde{x}_i P_I\right)g_i(z_{i,1})^T\right\|\\ &\le 6a_i n_i\sqrt{n_i}z_{i,1}^2\|g_i(z_{i,1})\|^2\|P_i\|^2\|\tilde{x}_i\|^2\\ &\le \frac{3a_i n_i\sqrt{n_i}}{c_{i,1}^2}z_{i,1}^4\|\bar{g}_i(z_{i,1})\|^4 + 3a_i n_i\sqrt{n_i}c_{i,1}^2\|P_i\|^4\|\tilde{x}_i\|^4\end{aligned} \tag{12}$$

$$\begin{aligned}&z_{i,1}^3\tilde{x}_{i,2}\\ &\le \frac{3}{4}c_{i,2}^{\frac{4}{3}}z_{i,1}^4 + \frac{1}{4c_{i,2}^4}\tilde{x}_{i,2}^4\\ &\le \frac{3}{4}c_{i,2}^{\frac{4}{3}}z_{i,1}^4 + \frac{1}{4c_{i,2}^4}\|\tilde{x}_i\|^4\\ &z_{i,1}^3 h_{i,1}(y)\\ &\le \left|z_{i,1}^3\right|p_{i,1}\sum_{l=1}^{N}\varphi_{i1l}(|y_l|)\\ &\le \frac{3}{4}c_{i,3}^{\frac{4}{3}}p_{i,1}^{\frac{4}{3}}z_{i,1}^4 + \left(\sum_{l=1}^{N}\varphi_{i1l}(|y_l|)\right)^4.\end{aligned} \tag{13}$$

As $\varphi_{i1l}$ is a smooth function, there exists a smooth nonnegative function $\eta_{i1l}^{(2)}(y_l)$, such that

$$\left(\sum_{l=1}^{N}\varphi_{i1l}(|y_l|)\right)^4 \le \sum_{l=1}^{N}\eta_{i1l}^{(2)}y_l^4 + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4.$$

Therefore

$$\begin{aligned}z_{i,1}^3 h_{i,1}(y) &\le \frac{3}{4}c_{i,3}^{\frac{4}{3}}p_{i,1}^{\frac{4}{3}}z_{i,1}^4 + \sum_{l=1}^{N}\eta_{i1l}^{(2)}y_l^4\\ &\quad + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4.\end{aligned} \tag{14}$$

Combining (9)–(14) and (8), we have

$$\begin{aligned}\mathcal{L}V_{i,1} \leq & -\Pi_i\|\tilde{x}_i\|^4 + z_{i,1}^3(\hat{x}_{i,2}+\bar{f}_{i,1}) + \sum_{j=1}^{n_i}\sum_{l=1,l\neq i}^{N}\eta_{ijl}^{(1)}y_l^4 \\ & + \sum_{l=1,l\neq i}^{N}\eta_{i1l}^{(2)}y_l^4 + \sum_{j=1}^{n_i}\sum_{l=1}^{N}4a_i\varphi_{ijl}^4(0) \\ & + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4 - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i - \frac{3}{4}z_{i,1}^4 \\ & + \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4\end{aligned}$$

where

$$\begin{aligned}\Pi_i =& a_i\lambda_{\min}(P_i) - 3a_i - \frac{3a_in_iN}{2}p_{i,j}^{\frac{4}{3}}\|P_i\|^{\frac{8}{3}} \\ & - 3a_in_i\sqrt{n_i}c_{i,1}^2\|P_i\|^4 - \frac{1}{4c_{i,2}^4}, \\ \bar{f}_{i,1}(X_{i,1}) =& f_{i,1}(z_{i,1}) \\ & + \left(\frac{3}{4}c_{i,2}^{\frac{4}{3}} + \frac{3}{4}(c_{i,3}p_{i,1})^{\frac{4}{3}} + \sum_{j=1}^{n_i}\eta_{iji}^{(1)}\right. \\ & + \eta_{i1i}^{(2)} + \frac{3a_in_i\sqrt{n_i}}{c_{i,1}^2}\|\bar{g}_i(z_{i,1})\|^4 \\ & \left.+\frac{3}{2}\bar{g}_{i,1}^T(z_{i,1})\bar{g}_{i,1}(z_{i,1}) + \frac{3}{4}\right)z_{i,1}.\end{aligned}$$

Taking the intermediate control signal $\hat{\alpha}_{i,1}(X_{i,1})$ as

$$\hat{\alpha}_{i,1}\left(X_{i,1}\right) = -\left(k_{i,1}z_{i,1} + \bar{f}_{i,1} + v_{i,1}\left(z_{i,1}^2\right)z_{i,1}\right)$$

where $k_{i,1} > 0$. Then, we have

$$\begin{aligned}\mathcal{L}V_{i,1} \leq & -\Pi_i\left\|\tilde{x}_i\right\|^4 + z_{i,1}^3(\hat{x}_{i,2}-\hat{\alpha}_{i,1}) - k_{i,1}z_{i,1}^4 \\ & - v_{i,1}\left(z_{i,1}^2\right)z_{i,1}^4 + \sum_{j=1}^{n_i}\sum_{l=1,l\neq i}^{N}\eta_{ijl}^{(1)}y_l^4 + \sum_{l=1,l\neq i}^{N}\eta_{i1l}^{(2)}y_l^4 \\ & + \sum_{j=1}^{n_i}\sum_{l=1}^{N}4a_i\varphi_{ijl}^4(0) + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4 - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\ & - \frac{3}{4}z_{i,1}^4 + \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4. \end{aligned} \tag{15}$$

From the definition of $\hat{\alpha}_{i,1}(X_{i,1})$, we know that it is an unknown nonlinear function because it contains $f_{i,1}(z_{i,1})$ and cannot be implemented in practice. Therefore, according to (3), for any given constant $\varepsilon_{i,1} > 0$, there exists $W_{i,1}^{*T}S_{i,1}(X_{i,1})$ such that

$$\begin{aligned}&\hat{\alpha}_{i,1}(X_{i,1}) = W_{i,1}^{*T}S_{i,1}(X_{i,1}) + \delta_{i,1}(X_{i,1}), \\ &|\delta_{I,1}(X_{i,1})| \leq \varepsilon_{i,1}.\end{aligned}$$

From the definition of $\theta_i$ and $\alpha_{i,1}$, we have

$$\begin{aligned}-z_{i,1}^3\hat{\alpha}_{i,1} =& -z_{i,1}^3W_{i,1}^{*T}S_{i,1}(X_{i,1}) - z_{i,1}^3\delta_{i,1}(X_{i,1}) \\ \leq& \frac{N_{i,1}}{2a_{i,1}^2}z_{i,1}^6\left\|W_{i,1}^*\right\|^2 + \frac{1}{2}a_{i,1}^2 + \frac{3}{4}z_{i,1}^4 + \frac{1}{4}\varepsilon_{i,1}^4 \\ \leq& \frac{1}{2a_{i,1}^2}z_{i,1}^6\theta_i + \frac{1}{2}a_{i,1}^2 + \frac{3}{4}z_{i,1}^4 + \frac{1}{4}\varepsilon_{i,1}^4\end{aligned} \tag{16}$$

$$z_{i,1}^3\alpha_{i,1} = -\frac{1}{2a_{i,1}^2}z_{i,1}^6\hat{\theta}_i \tag{17}$$

where the fact $S_{i,1}^TS_{i,1} \leq N_{i,1}$ can be used and $N_{i,1}$ is the dimension of $S_{i,1}$. Then, substituting (16) and (17) into (15) yields

$$\begin{aligned}\mathcal{L}V_{i,1} \leq & -\Pi_i\|\tilde{x}_i\|^4 + z_{i,1}^3(\hat{x}_{i,2}-\alpha_{i,1}) - k_{i,1}z_{i,1}^4 \\ & - v_{i,1}\left(z_{i,1}^2\right)z_{i,1}^4 + \frac{1}{r_i}\tilde{\theta}_i\left(\frac{r_i}{2a_{i,1}^2}z_{i,1}^6 - \dot{\hat{\theta}}_i\right) \\ & + \sum_{j=1}^{n_i}\sum_{l=1,l\neq i}^{N}\eta_{ijl}^{(1)}y_l^4 + \sum_{l=1,l\neq i}^{N}\eta_{i1l}^{(2)}y_l^4 + \Delta_{i,1}.\end{aligned} \tag{18}$$

where

$$\begin{aligned}\Delta_{i,1} =& \frac{1}{2}a_{i,1}^2 + \frac{1}{4}\varepsilon_{i,1}^4 + \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4 \\ & + \sum_{j=1}^{n_i}\sum_{l=1}^{N}4a_i\varphi_{ijl}^4(0) + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4.\end{aligned}$$

Due to $\hat{x}_{i,2} = z_{i,2} + \alpha_{i,2,f}$, (18) can be rewritten as

$$\begin{aligned}\mathcal{L}V_{i,1} \leq & -\Pi_i\|\tilde{x}_i\|^4 + z_{i,1}^3(z_{i,2} + \alpha_{i,2,f} - \alpha_{i,1}) \\ & - k_{i,1}z_{i,1}^4 - v_{i,1}(z_{i,1}^2)z_{i,1}^4 \\ & + \sum_{j=1}^{n_i}\sum_{l=1,l\neq i}^{N}\eta_{ijl}^{(1)}y_l^4 + \sum_{l=1,l\neq i}^{N}\eta_{i1l}^{(2)}y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i\left(\frac{r_i}{2a_{i,1}^2}z_{i,1}^6 - \dot{\hat{\theta}}_i\right) + \Delta_{i,1}.\end{aligned}$$

To avoid repeatedly differentiating $\alpha_{i,1}$, a new state variable $\alpha_{i,2,f}$ is introduced, and let $\alpha_{i,1}$ pass through a first-order filter with time constant $\kappa_{i,2}$ to obtain $\alpha_{i,2,f}$ as

$$\kappa_{i,2}\dot{\alpha}_{i,2,f} + \alpha_{i,2,f} = \alpha_{i,1}, \quad \alpha_{i,2,f}(0) = \alpha_{i,1}(0).$$

Let $\chi_{i,2} = \alpha_{i,2,f} - \alpha_{i,1}$ be the output error of this filter; then, one has $\dot{\alpha}_{i,2,f} = -\chi_{i,2}/\kappa_{i,2}$ and

$$\dot{\chi}_{i,2} = \dot{\alpha}_{i,2,f} - \dot{\alpha}_{i,1} = -\frac{\chi_{i,2}}{\kappa_{i,2}} + B_{i,2}(X_{i,1})$$

where

$$B_{i,2}(X_{i,1}) = \frac{3}{2a_{i,1}^2}z_{i,1}^2\dot{z}_{i,1}\hat{\theta}_i + \frac{1}{2a_{i,1}^2}z_{i,1}^3\dot{\hat{\theta}}_i.$$

Then, this implies

$$\begin{aligned}\mathcal{L}V_{i,1} \leq & -\Pi_i\|\tilde{x}_i\|^4 + z_{i,1}^3 z_{i,2} + z_{i,1}^3 \chi_{i,2} \\ & - k_{i,1} z_{i,1}^4 - v_{i,1}(z_{i,1}^2) z_{i,1}^4 \\ & + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left(\frac{r_i}{2a_{i,1}^2} z_{i,1}^6 - \dot{\hat{\theta}}_i\right) + \Delta_{i,1}.\end{aligned}$$

Step $i, m$: $(2 \leq m \leq n_i - 1)$. Choose the Lyapunov functional

$$V_{i,m} = V_{i,m-1} + \frac{1}{4} z_{i,m}^4 + \frac{1}{4}\chi_{i,m}^4.$$

Similarly, we have

$$\begin{aligned}\mathcal{L}V_{i,m} \leq & -\Pi_i\|\tilde{x}_i\|^4 + \sum_{j=1}^{m-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{m-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{m-1} k_{i,j} z_{i,j}^4 - v_{i,1}\left(z_{i,1}^2\right) z_{i,1}^4 \\ & + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left(\sum_{j=1}^{m-1} r_i 2a_{i,j}^2 z_{i,j}^6 - \dot{\hat{\theta}}_i\right) + \Delta_{i,m-1} \\ & - \sum_{j=1}^{m-1}\left(\frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j})\right) \\ & + z_{i,m}^3\left(\hat{x}_{i,m+1} + \bar{f}_{i,m}(X_{i,m})\right) - \frac{3}{4} z_{i,m}^4 \end{aligned} \tag{19}$$

where

$$\bar{f}_{i,m}(X_{i,m}) = l_{i,m}\tilde{x}_{i,1} - \dot{\alpha}_{i,m,f} + \frac{3}{4} z_{i,m}.$$

Take the intermediate control signal $\hat{\alpha}_{i,m}(X_{i,m})$ as

$$\hat{\alpha}_{i,m} = -(k_{i,m} z_{i,m} + \bar{f}_{i,m})$$

where $k_{i,m} > 0$; then, adding and subtracting $\hat{\alpha}_{i,m}(X_{i,m})$ in (19), it yields

$$\begin{aligned}\mathcal{L}V_{i,m} \leq & -\Pi_i\|\tilde{x}_i\|^4 + \sum_{j=1}^{m-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{m-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{m-1} k_{i,j} z_{i,j}^4 - v_{i,1}\left(z_{i,1}^2\right) z_{i,1}^4 \\ & + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left(\sum_{j=1}^{m-1} r_i 2a_{i,j}^2 z_{i,j}^6 - \dot{\hat{\theta}}_i\right) + \Delta_{i,m-1} \\ & - \sum_{j=1}^{m-1}\left(\frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j})\right) \\ & + z_{i,m}^3\left(\hat{x}_{i,m+1} + \hat{\alpha}_{i,m}(X_{i,m})\right) - \frac{3}{4} z_{i,m}^4. \end{aligned} \tag{20}$$

Similarly, $\hat{\alpha}_{i,m}(X_{i,m})$ can be approximated by the NN $W_{i,m}^{*T} S_{i,m}(X_{i,m})$ as

$$\begin{aligned}&\hat{\alpha}_{i,m}(X_{i,m}) = W_{i,m}^{*T} S_{i,m}(X_{i,m}) + \delta_{i,m}(X_{i,m}),\\ &|\delta_{i,m}(X_{i,m})| \leq \varepsilon_{i,m}.\end{aligned}$$

In addition,

$$-z_{i,m}^3\hat{\alpha}_{i,m} \leq 12a_{i,m}^2 z_{i,m}^6\theta_i + \frac{1}{2}a_{i,m}^2 + \frac{3}{4} z_{i,m}^4 + \frac{1}{4}\varepsilon_{i,m}^4 \tag{21}$$

$$z_{i,m}^3\alpha_{i,m} = -\frac{1}{2a_{i,m}^2} z_{i,m}^6\hat{\theta}_i. \tag{22}$$

Then, by substituting (21) and (22) into (20), we have

$$\begin{aligned}\mathcal{L}V_{i,m} \leq & -\Pi_i\|\tilde{x}_i\|^4 + \sum_{j=1}^{m-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{m-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{m-1} k_{i,j} z_{i,j}^4 - v_{i,1}\left(z_{i,1}^2\right) z_{i,1}^4 \\ & + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left(\sum_{j=1}^{m} \frac{r_i}{2a_{i,j}^2} z_{i,j}^6 - \dot{\hat{\theta}}_i\right) + \Delta_{i,m} \\ & - \sum_{j=1}^{m-1}\left(\frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j})\right) \\ & + z_{i,m}^3\left(\hat{x}_{i,m+1} + \hat{\alpha}\right) \end{aligned} \tag{23}$$

where

$$\begin{aligned}\Delta_{i,m} = & \frac{1}{2}\sum_{j=1}^{m} a_{i,j}^2 + \frac{1}{4}\sum_{j=1}^{m}\varepsilon_{i,j}^4 \\ & + \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4 \\ & + \sum_{j=1}^{n_i}\sum_{l=1}^{N} 4a_i\varphi_{ijl}^4(0) + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4.\end{aligned}$$

Next, introduce a new variable $\alpha_{i,m+1,f}$, and let $\alpha_{i,m}$ pass through a first-order filter with the constant $\kappa_{i,m+1}$ to obtain $\alpha_{i,m+1,f}$

$$\kappa_{i,m+1}\dot{\alpha}_{i,m+1,f} + \alpha_{i,m+1,f} = \alpha_{i,m}, \quad \alpha_{i,m+1,f}(0) = \alpha_{i,m}(0).$$

Then, define

$$\chi_{i,m+1} = \alpha_{i,m+1,f} - \alpha_{i,m} \tag{24}$$

as the output error of this filter; we have $\dot{\alpha}_{i,m+1,f} = -(\chi_{i,m+1}/\kappa_{i,m+1})$ and

$$\dot{\chi}_{i,_{m+1}} = \dot{\alpha}_{i,m+1,f} - \dot{\alpha}_{i,m} = -\frac{\chi_{i,m+1}}{\kappa_{i,m+1}} + B_{i,_{m+1}}(X_{i,m}).$$

where

$$B_{i,_{m+1}}(X_{i,m}) = \frac{3}{2a_{i,m}^2} z_{i,m}^2 \dot{z}_{i,m} \hat{\theta}_i + \frac{1}{2a_{i,m}^2} z_{i,m}^3 \dot{\hat{\theta}}_i.$$

Substituting (24) into (23) yields

$$\begin{aligned}\mathcal{L}V_{i,m} \le & -\Pi_i \|\tilde{x}_i\|^4 + \sum_{j=1}^{m-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{m-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{m-1} k_{i,j} z_{i,j}^4 - v_{i,1}\left(z_{i,1}^2\right) z_{i,1}^4 \\ & + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left( \sum_{j=1}^{m} \frac{r_i}{2a_{i,j}^2} z_{i,j}^6 - \dot{\hat{\theta}}_i \right) + \Delta_{i,m} \\ & - \sum_{j=1}^{m-1} \left( \frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j}) \right).\end{aligned}$$

Step $i, n_i$: Consider the following Lyapunov functional:

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{4} z_{i,n_i}^4 + \frac{1}{4}\chi_{i,n_i}^4.$$

Similarly, we obtain

$$\begin{aligned}\mathcal{L}V_{i,n_i} \le & -\Pi_i \|\tilde{x}_i\|^4 + \sum_{j=1}^{n_i-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{n_i-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{n_i-1} k_{i,j} z_{i,j}^4 + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 \\ & + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 - v_{i,1}(z_{i,1}^2) z_{i,1}^4 \\ & + \Delta_{i,n_i-1} + \frac{1}{r_i}\tilde{\theta}_i \left( \sum_{j=1}^{n_i-1} \frac{r_i}{2a_{i,j}^2} z_{i,j}^6 - \dot{\hat{\theta}}_i \right) \\ & - \sum_{j=1}^{n_i-1} \left( \frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j}) \right) \\ & + z_{i,n}^3 (u_i + \bar{f}_{i,n_i}) - \frac{3}{4} z_{i,n_i}^4\end{aligned} \tag{25}$$

where

$$\bar{f}_{i,n_i}(X_{i,n}) = l_{i,n_i}\tilde{x}_{i,1} - \dot{\alpha}_{i,n_i,f} + \frac{3}{4} z_{i,n_i}.$$

Take the intermediate control signal $\hat{\alpha}_{i,n_i}(X_{i,n_i})$ as

$$\hat{\alpha}_{i,n_i} = -(k_{i,n_i} z_{i,n_i} + \bar{f}_{i,n_i})$$

where $k_{i,n_i} > 0$; then, adding and subtracting $\hat{\alpha}_{i,n_i}(X_{i,n_i})$ in (25) yields

$$\begin{aligned}\mathcal{L}V_{i,n_i} \le & -\Pi_i \|\tilde{x}_I\|^4 + \sum_{j=1}^{n_i-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{n_i-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{n_i} k_{I,j} z_{i,j}^4 + \sum_{j=1}^{n_i} \sum_{l=1,l\neq I}^{N} \eta_{ijl}^{(1)} y_l^4 \\ & + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 - v_{i,1}(z_{i,1}^2) z_{i,1}^4 + \Delta_{i,n_i-1} \\ & + \frac{1}{r_i}\tilde{\theta}_i \left( \sum_{j=1}^{n_i-1} r_i 2a_{i,j}^2 z_{i,j}^6 - \dot{\hat{\theta}}_i \right) \\ & - \sum_{j=1}^{n_i-1} \left( \frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j} \right) \\ & + z_{i,n}^3 (u_i - \hat{\alpha}_{i,n_i}) - \frac{3}{4} z_{i,n_i}^4.\end{aligned} \tag{26}$$

Similar to the above steps, $\hat{\alpha}_{i,n_i}(X_{i,n_i})$ can be approximated by the NN $W_{i,n_i}^{*T} S_{i,n_i}(X_{i,n_i})$ as

$$\begin{aligned}&\hat{\alpha}_{i,n_i}(X_{i,n_i}) = W_{i,n_i}^{*T} S_{i,n_i}(X_{i,n_i}) + \delta_{i,n_i}(X_{I,n_i}), \\ &|\delta_{i,n_i}(X_{i,n_i})| \le \varepsilon_{i,n_i}.\end{aligned}$$

Following the similar procedure and by the definition of $u_i$, we have

$$-z_{i,n_i}^3 \hat{\alpha}_{i,n_i} \le \frac{1}{2a_{i,n_i}^2} z_{i,n_i}^6 \theta_i + \frac{1}{2} a_{i,n_i}^2 + \frac{3}{4} z_{i,n_i}^4 + \frac{1}{4}\varepsilon_{i,n_i}^4 \tag{27}$$

$$z_{i,n_i} u_i = -\frac{1}{2a_{i,n_i}^2} z_{i,n_i}^6 \hat{\theta}_i. \tag{28}$$

Then, by substituting (27) and (28) into (26), we have

$$\begin{aligned}\mathcal{L}V_{i,n_i} \le & -\Pi_i \|\tilde{x}_i\|^4 + \sum_{j=1}^{n_i-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{n_i-1} z_{i,j}^3 \chi_{i,j+1} \\ & - \sum_{j=1}^{n_i} k_{i,j} z_{i,j}^4 + \sum_{j=1}^{n_i} \sum_{l=1,l\neq i}^{N} \eta_{ijl}^{(1)} y_l^4 + \Delta_{i,n_i} \\ & - v_{i,1}(z_{i,1}^2) z_{i,1}^4 + \sum_{l=1,l\neq i}^{N} \eta_{i1l}^{(2)} y_l^4 \\ & + \frac{1}{r_i}\tilde{\theta}_i \left( \sum_{j=1}^{n_i} \frac{r_i}{2a_{i,j}^2} z_{i,j}^6 - \dot{\hat{\theta}}_i \right) \\ & - \sum_{j=1}^{n_i-1} \left( \frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j}) \right)\end{aligned}$$

where

$$\begin{aligned}\Delta_{i,n_i} =& \frac{1}{2}\sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{4}\sum_{j=1}^{n_i}\varepsilon_j^4 \\ &+ \frac{a_i}{2}\|P_i\|^8\theta^2 + \frac{a_i}{2}\|P_i\|^8\varepsilon_{i,0}^4 \\ &+ \sum_{j=1}^{n_i}\sum_{l=1}^{N} 4a_i\varphi_{ijl}^4(0) + 8\left(\sum_{l=1}^{N}\varphi_{i1l}(0)\right)^4.\end{aligned}$$

By the definition of $\dot{\hat{\theta}}_i$, we can get

$$\begin{aligned}\mathcal{L}V_{i,n_i} \le& -\Pi_i\|\tilde{x}_i\|^4 + \sum_{j=1}^{n_i-1} z_{i,j}^3 z_{i,j+1} + \sum_{j=1}^{n_i-1} z_{i,j}^3\chi_{i,j+1} \\ &- \sum_{j=1}^{n_i} k_{i,j} z_{i,j}^4 + \sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{ijl}^{(1)} y_l^4 + \Delta_{i,n_i} \\ &- v_{i,1}(z_{i,1}^2)z_{i,1}^4 + \sum_{l=1,l\ne i}^{N}\eta_{i1l}^{(2)} y_l^4 + \frac{k_{i,0}}{r_i}\tilde{\theta}_i\hat{\theta}_i \\ &- \sum_{j=1}^{n_i-1}\left(\frac{\chi_{i,j+1}^4}{\kappa_{i,j+1}} - \chi_{i,j+1}^3 B_{i,j+1}(X_{i,j})\right). \end{aligned} \tag{29}$$

By utilizing Lemma 1, we have

$$\begin{aligned} z_{i,j}^3 z_{i,j+1} &\le \frac{3}{4}z_{i,j}^4 + \frac{1}{4}z_{i,j+1}^4, \\ z_{i,j}^3\chi_{i,j+1} &\le \frac{3}{4}z_{i,j}^4 + \frac{1}{4}\chi_{i,j+1}^4, \\ \left|\chi_{i,j+1}^3 B_{i,j+1}\right| &\le \frac{3}{4}\pi_i^{\frac{4}{3}} B_{i,j+1}^{\frac{4}{3}}\chi_{i,j+1}^4 + \frac{1}{4\pi_i^4}, \\ \tilde{\theta}_i\hat{\theta}_i &= \tilde{\theta}_i(\theta_i - \tilde{\theta}_i) \le -\frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\theta_i^2 \end{aligned} \tag{30}$$

where $\pi_i > 0$ is a design constant. Substituting (30) into (29), one has

$$\begin{aligned}\mathcal{L}V_{i,n_i} \le& -\Pi_i\|\tilde{x}_i\|^4 - \sum_{j=1}^{n_i}(k_{i,j} - \frac{7}{4})z_{i,j}^4 \\ &- v_{i,1}(z_{i,1}^2)z_{i,1}^4 - \frac{k_{i,0}}{2r_i}\tilde{\theta}_i^2 + \bar{\Delta}_{i,n_i} \\ &+ \sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{ijl}^{(1)} y_l^4 + \sum_{l=1,l\ne i}^{N}\eta_{i1l}^{(2)} y_l^4 \\ &- \sum_{j=1}^{n_i-1}\left(\frac{1}{k_{i,j+1}} - \frac{1}{4} - \frac{3}{4}\pi_i^{\frac{4}{3}} B_{i,j+1}^{\frac{4}{3}}\right)\chi_{i,j+1}^4, \\ \bar{\Delta}_{i,n_i} =& \Delta_{i,n_i} + \frac{k_{i,0}}{2r_i}\theta_i^2 + \frac{1}{4\pi_i^4}.\end{aligned}$$

Choose the following Lyapunov functional candidate for the whole system:

$$V_N = \sum_{I=1}^{N} V_{i,n_i}.$$

According to Definition 1, one has

$$\begin{aligned}\mathcal{L}V_N \le& -\sum_{i=1}^{N}\Pi_i\|\tilde{x}_i\|^4 - \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(k_{i,j} - \frac{7}{4}\right)z_{i,j}^4 \\ &+ \sum_{i=1}^{N}\bar{\Delta}_{i,n_i} - \sum_{i=1}^{N} v_{i,1}(z_{i,1}^2)z_{i,1}^4 - \sum_{i=1}^{N}\frac{k_{i,0}}{2r_i}\tilde{\theta}_i^2 \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{ijl}^{(1)} y_l^4 + \sum_{i=1}^{N}\sum_{l=1,l\ne i}^{N}\eta_{i1l}^{(2)} y_l^4 \\ &- \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(\frac{1}{k_{i,j+1}} - \frac{1}{4} - \frac{3}{4}\pi_i^{\frac{4}{3}} B_{i,j+1}^{\frac{4}{3}}\right)\chi_{i,j+1}^4.\end{aligned}$$

As

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{ijl}^{(1)} y_l^4 + \sum_{i=1}^{N}\sum_{l=1,l\ne i}^{N}\eta_{i1l}^{(2)} y_l^4 \\ &= \sum_{l=1}^{N}\sum_{j=1}^{n_i}\sum_{i=1,i\ne l}^{N}\eta_{lji}^{(1)} y_i^4 + \sum_{l=1}^{N}\sum_{i=1,i\ne l}^{N}\eta_{l1i}^{(2)} y_i^4 \\ &= \sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{lji}^{(1)} y_i^4 + \sum_{l=1}^{N}\sum_{i=1,l\ne i}^{N}\eta_{l1i}^{(2)} y_i^4.\end{aligned}$$

Choose a smooth nonnegative function candidate $v_{i,1}$ such that

$$\begin{aligned}\sum_{i=1}^{N} v_{i,1}(z_{i,1}^2)z_{i,1}^4 &- \sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1,l\ne i}^{N}\eta_{lji}^{(1)} z_{i,1}^4 \\ &- \sum_{i=1}^{N}\sum_{l=1,l\ne i}^{N}\eta_{l1i}^{(2)} z_{i,1}^4 \ge 0.\end{aligned}$$

Then,

$$\begin{aligned}\mathcal{L}V_N \le& -\sum_{i=1}^{N}\Pi_i\|\tilde{x}_i\|^4 - \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(k_{i,j} - \frac{7}{4}\right)z_{i,j}^4 \\ &- \sum_{i=1}^{N} k_{i,0}2r_i\tilde{\theta}_i^2 + \sum_{i=1}^{N}\bar{\Delta}_{i,n_i} \\ &- \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(\frac{1}{k_{i,j+1}} - \frac{1}{4} - \frac{3}{4}\pi_i^{\frac{4}{3}} B_{i,j+1}^{\frac{4}{3}}\right)\chi_{i,j+1}^4.\end{aligned}$$

Let

$$\Pi_i > 0$$

and denote

$$\begin{aligned} c_I =& \min_{1\le j\le n_i}\left\{\frac{2\Pi_i}{a_I\lambda_{\max}^2(P_i)}, 4\left(k_{i,j} - \frac{7}{4}\right),\right. \\ &\left. 4\left(\frac{1}{k_{i,j+1}} - \frac{1}{4} - \frac{3}{4}\pi_i^{\frac{4}{3}} B_{i,j+1}^{\frac{4}{3}}\right), k_{i,0}\right\}; \\ c =& \min\{c_1, \ldots, c_N\}, \\ d =& \sum_{i=1}^{N}\bar{\Delta}_{i,n_i}\end{aligned}$$

then one has

$$\mathcal{L}V_N \le -cV_N + d.$$

Therefore, the signals $\tilde{x}_i$, $z_i$, and $\tilde{\theta}_i$ are bounded in probability. As $\theta_i$ is a constant, $\hat{\theta}_i$ is also bounded in probability. Now, it can be shown that all the signals in the closed-loop system are semiglobally uniformly ultimately bounded in probability, which are the desired results, and the proof is completed.

*Remark 3:* In the backstepping design procedure, the dynamic surface control technique is successfully applied to large-scale stochastic nonlinear systems. In the controller design process, the repeated differentiation of virtual control $\alpha_i$ is avoided by replacing $\alpha_i$ with $\dot{\alpha}_{i+1f}$, and $\alpha_{i+1f}$ is defined by a first-order filter with $\alpha_i$ as input.

*Remark 4:* In most of the neural adaptive control design process, the number of the adaptation parameters depends on the number of the NN nodes. Consequently, if a system contains a large number of unknown nonlinear functions or if more NN nodes are used to improve the approximation precision, a large number of adaptation parameters will be needed to be updated online. In this paper, we estimated the norm of all the weight vectors but not each weight vector; therefore, only one adaptation learning law is required to control each subsystem.

## IV. Simulation Results

In this section, we will exploit a simulation example to demonstrate the effectiveness of the proposed adaptive NN control method.

Consider the following large-scale stochastic nonlinear systems:

$$\begin{aligned} dx_{1,1} &= (x_{1,2} + f_{1,1} + h_{1,1})dt + g_{1,1}dw_1 \\ dx_{1,2} &= (u_1 + f_{1,2} + h_{1,2})dt + g_{1,2}dw_1 \\ y_1 &= x_{1,1} \\ dx_{2,1} &= (x_{2,2} + f_{2,1} + h_{2,1})dt + g_{2,1}dw_2 \\ dx_{2,2} &= (u_2 + f_{2,2} + h_{2,2})dt + g_{2,2}dw_2 \\ y_2 &= x_{2,1} \end{aligned}$$

where the nonlinear functions are $f_{1,1} = -100x_{1,1}$, $f_{1,2} = x_{1,2}\sin(x_{1,1})$, $f_{2,1} = 3x_{2,1}\cos(0.2/x_{2,1})$, and $f_{2,2} = -x_{2,2}\sin(x_{2,1})$; the interconnection functions are $h_{1,1} = \cos(y_1)y_2$, $h_{1,2} = -y_1\sin(y_2)$, $h_{2,1} = \sin(y_1)y_2$, and $h_{2,2} = \cos(y_1)y_2)$; the stochastic disturbance functions are $g_{1,1} = \sin(y_1)$, $g_{1,2} = \cos(y_1)$, $g_{2,1} = \sin(y_2)$, and $g_{2,2} = \cos(y_2)$; and the initial states are chosen as $x_{1,1}(0) = 0.2$, $x_{1,2}(0) = 0.3$, $x_{2,1}(0) = 0.5$, and $x_{2,2}(0) = 0.4$. The observer is designed as

$$\begin{aligned} \dot{\hat{x}}_{1,1} &= \hat{x}_{1,2} + l_{1,1}(x_{1,1} - \hat{x}_{1,1}) \\ \dot{\hat{x}}_{1,2} &= u_1 + l_{1,2}(x_{1,1} - \hat{x}_{1,1}) \\ \dot{\hat{x}}_{2,1} &= \hat{x}_{2,2} + l_{2,1}(x_{2,1} - \hat{x}_{2,1}) \\ \dot{\hat{x}}_{2,2} &= u_2 + l_{2,2}(x_{2,1} - \hat{x}_{2,1}). \end{aligned}$$

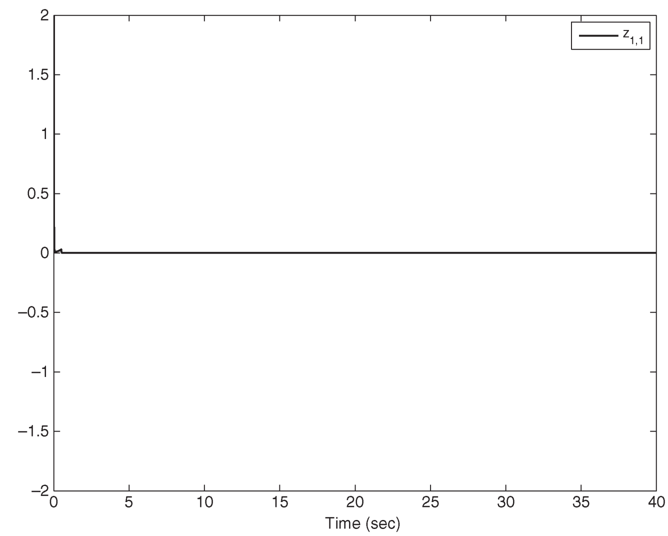


Fig. 1. Trajectory of system output $y_1$.

According to Theorem 1, the virtual control function $\alpha_{1,1}$, $\alpha_{2,1}$ and the true control law $u_1$, $u_2$ are chosen respectively as

$$\begin{aligned} \alpha_{1,1} &= -\frac{1}{2a_{1,1}^2} z_{1,1}^3 \hat{\theta}_1, \quad u_1 = -\frac{1}{2a_{1,2}^2} z_{1,2}^3 \hat{\theta}_1 \\ \alpha_{2,1} &= -\frac{1}{2a_{2,1}^2} z_{2,1}^3 \hat{\theta}_2, \quad u_2 = -\frac{1}{2a_{2,2}^2} z_{2,2}^3 \hat{\theta}_2 \end{aligned} \tag{31}$$

where $z_{1,1} = y_1$, $z_{1,2} = \hat{x}_{1,2} - \alpha_{1,2,f}$, $z_{2,1} = y_2$, and $z_{2,2} = \hat{x}_{2,2} - \alpha_{2,2,f}$. The adaptive laws are given as

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \sum_{j=1}^{2} \frac{r_1}{2a_{1,j}^2} z_{1,j}^6 - k_{1,0}\hat{\theta}_1 \\ \dot{\hat{\theta}}_2 &= \sum_{j=1}^{2} \frac{r_2}{2a_{2,j}^2} z_{2,j}^6 - k_{2,0}\hat{\theta}_2. \end{aligned} \tag{32}$$

In the simulation, the design parameters are chosen as $l_{1,1} = l_{1,2} = l_{2,1} = l_{2,2} = 50$, $a_{1,1} = a_{1,2} = 0.13$, $a_{2,1} = a_{2,2} = 0.115$, $r_{1,1} = r_{1,2} = 20.5$, $r_{2,1} = r_{2,2} = 30.5$, $k_{1,0} = k_{2,0} = 0.01$, $\kappa_{1,2} = 0.06$, and $\kappa_{2,2} = 0.001$. The simulation results are illustrated in Figs. 1–8, respectively. Figs. 1 and 2 show the system output $y_1$ and $y_2$. Fig. 3 and 4 illustrate the new state variables $\alpha_{1,2,f}$ and $\alpha_{2,2,f}$ of the first-order filters. Figs. 5 and 6 depict the trajectories of input $u_1$ and $u_2$, whereas Figs. 7 and 8 illustrate the trajectories of adaptive parameter $\hat{\theta}_1$ and $\hat{\theta}_2$.

*Remark 5:* From Figs. 1–8, we can see that the system output $y_1$ and $y_2$ can converge to a small neighborhood around the origin. In addition, the filter signals $\alpha_{1,2,f}$ and $\alpha_{2,2,f}$, control input $u_1$ and $u_2$, and the adaptive parameter $\hat{\theta}_1$ and $\hat{\theta}_2$ are all bounded. The presented simulation results illustrate the effectiveness of the adaptive dynamic surface control approach proposed in this paper.

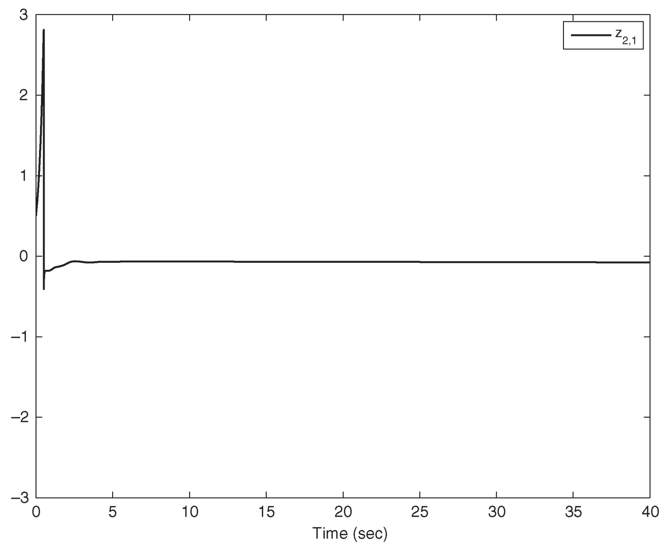


Fig. 2. Trajectory of system output $y_2$.

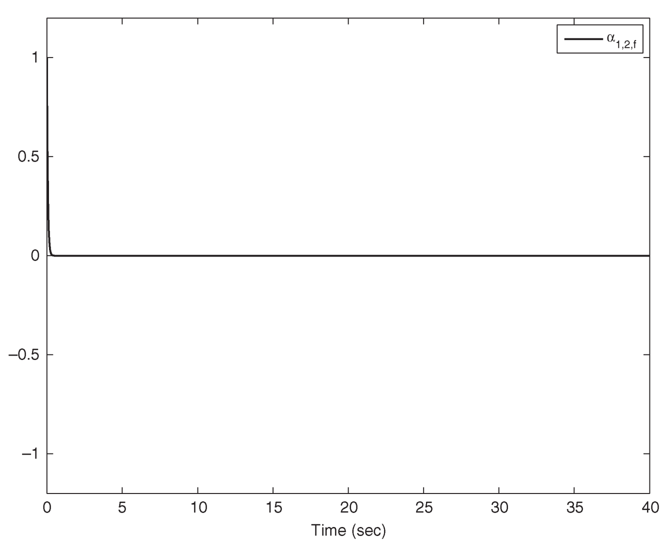


Fig. 3. Trajectory of a state variable of a first-order filter $\alpha_{1,2,f}$.

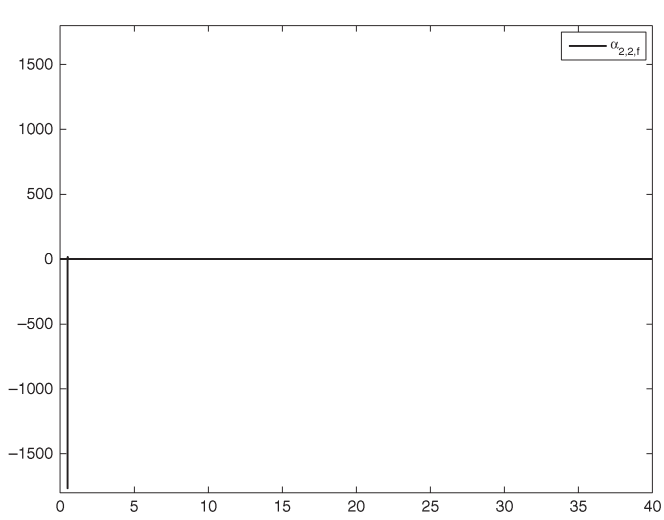


Fig. 4. Trajectory of a state variable of a first-order filter $\alpha_{2,2,f}$.

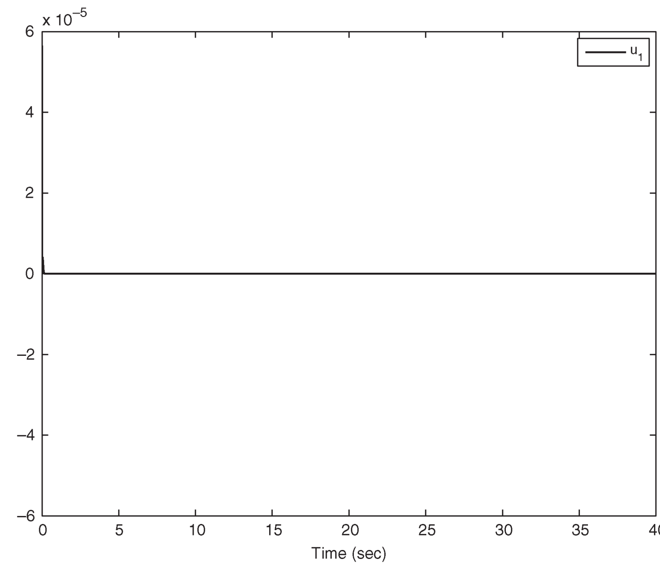


Fig. 5. Trajectory of control input $u_1$.

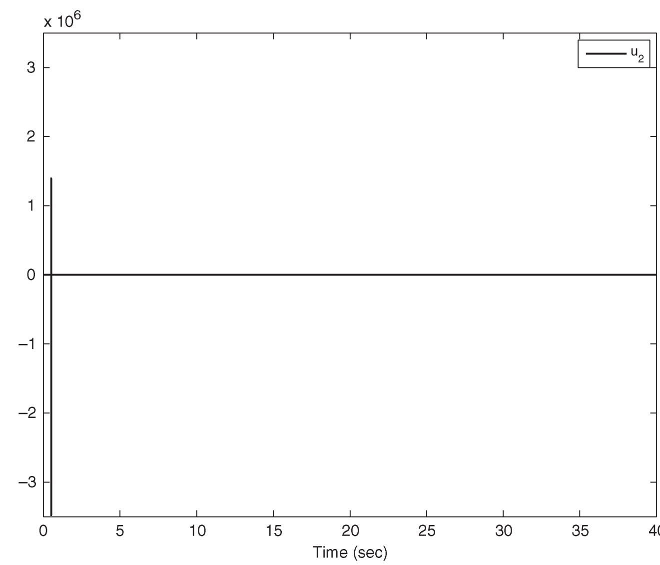


Fig. 6. Trajectory of control input $u_2$.

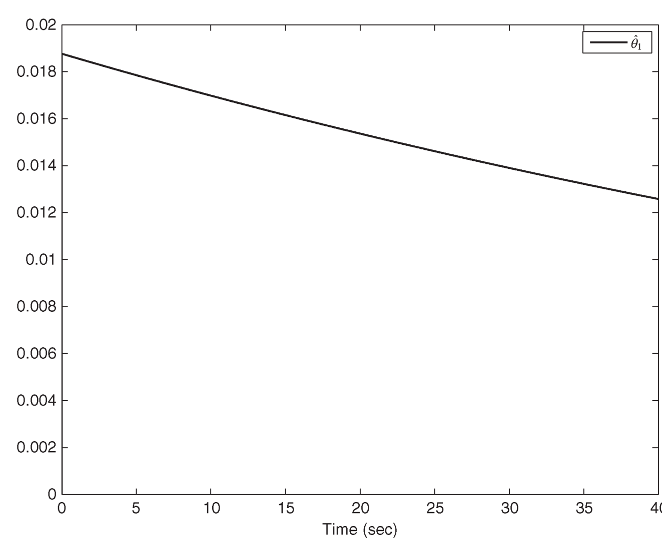


Fig. 7. Adaptive parameter $\hat{\theta}_1$.

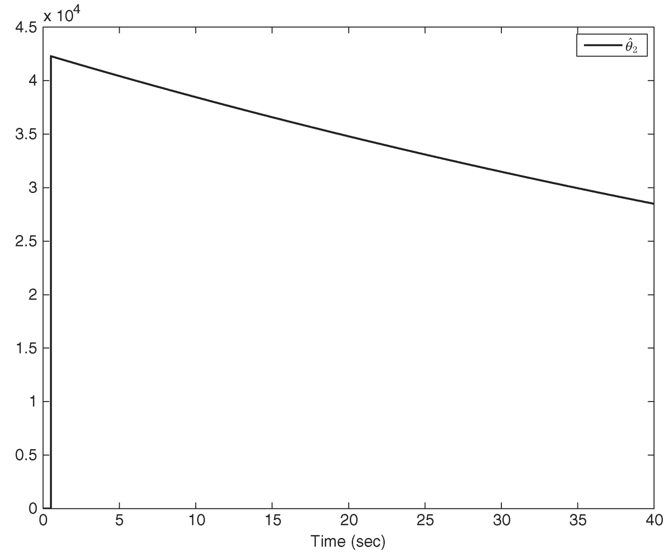


Fig. 8. Adaptive parameter $\hat{\theta}_2$.

## V. CONCLUSION

In this paper, a decentralized adaptive NN output-feedback control design method has been proposed for a class of large-scale stochastic nonlinear systems. Direct adaptive NN control method has been used to approximate the unknown nonlinear functions, in which the number of on-line adaptive parameters is only one; therefore, the computation burden can be significantly reduced. In addition, a state observer has been designed to estimate the unmeasured states. In order to overcome the problem of explosion of complexity, a dynamic surface control method has been applied in the large-scale stochastic nonlinear systems. It is shown that all the signals in the closed-loop system are semiglobally uniformly ultimately bounded. Finally, a numerical example has been given to illustrate the effectiveness of the proposed design technique.

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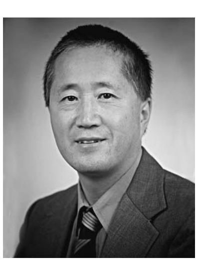

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