Inter-Cluster Multi-hop Routing in Wireless Sensor Networks employing Compressive Sensing

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Abstract—Compressive Sensing (CS) represents a new paradigm that addresses the problem of power consumption for collecting data over wireless sensor networks (WSN). Inter-cluster multi-hop routing, referred to as ICCS, is proposed as an extension to clustering in WSN utilizing CS to further reduce power consumption. With ICCS, CS measurements are relayed from each cluster head (CH) to the base station rather than being transmitted directly. A greedy algorithm is proposed to form a routing tree between the CHs and the base station. Total power consumption for networks supporting intra-cluster and inter-cluster transmission is formulated and compared to cluster based compressive sensing. Network characteristics are analyzed and optimal cases for least power consumption with ICCS are identified.

I. INTRODUCTION

Saving power consumption in wireless sensor networks (WSN) is always a critical problem that is highly related to network lifetime. The networks are envisioned as large ad-hoc collection of very small autonomous devices that can sense environmental conditions in their immediate surroundings while having limited processing, communication capacities and energy reserve. Since compressive sensing (CS) provides a new paradigm for collecting data in WSNs, the base station (BS) only needs $M$ CS measurements to recover all $N$ sensor readings, precisely ($M \ll N$). Then, many data collection methods are exploited in WSNs applying CS.

There are some CS-based data collection methods for WSNs tried to reduce transmission power. CCS paper [1] is the latest method that investigates the combination between clustering algorithms and CS. The idea is to partition a WSN into clusters, in which each cluster head (CH) collects the sensor readings within its cluster and generates CS measurements to be forwarded directly to the BS. This process creates block diagonal projection matrices that can be exploited to save more consumed power for the networks. The matrices are also well studied to satisfy CS recovery [2].

In order to save more energy, in this paper, we consider CCS as a baseline and propose inter-cluster multi-hop routing to forward CS measurements to the BS through CHs. The measurements are relayed through intermediate CHs based on a routing tree formed by our proposed greedy algorithm. We formulate the problem with all consumed power components for data transmission. Based on both analysis and simulation results, we analyze and suggest the optimal case for such network in collecting data.

The paper is organized as follows: We review CS theory and the CCS algorithm as the background and related work on section II. The problems are analyzed and formulated in section III and the simulations are presented in section IV. Finally, conclusion and suggestions for future work are provided.

II. BACKGROUND AND RELATED WORK

A. Compressive Sensing Basics

1) Sparse presentation of signals: Compressive sensing [3], [4], [5] provides a novel sensing/sampling paradigm for signals sparsely presented in a proper basis. A signal $\hat{x} = [x_1 x_2 \ldots x_N]^T \in \mathbb{R}^N$ is defined to be $k$-sparse if it has a sparse representation in basis $\psi = [\psi_{i,j}] \in \mathbb{R}^{N \times N}$, where $\hat{x} = \psi \hat{\theta}$ and $\hat{\theta}$ has only $k$ non-zero elements.

2) Signal sampling: Based on the CS paradigm, a $k$-sparse signal can be under-sampled and be recovered from only $M \ll N$ random measurements $\hat{y} = [y_1 y_2 \ldots y_M]^T \in \mathbb{R}^M$. These CS measurements are created by $\hat{y} = \phi \hat{x}$, where $\phi = [\phi_{i,j}] \in \mathbb{R}^{M \times N}$ is called the measurement matrix and is often a dense Gaussian matrix or a sparse binary matrix [6].

3) Signal recovery: we can recover $\hat{x}$ from $\hat{y}$ by solving the convex optimization problem based on a certain number of measurements $M = \mathcal{O}(k \log N/k)$ as follows

$$\hat{\theta} = \arg \min \| \theta \|_1, \text{ subject to } \hat{y} = \phi \hat{\theta}, \tag{1}$$

where $\| \theta \|_1 = \sum_{i=1}^{N} |\theta_i|$ and $\hat{x} = \psi \hat{\theta}$. The $l_1$ optimization problem can be solved with linear programming techniques such as Basis Pursuit (BP) [3].

B. CCS: Clustered-Based Compressive Sensing for Data Collection in WSNs

CCS [1] is divided into two parts. The first is the underlying clustering that can be based on different methods. We have chosen K-means [7] and LEACH [8] as two clustering methods for our simulations. The second is the CS-based data collection that is based on the following steps:

1) Non-CH sensors send their data once to the CH.
2) The $i^{th}$ CH ($i = 1, 2, \ldots, N_c$) generates an $M_i \times N_i$ block of Gaussian coefficients ($\phi_i$). The CH then generates $M_i$ CS measurements using $\hat{y}_i = \phi_i \mathbf{z}_i$.

3) Each CH sends the measurements and the seed that it used for generating the random matrix $\phi_i$ to the BS.

4) The BS implements a CS reconstruction algorithm to find sensor readings $\hat{\mathbf{z}}_i$ given the block diagonal matrix $\phi$ with blocks $\phi_i$ and $\mathbf{y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{N_c}]^T$.

In a real WSN each cluster may have different number of sensors and accordingly different numbers of measurements are required from each CH. The number of measurements required $M_i$ collecting from cluster $i^{th}$ should be linearly proportional to the number of sensors $N_i$ in that cluster for the lowest reconstruction error at the BS.

C. Related work

Clustering is proven to be an effective way to save energy consumption and upgrade the network lifetime. Many different clustering algorithms have been studied so far [8], [9], [10], [11], [12]. Each cluster has a cluster head (CH) and CHs can be pre-determined [9] or be selected while doing clustering as in the following algorithms. K-means [7] is a very well-known and simple clustering algorithm that chooses CHs for K clusters at the central point of each cluster. This helps to minimize the intra-cluster power consumption. In general, CHs drain power much more than other sensors as they transmit entire cluster’s data to the BS. In LEACH [8], sensor nodes randomly elect themselves to be CHs. This way, the high-energy dissipation in communicating with the BS will spread among the nodes in the network.

Applying CS in collecting data is also an effective way to reduce the number of required samples from a sparse signal. Due to the correlation between the sensor readings in WSNs, the monitored signal can have a sparse representation in a proper domain such as DCT or wavelet. Accordingly, CS have found applications in data collection in WSNs. In [13], [14], [15] CS is applied for tree-based multi-hop routing. In [16], [17], [18] CS based random walk routing reduce significant consumed power. Neighborhood based applying CS is mentioned in [19]. The approach in [20] is to optimize the transportation cost for multi-hop WSNs using CS. Despite many studies on application of CS for WSNs, none of them have investigated the application CS for clustered WSNs.

Papers [1], [21] investigate the integration of the CS and clustering in WSNs. In CCS sensors are sampled at each cluster only once. A certain number of CS measurements are generated at each cluster then transmitted directly to the BS. For further energy saving, we propose a multi-hop inter-cluster routing to relay CS measurements through CHs to the BS, called ICCS. The consumed power is reduced significantly based on shorter transmitting distances.

III. PROBLEM FORMULATION

A. Network model

To simplify the problem, in this model, we assume that the sensing area has a circular shape in which the BS is at the center. We also assume that CHs transmit data based on their transmission range, not the real distances between them. So with intra-cluster transmission, sensors still can adjust their power level to transmit data to the CHs, while all CHs only use transmission range, called $R$ to connect to other CHs and the BS. We deploy $N$ sensors uniformly distributed in the circular area with radius $R_0$, and also, the pass-loss exponent is assumed to be equal to 2 ($\alpha = 2$) for simplicity.

We assume the network is divided into $N_c$ non-overlapping clusters. In our simulations, we considered two well-known clustering mechanisms, K-means [7] and LEACH [8]. In our analysis part, we assume a random clustering similar to LEACH, in which first $N_c$ out of $N$ nodes in the network are selected uniformly at random as CHs and the other nodes find the closest CH to connect to. If the clustering is uniform, then the number of sensors in each cluster is about $N/N_c$ for large values of $N$.

For the reconstruction error related to CS signal recovery we considered the normalized reconstruction error $\frac{||\mathbf{e}||_2}{||\mathbf{y}||_2}$.

![Fig. 1. Transmissions in the network with inter-cluster multihop routing when the BS at the center](image)

As shown in Figure 1, CHs receive readings from their cluster members as non-CH sensors, then they generate and transmit measurements through other CHs or directly to the BS at the center depending on their positions.

B. Inter-cluster multi-hop routing in CCS (ICCS)

For networks have small number of clusters, inter-cluster multi-hop routing may not help because the routing paths may travel around that might cost more power than transmitting directly. But with a large number of CHs, it significantly helps reducing energy to transmit the measurements to the BS. So, we develop inter-cluster multi-hop routing for CCS for the purpose of energy saving as follows:

**Inter-cluster multi-hop routing in CCS (ICCS):** Since we already have clusters formed by K-means or LEACH, we develop a greedy distributed algorithm (GDA) to form inter-cluster routing: We assume all CHs have the same transmission range ($R$) that helps CHs communicate to each other within
range $R$. An appropriate $R$ should be chosen depending on the number of CHs formed to ensure that all CHs connected as an undirected geometry graph. Based on the graph, we can deploy GDA to form the routing paths for the CHs: All CHs broadcast their information about number of hops away from the BS to their neighbors. At the first iteration, only CHs close to the BS (their $Rs$ cover the BS) have the number of hops (NoH). They name their NoH as "1" and broadcast to their neighbors in the next iterations. After a few iterations, the routing paths may be formed but not completely done because a CH only choose one of its neighbors having NoH while the rest may not have one after a few iterations. So the algorithm keeps choosing the routing paths until there is no change between all CHs. This algorithm can be written shortly as below.

All cluster heads (CHs) connected as a graph with the same transmission range $R$.

1. While (the routing paths is changing)
2. NoH($BS$) = 0; $i \in N_c$ CHs
3. Nei = set of $i$’s neighbors
4. if distance[$i$, $j$] < $R$, where $j \in$ Nei
5. CH($i$) chooses CH($j$) when NoH($j$) = min{NoH(Nei)}
6. Name NoH($i$) = NoH($j$) + 1
7. end if
8. end while (Until no change of routing paths between CHs)

The algorithm is simple and distributed since CHs do not need global information from the network. We are going to formulate the problem in the next section.

C. Power consumption analysis

As stated in the previous section, non-CH sensors send their readings to their own CHs only once. We refer to the communication cost associated with the communication between the non-CH nodes to CHs as the intra-cluster power consumption and is denoted as $P_{intra-cluster}$. Next, the CHs create the CS measurements as the combinations of all reading data within each cluster ($y_i = \phi(x)$) and send the measurements to the BS. The corresponding power consumption is referred to as $P_{toBS}$. The total power consumption is formulated as

$$P_{total} = (P_{intra-cluster} + P_{toBS}).$$

1) Analysis of $P_{intra-cluster}$: We assume to have a uniformly distributed WSN divided into $N_c$ clusters with the same number of sensors as $N/N_c$, consisting of one CH and $(N/N_c - 1)$ non-CH nodes. We have

$$P_{intra-cluster} = N_c \left( \frac{N}{N_c} - 1 \right) E[r^2],$$

where $r$ is a random variable representing the distance of a non-CH sensor to its corresponding CH. We can calculate $E[r^2]$ as following

$$E[r^2] = \int \int r^2 \rho(r', \theta) r' \, dr' \, d\theta.$$

in which $\rho(r', \theta)$ is the node distribution. To make the analysis tractable, similar to [22], we assume each cluster area is a circle with radius $R = R_0/\sqrt{N_c}$ and the density of the nodes is uniform throughout the cluster area, i.e. $\rho(r', \theta) = 1/(\pi R_0^2/\sqrt{N_c})$. Hence

$$E[r^2] = \frac{1}{(\pi R_0^2/\sqrt{N_c})} \int_{\theta=0}^{2\pi} \int_{r'=0}^{R} r'^3 \, dr' \, d\theta = \frac{R_0^2}{2N_c},$$

and accordingly,

$$P_{intra-cluster} = \left( \frac{N}{N_c} - 1 \right) \frac{R_0^2}{2}.$$

As we see, the total intra-cluster power consumption is a decreasing function of the number of clusters.

2) Analysis of $P_{toBS}$: We need to formulate this inter-cluster transmission consumed power as follows

$$P_{toBS} = \sum_{i=1}^{N_c} NoH(i) \times R^2 \times M(i),$$

where $M(i)$ is the number of measurements required taken from $i$th cluster, and $R^2$ is the power consumption spent on each hop when we consider the path-loss exponent as $\alpha = 2$. In analysis case, we assume to have all equal size clusters. It means that all clusters have the same number of sensor nodes. According to [1], the number of measurements required taken from each cluster should be linearly proportional to the number of sensors in each cluster or $M(i) = \frac{M}{N_c}$. So, Equation (7) can be written as

$$P_{toBS} = R^2 \times \frac{M}{N_c} \sum_{i=1}^{N_c} NoH(i),$$

where, $M$ is the total number of measurements required collected from the network to satisfy an error-target. $N_c$ is the number of clusters. In [23], Chandler calculated the average number of relay hops in randomly located radio network. Based on the idea, Equation (8) can be written as

$$P_{toBS} = NoH_{ave} \times R^2 \times M,$$

where $NoH_{ave}$ is the average number of hops mentioned in [23] as $E[n]$. This expectation of the number of hops is calculated based on the probability of being able to make a connection between random nodes. These nodes have a same transmission range. If an area covered by a CH’s transmission range does not include its destination, there must be at least one CH exist in the area called A to relay data.

The number of CHs exist in the area $A$ follows Poisson distribution with the mean value $\lambda = \frac{N_c}{\pi R_0^2} \times A$. The probability of being able to make a connection between a source node and a destination node is

$$P(#ofCHs \geq 1) = 1 - P(#ofCHs = 0)$$

$$= 1 - e^{-\frac{N_c}{\pi R_0^2} \times A},$$

where $A = 2R(2\theta - \sin\theta\cos\theta)$ and $\theta = \cos^{-1}(x/2R)$.

Since CHs are randomly distributed and chosen in the sensing area, the distance between a CH and the BS is a random variable, shown as $x$. The probability of being able to make a connection at distance $x$ using $n$ or less hops is
can be calculated following the idea in [17].

Choosing an appropriate transmission range which results the smallest power consumption for the networks should be considered. For example, if we increase $R$, the number of hops is directly related to the transmission range $R$. Hence, to forward CS measurements, it may effect power consumption. For instance, if we increase $R$, the number of hops in each routing path might be reduced, but we have to deal with longer hop distances that definitely consume more energy.

In Figures 2 and 3, we have a 2000 sensor network formed in 500 clusters by K-means and LEACH. We choose different transmission ranges $R = \{10, 12, 14, 16, 18, 20\}$. Figure 2 shows the total number of hops reduced corresponding to the radius increased. In Figure 3, the total consumed power keeps increasing as we increase $R$. This is obviously explained in Equation (9).

Based on the Figures 2, 3 we should choose a possible smallest $R$ that results the least consumed power for the network.

* Additional Analysis for CCS in order to compare with ICCS: In CCS paper [1], the network is deployed in a square area that the mean square distance from CHs to the BS is calculated differently from this case. To compare fairly with ICCS, we provide additional analysis for CCS in this circular area network with the BS at the center. The mean value of consumed power to transmit data from any random CH to the BS $E[d_{toBS}^2]$ can be calculated following the idea in [17].

As shown in Figure 4, sensors are uniformly randomly distributed and the CHs are also chosen randomly. $d_{toBS}$ can be considered as a random variable. The maximum distance is the radius of the circular area $R_0$. $E[d_{toBS}^2]$ can be calculated in polar coordinates as follows

$$E[d_{toBS}^2] = \int_0^{2\pi} \int_0^{R_0} \rho(d_{toBS}, \theta) d_{toBS}^2 d\theta$$

where $\rho(d_{toBS}, \theta) = 1/(\pi R_0^2)$ is the joint probability function (pdf). Finally, we obtain

$$E[d_{toBS}^2] = \frac{1}{\pi R_0^2} \int_0^{2\pi} \int_{d_{toBS}=0}^{R_0} d_{toBS}^2 d\theta$$

$$= \frac{R_0^2}{2}.$$ 

**IV. SIMULATION RESULTS**

In this section, we work on 2000 sensors randomly deployed in a circular area with the radius of $R_0 = 50$. The BS is set at the center of the sensing area. We first work on real signals collected from [24] and then 50-sparse random signals. The sparsifying matrix $\psi$ is chosen as DCT. As addressed in [1] the total number of measurements required with different number of clusters in this case is a constant as $M = 500$ to satisfy the error-target of 0.1. With k-sparse signals, it increases linearly as we increase the number of clusters. The numbers of measurements collected from each cluster should be proportional to the size of the clusters as mentioned in [1]. We only consider the maximum number

Fig. 2. Total number of hops routing when changing the transmission range $R$.

Fig. 3. The total power consumption when change the transmission range $R$.

Fig. 4. Distances from CHs to the BS in a circular area arbitrary network.
of clusters up to \( N_c = 500 \). At each network divided into different numbers of clusters, we use different transmission ranges \( R = [50 \ 30 \ 25 \ 22 \ 18 \ 14 \ 11] \) corresponding to \( N_c = [10 \ 50 \ 100 \ 200 \ 300 \ 400 \ 500]. \)

We apply K-means and LEACH clustering algorithms to have two different clustered networks to compare with the uniform clustering mentioned in our analysis case. Figure 5 shows the total intra-cluster power consumption that calculate the consumed power to transmit data from non-CH sensors to their CHs. As shown in Figure 5, the consumed power becomes very small if the network is divided into that many clusters. Then, the total power consumption of the network mainly focuses on the inter-cluster routing paths.

Figure 6 shows the number of required measurements to reach a target reconstruction error is almost constant versus different values of \( N_c \). We call this constant value as \( M_0 \) and choose \( M_0 = 500 \) for next calculations.

The total inter-cluster routing power consumption shown in Figure 7 reduces as we increase the number of clusters for the networks.

Through Figures 5 and 7, the total power consumption is reduced by both intra-cluster and inter-cluster transmissions as we increase the number of clusters. In Figure 8, ICCS obviously outperforms CCS when the network is arranged into very huge number of clusters (\( N_c \geq 100 \)).

Working with 50-sparse signals provides us the number of measurements required increasing linearly as we increase the number of clusters as shown in Figure 9. As the number of clusters increased, we need to increase the number of CS measurements to satisfy an error-target that might increase the total power consumption for ICCS. As shown in Figure 10, the total power consumption provides us two options for the network to consume the least power as \( N_c = 50 \) or \( N_c = 500 \).

**V. Conclusion**

In this paper we proposed a method called multi-hop inter-cluster routing for cluster-based data collection in WSNs utilizing CS (ICCS). The idea is to relay CS measurements through short distances from CHs to the BS that significantly saves energy. We further propose an iterative greedy algorithm to form routing paths for CHs transmit data to the BS. All power consumptions are analyzed, formulated and simulated. Both real and sparse signals are applied. With the real data
working with DCT as the sparsifying matrix, since the number of measurements required stays, the total power consumption is much reduced as far as we increase the number of clusters. It is different with k-sparse signals in canonical basis since the number of measurement required increase linearly as the number of clusters increased. It gives us some specific cases for the networks to consume the least power.

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