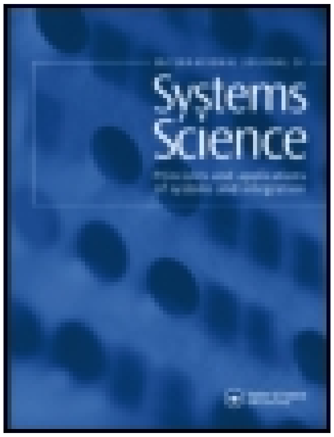

International Journal of Systems Science

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tsys20

# Fault-tolerant control for a class of non-linear systems with dead-zone

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Published online: 08 Aug 2014.

*International Journal of Systems Science*, 2014
http://dx.doi.org/10.1080/00207721.2014.945984



# Fault-tolerant control for a class of non-linear systems with dead-zone

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(*Received 14 November 2013; accepted 25 April 2014*)

In this paper, a fault-tolerant control scheme is proposed for a class of single-input and single-output non-linear systems with the unknown time-varying system fault and the dead-zone. The non-linear state observer is designed for the non-linear system using differential mean value theorem, and the non-linear fault estimator that estimates the unknown time-varying system fault is developed. On the basis of the designed fault estimator, the observer-based fault-tolerant tracking control is then developed using the backstepping technique for non-linear systems with the dead-zone. The stability of the whole closed-loop system is rigorously proved via Lyapunov analysis and the satisfactory tracking control performance is guaranteed in the presence of the unknown time-varying system fault and the dead-zone. Numerical simulation results are presented to illustrate the effectiveness of the proposed backstepping fault-tolerant control scheme for non-linear systems.

**Keywords:** non-linear systems; fault estimator; state observer; fault-tolerant control; tracking control



## 1. Introduction

With the development of the complex system, a key challenge is how to achieve the satisfactory control performance in the presence of the time-varying system fault (Zhang, Polycarpoub, & Parisini, 2010). As is well known, faults may lead to control system performance degradation and cause instability and even disastrous accidents (Alwi & Edwards, 2008a; Gao, Jiang, Shi, Liu, & Xu, 2012; Xu, Jiang, & Tao, 2011; Ye & Yang, 2006). Specially, to the safety critical systems, such as passenger aircraft and modern fighter aircraft, the control system must maintain stability and acceptable performance during faults including actuator fault, sensor fault, and so on. Thus, fault-tolerant control has become an important research topic in the control area (Alwi & Edwards, 2008b; Gao, Jiang, Qi, & Xu, 2011). In Li and Yang (2012), robust adaptive fault-tolerant control was developed for uncertain linear systems with actuator failures. Adaptive fuzzy fault-tolerant output-feedback control was proposed for uncertain non-linear systems with actuator faults in Huo, Tong, and Li (2013) and Tong, Hou, and Li (2014). In Wang, Xie, and Zhang (2012), sliding mode fault-tolerant control was studied to deal with modelling uncertainties and actuator faults. In practice, many control systems have non-linear characteristic. Thus, the tolerant control scheme should be further investigated for the non-linear system with the unknown time-varying system fault.

For the fault-tolerant control design of the non-linear system, the significant research activity is the fault-estimator design (Panagi & Polycarpou, 2011). To design the fault estimator, first the non-linear state observer should be developed (Wang, Zhou, & Gao, 2007). However, many research results of the state observer are effective for linear systems (Darouach, Zasadzinski, & Xu, 1994; Jo & Seo, 2000). For non-linear systems, the state observer design needs to be further investigated (Li, Cao, & Ding, 2011; Liu, Tong, & Li, 2011). In Ekramian, Sheikholeslam, Hosseinnia, and Yazdanpanah (2013), adaptive state observer was developed for Lipschitz non-linear systems. State observer-based fuzzy adaptive output tracking control was proposed for non-linear systems in Tong, Wang, and Tang (2000). In Dong and Mei (2011), state observer was presented for a class of multi-output non-linear dynamic systems. Recently, a new state observer design was developed for a class of systems with monotonic non-linearities (Zemouche, Boutayeb, & Bara, 2005). In this new design, the global Lipschitz restriction is removed. Moreover, the basic idea of observer design is to use the well-known differential mean value theorem (DMVT) (Zemouche, Boutayeb, & Bara, 2006). In Zemouche, Boutayeb, and Bara (2008), the state observers were proposed for a class of Lipschitz systems with extension to $H_\infty$ performance analysis using the DMVT technique. Furthermore, the fault estimator needs to be developed for the non-linear system based on the designed non-linear state observer for the appeared unknown time-varying system fault. Based on the output of the fault estimator, the observer-based fault-tolerant tracking control should be designed for the non-linear system to maintain the tracking control performance

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under the influence of time-varying unknown fault. In Huo, Li, and Tong (2012) and Tong, Hou, and Li (2014), observer-based fuzzy adaptive fault-tolerant output-feedback control schemes were proposed for single-input and single-output (SISO) and multi-input and multi-output (MIMO) non-linear systems in the strict-feedback form. Finite-time fault-tolerant control was developed for rigid spacecrafts with actuator saturations in Lu and Xia (2013). Considering the state observer and the fault estimator, the fault-tolerant control design will become more complicated for non-linear systems.

The coupled interaction between the fault estimator and the fault-tolerant controller in a closed-loop configuration is very complex, especially for non-linear systems. Backstepping control as an efficient control method of non-linear systems can provide a systematic framework for the tracking control, which is suitable for a class of special non-linear systems (Li, Qiang, Zhuang, & Kaynak, 2004). Due to the design flexibility, the robust adaptive backstepping control has been extensively studied in the non-linear control system design (Chen, Jiang, & Wu, 2008; Chen, Jiao, Li, & Li, 2010; Tong, Li, & Shi, 2012). Adaptive fuzzy output-feedback control was proposed for a class of MIMO non-linear uncertain systems with time-varying delays and unknown backlash-like hysteresis using the backstepping method in Li, Tong, and Li (2012a). In Tong, Li, Li, and Liu (2011), observer-based adaptive fuzzy backstepping control was developed for a class of stochastic non-linear strict-feedback systems. At the same time, some backstepping fault-tolerant control results have been studied for the non-linear system. In Jiang, Hu, and Ma (2010), adaptive backstepping fault-tolerant control was proposed for flexible spacecrafts with unknown bounded disturbances and actuator failures. Adaptive neural network output-feedback control was proposed for stochastic non-linear systems with unknown dead-zone and unmodelled dynamics in Tong, Wang, Li, and Zhang (2014). However, the actuator cannot promptly respond to the command signal resulting in an actuator dead-zone for a practical system. Thus, the backstepping fault-tolerant control scheme should be further developed for the non-linear system with the unknown time-varying system fault and the dead-zone.

Dead-zone as one of the non-smooth non-linearities can severely degrade system control performance, if it is ignored in the tolerant control design. Thus, the degradation of fault-tolerant control performance caused by the dead-zone of actuators needs to be considered in the control design stage (Corradini & Orlando, 2002; Luo, Wu, & Guan, 2010). To handle the actuator dead-zone problem, various control schemes have been proposed in Corradini and Orlando (2003), Zhou, Wen, and Zhang (2006), Li and Yang (2009), and Tong and Li (2013). Adaptive output control was proposed for uncertain non-linear systems with non-symmetric dead-zone input in Ma and Yang (2010). In Zhang and Ge (2008), adaptive dynamic surface control was proposed to handle a class of pure-feedback non-linear systems with unknown dead-zone and perturbed uncertainties. Robust adaptive control was developed for a class of non-linear systems with unknown dead-zone in Wang, Su, and Hong (2004). In Tong and Li (2012) and Tong, Wang, and Li (2014), observer-based adaptive fuzzy output-feedback tracking backstepping control approaches were first proposed for strict-feedback SISO/MIMO non-linear systems with the unknown dead-zones, and the stability of the control systems were proved. The presence of dead-zone and unknown time-varying system fault will further enhance the design difficulty of the fault-tolerant tracking control scheme for non-linear systems, which should be further derived.

This work is motivated by the design of fault-tolerant control scheme to follow the desired trajectories of non-linear systems with the unknown time-varying system fault and the dead-zone. The organisation of the paper is as follows. The problem statement is detailed in Section 2. The fault-tolerant control of the non-linear system is proposed in Section 3 including the state observer design and the fault estimator design. Simulation results of an example are presented in Section 4 to demonstrate the effectiveness of the developed fault-tolerant control of the non-linear systems, followed by some concluding remarks in Section 5.

## 2. Problem statement

To develop the fault-tolerant control, let us consider the following non-linear systems in the form of:

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x) + gu + \eta(t) \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T = [x_1, \dot{x}_1, \dots, x_1^{(n-1)}] \in R^n$ is the state vector, and $u \in R$ is the control input of the non-linear system. $f(x) \in R$ is a known continuous function, and $g \in R$ is a known control gain constant. In order to be controllable for (1), it is required that $g \neq 0$. $\eta(t) \in R$ denotes the (additive) unknown system time-varying fault.

In accordance with (1), the non-linear system with the unknown time-varying system fault can be equivalently written as

$$\begin{cases} \dot{x} = Ax + Ly + B(f(x) + gu + \eta(t)) \\ y = C^T x \end{cases}, \tag{2}$$

where $A = \begin{bmatrix} -l_1 & 1 & 0 & \dots & 0 \\ -l_2 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & \ddots & \ddots\ 0 \\ -l_{n-1} & 0 & 0 & \dots & 1 \\ -l_n & 0 & 0 & \dots & 0 \end{bmatrix}$, $L = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{n-1} \\ l_n \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

and $C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$. $l_i$, $i = 1, 2, \ldots, n$ are the designed constants. In the backstepping fault-tolerant control design, $L$ should be properly chosen, such that $A$ is a strict Hurwitz matrix.

In practice, the actuator usually exists the dead-zone non-linearity. Thus, the control input $u$ can be expressed as

$$u(t) = D(v(t)), \tag{3}$$

where $v$ is the input of the dead-zone and $D(.)$ denotes a dead-zone operator.

In this paper, the considered dead-zone is described as (Wang et al., 2004)

$$u(t) = D(v(t)) = \begin{cases} m(v(t) - b_r), & \text{for } v(t) \geq b_r \\ 0, & \text{for } b_l < v(t) < b_r \\ m(v(t) - b_l), & \text{for } v(t) \leq b_l, \end{cases} \tag{4}$$

where $m > 0$, $b_r > 0$, and $b_l < 0$ are the known dead-zone parameters.

To develop the fault-tolerant control scheme, the dead-zone model (4) is rewritten as (Wang et al., 2004)

$$D(v(t)) = mv(t) + d(v(t)), \tag{5}$$

where $d(v(t))$ is given by

$$d(v(t)) = \begin{cases} -mb_r, & \text{for } v(t) \geq b_r \\ -mv(t), & \text{for } b_l < v(t) < b_r \\ -mb_l, & \text{for } v(t) \leq b_l. \end{cases} \tag{6}$$

For a practical system, we know that the parameter $m$ of the dead-zone is bounded. Thus, from (6), we have

$$|d(v(t))| \leq d_m, \tag{7}$$

where $d_m = \max\{m_{\max} b_{r\max}, -m_{\max} b_{l\min}\}$, $m_{\max}$ is the upper bound of the parameter $m$, $b_{r\max}$ is the upper bound of the parameter $b_r$ and $b_{l\min}$ is the lower bound of the parameter $b_l$.

Invoking (2) and (4) yields

$$\begin{cases} \dot{x} = Ax + Ly + B(f(x) + gD(v(t)) + \eta(t)) \\ y = C^T x \end{cases}. \tag{8}$$

In this paper, the state observer and the fault estimator will be proposed for the non-linear system (1) with the unknown time-varying system fault and the dead-zone. Then, the backstepping fault-tolerant control design is presented for the studied non-linear system. The control objective is that the backstepping fault-tolerant control scheme can render the system output to follow a given desired output of the non-linear system in the presence of the unknown time-varying system fault. For the desired system output $y_d$, the proposed backstepping fault-tolerant control must ensure that all closed-loop system signals are convergent. To facilitate and proceed the backstepping fault-tolerant control design for the non-linear system (1), the following lemmas and assumptions are required:

**Lemma 1** (Ge & Wang, 2004)**:** *For bounded initial conditions, if there exists a $C^1$ continuous and positive definite Lyapunov function $v(x)$ satisfying $\pi_1(\|x\|) \leq V(x) \leq \pi_2(\|x\|)$, such that $\dot{V}(x) \leq -c_1 V(x) + c_2$, where $\pi_1$, $\pi_2$: $R^n \to R$ are the class K functions, and $c_1$, $c_2$ are the positive constants, then the solution $x(t)$ is uniformly bounded.*

**Lemma 2** (Zemouche et al., 2005, 2006)**:** *Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^q$ and $a, b \in \mathbb{R}^n$. Assume that $f$ is differentiable on $Co(a, b)$. Then, there exist constant vectors $c_1, \ldots, c_q \in Co(a, b)$, $c_i \neq a$, $c_i \neq b$ for $i = 1, \ldots, q$, such that*

$$f(a) - f(b) = \left( \sum_{i,j=1}^{q,n} e_q(i) e_n^T(j) \frac{\partial f_i}{\partial x_j}(c_i) \right)(a - b), \tag{9}$$

*where $e_q(i) = (0, \ldots, 0, \underbrace{1}_{ith}, 0, \ldots, 0) \in R^q$ and $Co(a, b) = \lambda a + (1 - \lambda)b$, $0 \leq \lambda \leq 1$.*

**Assumption 1** (Ge & Wang, 2004)**:** *There exists two positive constants $\underline{g}$ and $\bar{g}$ to make $\underline{g} \leq |g| \leq \bar{g}$ valid.*

**Assumption 2:** *For the desired system trajectory $y_d$, there exist unknown positive constants $\tau_i$, such that $|y_d^{(i)}| \leq \tau_i$, $i = 1, \ldots, n$.*

**Assumption 3** (Jiang, Wang, & Soh, 2002)**:** *For the unknown time-varying system fault $\eta(t)$, we assume that it satisfies $|\dot{\eta}| \leq \delta$ with $\delta > 0$.*

**Remark 1:** In this paper, the fault-tolerant control scheme will be developed for the non-linear system (1) in the presence of the unknown time-varying fault. Generally speaking, the studied system (1) can denote a large variety of non-linear systems, such as the single-link robot system, the chaotic system, and so on. To the system fault $\eta(t)$, it is important to develop the fault-tolerant control scheme for achieving the satisfactory tracking control performance. Although the states of non-linear system (1) are in a cascade form, the fault-tolerant control analysis of more general non-linear systems can be similarly developed. On the other hand, $\eta(t)$ in non-linear system (1) can denote the actuator/component fault of the system. Since the actuator/component fault $\eta_0$ appears for the subsystem $\dot{x}_n = f(x) + gu$, the subsystem can be rewritten as $\dot{x}_n = f(x) + gu + g\eta_0$. Defining $\eta = g\eta_0$, we have $\dot{x}_n = f(x) + gu + \eta(t)$.

## 3. Backstepping fault tolerant control for non-linear systems

### 3.1. Design of state observer and fault estimator

In this subsection, the state observer and the system fault estimator will be developed for the non-linear system, respectively. Motivated by the state observer in Li, Tong, and Li (2012b), the state observer of the non-linear system (1) is proposed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Ly + B(f(\hat{x}) + gD(v(t)) + \hat{\eta}(t)) \\ \hat{y} = C^T\hat{x}, \end{cases} \tag{10}$$

where $\hat{\eta}(t)$ is the estimate of the unknown system fault $\eta(t)$.

Define $\tilde{x} = x - \hat{x}$ and $\tilde{\eta} = \eta - \hat{\eta}$. Considering (10) and (2) yields

$$\dot{\tilde{x}} = A\tilde{x} + B(f(x) - f(\hat{x}) + \tilde{\eta}). \tag{11}$$

By invoking Lemma 2, there exists $\xi(t) \in Co(x(t), \hat{x}(t))$, such that (Zemouche et al., 2005, 2006),

$$f(x) - f(\hat{x}) = \left( \sum_{i=1}^{n} e_n^T(i) \frac{\partial f}{\partial x_i}(\xi) \right) \tilde{x}. \tag{12}$$

Define $h_i(t) = e_n^T(i)\frac{\partial f}{\partial x_i}(\xi)$ and $h(t) = \sum_{i=1}^n h_i(t)$. Then, (12) can be written as

$$f(x) - f(\hat{x}) = h(t)\tilde{x}. \tag{13}$$

Invoking (13), (11) can be expressed as

$$\dot{\tilde{x}} = \bar{A}\tilde{x} + B\tilde{\eta}, \tag{14}$$

where $\bar{A} = A + Bh(t)$.

For the matrix $\bar{A}$, by choosing a matrix $P = P^T > 0$, there always exists a positive definite matrix $Q = Q^T > 0$, such that the following matrix inequality hold:

$$\bar{A}^T P + P\bar{A} \le -Q. \tag{15}$$

To achieve the estimate $\hat{\eta}(t)$ of the unknown time-varying system fault $\eta$, we define $z = \eta - \gamma x_n$ with $\gamma > 0$. Then, considering (1), we have

$$\begin{aligned} \dot{z} &= \dot{\eta} - \gamma \dot{x}_n = \dot{\eta} - \gamma(f(x) + gD(v(t)) + \eta(t)) \\ &= \dot{\eta} - \gamma(f(x) + gD(v(t)) + z + \gamma x_n). \end{aligned} \tag{16}$$

Furthermore, the estimate of intermediate variable $z$ is designed as

$$\dot{\hat{z}} = -\gamma\hat{z} - \gamma(f(\hat{x}) + gD(v(t)) + \gamma\hat{x}_n). \tag{17}$$

According to $z = \eta - \gamma x_n$, the estimate of the unknown time-varying system fault $\eta$ can be written as

$$\hat{\eta} = \hat{z} + \gamma\hat{x}_n. \tag{18}$$

Defining $\tilde{z} = z - \hat{z}$ and invoking (18) yields

$$\tilde{z} = z - \hat{z} = \eta - \hat{\eta} - \gamma(x_n - \hat{x}_n) = \tilde{\eta} - \gamma\tilde{x}_n. \tag{19}$$

Differentiating (19), and considering (13), (16), and (17) yields

$$\begin{aligned} \dot{\tilde{z}} &= \dot{z} - \dot{\hat{z}} = \dot{\eta} - \gamma\tilde{z} - \gamma(f(x) - f(\hat{x})) - \gamma^2\tilde{x}_n \\ &= \dot{\eta} - \gamma\tilde{z} - \gamma h(t)\tilde{x} - \gamma^2\tilde{x}_n. \end{aligned} \tag{20}$$

Consider the Lyapunov candidate $V_0$ for the estimate errors of the state observer and the system fault estimator as follows:

$$V_0 = \tilde{x}^T P\tilde{x} + \frac{1}{2}\tilde{z}^2. \tag{21}$$

Considering (11) and (20), the time derivative of $V_0$ is given by

$$\begin{aligned} \dot{V}_0 &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{z}\dot{\tilde{z}} \\ &= \tilde{x}^T(\bar{A}^T P + P\bar{A})\tilde{x} + 2\tilde{x}^T PB\tilde{\eta} \\ &\quad + \tilde{z}(\dot{\eta} - \gamma\tilde{z} - \gamma h(t)\tilde{x} - \gamma^2\tilde{x}_n) \\ &\le -\tilde{x}^T Q\tilde{x} - \gamma\tilde{z}^2 + 2\tilde{x}^T PB\tilde{\eta} \\ &\quad + \tilde{z}(\dot{\eta} - \gamma h(t)\tilde{x} - \gamma^2\tilde{x}_n). \end{aligned} \tag{22}$$

Considering the following facts

$$2\tilde{x}^T PB\tilde{\eta} \le \beta_0\|\tilde{x}\|^2 + \tilde{\eta}^2 \tag{23}$$

$$\tilde{\eta}^2 = (\tilde{z} + \gamma\tilde{x}_n)^2 \le 2\tilde{z}^2 + 2\gamma^2\|\tilde{x}\|^2 \tag{24}$$

$$2\tilde{z}\dot{\eta} \le \tilde{z}^2 + \delta^2 \tag{25}$$

$$-2\gamma\tilde{z}h(t)\tilde{x} \le \tilde{z}^2 + \gamma^2 h^2\|\tilde{x}\|^2 \tag{26}$$

$$-2\gamma^2\tilde{z}\tilde{x}_n \le \tilde{z}^2 + \gamma^4\|\tilde{x}\|^2, \tag{27}$$

we have

$$\dot{V}_0 \le -\tilde{x}^T(Q - \beta I_{n\times n})\tilde{x} - (\gamma - 3.5)\tilde{z}^2 + 0.5\delta^2, \tag{28}$$

where $\beta_0 = \|PB\|^2$ and $\beta = \beta_0 + 2\gamma^2 + 0.5\gamma^2 h^2 + 0.5\gamma^4$.

### 3.2. Design of backstepping fault tolerant control

In this subsection, the backstepping fault-tolerant control scheme will be proposed, which is based on the developed the state observer and the system fault estimator using the standard backstepping technique. At the same time, the stability of the whole closed-loop system will be analysed via the Lyapunov method. To explicitly show the backstepping fault-tolerant control design, the detailed design process is described as follows.

**Step 1:** To design the fault-tolerant control strategy, let us define

$$e_1 = \hat{x}_1 - y_d \tag{29}$$

$$e_2 = \hat{x}_2 - \alpha_1, \tag{30}$$

where $\alpha_1$ is a virtual control law. $\hat{x}_1$ and $\hat{x}_2$ are the estimate values of $x_1$ and $x_2$, respectively.

Considering (10), (30), and differentiating $e_1$ with respect to time yields

$$\begin{aligned} \dot{e}_1 &= \hat{x}_2 + l_1(y - \hat{x}_1) - \dot{y}_d \\ &= e_2 + \alpha_1 + l_1(y - \hat{x}_1) - \dot{y}_d. \end{aligned} \tag{31}$$

The virtual control law $\alpha_1$ is designed as

$$\alpha_1 = -k_1 e_1 + \dot{y}_d - l_1(y - \hat{x}_1), \tag{32}$$

where $k_1 > 0$.

Substituting (32) into (31) yields

$$\dot{e}_1 = -k_1 e_1 + e_2. \tag{33}$$

Consider the Lyapunov function candidate as

$$V_1 = \frac{1}{2} e_1^2. \tag{34}$$

Invoking (33), the time derivative of $V_1$ is given by

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1(-k_1 e_1 + e_2) = -k_1 e_1^2 + e_1 e_2. \tag{35}$$

**Step i ($2 \le i \le n-1$):** Define

$$e_i = \hat{x}_i - \alpha_{i-1} \tag{36}$$

$$e_{i+1} = \hat{x}_{i+1} - \alpha_i, \tag{37}$$

where $\alpha_{i-1}$ and $\alpha_i$ are the virtual control laws. $\hat{x}_i$ and $\hat{x}_{i+1}$ are the estimates of $x_i$ and $x_{i+1}$, respectively.

Considering (10), (36), and differentiating $e_i$ with respect to time yields

$$\begin{aligned} \dot{e}_i &= \hat{x}_{i+1} + l_i(y - \hat{x}_1) - \dot{\alpha}_{i-1} \\ &= e_{i+1} + \alpha_i + l_i(y - \hat{x}_1) - \dot{\alpha}_{i-1}. \end{aligned} \tag{38}$$

The virtual control law $\alpha_i$ is designed as

$$\alpha_i = -k_i e_i - e_{i-1} + \dot{\alpha}_{i-1} - l_i(y - \hat{x}_1), \tag{39}$$

where $k_i > 0$.

Substituting (39) into (38), we have

$$\dot{e}_i = -k_i e_i - e_{i-1} + e_{i+1}. \tag{40}$$

Choose the Lyapunov function candidate as

$$V_i = \frac{1}{2} e_i^2. \tag{41}$$

Invoking (40), the time derivative of $V_i$ is given by

$$\begin{aligned} \dot{V}_i &= e_i \dot{e}_i = e_i(-k_i e_i - e_{i-1} + e_{i+1}) \\ &= -k_i e_i^2 - e_{i-1} e_i + e_i e_{i+1}. \end{aligned} \tag{42}$$

**Step *n*:** Define

$$e_n = \hat{x}_n - \alpha_{n-1}, \tag{43}$$

where $\alpha_{n-1}$ is the virtual control law of the $(n-1)$th step and $\hat{x}_n$ is the estimate of $x_n$.

Considering (5), (10), and differentiating $e_n$ with respect to time yields

$$\begin{aligned} \dot{e}_n &= l_n(y - \hat{x}_1) + f(\hat{x}) + gD(v(t)) + \hat{\eta}(t) - \dot{\alpha}_{n-1} \\ &= l_n(y - \hat{x}_1) + f(\hat{x}) + gmv(t) + gd(v(t)) \\ &\quad + \hat{\eta}(t) - \dot{\alpha}_{n-1}. \end{aligned} \tag{44}$$

The system control law $v$ is designed as

$$\begin{aligned} v = \frac{1}{mg}&(-k_n e_n - e_{n-1} - f(\hat{x}) + \dot{\alpha}_{n-1} - l_n(y - \hat{x}_1) \\ &- \hat{\eta} - \bar{g} d_m \operatorname{sign}\{e_n\}), \end{aligned} \tag{45}$$

where $k_n > 0$.

Substituting (45) into (44), we have

$$\dot{e}_n = -k_n e_n - e_{n-1} + gd(v(t)) - \bar{g} d_m \operatorname{sign}\{e_n\}. \tag{46}$$

Consider the Lyapunov function candidate

$$V_n = \frac{1}{2} e_n^2. \tag{47}$$

Invoking (46), the time derivative of $V_n$ is given by

$$\begin{aligned} \dot{V}_n &= e_n \dot{e}_n = e_n(-k_n e_n - e_{n-1} + gd(v(t)) - \bar{g} d_m \operatorname{sign}\{e_n\}) \\ &\le -k_n e_n^2 - e_n e_{n-1} + \bar{g} d_m |e_n| - \bar{g} d_m e_n \operatorname{sign}\{e_n\}) \\ &= -k_n e_n^2 - e_n e_{n-1}. \end{aligned} \tag{48}$$

The above design procedure and analysis of the fault-tolerant control can be summarised in the following theorem, which includes the fault-tolerant control results for the non-linear system (1) with unknown time-varying system fault and dead-zone based on the backstepping technique.

**Theorem 3.1:** *Considering the non-linear system* (1) *with unknown time-varying system fault and the dead-zone, the state observer is proposed as* (10)*, and the fault estimator is designed according to* (17) *and* (18)*. Based on the outputs of the state observer and the fault estimator, the backstepping tolerant control law is proposed as* (45)*. Then, all closed-loop system signals are semi-globally uniformly stable under the proposed backstepping tolerant control scheme.*

**Proof:** To analyse the convergence of all signals in the closed-loop system, the Lyapunov function candidate of the whole closed-loop fault-tolerant control system is chosen as

$$V = \sum_{i=0}^{n} V_i. \tag{49}$$

Differentiating $V$ and considering (28), (35), (42), and (48), we have

$$\begin{aligned}\dot{V} &\leq \leq -\tilde{x}^T(Q-\beta I_{n\times n})\tilde{x} - (\gamma-3.5)\tilde{z}^2 \\ &\quad - \sum_{i=1}^{n} k_i e_i^2 + 0.5\delta^2, \\ &\leq -\kappa V + C,\end{aligned} \tag{50}$$

where $\kappa$ and $C$ are given by

$$\begin{aligned}\kappa &:= \min\left(\tfrac{\lambda_{\min}(Q-\beta I_{n\times n})}{\lambda_{\max}(P)}, 2(\gamma-3.5), 2k_i\right) \\ C &:= 0.5\delta^2.\end{aligned} \tag{51}$$

To ensure the closed-loop stability, all design parameters $l_i$, $P$, $Q$, $\gamma$, and $k_i$ should be chosen, such that $Q - \beta I_{n\times n} > 0$ and $\gamma - 3.5 > 0$. From (50) and Lemma 1, we can know that the closed-loop system signals $e_i$, $\tilde{x}_i$, and $\tilde{\eta}$ are bounded. Since $e_1$ and $\tilde{\eta}$ are bounded, the tracking error and the estimate error of the unknown system fault are bounded. This concludes the proof. □

**Remark 2:** In the developed backstepping tolerant control law, all parameters of the dead-zone for the studied non-linear system is known. For a practical system, the dead-zone can be known when the system actuator is determined. It is our future work to study the backstepping tolerant control scheme for the non-linear system with unknown time-varying system fault and the unknown dead-zone.

**Remark 3:** In the $i$th step, the virtual control law $\alpha_i$ includes $\dot{\alpha}_{i-1}$. However, $\dot{\alpha}_{i-1}$ cannot be available due to the existing $\dot{\alpha}_{i-2}$ and $x_2$. However, $\alpha_{i-1}$ is known. Thus, the first-order sliding mode differentiator can be employed to approximate $\dot{\alpha}_{i-1}$. To produce the corresponding derivatives, the general form of the $n$th-order higher order sliding mode differentiator (HOSMD) is designed as (Levant, 2003)

$$\begin{aligned}\dot{z}_0 &= \zeta_0 = -\epsilon_0|z_0 - p(t)|^{n/(n+1)}\operatorname{sign}(z_0 - p(t)) + z_1 \\ &\vdots \\ \dot{z}_i &= \zeta_i = -\epsilon_i|z_i - \zeta_{i-1}|^{(n-i)/(n-i+1)}\operatorname{sign}(z_i - \zeta_{i-1}) + z_{i+1} \\ &\vdots \\ \dot{z}_{n-1} &= \zeta_{n-1} = -\epsilon_{n-1}|z_{n-1} - \zeta_{n-2}|^{1/2}\operatorname{sign}(z_{n-1} - \zeta_{n-2}) + z_n \\ \dot{z}_n &= -\epsilon_n \operatorname{sign}(z_n - \zeta_{n-1}),\end{aligned} \tag{52}$$

where $z_i$ and $\zeta_i$ are the states of the system (52), $\epsilon_0, \epsilon_1, \ldots, \epsilon_n$ are designed parameters, $p(t)$ is the known function. The aim of HOSMD is to make $\zeta_j$ approximate the differential term $p(t)^{(j+1)}$ to arbitrary accuracy, $i = 0, 1, \ldots, n$, $j = 0, 1, \ldots, n-1$. To obtain the estimate of differential term $\dot{\alpha}_{i-1}$, the order of HOSMD should be only 1 and $p(t) = \alpha_{i-1}$.

## 4. Simulation study

In this section, simulation results of an example are given to illustrate the effectiveness of the proposed backstepping fault-tolerant control scheme for a non-linear system with unknown time-varying system fault and the dead-zone. In the simulation study, let us consider the single-link robot system which is shown in Figure 1. Its motion dynamics can be described as (Wang, Chai, & Zhangh, 2010)

$$\begin{aligned}M\ddot{q} + 0.5m_0 gl\sin(q) &= \tau + \eta_0(t) \\ y &= q,\end{aligned} \tag{53}$$

where $g = 9.8\,\mathrm{m/s^2}$ is the acceleration due to gravity, $M$ is the inertia, $q$ is the angle position, $\dot{q}$ is the angle velocity, $\ddot{q}$

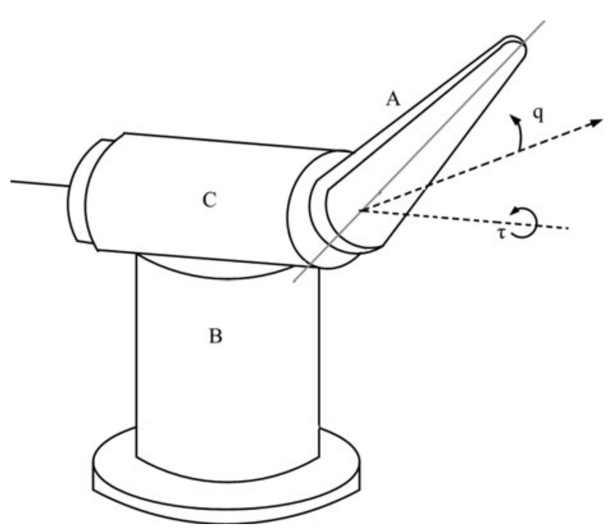

Figure 1. A single-link robot system.

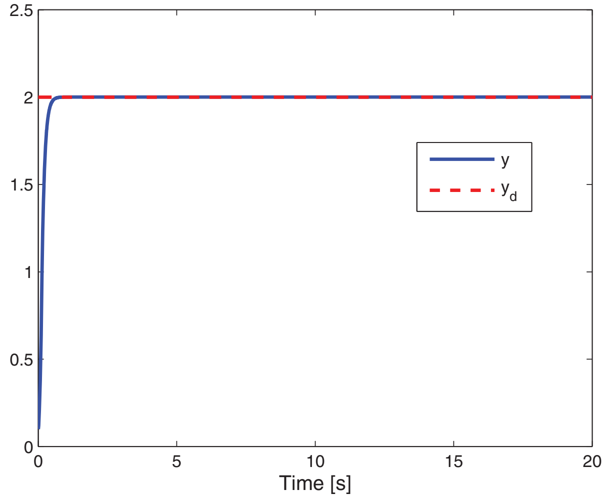


Figure 2. Tracking control result of case 1.

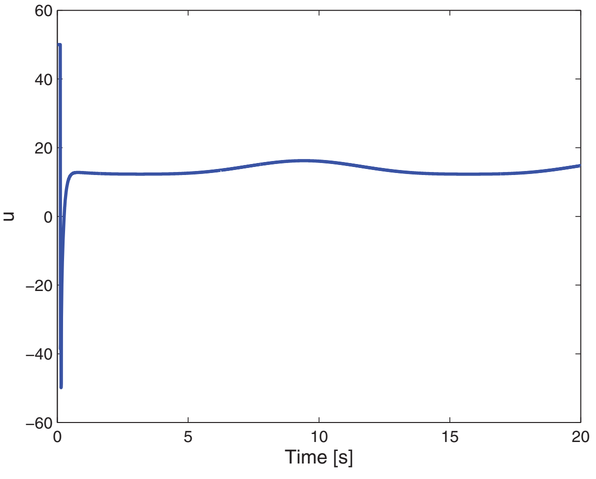


Figure 4. Control input of case 1.

is the angle acceleration, $l$ is the length of the link, $m_0$ is the mass of the link, $\tau$ is the control force, and $\eta_0(t)$ denotes the actuator/component fault in the system. Setting $x_1 = q$, $x_2 = \dot{q}$, and $u = \tau$, then Equation (53) can be rewritten in the following non-linear form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + gu + \eta \\ y &= x_1, \end{aligned} \tag{54}$$

where $f(x) = \frac{-0.5 m_0 g l \sin(x_1)}{M}$, $g = \frac{1}{M}$, $u = D(v(t))$, and $\eta = \frac{\eta_0(t)}{M}$ is the unknown time-varying system fault.

For the single-link robot system (54) with unknown system fault and the dead-zone, the state observer is proposed as (10) and the fault estimator is designed according to (17) and (18). The backstepping tolerant control law is proposed as (45). In the backstepping tolerant control law, the first-order sliding mode differentiator is employed to estimate $\dot{\alpha}_1$. In this simulation, all system parameters of the single-link robot are chosen as $m_0 = 1$ kg, $M = 0.5$ kg m$^2$, $l = 1$ m and $\eta = \sin(0.5t) + 0.2\cos(t)$. The parameters of dead-zone are chosen as $m = 1$, $b_r = 0.2$ and $b_l = -0.3$. At the time, the input saturation is considered for the single-link robot system (54) with $u_{\max} = 50$ and $u_{\min} = -50$. To proceed the design of backstepping fault-tolerant control, all design parameters are chosen as $l_1 = 8$, $l_2 = 15$, $\gamma = 5$, $k_1 = 10$ and $k_2 = 20$. The initial state conditions are chosen as $x_1(0) = 0.1$, $x_2(0) = 0.2$, and $\hat{\eta}(0) = 0$.

**Case 1. For the constant tracking signal**

Here, the desired trajectory of the single-link robot system (54) is taken as $y_d = 2$. Under the proposed fault-tolerant control (45), from Figures 2 and 3, the satisfactory tracking performance is noted and the tracking error is convergent for the single-link robot system (54) in the presence of the unknown system fault and the dead-zone. From Figure 4,

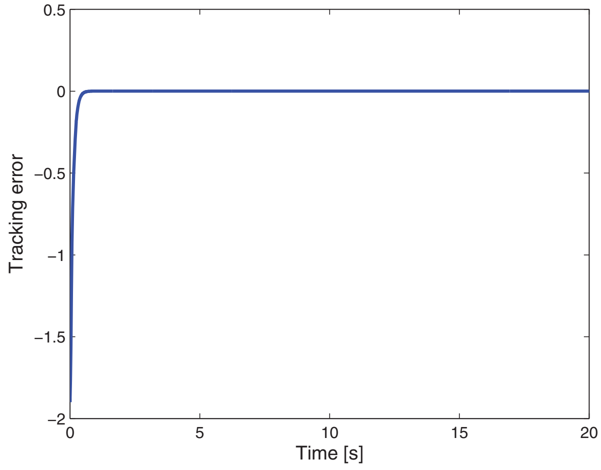


Figure 3. Tracking error of case 1.

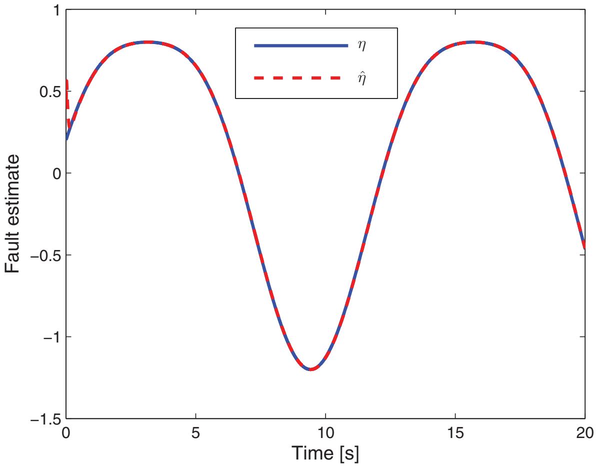


Figure 5. Fault estimate result of case 1.

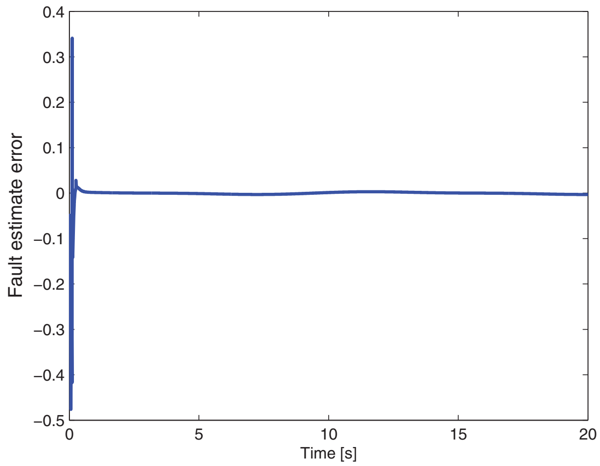


Figure 6. Fault estimate error of case 1.

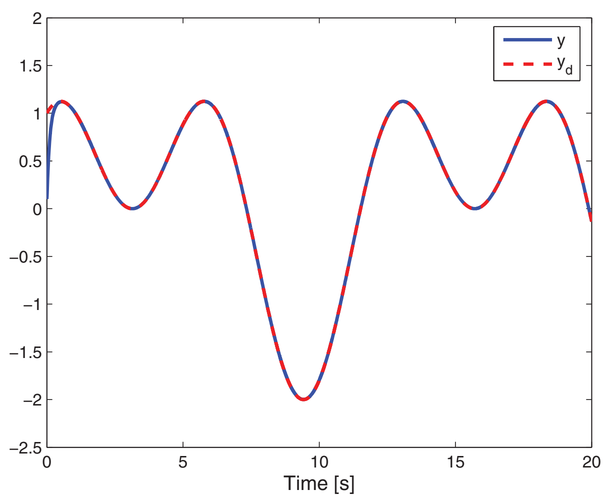


Figure 7. Tracking control result of case 2.

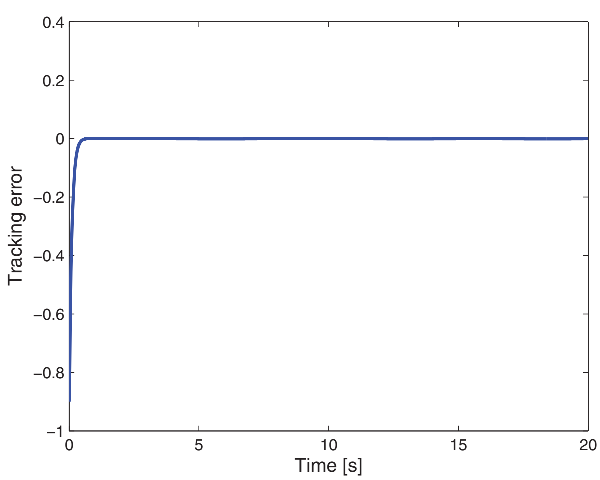


Figure 8. Tracking error of case 2.

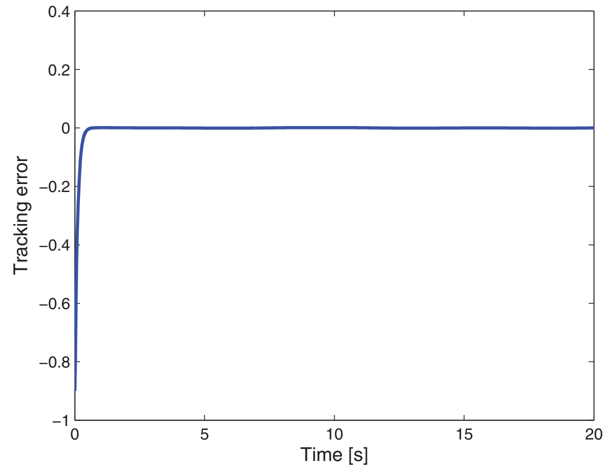


Figure 9. Control input of case 2.

the control input is bounded and convergent. The fault approximate ability of the developed fault estimator is shown in Figures 5 and 6. From Figure 5, the satisfactory estimate performance is obtained by using the developed fault estimator. The estimate error of the unknown system fault $\eta$ is presented in Figure 6, which converges to zero with time change. Thus, the developed fault-tolerant control (45) can accurately track the given constant desired trajectory.

**Case 2. For the time-varying tracking signal**

In this case, the desired trajectory is chosen as $y_d = \cos(t) + \sin(0.5t)$. From Figure 7, we note that the tracking performance is satisfactory for the single-link robot system (54) in the presence of the unknown system fault. The tracking error is presented in Figure 8 which is convergent. According to Figure 9, the bounded control input is obtained. The good fault estimate ability of the developed fault estimator is shown in Figures 10 and 11. According to Figures 10 and 11, we can see that the fault estimate error is bounded

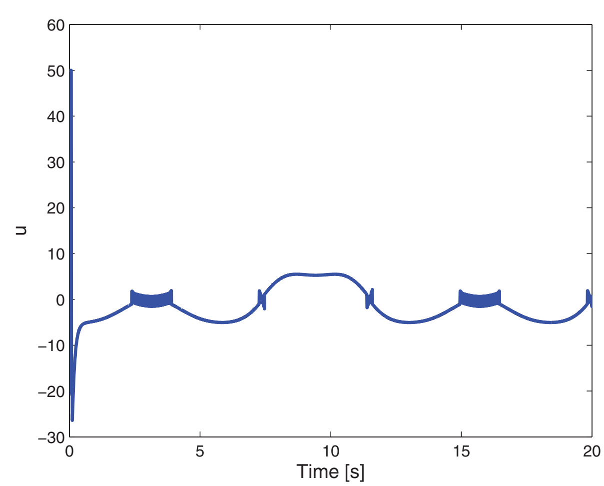


Figure 10. Fault estimate result of case 2.

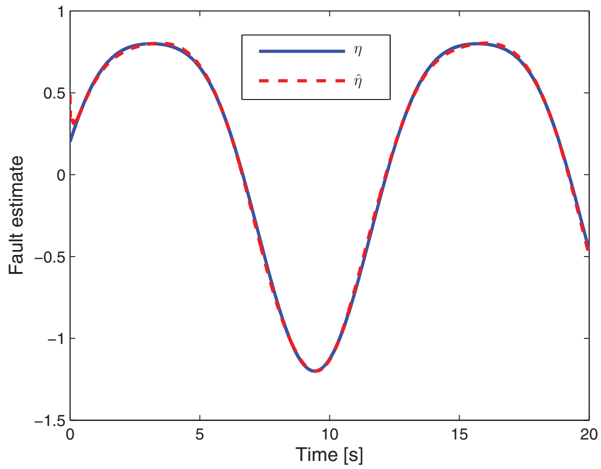


Figure 11. Fault estimate error of case 2.

and convergent. From these simulation results, we can conclude that the proposed backstepping fault-tolerant control scheme has a good tracking control performance for the non-linear system with the unknown system fault and the dead-zone.

In accordance with above simulation results, the satisfactory tracking control performances are achieved under the developed backstepping fault-tolerant control scheme of the single-link robot system with the unknown system fault and the dead-zone. Thus, the developed backstepping fault-tolerant control scheme is valid for the non-linear system.

## 5. Conclusion

Backstepping fault-tolerant control approach has been developed for a class of non-linear systems in the presence of the unknown time-varying system fault and the dead-zone in this paper. The state observer and the fault estimator have been proposed for the studied non-linear system. Then, the backstepping fault-tolerant control has been developed using the outputs of the state observer and the fault estimator. By utilising the Lyapunov method, the uniformly asymptotical convergence of all closed-loop signals have been proved. Finally, simulation results have been given to illustrate the effectiveness of the developed backstepping fault-tolerant control approach. In our developed fault-tolerant control scheme, the dead-zone is required as known. The future work is to study the backstepping tolerant control scheme for the non-linear system with unknown time-varying system fault and the unknown dead-zone. Furthermore, the input saturation can be considered for the studied non-linear system (1) and the developed fault-tolerant control can be extended to the MIMO non-linear system.

## Acknowledgements

The authors gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

## Funding

This work is partially supported by National Natural Science Foundation of China [grant number 61174102]; Program for New Century Excellent Talents in University of China [grant number NCET-11-0830]; Jiangsu Natural Science Foundation of China [grant number SBK20130033], [grant number SBK2011069]; Specialised Research Fund for the Doctoral Program of Higher Education [grant number 20133218110013].

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