Robust Adaptive Dynamic Surface Path Tracking Control for Dynamic Positioning Vessel with Big Plough

Fu Mingyu, Zhang Aihua, Xu Jinlong,Yu Lingling

College of Automation
Harbin Engineering University
Harbin, Heilongjiang Provance,150001,China

{fumingyu & zhangaihua & xujinlong & yulingling}@hrbeu.edu.cn

***Abstract* – To deal with the tracking control problem of the fully actuated dynamic positioning vessel with large disturbances from big ploughs, a novel robust adaptive controller based on dynamic surface is proposed. And in order to product reasonable expectation input for the controller, a guidance strategy for calculating the desired path for vessel is proposed. The mathematical model with uncertain parameters of the dynamic positioning vessel is built. Disturbances from environment and big ploughs are compensated for by the proposed adaptive estimator. By using Lyapunov method, uniformly bounded of the closed-loop system is proved via some assumptions. The effectiveness and the transient performance of the proposed nonlinear robust adaptive dynamic surface controller are illustrated by simulation results.**

Index Terms – Fully actuated dynamic positioning vessel. Robust adaptive dynamic surface. Tracking control. Large disturbances.

I. INTRODUCTION

In the modern ocean engineering, offshore pipe laying and cable laying jobs play important roles. With the improvement of the accuracy requirements of these operations, fully actuated dynamic positioning (DP) vessels, which can reduce time, cost and risks by means of active thrusters, have been widely used as effective alternatives. Meanwhile, in order to complete offshore pipe laying and cable laying operations, fully actuated DP vessels usually need to drag large laying equipments for burying pipelines or cables in the seabed to protect them from damage. While, the burying process is the so-called offshore ploughing, and the large laying equipments are underwater big ploughs. Fig.1 is the sketch map of offshore ploughing operation.

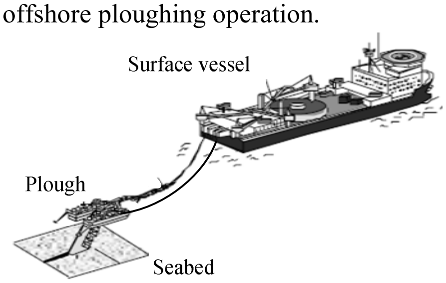

Fig. 1 Sketch map of submarine ploughing operation

During offshore ploughing operation, it is very important to maintain the stability and robustness of vessel's heading and position control, because the uncontrolled instantaneous movement may cause destroy to expensive pipelines and cables. Besides, control of the vessel’s path in accordance with the specified routes for the plough is necessary. Moreover, because of towing a big plough, the dynamic condition of the fully actuated DP vessel is changed, and the compensation for the disturbance of the big plough and the environment is required. Though force and torque sensors are applied to compensate for reactions caused by big ploughs in traditional projects, however, it will increase complexity and unreliability.

When taking above problems into account, the controller of the fully actuated DP vessel must have good instantaneous response, it should be adaptive to the uncertain parameters and unknown disturbances. In addition, for controlling the plough’s path, a good guidance strategy is need for giving a desire input to the controller of the vessel. So, a novel adaptive controller based on nonlinear dynamic surface is proposed for the tracking control of the DP vessel in this paper. And a guidance strategy is proposed for providing inputs to the controller from the desired plough’s path. In the controller, the uncertain nonlinear hydrodynamic damping and the unknown disturbance from the plough and the environment are compensated for by the adaptive estimator, the stability and robustness is proved by Lyapunov method, and the dynamic responses during path tracking is shown by simulations.

For vessels, especially for DP vessels, researches of tracking control attract attentions of many scholar, they have made important contributions in this area. The conventional controller of DP vessel uses the linear method, such as PID and LQ[1-2]. For improving the robustness, H∞ control technique is applied by Katebi et al.[3]. However, because the global stability can not be obtained by linear system, researchers pay their attentions to nonlinear control method. Fossen and his team use Lyapunov technique to prive the stability of the controller designed for DP vessel, they use nonlinear PID and backstepping method to solve the tracking control problem of DP vessel[4-5]. Recently, the concept of manoeuvring is proposed by Skjetne, manoeuvring control is suitable to the control problem of the uncertain system, an adaptive method is used to estimate the uncertain parameters[6]. Then Breivik takes the significance of the guidance system into consideration to deal with the motion control of vessels, a backstepping-inspired and cascaded-based approach is employed to get the stability of the closed loop tracking system[7-8]. But in the backstepping method, the computing expansion is a fatal problem, so Swaroop et al. improve the dynamic surface technique to solve the computing

978-0-933957-40-4 ©2013 MTS

expansion problem[9]. Based on the dynamic surface technique, Li et al. propose an adaptive dynamic surface technique to prove the semiglobally uniformly ultimately bound of a kind of strict-feedback nonlinear system[10], and Zhang et al. exhibit a disturbances compensative controller for DP dreger[11].

Inspired by the work of those researchers, to solve the tracking control problem for fully actuated DP vessel with big plough, a robust adaptive nonlinear path tracking controller based on dynamic surface is proposed.

II. Problem Formulation

Big ploughs suffer large friction forces when trenching on the seabed, so they will product large disturbances to fully actuated DP surface vessel, when designing the controller for the vessel, these disturbances must be considered. For making the research simplified, the following assumption is made.

Assumption 1. DP vessel and the plough locate in the same longitudinal plane, because the speed of vessel is low when during the ploughing operation, and the distance between the plough and vessel L is constant in the horizontal plane, as shown in Fig.2.

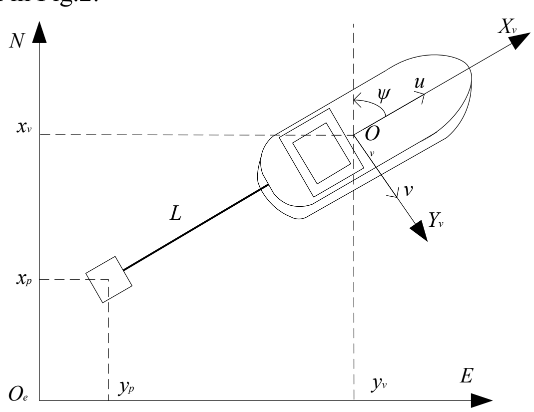

Fig. 2 Definition of frames in the horizontal plane

So, the mathematical model of the DP vessel in a horizontal plane can be described as follow[4]

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\psi)\boldsymbol{\upsilon} \\ \boldsymbol{M}\dot{\boldsymbol{\upsilon}} + \boldsymbol{C}(\boldsymbol{\upsilon})\boldsymbol{\upsilon} + \boldsymbol{D}(\boldsymbol{\upsilon})\boldsymbol{\upsilon} &= \boldsymbol{\tau} + \boldsymbol{\tau}_{env} + \boldsymbol{\tau}_{pl} \end{aligned} \tag{1}$$

where

$$\boldsymbol{\tau}_{pl} = \begin{bmatrix} \tau_{pl,x} & \tau_{pl,y} & \tau_{pl,N} \end{bmatrix}^{\mathrm{T}}$$

and

$$\boldsymbol{\tau}_{env} = \begin{bmatrix} \tau_{env,x} & \tau_{env,y} & \tau_{env,N} \end{bmatrix}^{\mathrm{T}}.$$

$\boldsymbol{\tau}_{env}$ is the plough and environmental disturbances vector individually, $\boldsymbol{\eta} = [x \ \ y \ \ \psi]^{\mathrm{T}}$ denotes the position and heading of the vessel in earth-fixed frame, $\boldsymbol{\upsilon} = [u \ \ v \ \ r]^{\mathrm{T}}$ is the velocities of the vessel in body-fixed frame, and r is the angular velocity of heading angle ψ.

Here, $\boldsymbol{J}(\psi)$ is the rotation matrix, $\boldsymbol{M}$ is the mass matrix, and $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, $\boldsymbol{C}(\boldsymbol{\upsilon})$ is the Coriolis and centripetal matrix, $\boldsymbol{D}(\boldsymbol{\upsilon})$ is the damping matrix. Because only position and heading of the surface vessel in the horizontal plane are considered in this paper, expressions of $\boldsymbol{J}(\psi)$, $\boldsymbol{M}$, $\boldsymbol{C}(\boldsymbol{\upsilon})$ and $\boldsymbol{D}(\boldsymbol{\upsilon})$ are as follows:

$$\boldsymbol{J}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix}$$

$$\boldsymbol{C}(\boldsymbol{\upsilon}) = \begin{bmatrix} 0 & 0 & c_{13}(\upsilon) \\ 0 & 0 & c_{23}(\upsilon) \\ -c_{13}(\upsilon) & -c_{23}(\upsilon) & 0 \end{bmatrix}$$

$$\boldsymbol{D}(\boldsymbol{\upsilon}) = \begin{bmatrix} d_{11}(\upsilon) & 0 & 0 \\ 0 & d_{22}(\upsilon) & d_{23}(\upsilon) \\ 0 & d_{32}(\upsilon) & d_{33}(\upsilon) \end{bmatrix}$$

where

$$\begin{aligned} c_{13}(\upsilon) &= -(m - Y_{\dot{v}})v - (mx_g - Y_{\dot{r}})r \\ c_{23}(\upsilon) &= (m - Y_{\dot{u}})u \\ d_{11}(\upsilon) &= -X_u - X_{|u|u}|u|, \\ d_{22}(\upsilon) &= -Y_v - Y_{|v|v}|v| - Y_{|r|v}|r|, \\ d_{23}(\upsilon) &= -Y_r - Y_{|v|r}|v| - Y_{|r|r}|r|, \\ d_{32}(\upsilon) &= -N_v - N_{|v|v}|v| - N_{|r|v}|r|, \\ d_{33}(\upsilon) &= -N_r - N_{|v|r}|v| - N_{|r|r}|r|. \end{aligned}$$

Notice that $\boldsymbol{D}(\boldsymbol{\upsilon}) = \boldsymbol{D}_k(\upsilon) + \boldsymbol{D}_u(\upsilon)$, $\boldsymbol{D}_k(\upsilon)$ is the known term and $\boldsymbol{D}_u(\upsilon)$ denotes the unknown term.

As for DP vessel with big plough, the uncertain constant parameter vector $\boldsymbol{\chi}$ is defined as

$$\begin{aligned} \boldsymbol{\chi} = [&X_{|u|u}, Y_{|v|v}, Y_{|r|v}, Y_{|v|r}, Y_{|r|r}, N_{|r|v}, N_{|v|r}, N_{|r|r}, \tau_{env,x} + \tau_{pl,x}, \\ &\tau_{env,y} + \tau_{pl,y}, \tau_{env,\psi} + \tau_{pl,\psi}]^{\mathrm{T}} \in \Re^{11}, \end{aligned}$$

Then, the mathematic model (1) is rewritten as

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}(\psi)\boldsymbol{\upsilon} \\ \boldsymbol{M}\dot{\boldsymbol{\upsilon}} + \boldsymbol{C}(\boldsymbol{\upsilon})\boldsymbol{\upsilon} + \boldsymbol{D}_k(\boldsymbol{\upsilon})\boldsymbol{\upsilon} &= \boldsymbol{\tau} + \boldsymbol{\Phi}(\boldsymbol{\upsilon})\boldsymbol{\chi} \end{aligned} \tag{2}$$

where

$$\boldsymbol{\Phi}(\upsilon) = \begin{bmatrix} |u|u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & |v|v & |r|v & |v|r & |r|r & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & |r|v & |v|r & |r|r & 0 & 0 & 1 \end{bmatrix}$$

is the regressor matrix, and $\boldsymbol{\Phi}(\upsilon)\boldsymbol{\chi}$ is constituted by unknown hydrodynamic damping forces $\boldsymbol{D}_u(\upsilon)$, environmental disturbances $\boldsymbol{\tau}_{env}$ and plough disturbances $\boldsymbol{\tau}_{pl}$.

Define the following variables

$$\begin{aligned} \boldsymbol{x}_1 &= \boldsymbol{\eta} \\ \boldsymbol{x}_2 &= \boldsymbol{\upsilon} \end{aligned} \tag{3}$$

Then, the model of the cable laying vessel can be rewritten as follow

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{g}_1(x_1)\boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 &= \boldsymbol{f}_2(x_2) + \boldsymbol{g}_2(x_2)\boldsymbol{u} + \Delta_2 \end{aligned} \tag{4}$$

where $\boldsymbol{g}_1(x_1) = \boldsymbol{J}(\psi)$, $\boldsymbol{f}_2(x_2) = -\boldsymbol{M}^{-1}\left(\boldsymbol{C(\upsilon)} + \boldsymbol{D}_k\boldsymbol{(\upsilon)}\right)$, $\boldsymbol{u} = \boldsymbol{\tau}$, $\boldsymbol{g}_2(x_2) = \boldsymbol{M}^{-1}$, $\Delta_2 = \boldsymbol{M}^{-1}\boldsymbol{\Phi(\upsilon)\chi}$. They are expressed as $\boldsymbol{g}_1$, $\boldsymbol{f}_2$, $\boldsymbol{g}_2$ simply in the following part.

We suppose following assumptions for equation (4) is built, and the control object is tracking the desired path $\boldsymbol{y}_d$.

Assumption 2. $\boldsymbol{f}_2$ is a known continuous function, and the gain $\boldsymbol{g}_i$ satisfy $0 < g_{\min} \le \|\boldsymbol{g}_i\| \le g_{\max}$, $g_{\min}$ and $g_{\max}$ are the upper and lower bound for $i = 1, 2$.

Assumption 3. The desired value $\boldsymbol{y}_d$ is smooth and bounded, and its second-order derivative is known and bounded. Which means that, there exist a positive constant B_0 satisfy $\Pi_0 = \left\{(\boldsymbol{y}_d, \dot{\boldsymbol{y}}_d, \ddot{\boldsymbol{y}}_d) : \|\boldsymbol{y}_d\|^2 + \|\dot{\boldsymbol{y}}_d\|^2 + \|\ddot{\boldsymbol{y}}_d\|^2 \le B_0\right\}$.

The environment disturbance $\boldsymbol{b}$ is modeled as the 1st-order Markov process:

$$\dot{\boldsymbol{b}} = -\boldsymbol{T}_{\mathrm{b}}^{-1}\boldsymbol{b} + \boldsymbol{E}\boldsymbol{w} \tag{5}$$

where $\boldsymbol{T}_{\mathrm{b}} = diag\{1000, 1000, 1000\}$ is selected as time constants, $\boldsymbol{E} = diag\{10, 10, 10\}$, $\boldsymbol{w}$ is a zero-mean white noise vector, and $\boldsymbol{\tau}_{env} = \boldsymbol{J}^{\mathrm{T}}(\psi)\boldsymbol{b}$. Furthermore, the plough disturbances are obtained from the plough's model in reference [12], only its effect on vessel longitudinal direction is considered.

III. Guidance Strategy And Controller Design

A. Guidance strategy for the tracking problem

The big plough must follow the route given by the operator, and the plough's share should direct toward the point which connect each path segment. Small perturbations to the plough may make breakage of pipelines or cables. Therefore, it is necessary to keep a low speed profile and keep the stability of the controller.

In the tracking control process, guidance strategy is used to transfer the plough's set route to the vessel's desired path $\boldsymbol{y}_d$. Guidance strategy can give the controller the required input. The so called Line-Of-Sight (LOS) guidance algorithm is introduced for solving this kind of tracking problem. And based on assumption 1, we use LOS algorithm calculates the vessel's expectation position and heading vector according the plough's desired position and heading vector.

For the path of straight-line segment, the vessel's expectation position and heading can be obtained as:

$$\begin{aligned} x_d &= x_p + Vt\cos\theta + L\cos\theta \\ y_d &= y_p + Vt\sin\theta + L\sin\theta \\ \psi_d &= \psi_p = \operatorname{at}\operatorname{an}2(y_k - y_p, x_k - x_p) \end{aligned} \tag{6}$$

where x_d y_d ψ_d are the vessel's desired positions and heading, x_p y_p ψ_p denote the plough's desired positions and heading, V is the set tracking speed, t is the sampling time, x_k and y_k are the coordinates of current way point, θ is the orientation angel of current path, L is the distance between the vessel and the plough in the horizontal plane, as shown in Fig.2.

For the path of circular-arc segment, the vessel's desired position and heading can be calculated as:

$$\begin{aligned} \psi_d &= \psi'_d + r\cdot t.\mathrm{sgn} \\ x_d &= x_R + R\cos(\alpha + r\cdot t.\mathrm{sgn}) + L\cos\psi_d \\ y_d &= y_R + R\sin(\alpha + r\cdot t.\mathrm{sgn}) + L\sin\psi_d \end{aligned} \tag{7}$$

where (x_R, y_R) is the centre of the heading change circular-arc, ψ'_d is the heading before turning, R is the turning radius, α is the initial angle of turning circle, r is the set turning rate. $\mathrm{sgn} = \begin{cases} 1 \\ -1 \end{cases}$ is a symbol function, 1 means turn right, -1 means turn lift.

B. Controller design and stability proof

The design process of the controller is presented as

Step 1: define the error vector

$$\boldsymbol{S}_1 = \boldsymbol{x}_1 - \boldsymbol{y}_d \tag{8}$$

Differentiating equation (8) with respect to time yields

$$\dot{\boldsymbol{S}}_1 = \dot{\boldsymbol{x}}_1 - \dot{\boldsymbol{y}}_d = \boldsymbol{g}_1\boldsymbol{x}_2 - \dot{\boldsymbol{y}}_d \tag{9}$$

Considering $\boldsymbol{x}_2$ as virtual controller, designing the following control law for $\boldsymbol{x}_2$

$$\bar{\boldsymbol{x}}_2 = -\boldsymbol{k}_1\boldsymbol{S}_1 + \boldsymbol{g}_1^{-1}\dot{\boldsymbol{y}}_d \tag{10}$$

where $\boldsymbol{k}_1 = \boldsymbol{k}_1^{\mathrm{T}} > \boldsymbol{0}$ is a constant to be specified later.

Next, we use the thought of dynamic surface method. Here a new variable z_2 is introduced to estimate the virtual control variable $\bar{\boldsymbol{x}}_2$ through a 1st-order filter

$$\boldsymbol{T}\dot{z}_2 + z_2 = \bar{\boldsymbol{x}}_2, z_2(0) = \bar{\boldsymbol{x}}_2(0) \tag{11}$$

where $\boldsymbol{T}$ is a time constant of the filter (11), and the estimating error is

$$\boldsymbol{y}_2 = z_2 - \bar{\boldsymbol{x}}_2 \tag{12}$$

Substituting (9), (10) and (11) into (12), the differential of the estimating error (12) is

$$\begin{aligned} \dot{\boldsymbol{y}}_2 &= \dot{z}_2 - \dot{\bar{\boldsymbol{x}}}_2 \\ &= -\boldsymbol{T}^{-1}\boldsymbol{y}_2 + (-\boldsymbol{k}_1\dot{\boldsymbol{S}}_1 + \dot{\boldsymbol{g}}_1^{-1}\dot{\boldsymbol{y}}_d + \boldsymbol{g}_1^{-1}\ddot{\boldsymbol{y}}_d) \\ &= -\boldsymbol{T}^{-1}\boldsymbol{y}_2 + (-\boldsymbol{k}_1\dot{\boldsymbol{S}}_1 - \boldsymbol{S}\boldsymbol{g}_1^{-1}\dot{\boldsymbol{y}}_d + \boldsymbol{g}_1^{-1}\ddot{\boldsymbol{y}}_d) \\ &= -\boldsymbol{T}^{-1}\boldsymbol{y}_2 + (-\boldsymbol{k}_1(\boldsymbol{g}_1\boldsymbol{x}_2 - \dot{\boldsymbol{y}}_d) - \boldsymbol{S}\boldsymbol{g}_1^{-1}\dot{\boldsymbol{y}}_d + \boldsymbol{g}_1^{-1}\ddot{\boldsymbol{y}}_d) \\ &= -\boldsymbol{T}^{-1}\boldsymbol{y}_2 + \boldsymbol{B}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_d, \dot{\boldsymbol{y}}_d, \ddot{\boldsymbol{y}}_d) \end{aligned} \tag{13}$$

where $\boldsymbol{B}_2(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{y}_d,\dot{\boldsymbol{y}}_d,\ddot{\boldsymbol{y}}_d)$ is a continuous function, and suppose B_0^* is its upper bound. $\boldsymbol{S}$ is a skew-symmetric matrix, the expression of $\boldsymbol{S}$ can be written as

$$\boldsymbol{S}=\begin{bmatrix}0 & -r & 0\\ r & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{14}$$

Notice that,

$$\left.\begin{array}{r}\boldsymbol{g}_1=\boldsymbol{J}(\psi)\\ \left.\begin{array}{r}\boldsymbol{J}(\psi)\boldsymbol{J}^{-1}(\psi)=\boldsymbol{I}\\ \dot{\boldsymbol{J}}(\psi)=\boldsymbol{J}(\psi)\boldsymbol{S}\end{array}\right\}\Leftrightarrow \dot{\boldsymbol{J}}^{-1}(\psi)=-\boldsymbol{S}\boldsymbol{J}^{-1}(\psi)\end{array}\right\}\Leftrightarrow \dot{\boldsymbol{g}}_1^{-1}=-\boldsymbol{S}\boldsymbol{g}_1^{-1} \tag{15}$$

We choose the first Lyapunov function candidate as

$$V_1=(1/2)\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1+(1/2)\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{y}_2 \tag{16}$$

Define a new error variable for $\boldsymbol{x}_2$

$$\boldsymbol{S}_2=\boldsymbol{x}_2-\boldsymbol{z}_2 \tag{17}$$

Time derivative of (16) is written as

$$\begin{aligned}\dot{V}_1&=\boldsymbol{S}_1^{\mathrm{T}}\dot{\boldsymbol{S}}_1+\boldsymbol{y}_2^{\mathrm{T}}\dot{\boldsymbol{y}}_2\\ &=\boldsymbol{S}_1^{\mathrm{T}}\left(\boldsymbol{g}_1\boldsymbol{x}_2-\dot{\boldsymbol{y}}_d\right)+\boldsymbol{y}_2^{\mathrm{T}}\left(-\boldsymbol{T}^{-1}\boldsymbol{y}_2+\boldsymbol{B}_2\right)\\ &=\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\left(\boldsymbol{S}_2+\boldsymbol{y}_2-\boldsymbol{k}_1\boldsymbol{S}_1\right)+\boldsymbol{y}_2^{\mathrm{T}}\left(-\boldsymbol{T}^{-1}\boldsymbol{y}_2+\boldsymbol{B}_2\right)\\ &=-\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\boldsymbol{k}_1\boldsymbol{S}_1+\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\boldsymbol{S}_2+\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\boldsymbol{y}_2+\boldsymbol{y}_2^{\mathrm{T}}\left(-\boldsymbol{T}^{-1}\boldsymbol{y}_2+\boldsymbol{B}_2\right)\end{aligned} \tag{18}$$

Notice that,

$$\begin{aligned}\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\boldsymbol{S}_2&\le\|\boldsymbol{y}_2\|^2+\left(\|\boldsymbol{g}_1\boldsymbol{S}_2\|/2\right)^2\\ &\le\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1+\left(g_{\max}^2\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2/4\right)\end{aligned} \tag{19}$$

and

$$\begin{aligned}\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{g}_1\boldsymbol{y}_2&\le\|\boldsymbol{y}_2\|^2+\left(\|\boldsymbol{g}_1\boldsymbol{y}_2\|/2\right)^2\\ &\le\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1+\left(g_{\max}^2\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{y}_2/4\right)\end{aligned} \tag{20}$$

According to (18), (19), (20) and assumption 2

$$\begin{aligned}\dot{V}_1\le&-\boldsymbol{S}_1^{\mathrm{T}}\left(\boldsymbol{g}_{\min}\boldsymbol{k}_1-2\boldsymbol{I}\right)\boldsymbol{S}_1+\left(g_{\max}^2\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2/4\right)\\ &+\left(g_{\max}^2\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{y}_2/4\right)-\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{T}^{-1}\boldsymbol{y}_2+\left|\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{B}_2\right|\end{aligned} \tag{21}$$

Step 2: Differentiating $\boldsymbol{S}_2$ in (17) with respect to time yields

$$\begin{aligned}\dot{\boldsymbol{S}}_2&=\dot{\boldsymbol{x}}_2-\dot{\boldsymbol{z}}_2\\ &=\boldsymbol{f}_2+\boldsymbol{g}_2\boldsymbol{u}+\Delta_2-\dot{\boldsymbol{z}}_2\end{aligned} \tag{22}$$

Define the estimate error of the uncertain parameter $\boldsymbol{\chi}$ as

$$\tilde{\boldsymbol{\chi}}=\boldsymbol{\chi}-\hat{\boldsymbol{\chi}}\ , \tag{23}$$

where $\hat{\boldsymbol{\chi}}$ is the parameter estimate of $\boldsymbol{\chi}$ through adaptive method, and $\tilde{\boldsymbol{\chi}}$ is the estimate error.

Choose a positive matrix $\boldsymbol{H}=\boldsymbol{H}^{\mathrm{T}}>\boldsymbol{0}$ as the adaptive gain coefficient matrix, and define the second Lyapunov function candidate as

$$V_2=V_1+(1/2)\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2+(1/2)\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{H}\tilde{\boldsymbol{\chi}} \tag{24}$$

whose time derivative is

$$\dot{V}_2=\dot{V}_1+\boldsymbol{S}_2^{\mathrm{T}}\dot{\boldsymbol{S}}_2-\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{H}^{-1}\dot{\hat{\boldsymbol{\chi}}} \tag{25}$$

Substituting (21), (22) and (23) into (24), we obtain

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+\boldsymbol{S}_2^{\mathrm{T}}\left(\boldsymbol{f}_2+\boldsymbol{g}_2\boldsymbol{u}+\Delta_2-\dot{\boldsymbol{z}}_2\right)-\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{H}^{-1}\dot{\hat{\boldsymbol{\chi}}}\\ &=\dot{V}_1+\boldsymbol{S}_2^{\mathrm{T}}\left(\boldsymbol{f}_2+\boldsymbol{g}_2\boldsymbol{u}+\boldsymbol{M}^{-1}\boldsymbol{\Phi}\boldsymbol{\chi}-\dot{\boldsymbol{z}}_2\right)-\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{H}^{-1}\dot{\hat{\boldsymbol{\chi}}}\\ &=\dot{V}_1+\boldsymbol{S}_2^{\mathrm{T}}\left(\boldsymbol{f}_2+\boldsymbol{g}_2\boldsymbol{u}+\boldsymbol{M}^{-1}\boldsymbol{\Phi}\hat{\boldsymbol{\chi}}-\dot{\boldsymbol{z}}_2\right)\\ &\quad+\tilde{\boldsymbol{\chi}}^{\mathrm{T}}(\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{M}^{-\mathrm{T}}\boldsymbol{S}_2-\boldsymbol{H}^{-1}\dot{\hat{\boldsymbol{\chi}}})\end{aligned} \tag{26}$$

To make the closed-loop system stable, the control law and the adaptive update law for $\hat{\boldsymbol{\chi}}$ are chosen as

$$\boldsymbol{u}=-\boldsymbol{k}_2\boldsymbol{S}_2+\boldsymbol{g}_2^{-1}\left(-\boldsymbol{f}_2-\boldsymbol{M}^{-1}\boldsymbol{\Phi}\hat{\boldsymbol{\chi}}+\dot{\boldsymbol{z}}_2\right) \tag{27}$$

$$\dot{\hat{\boldsymbol{\chi}}}=\boldsymbol{H}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{M}^{-\mathrm{T}}\boldsymbol{S}_2+\boldsymbol{H}\boldsymbol{\sigma}\left(\underline{\boldsymbol{\chi}}-\hat{\boldsymbol{\chi}}\right) \tag{28}$$

where $\boldsymbol{k}_2$ is a positive gain matrix, $\boldsymbol{k}_2=\boldsymbol{k}_2^{\mathrm{T}}>\boldsymbol{0}$. $\boldsymbol{\sigma}$ and $\underline{\boldsymbol{\chi}}$ are parameter matrixes need to be designed.

Notice that,

$$\begin{aligned}\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\left(\underline{\boldsymbol{\chi}}-\hat{\boldsymbol{\chi}}\right)&=\left(\|\tilde{\boldsymbol{\chi}}\|^2/2\right)+\left(\|\underline{\boldsymbol{\chi}}-\hat{\boldsymbol{\chi}}\|^2/2\right)-\left(\|\boldsymbol{\chi}-\hat{\boldsymbol{\chi}}\|^2/2\right)\\ &\ge\left(\|\tilde{\boldsymbol{\chi}}\|^2/2\right)-\left(\|\underline{\boldsymbol{\chi}}-\boldsymbol{\chi}^*\|^2/2\right)\end{aligned} \tag{29}$$

$$\Rightarrow -\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{\sigma}\left(\underline{\boldsymbol{\chi}}-\hat{\boldsymbol{\chi}}\right)\le-\left(\|\boldsymbol{\sigma}\|\|\tilde{\boldsymbol{\chi}}\|^2/2\right)+\left(\|\boldsymbol{\sigma}\|\|\underline{\boldsymbol{\chi}}-\boldsymbol{\chi}^*\|^2/2\right)$$

And

$$\begin{aligned}\left|\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{B}_2\right|&\le\left(\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{y}_2\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{B}_2/2\beta\right)+\left(\beta/2\right)\\ &\le\left(\left(B_0^*\right)^2\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{y}_2/2\beta\right)+\left(\beta/2\right)\end{aligned} \tag{30}$$

Let $\beta=\|\boldsymbol{\sigma}\|\|\underline{\boldsymbol{\chi}}-\boldsymbol{\chi}^*\|^2$, $\boldsymbol{\chi}^*$ is designed parameter matrix, Substitute (27) and (28) to (26), we get

$$\begin{aligned}\dot{V}_2\le&-\boldsymbol{S}_1^{\mathrm{T}}\left(\boldsymbol{g}_{\min}\boldsymbol{k}_1-2\boldsymbol{I}\right)\boldsymbol{S}_1-\boldsymbol{S}_2^{\mathrm{T}}\left(\boldsymbol{g}_{\min}\boldsymbol{k}_2-\left(g_{\max}^2/4\right)\boldsymbol{I}\right)\boldsymbol{S}_2\\ &-\boldsymbol{y}_2^{\mathrm{T}}\left(\boldsymbol{T}^{-1}-\left(\left(\beta g_{\max}^2+2\left(B_0^*\right)^2\right)/(4\beta)\right)\boldsymbol{I}\right)\boldsymbol{y}_2\\ &-\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\left(\boldsymbol{\sigma}/2\right)\tilde{\boldsymbol{\chi}}+\beta\end{aligned} \tag{31}$$

Choose the parameters as

$$\boldsymbol{\sigma}=\boldsymbol{\beta}_0 \tag{32}$$

$$\boldsymbol{k}_1=\boldsymbol{g}_{\min}^{-1}\left((1/2)\boldsymbol{\beta}_0+2\boldsymbol{I}\right) \tag{33}$$

$$\boldsymbol{k}_2=\boldsymbol{g}_{\min}^{-1}\left((1/2)\boldsymbol{\beta}_0+\left(g_{\max}^2/4\right)\boldsymbol{I}\right) \tag{34}$$

$$\boldsymbol{T}^{-1}=(1/2)\boldsymbol{\beta}_0+\left(\left(\beta g_{\max}^2+2\left(B_0^*\right)^2\right)/(4\beta)\right)\boldsymbol{I} \tag{35}$$

When substitute (32)(33)(34)(35) into (31), we can obtain

$$\begin{aligned}\dot{V}_2\le&-(1/2)\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{\beta}_0\boldsymbol{S}_1-(1/2)\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{\beta}_0\boldsymbol{S}_2\\ &-(1/2)\boldsymbol{y}_2^{\mathrm{T}}\boldsymbol{\beta}_0\boldsymbol{y}_2-(1/2)\tilde{\boldsymbol{\chi}}^{\mathrm{T}}\boldsymbol{\beta}_0\tilde{\boldsymbol{\chi}}+\beta\end{aligned} \tag{36}$$

Let $\beta_0=\|\boldsymbol{\beta}_0\|$, we have

$$\dot{V}_2\le-\beta_0V_2+\beta \tag{37}$$

According to above discussion, for a given vector $\boldsymbol{\beta}_0$, there exist vector $\boldsymbol{\sigma}$, $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{\chi}^*$, $\underline{\boldsymbol{\chi}}$, $\boldsymbol{T}$ and $\boldsymbol{H}$, such that the closed-loop system composed of (4), (10), (11), (27) and (28) is uniformly bounded.

According to above demonstrations, the robust adaptive nonlinear dynamic surface controller can be written as the following equality system

$$\begin{aligned} \boldsymbol{\tau} &= -\boldsymbol{k}_2\left(\boldsymbol{\upsilon}-\boldsymbol{z}_2\right)+\boldsymbol{C}(\boldsymbol{\upsilon})+\boldsymbol{D}_k(\boldsymbol{\upsilon})+\boldsymbol{\Phi}\hat{\boldsymbol{\chi}}+\boldsymbol{M}\dot{\boldsymbol{z}}_2 \\ \dot{\hat{\boldsymbol{\chi}}} &= \boldsymbol{H}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{M}^{-\mathrm{T}}\left(\boldsymbol{\upsilon}-\boldsymbol{z}_2\right)+\boldsymbol{H}\boldsymbol{\sigma}\left(\underline{\boldsymbol{\chi}}-\hat{\boldsymbol{\chi}}\right) \\ \boldsymbol{T}\dot{\boldsymbol{z}}_2+\boldsymbol{z}_2 &= -\boldsymbol{k}_1\left(\boldsymbol{\eta}-\boldsymbol{y}_d\right)+\boldsymbol{J}^{-1}\dot{\boldsymbol{y}}_d \end{aligned} \tag{38}$$

IV. Simulation Analysis

In this section, the results of computer simulation for the DP vessel with big plough are presented. In the simulation, a supply vessel in [4] with a plough in [12] is used, the simulation parameters of the vessel are shown in Table I.

TABLE I
System Parameter Data

L_v	76.2 (m)	I_Z	2.0903*10^9 (kgm^2)	$N_{\dot r}$	-0.8780*10^9 (kgm)
B	19.2 (m)	$X_{\dot u}$	-0.5096*10^6 (kg)	X_u	-0.05138*10^6 (kg/s)
G	6.0*10^6 (kg)	$Y_{\dot v}$	-3.5608*10^6 (kg)	Y_v	-0.1698*10^6 (kg/s)
x_g	0.0 (m)	$Y_{\dot r}$	-0.02268*10^9 (kgm)	Y_r	1.5081*10^6 (kg/s)
m	4.0*10^6 (kg)	$N_{\dot v}$	-0.02268*10^9 (kgm)	N_r	-0.2530*10^9 (kgm^2/s)

L_v and B are the vessel's length and width respectively, m is the vessel's mass, x_g denotes the distance between the vessel's gravity centre and geometry centre, G denotes the water displacement of the vessel, I_z is the vessel's moment of inertia. Other parameters are the hydrodynamic coefficients. as the mention in part II, only the plough's impact on the longitudinal is considered here. The plough' path is suppose to be assembled by straight lines and circular arcs, which are generated by the so called way points.

The adaptive parameter is selected as $\boldsymbol{H}=diag\{1,1,1,1,1,1,1,2,2,2\}$, the controller parameter matrixes are selected as $\boldsymbol{k}_1=diag\{0.0001,0.0015,0.01\}$, $\boldsymbol{k}_2=diag\{0.3,0.2,1\}$, $\boldsymbol{T}=diag\{10^5,2*10^5,10^6\}$. The initial position of the vessel is given as $\boldsymbol{\eta}_0=[80\ 10\ \mathrm{pi}/30]^{\mathrm{T}}$, the initial desired position of the plough is given as $\boldsymbol{\eta}_{\mathrm{pd}}=[0\ 0\ \mathrm{pi}/31]^{\mathrm{T}}$. And the set plough tracking path is given by four way-points (0, 0), (100, 1000), (500, 1000), (600, 50) and (1000, 800). The DP vessel's tracking performance with initial position vector η_0 is shown in Fig.3.

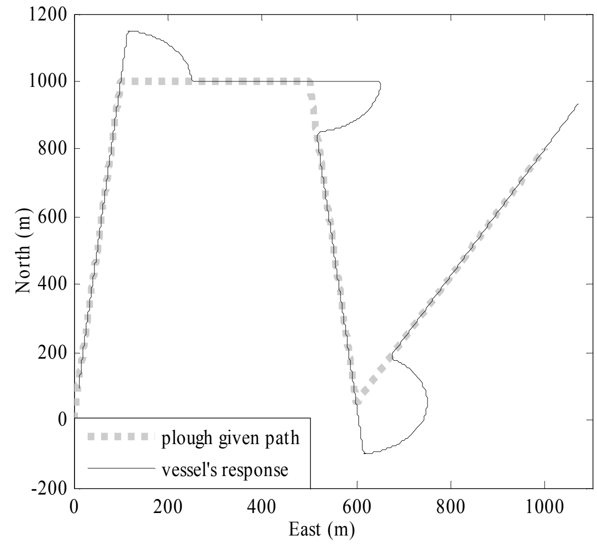

Fig. 3 Vessel position response and plough set path in earth-fixed frame.

From Fig. 3 we can see that, because of the distance existing between the vessel and the plough, at the veering point (100, 1000), (500, 1000) and (600, 50), the vessel goes through larger distances and turning arcs than the plough.

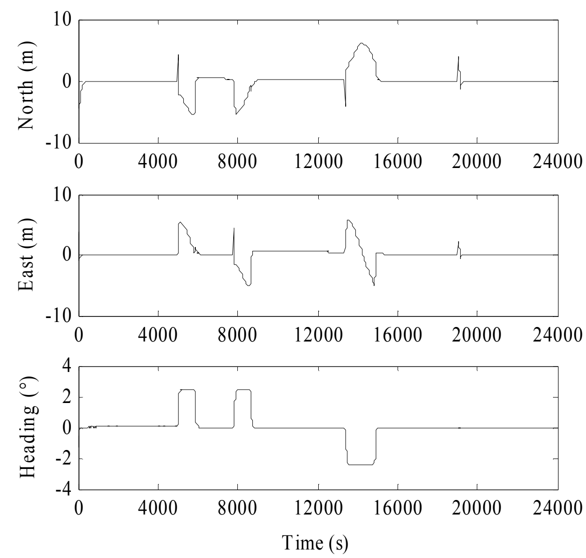

Fig. 4 Curves of position and heading deviations between the response and LOS calculated values of DP vessel in earth-fixed frame.

The error response waveforms of the DP vessel with the big plough are shown in Fig. 4. The proposed controller can drive the vessel tracking the desired path well under large disturbances from the plough and environment. This means that the closed system is uniformly bounded and robust to disturbances. Fig.5 shows the time series of the disturbances from the plough and environment, and also their estimate. It can be seen from the waveforms that, the estimate converge to the true values with the adaptive gain H.

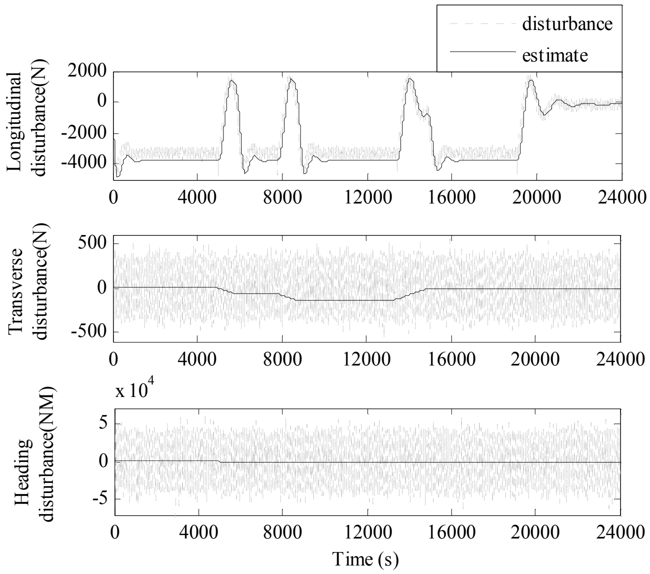

Fig. 5 Curves of disturbances and adaptive estimates in body fixed frame.

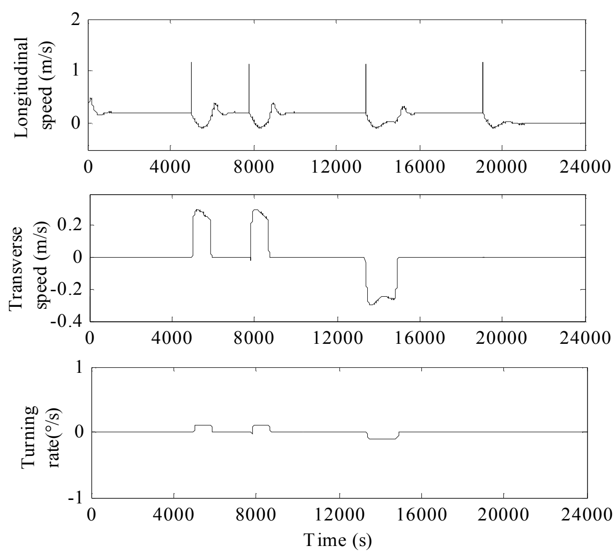

Fig. 6 Curves of speed response during tracking in body fixed frame.

The vessel's desired tracking speed is set as 0.2 m/s, and the heading is equal to the orientation of the path on the straight line paths. The turning rate is set as 0.1 deg/s when the vessel veers to the next way point. The response waveforms are shown in Fig. 6. The oscillation appears when the vessel switches to the next straight segment of the path, but the speed responses converge to the set values rapidly after the oscillation.

V. Conclusion

To avoid computing expansion, and make the controller be steady and robust to large disturbance and the uncertainty of the DP vessel control system, a novel nonlinear adaptive robust tracking control strategy is proposed for fully actuated DP vessels with big plough. Moreover, the vessel's desired path is obtained from the plough's given path by using the guidance strategy. To make the model more accurately, the mathematical model of the DP vessel is built by taking unknown hydrodynamic coefficients and disturbances from environment and big plough as the uncertain parameter vector. Then the uncertain parameter vector is dealt with by an adaptive estimator, so that the stability and the robustness can be ensured in the closed-loop system, and the semi-globally uniform bound is proved by Lyapunov theorem. Finally, simulations validate that, when using the proposed control strategy, the fully actuated DP vessel in reference [4] can tow the big plough track the desired path accurately.

Acknowledgment

The authors would like to thank the financial support of The National high-tech research for ships (GJCB09001).

References

[1] J. G. Balchen, N. A. Jenssen and S. Salid, "A dynamic positioning system based on kalman filtering and optimal control," *Modeling, Identification and Control,* Vol. 1, No. 3, pp. 135-163, 1980.

[2] J. G. Balchen, "A Modified LQG Algorithm (MLQG) for Robust Control of Nonlinear Multivariable Systems," *Modeling, Identification and Control,* Vol. 14, No. 3, pp. 175-180, 1993.

[3] M.R. Katebi, M.J. Grimble and Y. Zhang, "H∞ robust control design for dynamic ship positioning,", *IEEE Proceedings of Control Theory and Applications,* Vol. 144, No. 2, pp. 110-120, 1997.

[4] T. I. Fossen, *Handbook of marine craft hydrodynamics and motion control*, West Sussex, United Kingdom: John Wiley & Sons Ltd, 2011.

[5] T. I. Fossen and A. Grøvlen, "Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping," *IEEE Transaction on Control Systems Technology,* no.6, pp. 121-128, 1998.

[6] R. Skjetne. The maneuvering problem, Norway, CA: Norwegian University of Science and Technology, 2005.

[7] M. Breivik. *Topics in guided motion control of marine vehicles*, Trondheim, Norway, CA: Norwegian University of Science and Technology, 2010.

[8] M. Breivik and T. I. Fossen, "Motion control concepts for trajectory tracking of fully actuated ships," *Proceedings of the 7th IFAC MCMC*, Lisbon, Portugal, 2006.

[9] D. Swaroop, J. C. Gerdes, P. P. Yip. "Dynamic surface control of nonlinear systems," *Proceedings of the American Control Conference,* vol. 5, pp. 3028-3034, 1997.

[10] T. S. Li, G. Feng, Z. J. Zou. "DSC-backstepping based robust adaptive fuzzy control of a class of strict-feedback nonlinear systems," *Proceedings of the 2008 IEEE International Conference on Fuzzy Systems,* Hong Kong, China, pp. 1274-1281, 2008.

[11] Y. H. Zhang, J. G. Jiang, "Dynamic positioning of dredgers based on disturbances compensating," *Proceedings of the International Conference on E-Product, E-Service and E-Entertainment,* Henan, China, vol. 1, pp.1-4, 2010.

[12] T.Voldsund, *Modelling and control of offshore ploughing operations,* Trondheim, Norway, CA: Norwegian University of Science and Technology, 2007.