



# Mathematical modeling and simulation of an industrial rotary dryer: A case study of ammonium nitrate plant

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## ABSTRACT

Drying of solids is of great significance in fertilizer industry as the quality of final product should meet the commercial required standards. A rotary dryer which has been investigated in the present study is used in ammonium nitrate (AN) plant located at Shiraz Petrochemical Complex (SPC). This plant is designed to produce 650 metric tons of final products in a day. In this work, mathematical modeling of the concurrent rotary dryer for AN including heat and mass transfer equations between solid and air was developed. New correlations were proposed for AN equilibrium moisture and drying rate which were used in the modeling. The model was checked against the industrial data which showed a good agreement. The model predicts air and product moisture and temperature depending on working conditions of the rotary dryer. Regardless of the slope and speed of the dryer, inlet AN moisture and air temperature have been shown to be the variables that have the greatest effect, on the outlet moisture content of the product.

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## 1. Introduction

### 1.1. Ammonium nitrate plant

Ammonium nitrate (AN) is a chemical product which is frequently produced by neutralization of nitric acid with ammonia and mainly processed into high quality fertilizers. It is used straight for an oxidizing agent or blended for fertilizer or a constituent of many explosives [1].

The AN plant considered in this article located in Shiraz Petrochemical Complex (SPC) has been designed to produce, at will [2]:

- either 750 metric tons per day prills of AN containing 30% in weight of nitrogen and used as a fertilizer (AAN);
- or 650 metric tons per day porous prills of pure AN used to prepare an explosive (ANFO).

Many parts of the process are common to both products. The drying process is utilized in ANFO grade to dry wet particles and produce required porosity.

### 1.2. Drying process

Drying commonly describes the process of thermally removing moisture to yield a solid product. Despite its importance, in many cases the design and operation of dryers are done according to

empiricism, based on the experience of engineers [3]. However, observed progress has limited empiricism to a large extent.

Rotary drying is one of the many drying methods existing in unit operations of chemical engineering which is classified as direct, indirect–direct and indirect. This classification is based on the method of heat transfer between hot air and solid particles. The most commonly used rotary dryers in industry are direct contact type [4]. The wide use of rotary dryers is motivated by their ability to handle a wide range of solids such as fertilizers, pharmaceuticals, mineral concentrates, cement, sugar, soybean meal, corn meal, and plastics [5].

Several works were carried out in literature on the steady state modeling of the rotary drying process. Myklestad [6] was the first to obtain an expression to predict product moisture content throughout a rotary dryer. Sharples et al. [7] developed a steady state model of a concurrent fertilizer rotary dryer including material and energy balances. Thorpe [8] carried out an analysis similar to the one used by Sharples et al., but used another equation to calculate residence time. By subdividing the dryer into high enough elemental volumes, he obtained similar results to that of Sharples et al. However, in this case the results were neither compared with the experimental values. O'Donnell [9] developed a new equation to calculate retention time which was coupled with heat transfer equations to construct an overall complex and laborious dryer model. A simplified drying model was proposed by Kisakurek [10] who assumed a constant solid temperature and neglected sensible heat effects. Kamke and Wilson [11] developed a model to predict heat transfer using a retention time equation similar to that of Kelly and O'Donnell [12] and Ranz and Marshall [13] for wood. The model agreed with the experimental values and depicted that initial product moisture content and

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drying air temperature had the greatest effect on outlet product moisture content. In order to improve drying efficiency, another configuration of the rotary dryer known as roto-aerated dryer was presented by Lisboa [14] and Arruda [15,16]. Mathematical modeling of woody biomass drying was developed by Xu and Pang [17] and a new correlation between the theoretical maximum drying rate and the actual constant drying rate for the wood chips was proposed from the drying experiments. It was also found that the drying curve from the wood chips is within a falling rate drying period below the critical moisture of 55%. Iguaz et al. also assumed that during the drying process, there is no constant-rate period, which means that drying happens during the falling-rate only [18].

Generally, the performance of rotary dryers is dictated by three important transport phenomena, namely: solid transportation (Cao and Langrish [19]; Renaud [20]; Shahhosseini [21]; Song [22]; Sheehan [23]; Britton [24]), heat, and mass transfer (Arruda [15,16]; Lobato [25]; Kemp and Oakley [26]; Zabaniotou [27]; Iguaz [18]; Cao and Langrish [28]). The ability to estimate each of these transport mechanisms is essential for proper design and operation of rotary dryers (Krokida [29]). This approach requires constitutive equations for the heat transfer coefficients, drying kinetics, equilibrium moisture content and fluid dynamic characteristics.

Although several models have been proposed till present, there is not a general trend to best describe the mechanism of rotary drying. It seems that specific models for an equipment and material are more useful than general models. Moreover, there is a significant lack of information with respect to model validation with large scale (industrial) data.

The objective of this project is to develop a mathematical model to simulate the industrial rotary dryer for the AN particles and to provide information for the operation optimization. In this work, the concurrent rotary dryer will be modeled and the influence of different operating variables in the outlet moisture content as well as outlet temperature of the solid is studied. Industrial experiences suggest that the most significant parameters of AN exiting the rotary dryer which affect the commercial properties of the final product are material moisture and

temperature. These two variables have major effects on caking and size of particles. Therefore, the effect of various parameters on these variables was analyzed. Moreover, new correlations for equilibrium moisture and drying rate of the AN were proposed.

## 2. The AN drying description

As it was illustrated in Fig. 1, while melted AN is prilling from the showers at the top of the prilling tower down to the bottom, crystallization takes place along the tower height. At the bottom of the prilling tower, the ANFO prill temperature is around 90 °C which is then introduced to the entrance of rotary dryer by means of the belt conveyors. The grains commonly have sizes ranging from 1 to 3.15 mm and moisture content of about 2.5%. The rotary dryer used in this plant consists of a cylinder rotating upon bearings and slightly inclined to the horizontal which is simply illustrated in Fig. 1A. The feeding of the prills is ensured by a belt conveyor which is extended into the cylinder at the top end. The prills progress through the cylinder by means of rotation, head effect and slop, and discharged at the other end on the belt conveyor. According to Fig. 1B, the rotary dryer is equipped with flights on the interior for lifting and showering the prills through the hot air stream and partitions to increase the effectiveness of material distribution and reduce dusting during passage through the cylinder. The air is drawn from the atmosphere through a filter by means of a fan which is then heated up by an air heater.

As the AN prills are very heat sensitive and must be dried carefully, the hot air and prills flow co-currently to ensure that the hot air cools rapidly during the initial evaporation of surface moisture.

The full characteristics of material, service conditions and conditioned air in the current industrial rotary dryer are tabulated in Table 1.

Dust entrained in the exit air stream is recovered in a washing tower (scrubber) where it is dissolved with water and resulting solution is sent back to the neutralizer.

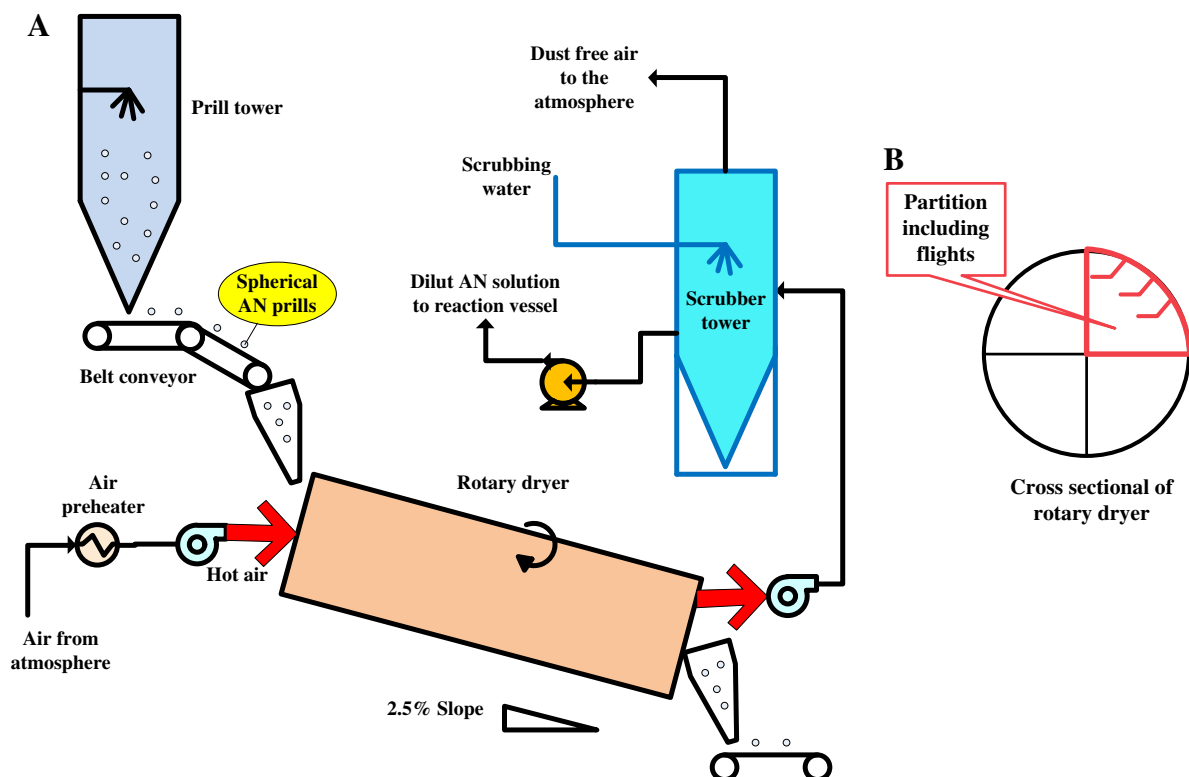


Fig. 1. A) Schematic diagram for solid crystallization and drying process. B) Cross sectional area of rotary dryer including flights and partitions.

### 3. Model development

#### 3.1. Energy and mass balances

A one-dimensional mathematical model considering a set of mass and energy balances was developed in order to predict the changes of water content and temperature of corresponding material along the dryer length. The mathematical model of rotary dryer consists of the following assumptions:

- The product particles have a spherical geometry and their dimensions remain unchanged along the drying process.
- The operation was assumed to be in steady state.
- The drying process takes place only in falling-rate period which will be supported by the laboratory experiments carried out to find drying rate. It means that the drying occurs below critical moisture content and there is no constant rate period.
- Both the flows of air and solid throughout the dryer follow a plug flow regime.
- Axial diffusion, dispersion or back-mixing of the solid is not taken into account.
- Effective contact time between solid and air was considered.
- The heat capacities of materials were assumed to be constant and the latent heat changes according to Clausius–Clapeyron's equation throughout the dryer length.

Fig. 2 shows the scheme of the infinitesimal volume element of the rotary dryer operating at a concurrent flow, where  $z$  is the non-dimensional length, given by the proportion between a given position ( $x$ ) and the total length of the dryer ( $L$ ).

From the conservation of the mass and energy around each element, a set of differential equations can be derived and expressed by:

$$\frac{dX}{dz} = -\frac{R \times M}{S} \quad (1)$$

$$\frac{dY}{dz} = \frac{R \times M}{G} \quad (2)$$

$$\frac{dT_s}{dz} = \frac{-Q - \lambda RM}{S(C_{pd} + XC_{pw})} \quad (3)$$

$$\frac{dT_g}{dz} = \frac{Q + C_{pv}RM(T_s - T_g) - Q_p}{G(C_{pg} + YC_{pv})} \quad (4)$$

The above Eqs. (1) to (4) can be solved numerically knowing the drying rate curve and the residence time to determine the changes in the air temperature and humidity, AN prill moisture content and temperature. In the above equations  $M$  is the total load (kg) which is defined as the product of solid flow rate and average residence

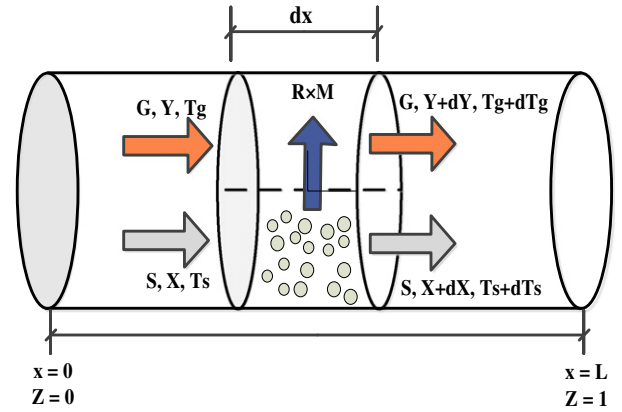


Fig. 2. The infinitesimal volume element of the rotary dryer.

time ( $M = \tau \times S$ ). The heat transfer within the control volume and lost through the shell wall were defined by the equations:

$$Q = U_{va}V(T_s - T_g) \quad (5)$$

$$Q_p = U_pA(T_g - T_{amb}). \quad (6)$$

The best correlations for global volumetric heat transfer coefficient ( $U_{va}$ ) and heat loss coefficient ( $U_p$ ) determined by Arruda [15] were used and expressed by:

$$U_{va} = 0.394(G/A)^{0.289}(S/A)^{0.541} \quad (7)$$

$$U_p = 0.022(G/A)^{0.879}. \quad (8)$$

#### 3.2. Equilibrium moisture and drying kinetic

Fig. 3 illustrates the experiments to characterize the product equilibrium isotherms through the thin layer drying experiments in order to derive a correlation that best fits the equilibrium moisture content of the simple AN. The following empirical correlation shows good agreement with the experimental data in the range of the experiments:

$$X_{eq} = RH(a \times RH^2 + b \times RH + c) \quad (9)$$

$$a = 2.39 \times 10^{-6} \times 0.987^{T_g} T_g^{-0.832} \quad (10)$$

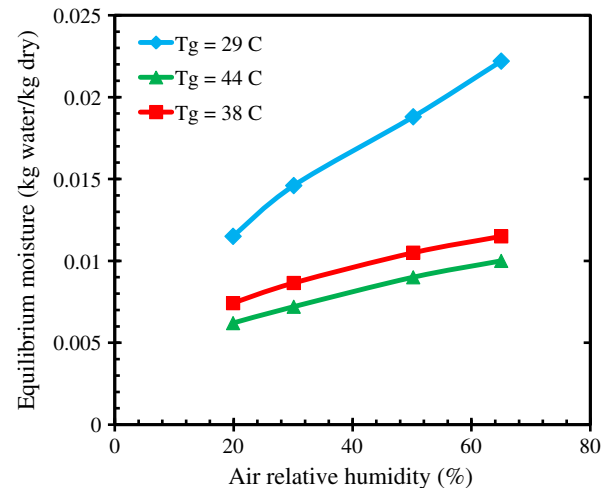


Fig. 3. Equilibrium isotherms for AN measuring by thin layer drying experiments.

**Table 1**  
Inlet properties of material and air to the rotary dryer [2].

Properties		Unit	AN		Air	
Operating temperature		°C	85		72.5	
Density		kg/m <sup>3</sup>	800		0.821	
Velocity		m/s	---		16.6	
Normal flow		kg/hr	31312		57372	
Dry	water	kg/hr	30122.1	1189.9	56100	1272
Normal flow		m <sup>3</sup> /hr	39.1		59522	

**Table 2**

Comparison of models for residence time based on industrial data.

Models	Residence time	Error %
Foust et al. [30]	31.113	+3.76
Friedman & Marshall [31]	10.581	−64.73
Perry & Green [32]	17.254	−42.49
Song et al. [22]	20.375	−32.086
Thibault et al. [5]	31.971	+6.56

$$b = -5.76 \times 10^{-5} + 1.306 \times 10^{-5} \times \ln(T_g) \quad (11)$$

$$c = 0.9715 \times 1.024^{T_s T_g^{-2.31}} \quad (12)$$

As the drying process is working in falling rate period, the drying rate expression can be assumed as following:

$$-\frac{dX}{dt} = R = kX + k' \quad (13)$$

Regarding the point that at equilibrium moisture the drying rate is equal to zero, the intersection ( $k'$ ) becomes:

$$k' = -k \times X_{eq} \quad (14)$$

Then,

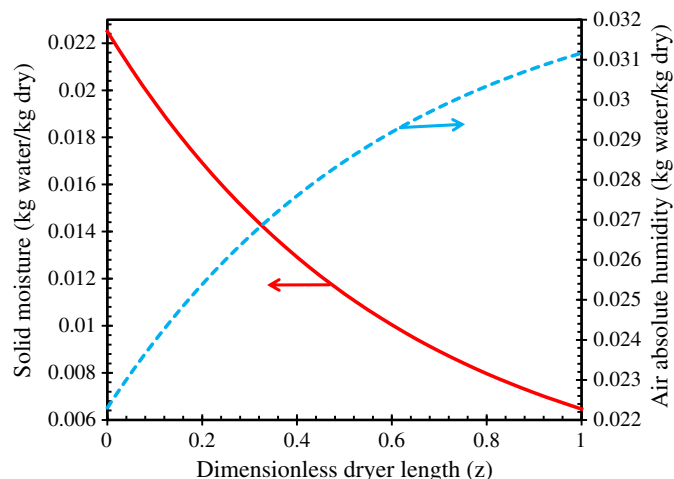
$$R = k(X - X_{eq}) \quad (15)$$

Drying experiments of AN sample of this plant and air temperatures similar to that used for equilibrium moisture experiments were made. It was found that the simple exponential equation [18] described adequately the drying kinetic of these particles. In Eq. (15),  $k$  is the drying constant and is related to the temperature of the drying air by:

$$k = 0.0349 \exp\left(\frac{-7.95}{T_a}\right) \quad (16)$$

### 3.3. Residence time

In order to express residence time as a function of dryer's characteristics, there are several surveys in the literature focused on this matter. Table 2 depicts some of the correlations and compares their deviation from that of real residence time measured for rotary dryer of AN plant. As can be seen, there is not a general model that best

**Fig. 4.** Solid moisture and air absolute humidity profiles along the dryer length.

describes the residence time of solids in the dryer yet. However, the relation proposed by Foust et al. [30] better fits the industrial rotary dryer residence time (30 min) and expressed as:

$$\tau = \frac{13.8L}{sn^{0.9}D} - \left(\frac{614.2}{D_p^{0.5}}\right) \frac{LG}{S} \quad (17)$$

### 4. Numerical solution

The model consists of four ordinary differential equations. The most widely used method of integration for ordinary differential equations are the series of methods called Runge–Kutta second, third and fourth orders. We used the fourth order method for solving the system of equations. An initial value problem is solved since all boundary conditions are at the same point by means of MATLAB code. The boundary conditions are:

$$X(0) = X_0; \quad Y(0) = Y_0; \quad T_s(0) = T_{s0}; \quad T_g(0) = T_{g0} \quad (18)$$

### 5. Model validation

The steady state model validation is performed between the plant data and the mathematical modeling of concurrent rotary dryer for AN. The model results and the corresponding observed data of the plant are presented in Table 3. The average absolute deviation (AAD) of the simulation results in relation to industrial data for the solid moisture was 4.04%, 1.33% for solid temperature, and 1.84% for

**Table 3**

Model validity for plant data.

Run no.	Dryer inlet variables			Dryer outlet variables					
	Product moisture (kg water/kg dry)	Product temperature (°C)	Air temperature (°C)	Product moisture (kg water/kg dry)		Product temperature (°C)		Air temperature (°C)	
				Meas.	Pred.	Meas.	Pred.	Meas.	Pred.
1	0.0256	80	73	0.0073	0.0071	73	71.91	69	68.55
2	0.0243	80	71	0.0068	0.0070	73	71.47	65	67.43
3	0.0214	81	73	0.0063	0.0062	72	71.90	69	68.54
4	0.0288	88	75	0.0081	0.0075	77	77.11	68	71.87
5	0.0225	82	73	0.0064	0.0065	72	73.65	69	69.24
6	0.0204	90	73	0.0056	0.0059	77	79.01	71	71.60
7	0.0277	89	75	0.0068	0.0072	77	77.91	72	72.20
8	0.0204	95	75	0.0055	0.0058	79	79.66	75	72.77

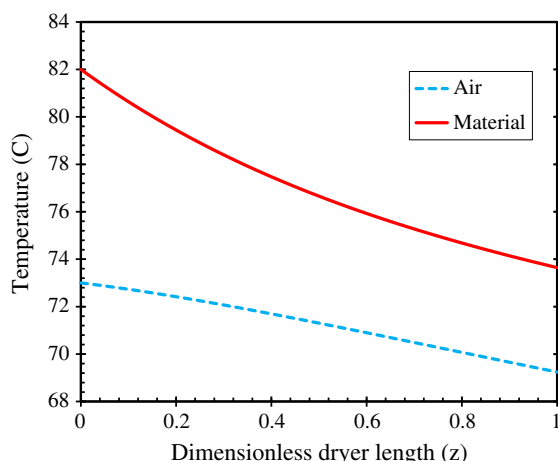


Fig. 5. Solid and air temperature profile along the dryer length.

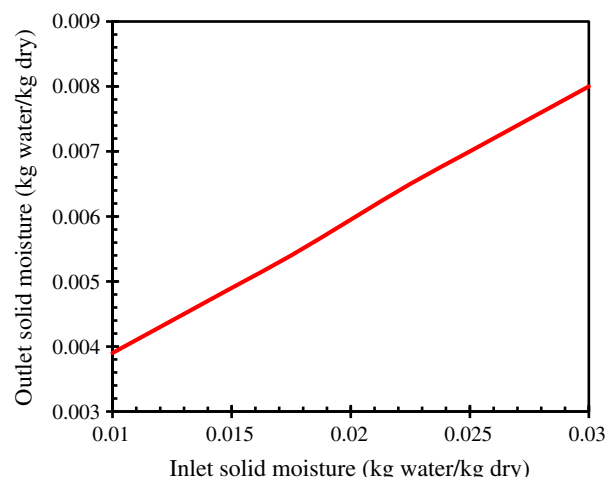


Fig. 6. Effect of inlet solid moisture variations on outlet solid moisture.

air temperature. Therefore, it can be said that the mathematical model performs well under the industrial condition.

## 6. Results and discussion

Fig. 4 shows the profile of material moisture and air absolute humidity and Fig. 5 shows the profile of material and air temperature along the length of the rotary dryer. The parameters used in the modeling are listed in Table 4 which is based on AN plant data of SPC.

Fig. 6 illustrates the effect of inlet moisture content of solid materials on that of outlet at constant operating condition. As can be seen, any changes in inlet moisture of solids have serious effect on outlet one in the dryer operating zone. Moreover, it can be concluded that the outlet moisture has approximately linear relation with inlet moisture under constant conditions.

Figs. 7 and 8 depict the behavior of outlet solid moisture and temperature with respect to any variations of inlet solid and air temperature respectively. The figures suggest that while inlet air temperature has more effect on the outlet solid moisture, the inlet solid temperature affects the outlet solid temperature more seriously.

Figs. 9 and 10 summarize the predicted effects of some selected input variables and rotary dryer parameters on outlet product moisture content and temperature respectively. The reference conditions for all the comparisons are the same as those tabulated in Table 4. Simulations were performed for each variation of a reference condition of plus and minus 30%, while all other conditions were held constant.

Within the range of conditions examined, the dryer inclination and rotation speed have the greatest effect on outlet product moisture

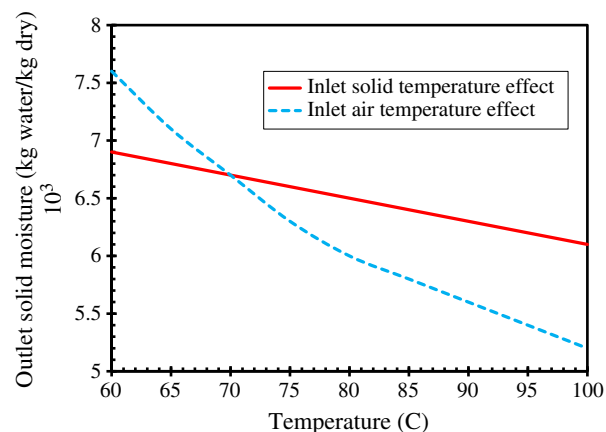


Fig. 7. Effect of inlet air and solid temperature variations on outlet solid moisture.

content while for outlet product temperature, the inlet solid temperature is the most effective parameter. Among the other operating variables affecting solid moisture, inlet air temperature and solid moisture are the next order of importance. Inlet air humidity was found as a variable that has no effect on outlet solid moisture content.

Considering the relations used for heat transfer and residence time, it can be found that inlet air flow rate has two opposed effects

Table 4

The industrial rotary dryer characteristics used in the modeling.

Properties	Unit	Value
Inlet solid temperature	°C	82
Inlet air temperature	°C	73
Inlet solid moisture	kg water/kg dry solid	0.0225
Inlet air absolute humidity	kg water/kg dry air	0.0223
Solid flow	kg/h	32,251
Air flow	kg/h	60,979
Dryer length	m	18
Dryer inside diameter	m	3.324
Dryer slope	m/m	0.025
Dryer rotational speed	RPM	3
Average solid diameter	mm	2
Solid heat capacity	kJ/kg °C	1.56
Air heat capacity	kJ/kg °C	1.009
Water heat capacity	kJ/kg °C	4.18
Vapor heat capacity	kJ/kg °C	1.88

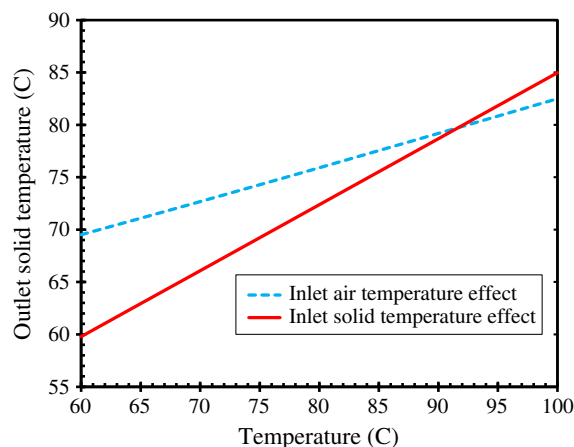


Fig. 8. Effect of inlet air and solid temperature variations on outlet solid temperature.





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