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Third Order Sliding Mode Control with Box State Constraints

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***Abstract*— This paper deals with the design of a third order Sliding Mode Control (SMC) algorithm for a perturbed chain of integrators with box constraints on state variables. The proposed strategy takes into account a robust generalization of the so-called Fuller's Problem, which is a standard optimal control problem for a chain of integrators under critical uncertainty condition, and proves to steer the system trajectories to the origin of the state space in finite time, while satisfying the imposed constraints. The proposed algorithm is tested in simulation to solve a trajectory-tracking problem for a nonholonomic car.**

I. INTRODUCTION

Control systems often need to cope with constraints on state variables, in order to avoid failures or critical conditions of the process. Such constraints are typical of various application contexts, including robots [1], automotive systems [2], and industrial processes like chemical plants or oil refineries [3].

The most well-known control methodology able to manage state constraints is Model Predictive Control (MPC) [4], [5]. MPC algorithms can provide an optimal control solution while satisfying constraints, and can be easily applied to multivariable plants with coupled dynamics. The drawback is typically an increased computation time with respect to explicit control laws, which in some cases makes MPC not suitable when very fast systems need to be controlled, at the same time using inexpensive hardware for implementing the control law. In order to guarantee the robustness of the control systems with respect to unavoidable modeling uncertainties or external disturbances, robust MPC laws have also been developed [6]–[10].

In many nonlinear control problems, especially those considering discontinuous control laws such as Sliding Mode Control (SMC), the objective is often to steer the state to the origin in finite time, in spite of uncertain terms affecting the systems dynamics, but the possible presence of state constraints is rarely taken into account. SMC is a powerful tool which can ensure the convergence of the system trajectories in finite time onto a suitably defined surface (the so-called sliding manifold), and, under suitable design conditions, make the origin of the state space an asymptotically stable equilibrium point for the closed-loop system. Moreover, SMC guarantees beneficial effects by making the systems, while in sliding mode, insensitive with respect to the so-called matched disturbances [11], [12]. Finally, SMC laws typically have a very low computation burden.

The main drawback of SMC is the so-called chattering phenomenon, which is the high-frequency oscillatory motion around the sliding manifold due to the discontinuity of the control law. Higher Order Sliding Mode (HOSM) proves to strongly reduce the chattering phenomenon: also for this reason, several HOSM algorithms have been proposed [13]–[19]. Among these, the algorithm obtained as solution of the so-called Fuller's Problem is presented in [20], where different HOSM algorithms are discussed to stabilize in finite time a perturbed chain of integrators with bounded control, while guaranteeing an optimal reaching of the sliding manifold. In case of second order HOSM algorithms, a solution to the problem of stabilizing the system to the origin, while satisfying box state constraints is proposed in [21], while a more general formulation in case of polyhedral constraints has been provided in [22]. To the best of our knowledge, for the third-order case no analogous solutions have been provided to cope with state constraints.

In this paper, the connection between the construction of a third order SMC with optimal reaching and the problem of satisfying box state constraints is studied. The sliding manifold is suitably designed and a switched control law is applied in order to maintain the state trajectory within the admissible region of the state space. Preliminary theoretical results are presented and the performance of the proposed algorithm are assessed in simulation on a benchmark example.

The paper is organized as follows. Section II reports basic elements of HOSM control theory. In Section III the control problem is formulated, while in Section IV the proposed solution is described. Section V describes the application of the proposed control law in simulation, and conclusions are discussed in Section VI. For the sake of readability, the proof of the main theoretical result of the paper is moved to the Appendix section.

II. BASIC CONCEPTS

Consider the nonlinear uncertain dynamics

$$\begin{cases} \dot{x}(t) = a(x,t) + b(x,t)u(t) \\ y(t) = \sigma(x(t)) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control variable, $\sigma : \mathbb{R}^n \to \mathbb{R}$ is a smooth output function, denoted as *sliding variable*, while $a(\cdot)$ and $b(\cdot)$ are unknown vector functions. The *relative degree* of the system, i.e. the minimum order r of the time derivative $\sigma^{(r)}$ of the sliding variable in which the control u explicitly appears, is assumed well defined, uniform

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and time invariant. In the following, the dependence of σ on $x(t)$ and of all the variables on t is omitted in some cases, when obvious, for the sake of simplicity.

For the readers' convenience, some basic notions of higher order sliding mode control theory are hereafter reported, along with a brief overview of the Robust Fuller's Problem and the third order sliding mode with optimal reaching published in the literature.

A. Higher Order Sliding Modes

The Higher Order Sliding Mode (HOSM) control problem is based on the regularization of an *auxiliary system* associated with the uncertain system (1), i.e., by posing $\sigma^{(i)} \triangleq d^i y/dt^i$ and $z_i \triangleq \sigma^{(i-1)}$,

$$\begin{cases} \dot{z}_i(t) = z_{i+1}(t), \quad i = 1, \dots, r-1 \\ \dot{z}_r(t) = f(x,t) + g(x,t)u(t) \end{cases} \tag{2}$$

which is a perturbed chain of integrators built starting from the sliding variable and its time derivatives, with $f(\cdot) = \sigma^{(r)}|_{u=0}$ and $g(\cdot) = (\partial\sigma^{(r)}/\partial u) \neq 0$ being unknown functions. Moreover, it is assumed that there exist positive constants $G_m,\ G_M,\ F$, such that

$$0 < G_m \leq g(x(t)) \leq G_M \tag{3}$$

$$|f(x(t),u(t))| \leq F \tag{4}$$

The control objective for an r-th order sliding mode control law is to force the system state to reach in finite time and remain on the r-*sliding manifold* $\sigma = \dot{\sigma} = \cdots = \sigma^{(r-1)} = 0$. The latter can be made finite-time attractive by using any $r-th$-order sliding mode controller of the type

$$u(t) = \alpha\Psi\left(\sigma, \dot{\sigma}, \dots, \sigma^{(r-1)}\right) \tag{5}$$

where Ψ is a discontinuous function, and $\alpha > 0$ is chosen so as to enforce a sliding mode [15]–[17].

B. Third Order Sliding Modes with Optimal Reaching

Taking into account the results presented in [20] and referring to the auxiliary system (2), the so-called Robust Fuller's Problem can be formulated as follows

$$\min_{u(\cdot),\ T\in\mathbb{R}^+} \left[\max_{f(\cdot),\ g(\cdot)} \int_{t_0}^{T} |\sigma(t)|^{\nu} dt \right]$$

subject to (2)-(4), $\sigma(t_0) = \sigma_0$, and bounded control $\|u\|_\infty \leq \alpha$. The time interval $[t_0,\ T]$ is compact, ν is a positive constant, and $\|\cdot\|_\infty$ denotes the norm of the Banach space L^∞. The underlying idea is that the control law guarantees the best control action for the worst-case realization of the uncertain terms.

Let $\alpha_r \triangleq (\alpha G_m - F) > 0$ denote the so-called "reduced control amplitude", i.e. the minimum possible amplitude $|\dot{z}_r|$, for any possible realization of the uncertain terms. The solution to the Robust Fuller's Problem with $\nu = 0$ in the third order case is given by

$$u_{or} = -\alpha \begin{cases} u_0 \triangleq 0, & (\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathcal{M}_0 \\ u_1 \triangleq \mathrm{sgn}(\ddot{\sigma}), & (\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathcal{M}_1 \setminus \mathcal{M}_0 \\ u_2 \triangleq \mathrm{sgn}\left(\dot{\sigma} + \frac{\ddot{\sigma}^2 u_1}{2\alpha_r}\right), & (\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathcal{M}_2 \setminus \mathcal{M}_1 \\ u_3 \triangleq \mathrm{sgn}(s(\sigma,\dot{\sigma},\ddot{\sigma})), & \text{otherwise} \end{cases} \tag{6}$$

where

$$s(\sigma,\dot{\sigma},\ddot{\sigma}) \triangleq \sigma + \frac{\ddot{\sigma}^3}{3\alpha_r^2} + u_2\left[\frac{1}{\sqrt{\alpha_r}}\left(u_2\dot{\sigma} + \frac{\ddot{\sigma}^2}{2\alpha_r}\right)^{\frac{3}{2}} + \frac{\dot{\sigma}\ddot{\sigma}}{\alpha_r}\right] \tag{7}$$

while $\mathcal{M}_0,\ \mathcal{M}_1,\ \mathcal{M}_2$ are defined as

$$\begin{aligned} \mathcal{M}_0 &\triangleq \left\{(\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathbb{R}^3 : \sigma = \dot{\sigma} = \ddot{\sigma} = 0\right\} \\ \mathcal{M}_1 &\triangleq \left\{(\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathbb{R}^3 : \sigma - \tfrac{\ddot{\sigma}^3}{6\alpha_r^2} = 0,\ \dot{\sigma} + \tfrac{\ddot{\sigma}|\ddot{\sigma}|}{2\alpha_r} = 0\right\} \\ \mathcal{M}_2 &\triangleq \left\{(\sigma,\dot{\sigma},\ddot{\sigma}) \in \mathbb{R}^3 : s(\sigma,\dot{\sigma},\ddot{\sigma}) = 0\right\} \end{aligned} \tag{8}$$

The surface $\mathcal{M}_2$ is referred to as *switching manifold*.

III. Problem Formulation

Consider the auxiliary third order system (2), expressed as

$$\begin{cases} \dot{z}_1(t) = z_2(t) \\ \dot{z}_2(t) = z_3(t) \\ \dot{z}_3(t) = f(x,t) + g(x,t)u(t) \end{cases} \tag{9}$$

where $z^T = [z_1\ z_2\ z_3] \in \mathbb{R}^3$ is the auxiliary state vector, the initial condition of which is $z(t_0) = z_0$. Note that the uncertain functions $f(\cdot)$ and $g(\cdot)$ are bounded as stated in (3)-(4). Moreover, the state z satisfies the constraints

$$z(t) \in \mathcal{Z}\ \forall t \geq t_0 \tag{10}$$

$\mathcal{Z}$ being a compact set containing the origin defined as

$$\mathcal{Z} \triangleq \left\{ (z_1,z_2,z_3) \in \mathbb{R}^3 : z_{1,m} \leq z_1 \leq z_{1,M},\ z_{2,m} \leq z_2 \leq z_{2,M},\ z_{3,m} \leq z_3 \leq z_{3,M} \right\} \tag{11}$$

where $z_{1,m},\ z_{2,m},\ z_{3,m} < 0$ and $z_{1,M},\ z_{2,M},\ z_{3,M} > 0$ are constant values. Then, with reference to (1), the problem faced in the present paper is that of zeroing the state of the auxiliary system in finite time while satisfying the state constraints (10) and (11), in spite of the presence of the uncertain terms.

IV. The Proposed Solution

In order to solve the problem formulated in Section III, consider a switching manifold $\mathcal{M}_2$ defined as in (8), and represented in Fig. 1 together with the constrained subspace. The proposed control law is defined as a modification of the third order sliding mode law with optimal reaching (6), as follows

$$u_{sc} = -\alpha \begin{cases} u_0 \triangleq 0, & z \in \mathcal{Z}_0 \\ u_1 \triangleq -1, & z \in (\mathcal{Z}_1 \cap \mathcal{Z}_3) \setminus \mathcal{Z}_0 \\ u_2 \triangleq 1, & z \in (\mathcal{Z}_2 \cap \mathcal{Z}_4) \setminus \mathcal{Z}_0 \\ u_3 \triangleq \mathrm{sgn}(s(z_1,z_2,z_3)), & \text{otherwise} \end{cases} \tag{12}$$

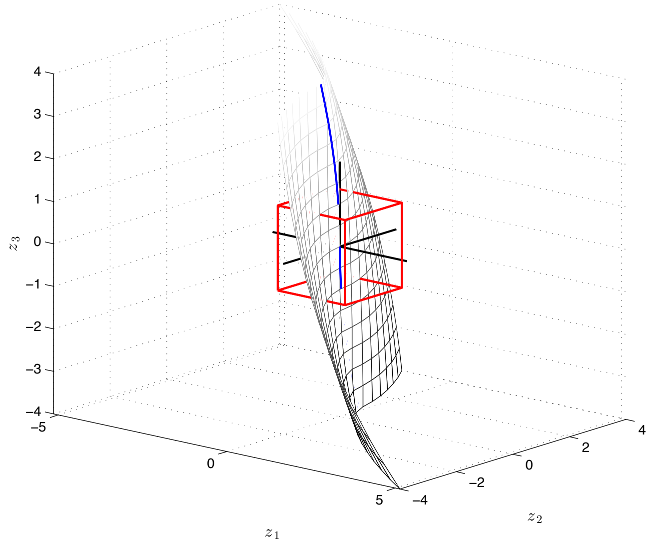

Fig. 1. Switching manifold $s(z_1, z_2, z_3)$ for the proposed third order controller with the state constraints.

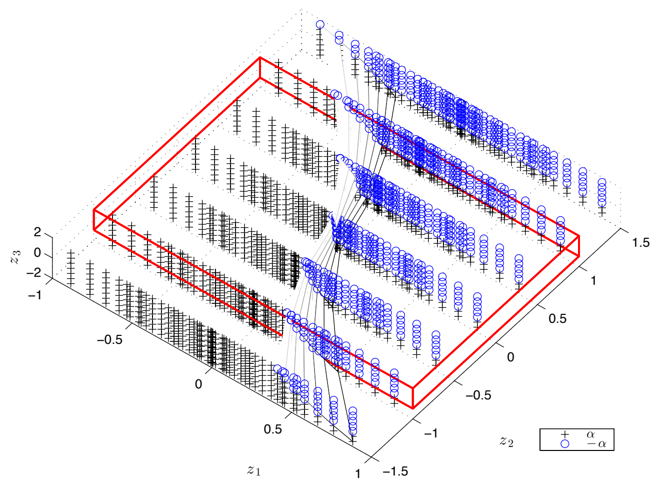

Fig. 2. Switching regions according to the proposed third order controller with the state constraints.

where the regions (see Fig. 2) in the auxiliary state space are the following

$$
\begin{aligned}
\mathcal{Z}_0 &\triangleq \left\{(z_1,\ z_2,\ z_3) \in \mathbb{R}^3 : z_1 = z_2 = z_3 = 0\right\} \\
\mathcal{Z}_1 &\triangleq \left\{(z_1,\ z_2,\ z_3) \in \mathbb{R}^3 : z_3 \leq z_{3,m}\right\} \\
\mathcal{Z}_2 &\triangleq \left\{(z_1,\ z_2,\ z_3) \in \mathbb{R}^3 : z_3 \geq z_{3,M}\right\} \\
\mathcal{Z}_3 &\triangleq \Bigg\{(z_1,\ z_2,\ z_3) \in \mathbb{R}^3 : z_3 \geq z_{3,m},\ z_1 < -\frac{z_3^3}{3\alpha_r^2} \\
&\qquad -u_2\left[\frac{1}{\sqrt{\alpha_r}}\left(u_2 z_2 + \frac{z_3^2}{2\alpha_r}\right)^{\frac{3}{2}} + \frac{z_2 z_3}{\alpha_r}\right]\Bigg\} \\
\mathcal{Z}_4 &\triangleq \Bigg\{(z_1,\ z_2,\ z_3) \in \mathbb{R}^3 : z_3 \leq z_{3,M},\ z_1 > -\frac{z_3^3}{3\alpha_r^2} \\
&\qquad -u_2\left[\frac{1}{\sqrt{\alpha_r}}\left(u_2 z_2 + \frac{z_3^2}{2\alpha_r}\right)^{\frac{3}{2}} + \frac{z_2 z_3}{\alpha_r}\right]\Bigg\}
\end{aligned}
\tag{13}
$$

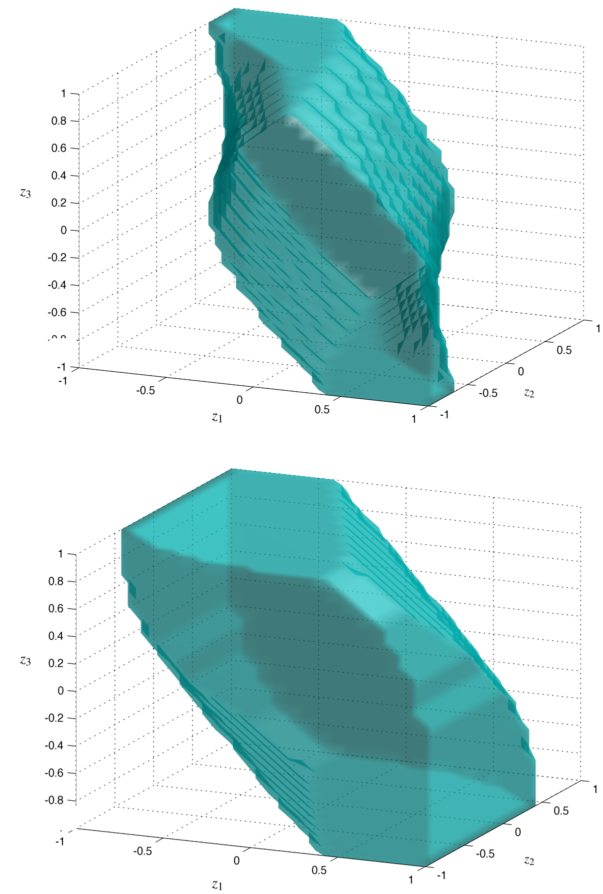

Fig. 3. Region of attraction applying the unconstrained control law (top) and the constrained control law (bottom).

In order to address the stability properties of the proposed control law, it is necessary to introduce the definition of the following sets.

Definition 1: $\Omega_{or} \triangleq \{\bar{z} \in \mathcal{Z} \ : \ (z(t_0) = \bar{z}) \wedge (u(t) \equiv u_{or}(t)\ \forall\ t \geq t_0) \Rightarrow z(t) \in \mathcal{Z}\ \forall\ t \geq t_0\}$, where the state-space equations and the definition of the control law are defined in (9) and (6), respectively.

Definition 2: $\Omega_{sc} \triangleq \{\bar{z} \in \mathcal{Z} \ : \ (z(t_0) = \bar{z}) \wedge (u(t) \equiv u_{sc}(t)\ \forall\ t \geq t_0) \Rightarrow z(t) \in \mathcal{Z}\ \forall\ t \geq t_0\}$, where the state-space equations and the definition of the control law are defined in (9) and (12), respectively.

Notice that, by construction, the sets Ω_{or} and Ω_{sc} are invariant.

Proposition 1: Given system (9), with the state constraints (10)-(11), then, applying the control law (12), one has $\Omega_{or} \subset \Omega_{sc}$, $\forall\ t \geq t_0$. Moreover, applying the control law (12), given any initial condition $z(t_0) \in \Omega_{sc}$ there exists a finite time instant $\bar{t} \geq t_0$ such that $z(\bar{t}) = 0$ for all $t \geq \bar{t}$.

Sketch of the proof: see Appendix.

Remark 1: Note that, because of the complexity of the problem, we did not focus on obtaining analytical expressions of sets Ω_{or} and Ω_{sc}. However, approximated numerical evaluations of such sets can be easily obtained, as shown in Fig. 3. It is possible to compute that, for an example system, the volume of Ω_{sc} is larger than the volume of Ω_{or} by 50%.

V. Simulation Example

In this section an illustrative example is presented to show the effectiveness of the proposed control law. The system models the kinematics of a nonholonomic car (see Fig. 4), and has already been considered as a benchmark for the application of higher-order SMC laws [16], [20], [23], [24]. The nonlinear state-space model of the system is

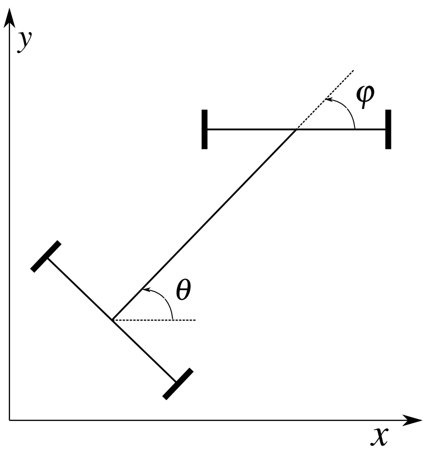

Fig. 4. Schematic view of the car model.

$$\begin{cases} \dot{x}(t) = v\cos\varphi(t) \\ \dot{y}(t) = v\sin\varphi(t) \\ \dot{\varphi}(t) = \frac{v}{l}\tan\theta(t) \\ \dot{\theta}(t) = u(t) \end{cases} \tag{14}$$

where x and y denote the cartesian coordinates of the rear-axle middle point, φ is the orientation angle, θ is the steering angle and u is the control variable. The longitudinal velocity is chosen as $v = 10\,\mathrm{m\,s^{-1}}$, while the distance between the two axles is $l = 5\,\mathrm{m}$. The sliding variable σ is defined as

$$\sigma(t) = y(t) - y_d(t) \tag{15}$$

with $y_d(t) = 10\sin(x(t)/20) + 5$ being the desired trajectory, and it can be shown that the relative degree of the system is $r = 3$ (see, e.g., [20]). Following the same approach as in [20] for the sake of simplicity, we consider the case in which $\varphi \approx 0$ and $\theta \approx 0$, so that

$$\sigma^{(3)}(t) \approx 50\cos\left(\frac{z_1(t)}{20}\right) + 20u(t) \tag{16}$$

and by posing $z_i = \sigma^{(i-1)}$, $i = 1,2,3$ one has

$$\begin{cases} \dot{z}_1(t) = z_2(t) \\ \dot{z}_2(t) = z_3(t) \\ \dot{z}_3(t) = 50\cos\left(\frac{z_1(t)}{20}\right) + 20u(t) \end{cases} \tag{17}$$

subject to the following constraints

$$|z_1| \le 7, \; |z_2| \le 7, \; |z_3| \le 7 \tag{18}$$

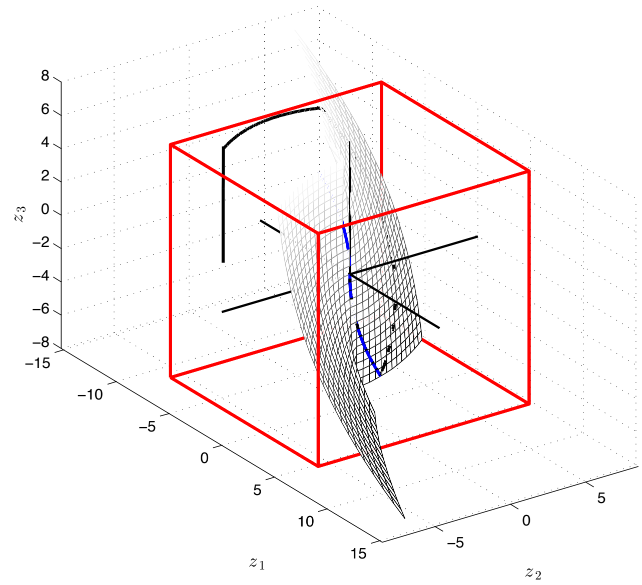

Fig. 5. State trajectories in the state space $\{z_1, z_2, z_3\}$.

The bounds in (3) and (4) are $F \approx 50$ and $G \approx 20$. We conservatively consider $F = 90$ and $G_m = 5$, $G_M = 5000$, and choose the control amplitude as $\alpha = 20$, from which $\alpha_r = 10$. The evolution of the system states has been simulated over a time interval $T_s = 30\,\mathrm{s}$, with initial condition $z(0) = \begin{bmatrix} -5 & -5 & 0 \end{bmatrix}^T$.

Fig. 5 reports the auxiliary state trajectories, which are steered to zero while satisfying the constraints, in the state space. In Fig. 6 the time evolution of the auxiliary state is reported. Fig. 7, Fig. 8 and Fig. 9 report the projections of the state trajectories on planes $\{z_1, z_3\}$, $\{z_2, z_3\}$ and $\{z_1, z_2\}$, respectively. Fig. 10 illustrates the time evolution of the car trajectory y.

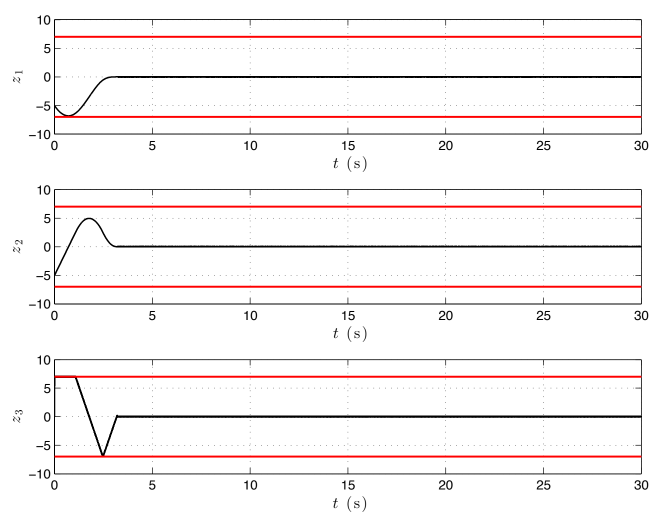

Fig. 6. Time evolution of the auxiliary state trajectories (z_1, z_2, z_3: solid black line) and the corresponding constraints (solid red line).

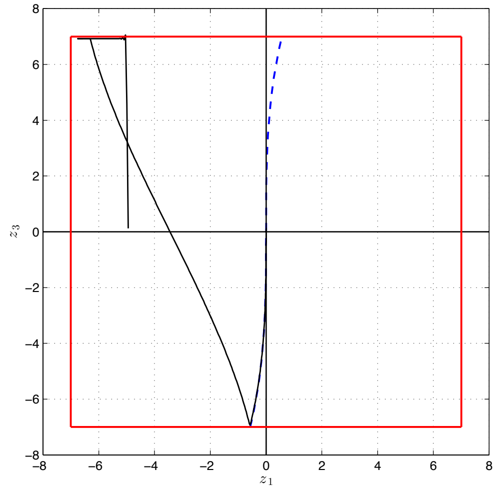

Fig. 7. Time evolution of the auxiliary state in the switching regions on the plane $\{z_1, z_3\}$ with $z_2 = -7$.

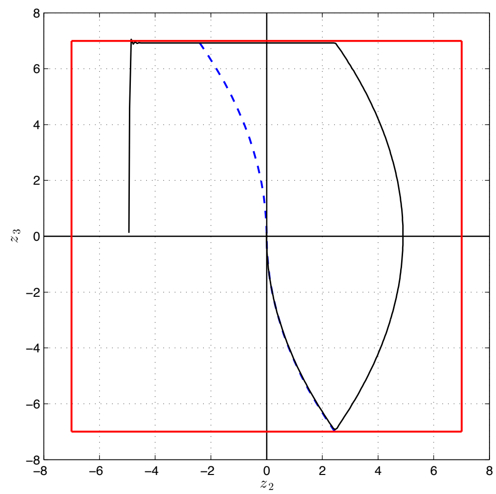

Fig. 8. Time evolution of the auxiliary state in the switching regions on the plane $\{z_2, z_3\}$ with $z_1 = 7$.

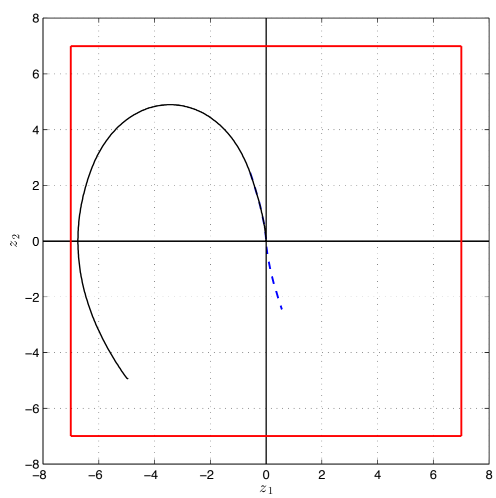

Fig. 9. Time evolution of the auxiliary state in the switching regions on the plane $\{z_1, z_2\}$ with $z_3 = 7$.

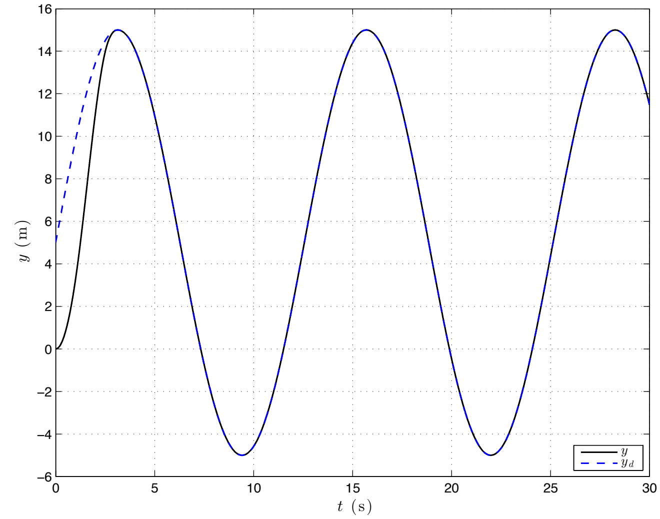

Fig. 10. Time evolution of car trajectory y (solid black line) with respect to the desired trajectory y_d (dashed blue line).

VI. Conclusions

This paper has addressed the problem of finite-time regulation to the origin of a perturbed third order chain of integrators, while satisfying box state constraints. Also taking into account recently-published results on sliding mode control with optimal reaching, it is shown that the set of points that can be steered to the origin without violating the constraints is enlarged with respect to using the original (unconstrained) control law. The effectiveness of the proposed strategy is finally assessed in simulation on a benchmark problem.

Appendix

Sketch of the proof of Proposition 1. As a preliminary consideration, one can notice that, when $z(t_0) \in \Omega_{or} \subset \mathcal{Z}$, then $u_{or}(z) = u_{sc}(z)$ by construction. As a consequence, $\Omega_{or} \subseteq \Omega_{sc}$. Therefore, according to [20], if $z(t_0) \in \Omega_{or}$, the convergence in finite time to the origin is ensured also satisfying the state constraints. In the following, we prove that there exists a set of points which belong to Ω_{sc} but not to Ω_{or}, and for which the finite-time convergence to the origin is obtained, which would prove the proposition.

Define Π as the plane perpendicular to the z_3 axis with $z_3 = z_{3,m}$, i.e., Π includes the lower face of the box which defines the constraints (the case with $z_3 = z_{3,M}$ is specular). Assume, without loss of generality, that $z_{3,m} = -1$. The intersection of Π with the switching surface defined in (7) is a curve (referred to as S^*), each point of which satisfies

$$z_1 - \frac{1}{3\alpha_r^2} + u_2\left[\frac{1}{\sqrt{\alpha_r}}\left(u_2 z_2 + \frac{1}{2\alpha_r}\right)^{\frac{3}{2}} - \frac{z_2}{\alpha_r}\right] = 0 \quad (19)$$

u_2 being defined in (6). The intersection of the switching line defined by $\mathcal{M}_1$ in (8) with Π is a point, with coordinates

$$z_1^* = -\frac{1}{6\alpha_r^2}, \quad z_2^* = \frac{1}{2\alpha_r}, \quad z_3^* = -1 \quad (20)$$

Now, consider the set $P \triangleq \{z \in \mathbb{R}^3 \;:\; z \in s^*(z) \cap \mathcal{Z} \wedge z_2 > z_2^*\}$. One can observe that, if $z \in P$, then $u_{or} = -\alpha$. As a consequence, $\dot{z}_3 \leq -\alpha_r < 0$, which implies that the vector field at z is directed out of $\mathcal{Z}$. Therefore, $P \cap \Omega_{or} = \emptyset$. By definition of u_{sc}, $z \in P$ implies $u_{sc}(z) = \alpha$. The components of the vector field are such that $\dot{z}_3 \geq \alpha_r > 0$, and $\dot{z}_2 = z_3 = -1 < 0$. However, as z_3 increases of an infinitesimal quantity, one can observe that the value of $u_{sc}(z)$ switches to $u_{or} = -\alpha$, which implies $\dot{z}_3 \leq \alpha_r < 0$, and $\dot{z}_2 = z_3 = -1 < 0$. The direction of the resulting vector field, obtained by merging these two components can be generated as the Filippov solution of the state-space equation, which is a vector field having the same components of the two component vectors as for $\dot{z}_1$ and $\dot{z}_2$, while $\dot{z}_3 = 0$. As a consequence, the state will move along the plane Π with decreasing values of z_2: by definition of u_{sc}, the state will move along P. If we consider equation (19) as a function relating z_2 to z_1, it is possible to observe that such function has a monotonic behavior. As a consequence, also due to the fact that $\alpha_r > 0$, the point $Q \triangleq \mathcal{M}_1 \cap \Pi$ is reached in finite time without violating the box constraints. After Q is reached, since $Q \in \Omega_{or}$, the state vector will be steered to the origin in finite time, again without violating the box constraints. ■

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