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A NOVEL TECHNIQUE FOR OPTIMAL CAPACITOR PLACEMENT AND SIZING IN DISTRIBUTION SYSTEMS WITH NON-LINEAR LOADS BASED ON HARMONIC SOURCE IDENTIFICATION

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Abstract: *With increasing nonlinear loads, harmonic currents are injected into networks which distort all of the voltages and currents. This is an issue which must be considered when making a decision regarding capacitor placement so that resonance will not occur, and therefore losses and severe stress on the network equipment is prevented. In this paper, a new method is proposed based on the harmonic source identification and current separation to cope with nonlinear loads in the capacitor placement problem. Intrinsic difference between linear and nonlinear loads in their V-I characteristics can be used to model harmonic sources. Also, genetic algorithm (GA) is used to solve the related optimization problem. To validate the proposed method, an IEEE 33-bus test system is simulated. Based on the simulation results, by using the proposed method, the influence of harmonics is considered properly. Consequently, corresponding savings is increased and also the solution will not cause any resonance.*

Keywords: *Optimal capacitor placement, non-linear load, harmonic analysis, genetic algorithm (GA), harmonic source identification and current separation*

1. Introduction

Capacitors are very important for improvement of power factor and reducing network losses in the power systems. In recent years, the problem has been reformulated to account for the discrete nature of capacitor sizes and locations. One assumption that is still being made in formulating the capacitor placement problem is that all loads are considered to be linear [1]-[8]. In practice, a portion of the electric loads are nonlinear due to the widespread use of fluorescent lamps and power electronic devices. Recent measurements on typical distribution systems show that there is a significant amount of harmonic distortion in voltage and current waveforms. Consequently, if harmonic currents injection into power systems is not considered, shunt capacitors can amplify harmonic currents and voltages due to possible resonance at one or several harmonic frequencies. This condition could lead to dangerous magnification of harmonic signals, additional stress on equipment insulation, increased capacitor failure and interference with communication systems [9]-[11].

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Some efforts have recently been directed towards considering the effects of harmonics on capacitor placement problem and load models at high frequencies [12]-[19].

In some publication such as [17], it has been assumed that harmonics are generated only at the substation supply and also all electrical loads are linear. Whereas, a portion of the electric loads are nonlinear; and distorted voltages and currents. In other methods, without attention to the types of loads (harmonic producing or consuming), harmonic contents of busses are determined and included in the capacitor placement problem. Existence of at least one harmonic source in the power system can distort all voltages and currents. Therefore, measuring harmonic contents without paying attention to loads type will result in improper nonlinear effect considerations.

This paper outlines a new procedure to take the nonlinear effect into account in the capacitor placement problem, so that it included harmonic sources locations and magnitudes. The difference between linear and nonlinear loads in their harmonic $V-I$ characteristic is used for harmonic source modeling and identification. Also, as the cost function is not explicitly available; therefore the GA is used for optimization of the problem. Based on the case study undertaken in this work, it has been observed that by

the proposed method more saving is obtained as compared to other methods. Moreover, using the capacitors obtained by this method, the resonance is not occurred at all possible frequencies.
This paper is organized as follows: In section 2 the principles of harmonic source identification and current separation is described. In section 3 determination of the model parameters by using least squares estimation is presented, and in section 4 the mathematical formulation of optimal capacitor placement is outlined. In section 5 solving the optimization problem based on genetic algorithm is described. In section 6 the results obtained in an IEEE 33-bus test system are given. Finally, concluding remarks are made in section 7.

2. Using V-I Characteristic of Loads for Harmonic Source Identification and Current Separation

Consider a composite load to be studied in a distribution system, which may represent an individual consumer or a group of customers supplied by a common feeder in the system. In this section, the following assumptions are made [20]:

a) The supply voltage and the load currents are both periodical waveforms with period T, so that they can be expressed by Fourier series as:

$$v(t)=\sum_{h=1}^{\infty}\sqrt{2}V_h\sin(2\pi ht/T+\theta_h)$$
$$i(t)=\sum_{h=1}^{\infty}\sqrt{2}I_h\sin(2\pi ht/T+\phi_h) \quad (1)$$

The fundamental frequency and harmonic components can further be presented by corresponding phasors:

$$V_{hr}+jV_{hi}=V_h\angle\theta_h$$
$$I_{hr}+jI_{hi}=I_h\angle\Phi_h,\quad h=1,2,3,\ldots,n \quad (2)$$

b) During the period of identification, the composite load is stationary, i.e. both its composition and circuit parameters of all individual loads keep unchanged.

Under the above assumptions, the relationship between the total harmonic currents of the harmonic sources (denoted by subscript N) in the composite load and the supply voltage, i.e. the $V-I$ characteristics, can be described by the following nonlinear equation:

$$i_N(t)=f(v(t)) \quad (3)$$

and can also be represented in terms of phasors as:

$$I_{Nh}=\begin{bmatrix}I_{Nhr}(V_1,V_{2r},V_{2i},\ldots,V_{nr},V_{ni})\\ I_{Nhi}(V_1,V_{2r},V_{2i},\ldots,V_{nr},V_{ni})\end{bmatrix},\quad h=2,3,\ldots,n \quad (4)$$

Note that in (4), the initial time (reference time) of the voltage waveform has been properly selected such that the phase angle θ_1 becomes 0 and $V_{1i}=0$, $V_{1r}=V_1$ in (2) for simplicity.
The $V-I$ characteristics of the linear part (denoted by subscript L) of the composite load can be represented by its equivalent harmonic admittance $Y_{Lh}=G_{Lh}+jB_{Lh}$, and the total harmonic currents absorbed by the linear part can be described as:

$$I_{Lh}=\begin{bmatrix}I_{Lhr}\\ I_{Lhi}\end{bmatrix}=\begin{bmatrix}G_{Lh} & -B_{Lh}\\ B_{Lh} & G_{Lh}\end{bmatrix}\begin{bmatrix}V_{hr}\\ V_{hi}\end{bmatrix},\quad h=2,3,\ldots,n \quad (5)$$

From Eqs. (4) and (5), the whole harmonic currents absorbed by the composite load can be expressed as:

$$I_h=\begin{bmatrix}I_{hr}\\ I_{hi}\end{bmatrix}=\begin{bmatrix}I_{Lhr}\\ I_{Lhi}\end{bmatrix}-\begin{bmatrix}I_{Nhr}(V_1,V_{2r},V_{2i},\ldots,V_{nr},V_{ni})\\ I_{Nhi}(V_1,V_{2r},V_{2i},\ldots,V_{nr},V_{ni})\end{bmatrix},\quad h=2,3,\ldots,n \quad (6)$$

As the $V-I$ characteristics of harmonic source are nonlinear, (6) can neither be directly used for harmonic source identification nor for harmonic current separation. To facilitate practical work, simplified methods should be used. The common approach in harmonic studies is to represent nonlinear loads by means of current harmonic sources or equivalent Norton models [21]-[22]. However, these models are imprecise precision and new simplified model is needed.
From the engineering point of view, the variations of V_{hr} and V_{hi}, usually fall into $\pm 3\%$ bound of the rated bus voltage, while the change of V_1 is usually less than $\pm 5\%$. Within such a range of supply voltages, the following simplified linear relation is used to approximate the harmonic source characteristics, i.e. (4)

$$I_{Nh}=\begin{bmatrix}a_{h0}+a_{h1}V_1+a_{h2r}V_{2r}+a_{h2i}V_{2i}+\ldots+a_{hnr}V_{nr}+a_{hni}V_{ni}\\ b_{h0}+b_{h1}V_1+b_{h2r}V_{2r}+b_{h2i}V_{2i}+\ldots+b_{hnr}V_{nr}+b_{hni}V_{ni}\end{bmatrix}$$
$$h=2,3,\ldots,n \quad (7)$$

The total harmonic current (equation (6)) then becomes:

$$I_h=\begin{bmatrix}G_{Lh} & -B_{Lh}\\ B_{Lh} & G_{Lh}\end{bmatrix}\begin{bmatrix}V_{hr}\\ V_{hi}\end{bmatrix}-\begin{bmatrix}a_{h0}+a_{h1}V_1+a_{h2r}V_{2r}+a_{h2i}V_{2i}+\ldots+a_{hni}V_{ni}\\ b_{h0}+b_{h1}V_1+b_{h2r}V_{2r}+b_{h2i}V_{2i}+\ldots+b_{hni}V_{ni}\end{bmatrix},\quad (8)$$
$$h=2,3,\ldots,n$$

It can be seen from the above equations that the harmonic currents of the harmonic sources (nonlinear loads) and the linear loads differ from each other intrinsically in their $V-I$ characteristics. The harmonic current component drawn by the linear loads is uniquely determined by the harmonic voltage component with same order in its supply voltage (corresponding bus voltage). On the other hand, the harmonic current component of the nonlinear loads contains not only a term caused by the same order harmonic voltage but also a constant term and the terms caused by fundamental and harmonic voltages of all other orders. This property will be used for identifying the existence of harmonic sources in composite load.

In this paper, further approximation for (7) can be made as follows:

Let

$$I'_{Nh} = \begin{bmatrix} a_{h0} + a_{h1}V_1 + \sum_{\substack{k=2\\k\neq h}}^{n} (a_{hkr}V_{kr} + a_{hki}V_{ki}) \\ b_{h0} + b_{h1}V_1 + \sum_{\substack{k=2\\k\neq h}}^{n} (b_{hkr}V_{kr} + b_{hki}V_{ki}) \end{bmatrix}$$

$$I''_{Nh} = \begin{bmatrix} a_{hhr} & a_{hhi} \\ b_{hhr} & b_{hhi} \end{bmatrix} \begin{bmatrix} V_{hr} \\ V_{hi} \end{bmatrix}$$

$$I'_{Lh} = I_{Lh} - I''_{Nh} = \begin{bmatrix} a'_{hhr} & a'_{hhi} \\ b'_{hhr} & b'_{hhi} \end{bmatrix} \begin{bmatrix} V_{hr} \\ V_{hi} \end{bmatrix},$$

$$\begin{bmatrix} a'_{hhr} & a'_{hhi} \\ b'_{hhr} & b'_{hhi} \end{bmatrix} = \begin{bmatrix} G_{Lh} & -B_{Lh} \\ B_{Lh} & G_{Lh} \end{bmatrix} - \begin{bmatrix} a_{hhr} & a_{hhi} \\ b_{hhr} & b_{hhi} \end{bmatrix},$$

$$h = 2,3,\ldots,n$$

The total harmonic current of the composite load becomes:

$$I_h = I_{Lh} - I_{Nh} = I'_{Lh} - I'_{Nh} = \begin{bmatrix} a'_{hhr} & a'_{hhi} \\ b'_{hhr} & b'_{hhi} \end{bmatrix} \begin{bmatrix} V_{hr} \\ V_{hi} \end{bmatrix} - \begin{bmatrix} a_{h0} + a_{h1}V_1 + \sum_{\substack{k=2\\k\neq h}}^{n} (a_{hkr}V_{kr} + a_{hki}V_{ki}) \\ b_{h0} + b_{h1}V_1 + \sum_{\substack{k=2\\k\neq h}}^{n} (b_{hkr}V_{kr} + b_{hki}V_{ki}) \end{bmatrix}, \quad h = 2,3,\ldots,n \tag{9}$$

By ignoring I''_{Nh} in the harmonic current of nonlinear load and adding it to the harmonic current of linear load, I'_{Nh} can then be deemed as harmonic current of the nonlinear load while I'_{Lh} can be taken as harmonic current of linear load. $I'_{Nh} = 0$ means the composite load contains no harmonic sources, while $I'_{Nh} \neq 0$ signify that harmonic sources may exist in this composite load.

3. Determination of Model Parameters Based on Least Squares Estimation

In order to identify the existence of harmonic sources in a composite load, the parameters in (9) should be determined primarily. These parameters can be considered as follows:

$$C_{hr} = \left[a_{h0}\, a_{h1}\, a_{h2r}\, a_{h2i} \cdots a'_{hhr}\, a'_{hhi} \cdots a_{hnr}\, a_{hni}\right]$$
$$C_{hi} = \left[b_{h0}\, b_{h1}\, b_{h2r}\, b_{h2i} \cdots b'_{hhr}\, b'_{hhi} \cdots b_{hnr}\, b_{hni}\right]$$

For this purpose, measurement of different supply voltages and corresponding harmonic currents of the composite load should be repeatedly performed several times in some short period while keeping the composite load stationary. The change of supply voltage can for example be obtained by switching in or out some shunt capacitors, disconnecting a parallel transformer or changing the tap position of transformer. Then, the least squares approach can be used to estimate the parameters by the measured voltages and currents. The identification procedure will be explained as follows [20]:

1) Perform the test for m $(m \geq 2n)$ times to get measured fundamental frequency and harmonic voltage and current phasors $V_h^{(k)} \angle \theta_h^{(k)}$, $I_h^{(k)} \angle \phi_h^{(k)}$ ($k = 1,2,\ldots,m$, and $h = 1,2,\ldots,n$).
2) For $k = 1,2,\ldots,m$, transfer the phasors corresponding to zero fundamental voltage phase angle $(\theta_1^{(k)} = 0)$ and change them into orthogonal components, i.e.

$$V_{1r}^{(k)} = V_1^{(k)}, \quad V_{1i}^{(k)} = 0$$
$$V_{hr}^{(k)} = V_h^{(k)} \cos(\theta_h^{(k)} - h\theta_1^{(k)})$$
$$V_{hi}^{(k)} = V_h^{(k)} \sin(\theta_h^{(k)} - h\theta_1^{(k)})$$
$$I_{hr}^{(k)} = I_h^{(k)} \cos(\phi_h^{(k)} - h\theta_1^{(k)})$$
$$I_{hi}^{(k)} = I_h^{(k)} \sin(\phi_h^{(k)} - h\theta_1^{(k)}), \quad (h = 2,3,\ldots,n)$$

3) Let

$$\mathbf{V}^{(k)} = [1 \quad V_1^{(k)} V_{2r}^{(k)} V_{2i}^{(k)},\ldots,V_{hr}^{(k)} V_{hi}^{(k)},\ldots,V_{nr}^{(k)} V_{ni}^{(k)}]^T,$$
$$k = 1,2,\ldots,m$$
$$\mathbf{X} = \left[\mathbf{V}^{(1)} \quad \mathbf{V}^{(2)} \quad \ldots \quad \mathbf{V}^{(m)}\right]^T$$
$$\mathbf{W}_{hr} = \left[I_{hr}^{(1)} \quad I_{hr}^{(2)} \quad \ldots \quad I_{hr}^{(m)}\right]^T$$
$$\mathbf{W}_{hi} = \left[I_{hi}^{(1)} \quad I_{hi}^{(2)} \quad \ldots \quad I_{hi}^{(m)}\right]^T$$

Minimize $\sum_{k=1}^{m} (I_{hr}^{(k)} - \mathbf{C}_{hr}\mathbf{V}^{(k)})^2$, and $\sum_{k=1}^{m} (I_{hi}^{(k)} - \mathbf{C}_{hi}\mathbf{V}^{(k)})^2$, and determine the parameters $\mathbf{C}_{hr}$ and $\mathbf{C}_{hi}$ by least squares approach as [23]:

$$\mathbf{C}_{hr} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}_{hr}$$
$$\mathbf{C}_{hi} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}_{hi} \tag{10}$$

4) By using (9), calculate I'_{Lh}, I'_{Nh} with the obtained $\mathbf{C}_{hr}$ and $\mathbf{C}_{hi}$, then the existence of harmonic source is identified and the harmonic current is separated.

4. Formulation of Capacitor Placement Problem

The load model

A forecasted load duration curve is approximated in a number of discrete load levels. In order to bring the problem under consideration to manageable proportions, the following assumptions are made [18]:

- The system is balanced.
- All loads vary in a conforming way, i.e., the k^{th} load level at the i^{th} bus can be expressed in terms of peak load by $(P_{ik}, Q_{ik}) = x_k(P_{il}, Q_{il})$ where $x_k \le 1$.
- The load at bus i is partitioned into w_i nonlinear loads and $(1-w_i)$ linear loads. This separation can be achieved by the mentioned method in previous section.
- For simplicity, let both clusters of linear and nonlinear loads at bus i have the same power factor.
- The n^{th} harmonic current injected at the i^{th} bus during the k^{th} load level is given by:
-

$$I_{ik}^{n} = \frac{w_i I_{ik}^{1}}{n}, n = 3,5,7,..., N \tag{11}$$

where N is the upper harmonic order of interest, and I_{ik}^1 is the fundamental current that is computed by:

$$I_{ik}^1 = \frac{P_{ik} - jQ_{ik}}{(V_{ik}^1)^*} \tag{12}$$

where $(V_{ik}^1)^*$ is the complex conjugate of the fundamental voltage.

Cost function and utilized constraints

The total peak power loss (k=1), including losses at harmonic frequencies, is expressed by [18]:

$$P_{loss,1} = \sum_{n=1}^{N}\sum_{j=1}^{m} r_j (I_j^n)^2 \tag{13}$$

where,

m is the total number of the system sections or buses,

I_j^n is the n^{th} harmonic current flowing in the j^{th} section, and is given by:

$$I_j^n = \frac{\Delta v_j^n}{z_j^n} \tag{14}$$

Δv_j^n is the n^{th} harmonic voltage drop across the j^{th} section

z_j^n is the n^{th} harmonic impedance of j^{th} section

r_j is the j^{th} section resistance

While the total energy loss at the k^{th} load level, which lasts T_k hours is simply given by:

$$E_{loss,k} = T_k P_{loss,k} \tag{15}$$

The annual shunt capacitor cost is determined by:

$$K^c = \sum_{i=1}^{m}(K_f^c u_{fi}^1 + K_s^c u_{si}^*) \tag{16}$$

where,

k_f^c, k_s^c the annual equivalent costs of fixed and switched type capacitors, respectively,

u_{fi}^1, u_{si}^k the numbers of fixed and switched capacitors, respectively,

$$u_{si}^* = \max\{u_{si}^k, k = h-1, h-2, ..., 2\} \tag{17}$$

Also, for considering capacitors effects in resonance occurrence, index VI^{rh} is defined as follows:

$$VI^{rh} = \sum_{j=1}^{m}\sum_{h=1}^{N}(V_{jh}^{(2)} - V_{jh}^{(1)}) + \sum_{j=1}^{m}\sum_{h=1}^{N}(I_{jh}^{(2)} - I_{jh}^{(1)}) \tag{18}$$

where, $V_{jh}^{(1)}$ and $I_{jh}^{(1)}$ are the h^{th} harmonic voltages and currents respectively in j^{th} section before capacitor placement; $V_{jh}^{(2)}$ and $I_{jh}^{(2)}$ are these parameters after capacitor placement.

The net savings resulting from peak power and total energy loss reduction, while taking capacitor cost into account, is computed by:

$$Saving = K^p(\Delta P_{loss,1}) + \sum_{k=1}^{h} K^e(\Delta E)_{loss,k} - K^c - K^{rh} \times VI^{rh} \tag{19}$$

where K^p, and K^e are the respective constants to convert power and energy into dollars, K^{rh} is the constant to convert voltage into dollars, while $\Delta P_{loss,1}$ and $(\Delta E)_{loss,k}$ represent the differences between peak power and energy losses before and after capacitor placement. In the capacitor selection process, the following constraints must be taken into account:

$$V_{\min} \le |V_{ik}| \le V_{\max} \tag{20}$$

$$THD_{ik} \le THD_{\max} \tag{21}$$

where $V_{\min}$, $V_{\max}$, and $THD_{\max}$ represent the allowed minimum rms, maximum rms, and maximum THD of node voltages, also

$|V_{ik}|$ is the overall rms voltage at the i^{th} bus and k^{th} load level which can be calculated by:

$$|V_{ik}| = \sqrt{\sum_{n=1}^{N} \left|V_{ik}^{n}\right|^2} \tag{22}$$

THD_{ik} is the total harmonic distortion which is defined by:

$$THD_{ik}(\%) = \frac{100}{\left|V_{ik}^{1}\right|} \sqrt{\sum_{n\neq 1}^{N} \left|V_{ik}^{n}\right|^2} \tag{23}$$

It is desired to find which combination of capacitors, will result in maximum savings according to (19), at different loads considering harmonic sources, and subject to constrains (20) and (21).

Model of Load Variation

Usually the load variations throughout the year are not known at every distribution feeder and every load point (node) to the power utility. Measurements of power variations are recorded only in selected locations and often the distribution engineers are primarily interested in the maximum power levels which are measured in main feeders and in major load points.

Hence a model is required to predict the load levels throughout the year in all feeders and in all load points from the measured values of the maximum power in each node on the feeder. The load forecast will then be included in the design of the capacitive compensation to satisfy the condition of cost effectiveness throughout the year.

It has been suggested that the year can be divided into 7 periods to accommodate the load variations due to season, the week (weekdays and weekend), and the daily load level (peak load period, and off-peak load period). Denoting the time for each period as T_P so that [18]:

$$\sum_{P=1}^{7} T_P = 8760 \tag{24}$$

Where $8{,}760 = 24h \times 365\, days$ is the total number of hours per year, and p is the number of period which varies from 1 to 7.

Capacitor Selection

Generally, utilities stock very few capacitors sizes, each of which is an integral multiple of the smallest size. In addition, the cost per *kVAr* varies with capacitor size. For the sake of brevity, it is assumed that the selection of capacitors is limited to only one standard size Q_c (e.g., 300 *kVAr*) with an annual equivalent cost of k_f^c for fixed-type and k_s^c for switched-type. When solving the shunt capacitor placement problem, optimal capacitor sizes and locations are first determined for the base load. The resulting capacitors are chosen to be of fixed-type. Switched capacitors are then determined for each incremental load level until the peak load is reached.

Consider now that the capital and installation costs of a standard unit of a compensating capacitor is C_c, with amortization rate of A_c per year, and the life time of the capacitor is N_{LT} years, so the annual cost of the capacitor is k^c, where [18]:

$$k^c = \frac{C_c A_c}{N_{LT}} \tag{25}$$

5. Genetic Algorithm

GA searches for an optimal solution using the principles of evolution and heredity. A certain strings are judged and propagated to form the next generation. It is designed such that the “fitter” strings survive and propagate into the later generations. The major advantage of using the GA is that the solution obtained is globally optimal. Also GA is capable of obtaining the global solution of a wide variety of functions such as differentiable or non-differentiable, linear or nonlinear, continuous or discrete, and analytical or procedural [24].

In this work the representation by means of strings of integers was chosen. Each gene i of the chromosome can store a 0, which indicates absence of capacitors on the corresponding bus or an integer different from 0 ($0 \le m$) that indicates the number of added capacitors sizes that is added in the bus i. Therefore a chromosome can be represented as in Fig. 1.

Bus number	1	2	… …	b-1	b
Gene	0	3	… …	2	1

Fig. 1. A GA chromosome of the capacitor placement problem

When the GA is implemented, it is usually done in a manner that involves the following cycle:

- Evaluate the fitness of all of the individuals in the population.

- Create a new population by performing mating on the individuals whose fitness has just been measured and ranked. For the selection of parents' chromosomes, the tournament method is used. Also, the crossover method is used for implement of mating [18].
- If the best string does not change for two successive generations, then the mutation operator is applied on it. Over the first chromosome, which has the least amount of costs, mutation won't be done and is directly transferred to the next generation. After selecting the location of bits which mutation is done; then integer numbers randomly are created and replaced.
- Discard the old population and the whole process is then iterated using the new population until convergence is obtained.

6. Simulation Results

In this section, an IEEE 33-bus test system is simulated to illustrate the performance of the proposed method. A schematic of this test system is shown in Fig. 2, and its data are provided from [18]. This test system contains two harmonic sources. One is a twelve-pulse HVDC terminal at bus 16 and the other is a SVC at bus 24. The total substation loads are 5084.26 *kW* and 2547.32 *kVAr*. The system is not well compensated and has large loss (the total loss is about 8% of the total load). A system with too much loss is selected because the loss reduction is expected to be appreciable. The load duration curve is considered as given in Table 1. The proposed algorithm has been implemented in MATLAB™ code [25]. Three cases are considered:

- *Case I: optimal capacitor placement without harmonics consideration.*
- *Case II: optimal capacitor placement with harmonics consideration based on [18].*
- *Case III: optimal capacitor placement with harmonics consideration based on the proposed method in this paper.*

These three cases are compared with a reference case in which no capacitor placement is used in the test system.

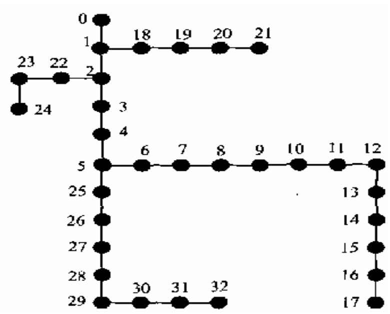

Fig. 2. Network structure of the test system

Tab. 1. Load curve

P	Load (%)	(hours) T_p
1	40	2000
2	50	2000
3	60	2000
4	70	1000
5	80	2000
6	90	700
7	100	60

The results of case I have been shown in Table 2. The average of losses for all of the load levels is 6.5% and the obtained saving is $9303. The results of case II have been shown in Table 3. The average of losses for all of the load levels is 5.6% and the obtained saving is $10030. It is observed that the total losses of case II is reduced only a little amount in comparison with case I; Also, in case II the saving is increased a little amount in comparison with case I (about $727).

The results of case III have been shown in Table 4. The average of losses for all of the load levels is 3.7% and the obtained saving is $10710. It is observed that the total losses of case III is reduced much more in comparison with case II; Also, in case III the saving is increased more amount in comparison with case II (about $680). It should be noted that the loss of each load level has been stated as percent of the total corresponding load level.

Tab. 2. The results for case I

Load Level [%]	Capacitors Locations	Loss [%]	Savings [$]
40	(Fixed) 7,3,29	6.6	1207.3
50	1,2,5,6	6.3	1095
60	3,4	6.5	1282.5
70	-	6.55	1376
80	-	6.57	2112
90	28	6.43	2003
100	-	6.44	226
For All of the Load Levels		6.5	9303

Tab. 3. The results for case II

Load Level [%]	Capacitors Locations	Loss [%]	Savings [$]
40	(Fixed) 7,23,29	5.5	1344
50	1,2,4,5,6	5.6	836
60	3	5.7	1693
70	-	5.6	1582
80	-	5.55	2318
90	11	5.65	2025
100	-	5.6	231
For All of the Load Levels		5.6	10030

Tab. 4. The results for case III

Load Level [%]	Capacitors Locations	Loss [%]	Savings [$]
40	(Fixed) 7,12,26	3.7	1450
50	1,4,5,8,10	3.55	930
60	3	3.8	1770
70	-	3.75	1670
80	-	3.7	2395
90	11	3.75	2095
100	28	3.65	400
For All of the Load Levels		3.7	10710

In Figs. 3 to 5 the differences between harmonic voltages magnitudes for all cases with the reference case have been shown for the above mentioned cases to investigate the shunt resonance occurrence.

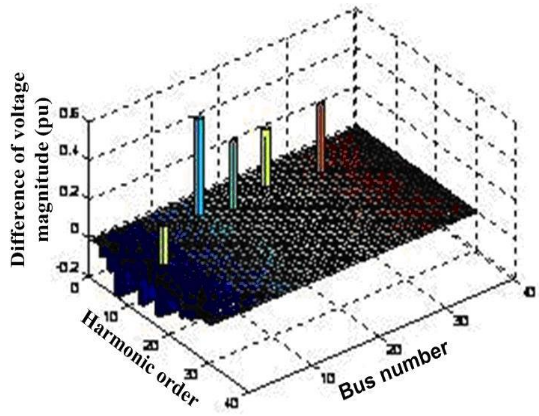

Fig. 3. Differences between harmonic voltages magnitudes in case I and reference case

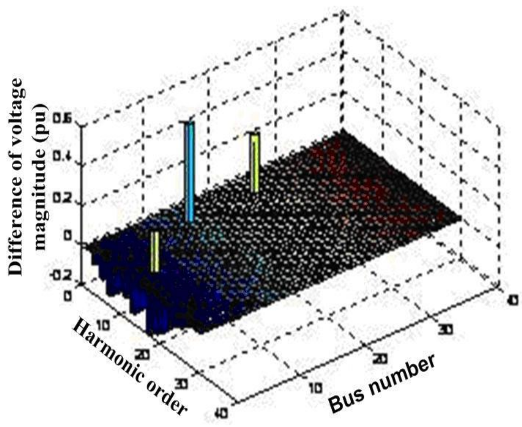

Fig. 4. Differences between harmonic voltages magnitudes in case II and reference case

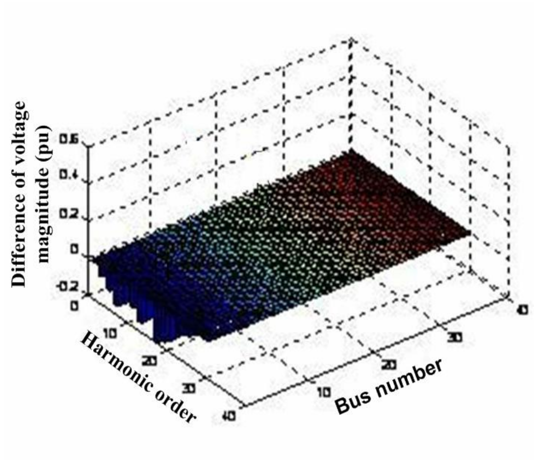

Fig. 5. Differences between harmonic voltages magnitudes in case III and reference case

The following results are obtained from Figs. 3 to 5:

- *In case I, the capacitors amplify harmonic voltages due to shunt resonance at harmonic 5^{th} in busses 14 and 24, at harmonic 7^{th} in busses 18 and 31, and at harmonic 13^{th} in bus 4.*
- *In case II, in spite of harmonics consideration based on [18], the capacitors amplify harmonic voltages due to shunt resonance at harmonic 5^{th} in busses 14 and 24, and at harmonic 13^{th} in bus 4.*
- *In case III, the capacitors do not amplify any of the harmonic voltages.*

It should be noted that, amplification of harmonic currents should be considered in investigation of the series resonance occurrence. In this test system, the obtained capacitors do not amplify harmonic currents in all cases; therefore corresponding figures have not been shown.

7. Conclusion

In this paper, a new method is presented to incorporate nonlinear load effects in the problem of finding optimal shunt capacitors in distribution systems. The problem was formulated so that the optimal solution does not result in severe resonant conditions at harmonic frequencies. The harmonic *V-I* characteristics are used for modeling harmonic sources. The major conclusions of this work are:

- *Without harmonics consideration in the problem of optimal capacitor placement, the amount of obtained saving is low; and capacitors may amplify harmonic voltages and currents due to possible resonance at harmonic frequencies.*
- *With harmonics consideration based on [18], in spite of reducing the losses and increasing the obtained saving; the optimal solution results in resonant conditions at harmonic frequencies.*

- *Using the proposed method in this paper, not only the loss is reduced much amount; but also the optimal solution does not result in resonant conditions at harmonic frequencies.*

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