Optimal Placement and Sizing Method to Improve the Voltage Stability Margin in a Distribution System Using Distributed Generation

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***Abstract*—Recently, integration of distributed generation (DG) in distribution systems has increased to high penetration levels. The impact of DG units on the voltage stability margins has become significant. Optimization techniques are tools which can be used to locate and size the DG units in the system, so as to utilize these units optimally within certain limits and constraints. Thus, the impacts of DG units issues, such as voltage stability and voltage profile, can be analyzed effectively. The ultimate goal of this paper is to propose a method of locating and sizing DG units so as to improve the voltage stability margin. The load and renewable DG generation probabilistic nature are considered in this study. The proposed method starts by selecting candidate buses into which to install the DG units on the system, prioritizing buses which are sensitive to voltage profile and thus improve the voltage stability margin. The DG units' placement and sizing is formulated using mixed-integer nonlinear programming, with an objective function of improving the stability margin; the constraints are the system voltage limits, feeders' capacity, and the DG penetration level.**

***Index Terms*—Distributed generation (DG), distribution system, optimum power flow, voltage profile, voltage stability.**

NOMENCLATURE

$P_{o,i}$	Base case of the real power demand.
$Q_{o,i}$	Base case of the reactive power demand.
ΔP	Incremental change in bus real power.
ΔQ	Incremental change in bus reactive power.
ΔV	Incremental change in bus voltage magnitude.
$\Delta \theta$	Incremental change in bus voltage angle.
J	Jacobian matrix.
P_{G1}	Substation active power injected.
Q_{G1}	Substation reactive power injected.
P_{DGDi}	Rated power of the dispatchable DG connected at bus i.
P_{DGWi}	Rated power of the wind-based DG connected at bus i.
P_{DGSi}	Rated power of the solar DG connected at bus i.
Q_{DGDi}	Rated reactive power of the dispatchable DG connected at bus i
Q_{DGWi}	Rated reactive power of the wind-based DG connected at bus i.
Q_{DGSi}	Rated reactive power of the solar DG connected at bus i.
P_{Di}	Peak active load at bus i.
Q_{Di}	The peak reactive load at bus i
$V_{n,i}$	Voltage at bus i during state n.
C	Matrix of four columns that include all possible combinations of the wind output power states, solar output power states, and load states (i.e., columns 1, 2, and 3 represent the output power of the dispatchable DG, wind-based DG, and the solar DG as a percentage of their rated power, and column 4 represents the different load levels [1].
N	Number of load and DG generation combination.
Prn	Probability of each combination.
96	Time segments (hours) for the year (24 for each season).
$V_{P,\text{with DG}}$	Voltage profile of the system with DG units.
$V_{P,\text{without DG}}$	Voltage profile of the system without DG units.
V_P	$\sum_{i=1}^{m} V_i L_i k_i$.
V_i	Voltage magnitude at bus i.
L_i	Load demand at bus i.
k_i	Weighting factor for load bus i.
m	Ttal number of load buses in the distribution system.

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I. Introduction

INTERESTING distributed generation (DG) in power system networks has rapidly increased. This increase can be justified by factors such as environmental concerns, the restructuring of electricity market, and the development in technologies for small-scale power generation.

DG units are typically connected so that they work in parallel with the utility grid, and they are placed depending on availability of the resources. Further, DG units are not, so far, permitted without a utility grid or what is known as microgrid operation. Integrating DG units can have an impact on the practices used in distribution systems, such as the voltage profile, power flow, power quality, stability, reliability, and protection. Since DG units have a small capacity compared to central power plants, the impacts are minor if the penetration level is low (1%–5%). However, if the penetration level of DG units increases to the anticipated level of 20%–30%, the impact of DG units will be profound [2].

Voltage instability in distribution systems has been understood for decades and was referred to as load instability [3]. For example, a voltage instability problem in a distribution network, which was widespread to a corresponding transmission system, caused a major blackout in the S/SE Brazilian system in 1997 [4].

With the development of economy, load demands in distribution networks are sharply increasing. Hence, the distribution networks are operating more close to the voltage instability boundaries. The decline of voltage stability margin is one of the important factors which restricts the increase of load served by distribution companies [5]. Therefore, it is necessary to consider voltage stability with the integration of DG units in distribution systems. The literature has covered this impact from different points of view. For example, the work in [6]–[9] studied the impact of induction generators to small and large disturbances. The authors of [10] and [11] investigated the impact of DG technology (such as synchronous, induction generators, and high- or low-speed generators that are grid coupled through a power electronic converter). A practical investigation of the impacts of DG units on system stability can be found in [12]. Reference [13] presented an assessment of the impact of the DG units size and location under a change in the loading conditions due to a contingency on unbalanced distribution systems. In [14], the effect of DG units' capacity and location on voltage stability enhancement of distribution networks was also investigated. The DG units were allocated and sized based on minimizing overall cost. This paper [14] recommended considering the voltage stability as an objective function when dealing with optimum location of DG units. Recently, the work in [15] and [16] proposed methods to locate distributed generation units to improve the voltage profile and voltage stability of a distribution system. The author in [15] placed DG units at the buses most sensitive to voltage collapse, and resulted in improvement in voltage profile, as well as decline in the power losses. The author in [16] developed the work in [15] to maximize the loadability conditions in normal and contingency situations.

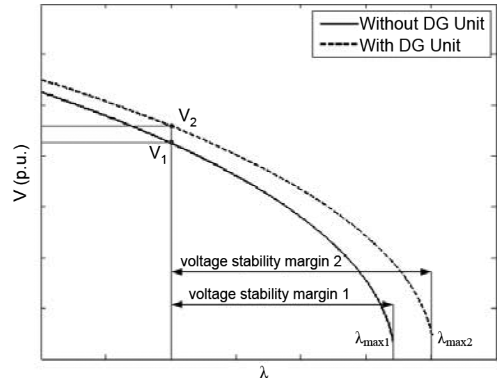

Fig. 1. Impact of a DG unit on maximum loadability and voltage stability margin.

Within the above-mentioned literature, the problem of voltage stability was tackled with the assumption that all connected DG units are dispatchable. However, this paper introduces the probabilistic nature of both the renewable energy resources and the load demand as vital factors to be considered for improving the voltage stability. Therefore, this paper will tackle placing and sizing of the DG units to improve the voltage stability margin and consider the probabilistic nature of the renewable energy resources and the load. In fact, placing and sizing DG units with an objective of improving the voltage stability margin while considering renewable DG generation and load probability might be a complicated problem due to the complexity of running continuous load flow and at the same time considering the probabilistic nature of the load and the DG unit's resources. Therefore, this paper proposes a modified voltage index method to place and size the DG units to improve the voltage stability margin, with conditions of both not exceeding the buses' voltage, and staying within the feeder current limits. The probability of the load and DG units are modeled and included in the formulation of the sizing and placing of the DG units. The remainder of this paper is arranged as follows. Section II presents the impact of the DG units on voltage stability, Section III carries out a method to select candidate buses for DG units installation, Section IV formulates a method to sit and size DG units, Section V describes the system under study, and Sections VI and VII demonstrates the results and conclusion, respectively.

II. Impact of the DG Size on Voltage Stability

Voltage stability analysis has been presented by many techniques, including static and dynamic methods. The static technique can be analyzed by using the relation between the receiving power (P) and the voltage (V) at a certain bus in a system which is known as a P–V curve or nose curve (see Fig. 1). The P–V curve is obtained by applying continuous power flow method [17]. The critical point $\lambda_{\max}$ (saddle-node bifurcation point) in the P–V curve represents the maximum loading of a system. This point corresponds to a singularity of the Jacobian of the power flow equations. The stability margin can be defined by the MW distant from the operating point to the critical point. The penetration of the DG units in a distribution system can increase or decrease the voltage stability margin depending on their operation at unity, lead or lag power factors. Currently most of the installed DGs are commonly connected to operate at unity power factor to avoid interference with the voltage regulation devices connected to the system [18], [19]. For this reason, this study assumes that all of the DG units are operating at unity power factor. In addition, some utilities allow the DG units to operate in fixed power factor mode ranging from 0.95 lagging to 0.95 leading, a case study representing this condition is also considered.

Fig. 1 visualize the impact of a DG unit on voltage stability margin and maximum loadability. The x-axis represents λ, which is the scaling factor of the load demand at a certain operating point (see (1)). λ varies from zero to the maximum loading ($\lambda_{\max}$). Due to real power injection of a DG unit, the normal operating point of the voltage increases from V_1 to V_2, and at the same time the maximum loadability increases from $\lambda_{\max 1}$ to $\lambda_{\max 2}$ as

$$\begin{aligned} P_i &= \lambda P_{o,i} \\ Q_i &= \lambda Q_{o,i}. \end{aligned} \tag{1}$$

III. Selection of the Candidate Buses

In the literature, the candidate buses for the DG installation can be selected randomly, by recommended location, or by selecting sensitive buses to the voltage profile. Because this study is focusing on improving the voltage stability of the system, it uses voltage sensitivity analysis to select the candidate buses. In addition, the candidate buses should be located on the main feeders of the system. The method is conducted by testing the voltage's sensitivity to the change of the DG injected power, and it can be explained as follows.

Power systems are typically modeled with nonlinear differential algebraic equations [20]. The system model can be linearized as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}. \tag{2}$$

With the assumption that the reactive load power (Q) is constant, the incremental change in bus reactive power ΔQ equals to zero. Then, using the partial inversion of (1) gives

$$\Delta P = (J_{PV} - J_{P\theta} {J_{Q\theta}}^{-1} J_{QV}) \Delta V \tag{3}$$

or

$$\Delta V = (J_{RPV})^{-1} \Delta P \tag{4}$$

where J_{RPV} is a reduced Jacobian matrix, which gives the voltage magnitude variations due to DG active power injection variations. If the buses are modeled as PQ buses, ${J_{Q\theta}}^{-1}$ is a feasible and square matrix. Therefore, this situation normally occurs in distribution systems, where the slack bus is the only bus that keeps the voltage magnitude at a fixed point. This situation normally occurs in distribution systems, where the slack bus is the only bus that keeps the voltage magnitude at a fixed point. This study focused on radial distribution systems. The load buses are considered as PQ. However, for the DG unit placement, there are three types of DG control, namely PV control, current control, and PQ control. For the DG with PV controller, the connected bus can be modeled as a PV bus. Further, for DG units equipped with either a current or a PQ controller, the connected bus is modeled as a PQ bus. However, the IEEE P1547 Standard [21] specified that the DG units should not regulate distribution system voltages. An attempt by a DG unit to regulate distribution system voltage can conflict with existing voltage regulation schemes applied by the utility to regulate the same or a nearby point to a different voltage [18]. Thus, DG units with PV controller are not recommended. Therefore, this research focused on modeling the DG units buses as PQ buses. It is worth mentioning that modeling DG units as PQ busses with controlled reactive power injection might be valid for electronically-coupled DG units where reactive power contribution is independent from the interface bus voltage. Since the most dominant type of DG units in the market are electronically coupled such as type C and type D for the wind turbines [22], and PV units, then this paper assumes the DG units are electronically interfaced (inverter-based DG). However, for fixed and semi-variable speed wind units (namely Type-A and Type-B [22]) , this assumption is not valid since the reactive requirements of these two units depends on the value of the voltage at the interface bus. Usually, the RX model, instead of the PQ model, is used to address these two points.

In this case, (4) can be used to study the impact of the DG units on voltage profile. This equation is valid if the DG units are operating at unity power factor, otherwise the V–Q sensitivity should be considered. Therefore, the system load values at an operating point can be analyzed using (4) to determine the buses most sensitive to the voltage profile. The most sensitive buses should be selected as the candidate buses for the DG installation.

IV. DG Placement Problem Formulation

After the candidate buses are selected in Section III, allocating DG units within the system requires investigation in terms of DG resources and their uncertainties. It also requires modeling the types of load and their criticality at each bus. In addition, placing the DG units in the most sensitive buses might violate the voltage limits or the capacity of the feeders, depending on the size of the DG units and the load demand of the system. Accordingly, this section proposes a method to place DG units with an objective of improving the voltage stability of the system. This study is demonstrated in five scenarios.

- Scenario #1: this is a reference scenario, in which no DG units are connected to the system (base case)
- Scenario #2: only dispatchable (non-renewable) DG units are connected.
- Scenario #3: only wind-based DG units are connected.
- Scenario #4: only PV DG units are connected.
- Scenario #5: a mix of dispatchable, wind-based, and PV DG units are connected.

In this formulation, the following assumptions are considered.

- More than one type of DG can be installed at the same candidate bus.
- The DG units are assumed to operate at unity power factor. In addition, a simulation for DG units that operates between 0.95 lead or lead power factor is presented

- All buses in the system are subjected to the same wind speed and solar irradiance. This assumption greatly simplifies the analysis.
- The penetration level is equal or less than 30%; referring to Ontario's standard program, the maximum penetration level is 30% of the maximum load [23].

The selected wind turbine is 1.1 MW, and the photovoltaic module is 75 W, however, other wind and PV ratings can be considered without loss of generality. The utilized DG units' ratings and characteristics are obtained from [1]. The annual capacity factor of the wind turbine is found to be 0.22, while the one for the PV module generator is found to be 0.174. The annual capacity factor is only used to formulate the maximum penetration level as in (16). The characteristics of both types are given in the Appendix. The penetration of the wind turbines can be a multiple number of the selected rating. For example, if the result shows the penetration level at a certain bus is 6.6 MW, it means six turbines of 1.1 MW are recommended to be installed at this bus. On the other hand, solar generators can be modeled using photovoltaic modules (PV modules). Since the ratings of PV modules are small, they are unlike wind turbines, and the solar generators can be modeled to the required sizes. For example, if the required size of the solar generator is 1.55 MW, it requires 206667 modules of 75 W. The dispatchable DG unit is selected to be 0.5 MW. It is assumed to generate a constant power at its rating. For example, if the required size is 4.5 MW, it requires nine dispatchable DG units. Since, the dispatchable generator generates constant power during the year, it does not have uncertainty, and hence its annual capacity factor is 1.

The DG placement method is carried out as follows.

Step 1: Load and DG Units Modeling

This paper is using the models proposed in [1]. The load is modeled by the IEEE-RTS system. For the renewable DG units, three years of historical data have been provided from the site under study. These data are used to model the solar irradiance and wind speed by Beta and Weibull probability distribution functions, respectively. The model is conducted as follows.

- Each year is divided into four seasons, and each season is being represented by any day within that season. These data are then utilized to generate for each season a typical day's frequency distribution of the irradiance and wind speed measurements.
- The day which is representing the season is further subdivided into 24 1-h segments (time segments) each referring to a particular hourly interval of the entire season. As a result, there are 96 time segments for the year (24 for each season). Considering a month to be 30 days, each time segment then has 270 irradiance and wind speed level data points (3 years $\times$ 30 days per month $\times$ 3 months per season).
- The mean and standard deviation for each time segment are calculated.
- The Beta and Weibull probability density functions are generated for each hour using the mean and standard deviation for each segment.
- The Beta and Weibull probability density functions are divided into states (periods) to incorporate the output power of the solar DG and wind-based DG units. The number of states is chosen carefully as a small number of states will affect the accuracy, while a large number will increase the problem's complexity. In this paper, the state is adjusted to be 0.1 kW/m^2 for solar irradiance and 1 m/s for wind speed.
- The corresponding output power of the PV module and wind turbine in each state are calculated using the PV module characteristics and wind turbine power performance curve.

Step 2: Load and DG Units Modeling

A. Objective Function

Based on Section III, the DG placement and sizing with an objective of increasing the voltage stability margin can be formulated by increasing the voltage of the system using DG units. The following equation is obtained from [24] and is used to improve the voltage profile of the system:

$$V_n = \frac{V_{P,\text{with DG}}}{V_{P,\text{without DG}}}. \tag{5}$$

Thus, it can be used to improve the voltage stability margin of the system. This equation is modified to include the probabilistic nature of the DG generation as in

$$\text{Maximize } V_{\text{index}} = \frac{\left(\sum_{n=1}^{N} V_n \, pr_n\right)}{96} \quad n = 1, 2, \ldots N. \tag{6}$$

The highest V_{index} implies the best location for the installation of the DG units in term of improving the voltage profile. The following attributes show the impact of the DG units:

$$V_{\text{index}} \begin{cases} < 1, & \text{DG units will worsen the voltage profile} \\ = 1, & \text{DG units will not impact on the voltage profile} \\ > 1, & \text{DG units will improve the voltage profile} \end{cases}. \tag{7}$$

A weighting factor k_i is chosen based on the importance and criticality of different loads. In this paper, the weighting factor is designed to be a ratio of the load demand at a specific bus to total demand

$$k_i = \frac{p_{i,n}}{p_{TD,n}}. \tag{8}$$

This means the bus that has highest load demand will have the highest k_i factor. The rationale behind this design is to improve the voltages in the buses that have high power demand, and consequently improve the voltage stability margin, where $p_{i,n}$ is the power demand at bus i at state n, and $p_{TD,n}$ is the total power demand of the system at state n. Starting with a set of equal weighting factors, modifications can be made and, based on an analysis of the results, the set that will lead to the most acceptable voltage profile on a system-wide basis can be selected. It should be noted that if all the load buses are equally weighted, the value of k_i is given as $\mathrm{k}_1 = \mathrm{k}_2 = \mathrm{k}_3 = \mathrm{K}_\mathrm{m} = 1/\mathrm{m}$ [24]. This voltage profile expression allows the important load to have a strong impact, because the weighting factor can be based on the important bus.

B. Constraints

Power flow equations:

$$P_{G_{n,1}} + C(n,1) * P_{DG\,D_i} + C(n,2) * P_{DG\,W_i} + C(n,3) * P_{DG\,S_i} - C(n,4) * P_{D_i} = \sum_{j=1}^{m} V_{n,i} * V_{n,i} * Y_{ij} * \cos(\theta_{ij} + \delta_{n,j} - \delta_{n,i}) \quad \forall i, n \tag{9}$$

$$Q_{G_{n,1}} - C(n,4) * Q_{D_{n,i}} = -\sum_{j=1}^{m} V_{n,i} * V_{n,i} * Y_{ij} * \sin(\theta_{ij} + \delta_{n,j} - \delta_{n,i}) \quad \forall i, n \tag{10}$$

Branch current equations:

$$I_{n,ij} = |Y_{ij}| * \left[(V_{n,i})^2 + (V_{n,j})^2 - 2 * V_{n,i} * V_{n,j} * \cos(\delta_{n,j} - \delta_{n,i})\right]^{1/2} \quad \forall n, i, j \tag{11}$$

where $I_{\mathrm{n,i,j}}$ is the current in the feeder connecting buses i and j during state n.

Slack bus voltage and angle (assumed to be bus 1):

$$V_{n,1} = 1.025$$
$$\delta_{n,1} = 0.0. \tag{12}$$

Voltage limits at the other buses:

$$0.95 \leq V_{n,i} \leq 1.05 \quad \forall\, i \notin substation\ bus,\ n. \tag{13}$$

Feeder capacity limits:

$$0 \leq I_{n,ij} \leq I_{ij_{\max}} \quad \forall i, j, n. \tag{14}$$

Maximum penetration on each bus:

$$P_{DG\,D_i} + P_{DG\,W_i} + P_{DG\,S_i} \leq 10\ \mathrm{MW}. \tag{15}$$

The maximum penetration of DG capacity should not exceed 10 MW at each bus of the candidate buses.

Maximum penetration of DG units on the system

$$\sum_{i=1}^{m} P_{\mathrm{DG}\ D_i} + \sum_{i=1}^{m} CF_w P_{\mathrm{DG}\,W_i} + \sum_{i=1}^{m} CF_s P_{\mathrm{DG}\,S_i} \leq y * \sum_{i=1}^{m} P_{D_i} \tag{16}$$

where y is the maximum penetration limit as a percentage of the peak load. For the penetration level not to exceed 30%, y equals 0.3.

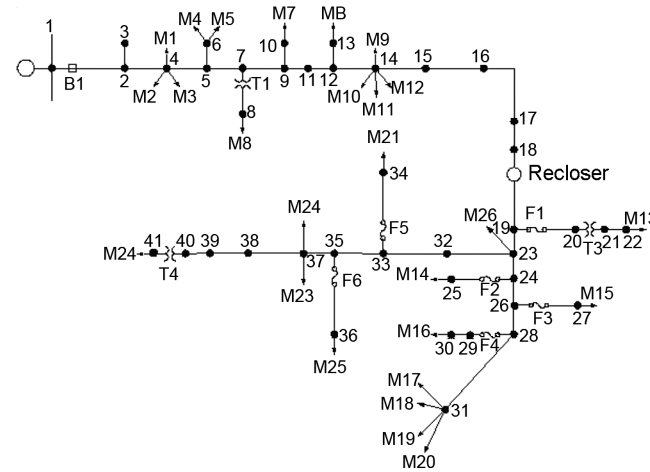

Fig. 2. Sistribution test system of 41 buses.

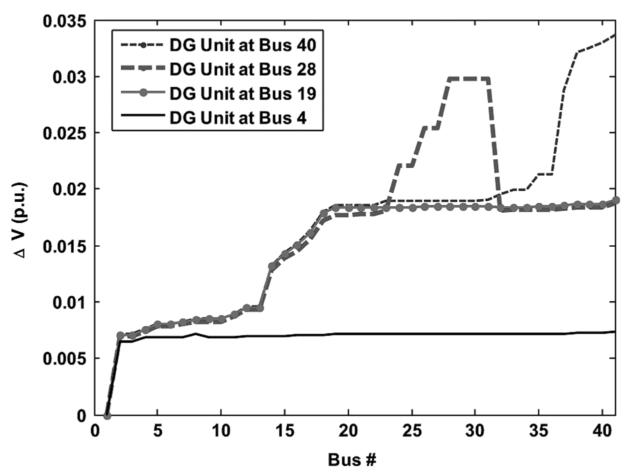

Fig. 3. Results of voltage sensitivity analysis (the penetration level is 30%).

V. System Under Study

Fig. 2 shows a single-line diagram of a rural distribution system of 41 buses. The system peak load is 16.18 MVA and the substation at bus 1 is used to feed the system with a capacity of 300 A. The system's detailed line and load data is obtained from [1]. The voltage at the substation is set to 1.025 p.u.

VI. Results

Here, we outline the results.

A. Results of Candidate Buses for the DG Units Installation

In Fig. 3, the selection is achieved by developing 26 case studies (the cases are equal to the number of the system buses which are located in the main feeders). In each case, a DG unit is installed at a certain bus, and the changes of the system voltages (ΔV) are observed. The installed DG unit is assumed to generate constant power of 4.5 MW at unity power factor (about 30% of the penetration level), and the system load demand is taken at the peak value. In addition, analysis for penetration level of 10% and 20% are shown in the Appendix.

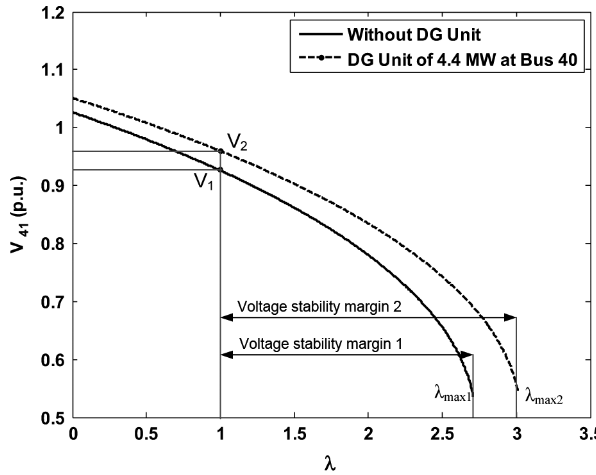

Fig. 4. Impact of DG unit on maximum loadability and voltage stability margin at bus 41.

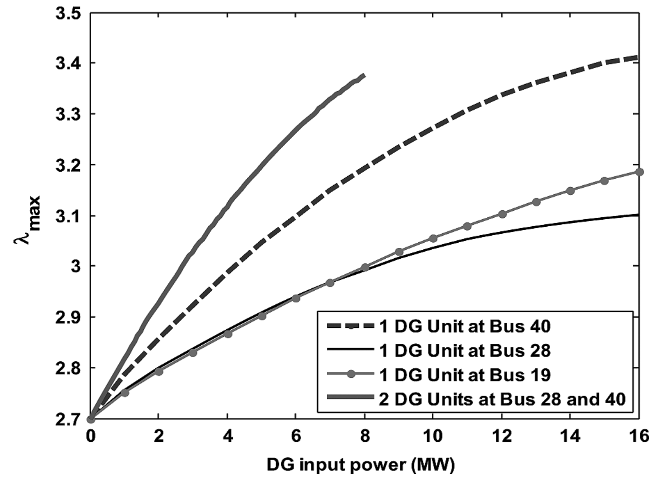

Fig. 5. Impact of the size of the DG units on maximum loading.

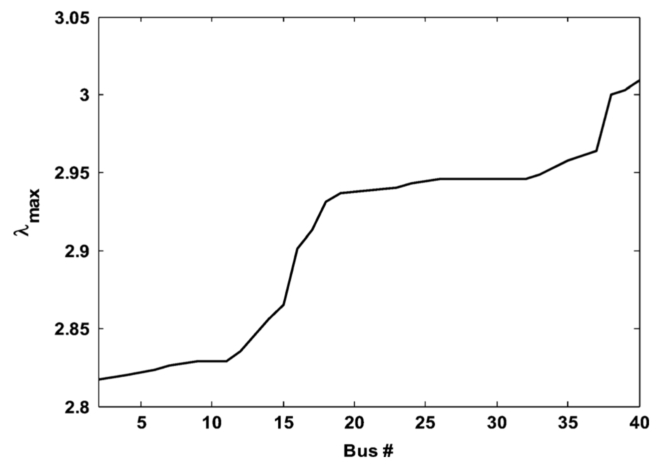

Fig. 6. Impact of the location of the DG units on maximum loading.

Fig. 3 represents the impact of $\Delta V/\Delta P$ of selected buses: 40, 28, 19, and 4. The x-axis shows the buses numbers and the y-axis shows the changes in ΔV, due to the injection of the DG unit. This figure shows that the buses from 19 to 41 can improve the voltage profile better than the buses from 1 to 18. Moreover, the order of the most sensitive buses can be determined using (5), and considering the k_i as in (8). The most sensitive buses are 40, 39, 38, 37, 35, 33, 32, 23, 24, 19, 26, and 28.

B. Results of the Impact of the DG Units on Voltage Stability

Fig. 4 shows the impact of the DG unit on voltage stability margin and maximum loadability. The DG unit is installed at bus 40. This PV curve represents the voltage stability at bus 41. The load demand in bus $41(P_{0,41})$ is 2.166 MW, which corresponds to $\lambda = 1$ in Fig. 4. When the DG unit generates 4.4 MW, the voltage moves from V_1 (0.924 p.u) to V_2 (0.959 p.u.), and $\lambda_{\max}$ moves from $\lambda_{\max 1}$ (2.71) to $\lambda_{\max 2}$ (3.09). Thus the voltage stability margin is improved by 0.82 MW.

Fig. 4 shows the impact of the DG units on voltage (V_1 moves to V_2) and the maximum loadability ($\lambda_{\max 1}$ moves to $\lambda_{\max 2}$). This results only represents one size and location. However, the size and location can also have an impact on the voltage stability. The rest of this section presents the impact of the size and location of the DG units on both the voltage and the maximum loadability. The study of the impact of the DG size is conducted by installing one DG unit in one of the candidate buses, and then finding the maximum loadability and the voltage of the system. The DG unit is varied from 0 to 16 MW. [Note that the DG unit is varied to approximately 100% of the penetration level in order to study the impacts of the DG size on voltage stability margin in different location. However, for the DG placement and sizing "Section IV," the penetration level is restricted to (15) and (16)]. Then, the same method is applied for the other candidate buses. Alternatively, the impact of the DG location study is achieved by developing 26 cases (the cases are equal to the number of the system buses which are located in the main feeders). In each case, a DG unit is installed at a certain bus, and the maximum loadability $(\lambda_{\max})$ is observed. The installed DG unit is assumed to generate constant power of 4.4 MW. In both studies, the DG unit is operated at unity power factor; the system load demand is taken at the peak value. Figs. 5 and 6 show the changes of the maximum loadability of both studies. Placing a DG unit in bus 40 improves the voltage stability margin more than the other candidate buses because the voltage at bus 40 is more sensitive to the real power (see Fig. 3). Also, bus 41 has high load demand, therefore, placing the DG unit on bus 40 makes the upper stream feeder gain more capacity for power loading. However, if the DG unit is placed on bus 28, the feeders will gain less capacity because the load demand in its down stream low compared to bus 41. Therefore, when the DG power at bus 28 increases, the current reverses to the upper stream. In this situation, the feeder between buses 23 to 41 will not gain extra capacity for power loading. Furthermore, Fig. 5 shows the impact of two DG units. They are placed in bus 40 and 28. Both DG units are varied from 0 to 8 MW; thus, the total of generation for both units is 16 MW (approximately 100% of the penetration level). When the two DG units are added, the downstream feeders (from bus 23 to 31, and from buses 23 to 41), and the upstream feeder (from buses 1 to 23), gain more capacity. Therefore, this capacity is reflected in the increase of the voltage stability margin to $(\lambda_{\max} = 3.37)$. However, this result is still lower than that of installing one DG unit

TABLE I
RESULTS OF THE DG LOCATION AND SIZE, SCENARIOS (1–4)

Candidate Buses	Scenarios			
	1	2	3	4
19	0	0	0	0
23	0	0	0	0
24	0	0	0	0
26	0	0	0	0
28	0	0	1.1MW	1.55MW
32	0	0	0	0
33	0	0	0	0
35	0	0	0	0
37	0	0	0	0
38	0	0	2.2MW	1.92MW
39	0	0	0	0
40	0	4.5MW	6.6MW	9.47MW
Total size	0	4.5MW	9.9MW	12.94

TABLE II
RESULTS OF THE DG LOCATION AND SIZE, SCENARIO (5)

Candidate Buses	Scenarios		
	5		
	Wind (MW)	Solar (MW)	Dispatchab le (MW)
19	0	0	0
23	0	0	0
24	0	0	0
26	0	0	0
28	0	0.87	0
32	0	0	0
33	0	0	0
35	0	0	0
37	0	0	0
38	0	0	0
39	0	0	0
40	3.3 MW	3.38	1.2
Total size	3.3MW	4.25MW	1.2MW

at bus 40 ($\lambda_{\max} = 3.404$). Thus, applying optimization method can solve the problem of placement and sizing of the DG units to improve the voltage stability margin.

C. Results of the DG Sizes and Locations

The results for the five scenarios which are presented in Section IV are given in Tables I and II. These results are obtained by the optimization formulation which is proposed in Section IV. The first column demonstrates the candidate buses for the DG installation. These candidate buses are obtained by the sensitivity analysis in Sections III and VI-A. The other columns show the sizing and sitting of DG units in each scenario.

In Tables I and II, the simulation of the optimization formulation placed and sized the DG units in buses 40, 38, and 28. In all scenarios, the highest DG rating is placed in bus 40. This placement is reasonable because bus 40 is located at the far end of the distribution system and has low voltage profile. However, if the optimization constraints of the voltage and current are violated, then the second option will be bus 38. Bus 28 is also sensitive to the DG penetration as shown from Fig. 3. As results, the simulation has considered this bus for the DG placement and sizing.

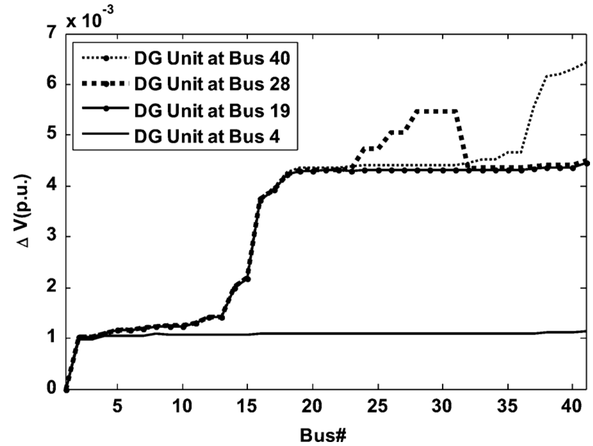

Fig. 7. Results of voltage sensitivity analysis $\Delta V/\Delta Q$.

The total size of the wind DG units in scenarios 3 and 5 is lower than the solar units in scenarios 4 and 5. This result is logical since the capacity factor of the wind turbine is higher than the solar photovoltaic generator.

Tables I and II show the results when the DG units are operating at unity power factor. On the other hand, the utilities that allow the DG units to operate in fixed power factor mode (0.95 lagging to 0.95 leading), need more elaboration. In this case, the DG units will have more chance to improve the voltage stability margin if they are operating in leading power factor and supporting the system with reactive power. The sensitivity analysis of $\Delta V/\Delta Q$ is conducted to test the most sensitive buses as in

$$\Delta V = (J_{RQV})^{-1}\Delta Q \tag{17}$$

where

$$\begin{aligned} &Q_{G_{n,1}} + C(n,1) * Q_{\mathrm{DG}\,D_i} + C(n,2) * Q_{\mathrm{DG}\,W_i} \\ &\quad + C(n,3) * Q_{\mathrm{DG}\,S_i} - C(n,4) * Q_{D_{n,i}} \\ &\qquad = -\sum_{j=1}^{m} V_{n,i} * V_{n,i} * Y_{ij} * \sin(\theta_{ij} + \delta_{n,j} - \delta_{n,i}) \quad \forall i, n. \end{aligned} \tag{18}$$

The results show that the most sensitive buses are the same buses as of $\Delta V/\Delta P$ sensitivity as shown in Fig. 7, but with higher magnitude of ΔV due to the high sensitivity of reactive power changes to the voltage profile of the system. In addition, the condition of operating DG units in fixed power factor mode (0.95 lagging to 0.95 leading) should be considered as constraints in the formulation of the placement and sizing of the DG units. In addition, (10) should be modified to include the reactive power generation as in (18).

The simulation results of DG units that operate in fixed power factor mode (0.95 lagging to 0.95 leading) are included in this paper. as given in Table III. These results show that the dispatchable DG units in scenario 1 are placed in bus 40. However, for the wind in scenario 3 and solar in scenario 4, the higher ratings of the DG units are placed in bus 19. These results in (scenario 3

TABLE III
RESULTS OF THE DG LOCATION AND SIZE, SCENARIOS WHEN DG UNITS OPERATES BETWEEN 0.95 LEAD OR LAG POWER FACTOR

Scenario	Type of DG	Location	Rating MVA	Power factor
1	Base case : No DG installed			
2	dispatchable	Bus 40	4.5	0.95 leading
3	wind	bus 19	8.8	0.95 leading
		bus 40	1.1	Unity
4	Solar	Bus 19	9.7	0.95 leading
		Bus 28	1.06	Unity
		Bus 40	2.38	Unity
5	dispatchable	Bus 40	0.82	0.95 leading
(mix)	wind	Bus 19	3.3	0.95 leading
	solar	Bus 19	4.2	0.95 leading

and 4) are reasonable because the sensitivity analysis shows that bus 19 is less sensitive to the injection of real and reactive power compared to bus 40. In addition, the voltage is more sensitive to the change in reactive power than real power. As a result placing a DG unit operating in leading power factor is better in upper stream to avoid the violation of the voltage constraints.

In scenario 5 (Table III), all of the DG units are operating at 0.95 leading power factor. The renewable DG units are sized and placed in bus 19, while the dispatchable DG unit is sized and sitted in bus 40. In this scenario, the ratings of the DG units are smaller compared to the other scenarios, because the dispatchable DG units are operating at constant real and reactive power (their capacity factor equal 1), therefore it improves the voltage stability constantly during the year. Thus, the constant operation of the dispatchable DG unit is less dependent on renewable energy DG units in improving the voltage stability margin, and hence their ratings are small.

VII. CONCLUSION

In this paper, a method of DG units allocation is proposed. This method targets utilizing the DG units to improve the voltage stability margin. It considers the probabilistic nature of both loads and renewable DG generation. The load is modeled by the IEEE-RTS system, while the renewable DG resources are modeled by using three years of historical data that have been provided from the site under study. These data are used to model the solar irradiance and wind speed by Beta and Weibull probability distribution functions, respectively. The candidate buses for the DG units' installation are selected based on the sensitivity to the voltage. Simulation results indicate that DG size and location can have positive impacts on the voltage stability margin. Therefore, an optimization method can be used to determine the locations and sizes of the DG units, to achieve the target of improving the voltage stability margin. Furthermore, formulating the problem using an optimization method helps to avoid any violation of the system limits, such as buses' voltage and feeders' current. Simulation shows that placing and sizing DG units is affected by the operating condition of the DG units (unity power factor or between 0.95 lead or lag). When the DG units operate at unity power factor, they are recommended to be placed in the most sensitive voltage buses in order to improve the voltage stability margin with a condition of not violating the system voltage and current limits. However, if the utility allows operating the DG units between 0.95 lead and 0.95 lag, the reactive power during leading power factor could improve the voltage stability margin due to the more sensitivity between $\Delta V/\Delta Q$ than $\Delta V/\Delta P$. Therefore, the DG units with higher rating might be placed in upper stream of a radial distribution system in order to keep the system operating within the allowed limits of voltage and currents.

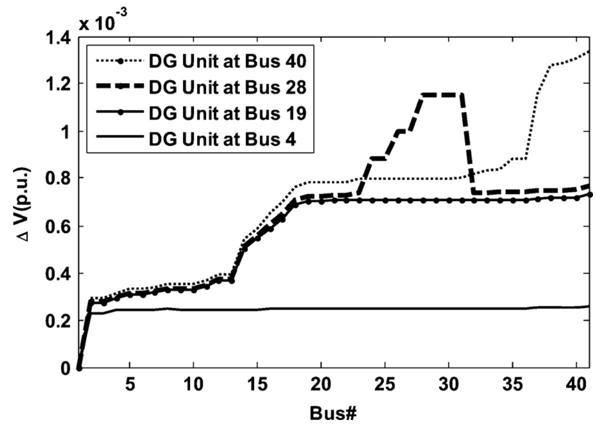

Fig. 8. Results of voltage sensitivity analysis (the penetration level is 10%).

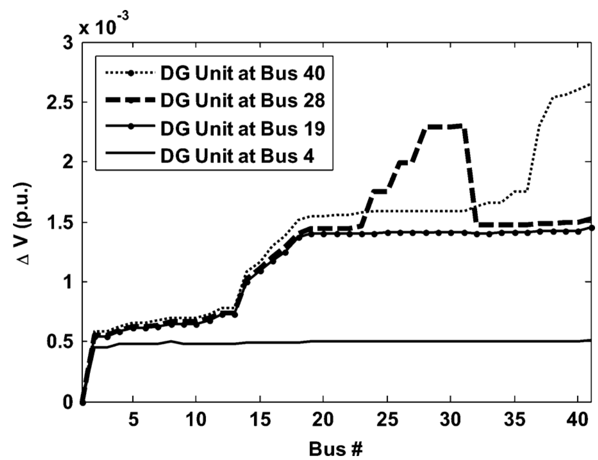

Fig. 9. Results of voltage sensitivity analysis (the penetration level is 20%).

APPENDIX

Figs. 8 and 9 shows the sensitivity analysis $\Delta V/\Delta P$ for in all buses of the radial distribution system when the penetration level is 10% and 20%, respectively.

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