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MIMO Tracking PI/PID Controller Design for Nonlinear Systems based on Singular Perturbation Technique ⋆

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Abstract: Problem of the tracking PI/PID multivariable controller design based on time-scale separation technique (singular perturbation technique) for MIMO nonlinear systems is discussed. The presented design methodology guarantees almost perfect rejection of nonlinearities, unknown external disturbances, and interactions between loops due to increase of time-scale separation degree between the fast and slow modes that are artificially forced in the closed-loop system. Simulation results for trajectory tracking control of two-link robot manipulator are presented as an example of the proposed design methodology application.

Keywords: Multivariable control, PI control, PID control, MIMO nonlinear system, singular perturbation method.

1. INTRODUCTION

The existing approaches for multi-input multi-output (MIMO) control system design were mostly developed for linear systems, for example, based on matrix technique and polynomial representation of multivariable systems (Wolovich (1974)), Nyquist techniques (Postlethwaite and MacFarlane (1979)), geometric approach (Wonham (1979)), frequency domain methods (Skogestad and Postlethwaite (2005)), state-space approach for multivariable controller design and decoupling control (Morgan (1964)). Problems of multivariable controller design, decoupling of control channels and disturbance rejection are the areas of the lively interests for many researchers at present (Chang and Davison (1995); Astrom et al. (2001); Vu et al. (2007)).

The problem of output regulation in order to achieve asymptotic tracking of prescribed trajectories and/or asymptotic disturbance rejection for fixed nonlinear systems, in the presence of exosystem-generated commands and disturbances is highlighted by Byrnes and Isidori (1998). The output regulation for a certain class of nonlinear MIMO systems with nonlinear internal model and high gain feedback was discussed by McGregor et al. (2006). Asymptotic regulation of minimum phase nonlinear systems using output feedback and high-gain observer was discussed by Mahmoud and Khalil (1996).

The decoupling of control channels and disturbance rejection for multivariable nonlinear control systems can be provided based on sliding mode control (Utkin (1992); Young (1978); Slotine and Sastry (1983); Piltan and Sulaiman (2012)), control with a high gain in feedback (Meerov (1965); Young et al. (1977); Krutko (1995)). The special feature of such advanced techniques of control system design is the presence of two-time-scale motions in the closed-loop system. Therefore, the singular perturbation technique (or time-scale separation technique) may be used for analysis of closed-loop system properties (Kokotović et al. (1999)).

In this paper, the discussed design methodology for MIMO tracking control is the development of the results presented in (Błachuta et al. (1999); Yurkevich (1995, 2004)) where a distinctive feature is that two-time-scale motions are artificially forced in the closed-loop system. Stability conditions imposed on the fast and slow modes, and a sufficiently large mode separation rate, can ensure that the full-order closed-loop system achieves desired properties: the output transient performances are as desired, and they are insensitive to parameter variations and external disturbances. The paper is organized as follows. At the beginning, the design of tracking PI controllers is discussed for MIMO nonlinear systems. Second, the design of tracking PID controllers is treated for MIMO nonlinear systems as well. Third, simulation results of a two-link manipulation robot tracking control system are presented.

2. MIMO PI CONTROLLER DESIGN

2.1 Control problem statement

Consider a MIMO nonlinear system of the form

$$y^{(1)} = f(y) + G(y)u + w, \qquad (1)$$

where $y^{(1)} = dy/dt$, $y = [y_1, \dots, y_p]^T$ is the measurable output, $u = [u_1, \dots, u_p]^T$ is the control, $w = [w_1, \dots, w_p]^T$ is the vector of unknown bounded external disturbances.

Let us assume that the condition

$$\det G(y) \neq 0 \ \forall \ y \in \Omega_y \qquad (2)$$

is satisfied, where Ω_y is the bounded working set of the nonlinear system (1).

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A control system is being designed so that

$$\lim_{t\to\infty} \|e(t)\| = \delta, \tag{3}$$

where $e(t)$ is an error of the reference input realization, $e(t) = r(t) - y(t)$, $r(t) = [r_1(t), \ldots, r_p(t)]^T$ is the reference input, $r(t)$ is a smooth vector function of the time t such that $\|dr(t)/dt\| \le \bar{r}_{max} < \infty \; \forall t$, and δ is an arbitrary small positive value.

2.2 MIMO tracking PI controller

Let us consider the control law given by

$$u = K_0\tilde{u}, \tag{4}$$

$$\mu\tilde{u}^{(1)} = T^{-1}e + e^{(1)}, \tag{5}$$

where $\mu = \mathrm{diag}\{\mu_1, \ldots, \mu_p\}$, $T = \mathrm{diag}\{T_1, \ldots, T_p\}$, $\mu_i > 0$, $T_i > 0$ for all $i = 1, \ldots, p$, and K_0 is the nonsingular matrix which will be specified below.

Note, due to diagonal structure of the matrices μ and T, the MIMO tracking controller (4),(5) consists of p linear controllers $C_1, \ldots, C_p$ generating the auxiliary control vector $\tilde{u}$ and accompanied by the matching matrix K_0. This structure is called as the centralized output feedback tracking controller, where the generalized block diagram of the discussed MIMO control system is represented by Fig. 1.

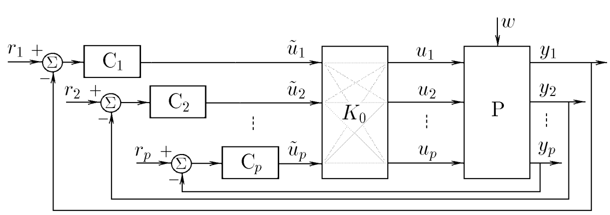

Fig. 1. Block diagram of the discussed MIMO control system

The each i-th linear controller C_i has the form

$$\mu_i\tilde{u}_i^{(1)} = T_i^{-1}e_i + e_i^{(1)}$$

and one may be expressed in terms of transfer functions, that is the structure of the conventional PI controller given by

$$\tilde{u}_i(s) = \left[\frac{1}{\mu_i T_i s} + \frac{1}{\mu_i}\right] e_i(s).$$

2.3 Two-time-scale motion analysis

In accordance with (1),(4), and (5), the equations of the closed-loop system are given by

$$\frac{dy}{dt} = f(y) + G(y)K_0\tilde{u} + w, \tag{6}$$

$$\mu\frac{d\tilde{u}}{dt} = T^{-1}e + \frac{de}{dt}, \tag{7}$$

where $e^{(1)} = r^{(1)} - y^{(1)}$. Substitution of (6) into (7) yields the closed-loop system equations in the form

$$\frac{dy}{dt} = f(y) + G(y)K_0\tilde{u} + w, \tag{8}$$

$$\mu\frac{d\tilde{u}}{dt} = -K_0G(y)\tilde{u} + \frac{dr}{dt} + T^{-1}[r - y] - f(y) - w. \tag{9}$$

If $\mu \to 0$, the closed-loop system equations (8),(9) have the standard singular perturbation form, then fast and slow modes are artificially forced in the system (8),(9) where the time-scale separation between these modes depends on the parameters of the matrix μ. Accordingly, the singular perturbation method may be used to analyze the closed-loop system properties (Kokotović et al. (1999)).

From (8),(9), we obtain the fast-motion subsystem (FMS) given by

$$\begin{aligned}\mu\frac{d\tilde{u}}{dt} &= -K_0G(y)\tilde{u} \\ &\quad + \frac{dr}{dt} + T^{-1}[r - y] - f(y) - w,\end{aligned} \tag{10}$$

where $r(t)$, $r^{(1)}(t)$, $y(t)$ and $w(t)$ are treated as the frozen variables during the transients in (10).

Since $\mu_i > 0 \; \forall \; i$ and in accordance with (2), the FMS stability can be provided by selection of K_0 such that all eigenvalues of the matrix $G(y)K_0$ have strictly positive real part for all $y \in \Omega_y$. The characteristic polynomial of the FMS (10) is

$$A_{FMS}(s) = \det[\mu s + G(y)K_0]. \tag{11}$$

Take for simplicity, the matching matrix K_0 such that $K_0 = G^{-1}(y)$. As the result, the FMS characteristic polynomial is factorized such that

$$A_{FMS}(s) = \prod_{i=1}^{p}(\mu_i s + 1). \tag{12}$$

If the FMS (10) is stable and $\mu \to 0$, then after the rapid decay of transients in (10), we have the steady state (more precisely, quasi-steady state) for the FMS, where $\tilde{u} = \tilde{u}^s$ and

$$\tilde{u}^s = [K_0G(y)]^{-1}\left[\frac{dr}{dt} + T^{-1}[r - y] - f(y) - w\right].$$

From (8),(9), the steady state of the FMS (10) yields the following reduced order system:

$$\begin{aligned}\frac{dy}{dt} &= f(y) + G(y)K_0\tilde{u}^s + w, \\ 0 &= -K_0G(y)\tilde{u}^s + \frac{dr}{dt} + T^{-1}[r - y] - f(y) - w.\end{aligned} \tag{13}$$

By eliminating $\tilde{u}^s$ from (13), we obtain the equations of the slow motion subsystem (SMS)

$$e^{(1)} + T^{-1}e = 0. \tag{14}$$

The characteristic polynomial of the SMS (14) is

$$A_{SMS}(s) = \det(sI_p + T^{-1}) = \prod_{i=1}^{p}(s + T_i^{-1}).$$

So, if the steady state of the FMS (10) takes place, then the closed-loop system equations (8),(9) imply the SMS (14). As the result, if a sufficient time-scale separation between the fast and slow modes in the closed-loop system is maintained and exponential convergence of FMS transients to equilibrium is provided, then after the damping of fast transients the behavior of error prescribed by (14) is fulfilled. Note, due to diagonal structure of matrix T in (14), the almost perfect decoupling of control channels is provided as well.

The almost perfect rejection of interaction between loops, nonlinearities, and unknown external disturbances is provided due to increase of time-scale separation degree between the fast and slow modes in the closed-loop system (8),(9) by selection μ_i such that

$$\mu_i \le T_i/\eta,$$

where η is the time-scale separation degree, for example, $\eta \ge 10$.

3. MIMO PID CONTROLLER DESIGN

3.1 Control problem statement

Consider a MIMO nonlinear system given by

$$y^{(2)} = f(y^{(1)}, y) + G(y^{(1)}, y)u + w, \qquad (15)$$

$y = [y_1, \dots, y_p]^T$ is the measurable output, $u = [u_1, \dots, u_p]^T$ is the control, $w = [w_1, \dots, w_p]^T$ is the vector of unknown bounded external disturbances.

Let us assume that the condition

$$\det G(y^{(1)}, y) \ne 0 \ \forall\ y \in \Omega_y \qquad (16)$$

is satisfied, where $\Omega_{y,\dot{y}}$ is the bounded working set of the nonlinear system (15).

A control system is being designed so that the condition (3) holds, where $e(t) = r(t) - y(t)$ and $r(t)$ is a smooth vector function of time t such that $\|r^{(j)}(t)/dt\| \le \bar{r}_{max} < \infty \ \forall\, t, \forall\ j = 1, 2$.

3.2 MIMO tracking PID controller

Consider the control law in the form

$$u = K_0\tilde{u}, \qquad (17)$$

$$\mu^2\tilde{u}^{(2)} + D\mu\tilde{u}^{(1)} = T^{-2}e + AT^{-1}e^{(1)} + e^{(2)}, \qquad (18)$$

where $\mu = \mathrm{diag}\{\mu_1, \dots, \mu_p\}$, $T = \mathrm{diag}\{T_1, \dots, T_p\}$, $A = \mathrm{diag}\{a_1, \dots, a_p\}$, $D = \mathrm{diag}\{d_1, \dots, d_p\}$, $\mu_i > 0$, $T_i > 0$, $a_i > 0$, $d_i > 0$ for all $i = 1, \dots, p$, and K_0 is the nonsingular matrix which will be specified below.

Due to the diagonal structure of matrices μ, T, D, and A the MIMO tracking controller (17),(18) consists of p linear controllers generating the auxiliary control vector $\tilde{u}$ and the matching matrix K_0 (Fig. 1), where the each i-th linear controller C_i has the form

$$\mu_i^2\tilde{u}_i^{(2)} + d_i\,\mu_i\tilde{u}_i^{(1)} = T_i^{-2}e_i + a_iT_i^{-1}e_i^{(1)} + e_i^{(2)}. \qquad (19)$$

The controller given by (19) can be expressed in terms of transfer functions such that

$$\tilde{u}_i(s) = \frac{s^2 + a_iT_i^{-1}s + T_i^{-2}}{\mu_i^2 s + d_i\mu_i s}e_i(s). \qquad (20)$$

3.3 Two-time-scale motion analysis

By introducing the new variables

$$\bar{y}_1 = y,\ \bar{y}_2 = y^{(1)},$$

the MIMO nonlinear system (15) can be rewritten such that

$$\bar{y}_1^{(1)} = \bar{y}_2, \qquad (21)$$

$$\bar{y}_2^{(1)} = f(\bar{y}_1, \bar{y}_2) + G(\bar{y}_1, \bar{y}_2)u + w. \qquad (22)$$

Denote

$$\bar{u}_1 = \tilde{u},\ \bar{u}_2 = \mu\tilde{u}^{(1)},\ \bar{e}_1 = e,\ \bar{e}_2 = e^{(1)}.$$

$$\bar{e}_1 = \bar{r}_1 - \bar{y}_1,\ \bar{e}_2 = \bar{r}_2 - \bar{y}_2,$$

$$\bar{r}_1 = r,\ \bar{r}_2 = r^{(1)},\ \bar{r}_3 = r^{(2)}.$$

As the result, the controller can be rewritten as

$$\begin{aligned} u &= K_0\bar{u}_1,\\ \mu\bar{u}_1^{(1)} &= \bar{u}_2, \\ \mu\bar{u}_2^{(1)} &= -D\bar{u}_2 + T^{-2}\bar{e}_1 + AT^{-1}\bar{e}_2 + \bar{e}_2^{(1)}. \end{aligned} \qquad (23)$$

In accordance with (21),(22), and (23), the equations of the closed-loop system are given by

$$\begin{aligned} \bar{y}_1^{(1)} &= \bar{y}_2,\\ \bar{y}_2^{(1)} &= f(\bar{y}_1, \bar{y}_2) + G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1 + w,\\ \mu\bar{u}_1^{(1)} &= \bar{u}_2, \\ \mu\bar{u}_2^{(1)} &= -D\bar{u}_2 + T^{-2}\bar{e}_1 + AT^{-1}\bar{e}_2 + \bar{e}_2^{(1)}. \end{aligned} \qquad (24)$$

Replace $\bar{e}_2^{(1)}$ in the last equation of (24) by $\bar{e}_2^{(1)} = e^{(2)} = \bar{r}_2^{(1)} - \bar{y}_2^{(1)}$ and, by taking into account the second equation of (24), the closed-loop system (24) can be rewritten as

$$\begin{aligned} \bar{y}_1^{(1)} &= \bar{y}_2,\\ \bar{y}_2^{(1)} &= f(\bar{y}_1, \bar{y}_2) + G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1 + w,\\ \mu\bar{u}_1^{(1)} &= \bar{u}_2, \\ \mu\bar{u}_2^{(1)} &= -G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1 - D\bar{u}_2 + \phi(\bar{r}_1, \bar{y}_1, \bar{r}_2, \bar{y}_2, \bar{r}_3, w), \end{aligned} \qquad (25)$$

where

$$\phi(\bar{r}_1, \bar{y}_1, \bar{r}_2, \bar{y}_2, \bar{r}_3, w)$$
$$= T^{-2}[\bar{r}_1 - \bar{y}_1] + AT^{-1}[\bar{r}_2 - \bar{y}_2] + \bar{r}_3 - f(\bar{y}_1, \bar{y}_2) - w.$$

The closed-loop system (25) has the standard singular perturbation form where fast and slow modes are artificially forced if $\mu \to 0$. Accordingly, similar as above, the singular perturbation method may be used to analyze the closed-loop system properties (Kokotović et al. (1999)). From (25), we obtain the FMS equation given by

$$\begin{aligned} \mu\bar{u}_1^{(1)} &= \bar{u}_2, \\ \mu\bar{u}_2^{(1)} &= -G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1 - D\bar{u}_2 + \phi(\bar{r}_1, \bar{y}_1, \bar{r}_2, \bar{y}_2, \bar{r}_3, w), \end{aligned} \qquad (26)$$

where $\phi(\cdot)$ is treated as the function of the frozen variables $\bar{r}_1$, $\bar{y}_1$, $\bar{r}_2$, $\bar{y}_2$, $\bar{r}_3$, w during the transients in (26).

The characteristic polynomial of the FMS (26) is

$$A_{FMS}(s) = \det\begin{bmatrix} \mu s & -I_p \\ GK_0 & (\mu s + D) \end{bmatrix}.$$

Take for simplicity, the matching matrix K_0 in the form $K_0 = G^{-1}$. As the result, the FMS characteristic polynomial is factorized such that

$$A_{FMS}(s) = \prod_{i=1}^{p}(\mu_i^2 s^2 + d_i\mu_i s + 1). \qquad (27)$$

If the FMS (26) is stable and $\mu \to 0$, then after the rapid decay of transients in (26), we have the steady state (more precisely, quasi-steady state) for the FMS, where $\bar{u}_1 = \bar{u}_1^s$, $\bar{u}_2 = \bar{u}_2^s = 0$, and

$$\bar{u}_1^s = [G(\bar{y}_1, \bar{y}_2)K_0]^{-1}\phi(\bar{r}_1, \bar{y}_1, \bar{r}_2, \bar{y}_2, \bar{r}_3, w).$$

From (25), the steady state of the FMS (26) yields the following reduced order system:

$$\begin{aligned} \bar{y}_1^{(1)} &= \bar{y}_2,\\ \bar{y}_2^{(1)} &= f(\bar{y}_1, \bar{y}_2) + G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1^s + w,\\ 0 &= \bar{u}_2^s, \\ 0 &= -D\bar{u}_2^s - G(\bar{y}_1, \bar{y}_2)K_0\bar{u}_1^s + \phi(\bar{r}_1, \bar{y}_1, \bar{r}_2, \bar{y}_2, \bar{r}_3, w). \end{aligned} \qquad (28)$$

By eliminating $\bar{u}_1^s$ and $\bar{u}_2^s$ from (28), we obtain the SMS equations

$$\bar{y}_1^{(1)} = \bar{y}_2,$$
$$\bar{y}_2^{(1)} = T^{-2}[\bar{r}_1 - \bar{y}_1] + AT^{-1}[\bar{r}_2 - \bar{y}_2] + \bar{r}_3. \quad (29)$$

As far as $\bar{r}_3 - \bar{y}_2^{(1)} = e^{(2)}$, $\bar{r}_2 - \bar{y}_2 = e^{(1)}$, and $\bar{r}_1 - \bar{y}_1 = e$, the SMS (29) takes the following form:

$$e^{(2)} + AT^{-1}e^{(1)} + T^{-2}e = 0. \quad (30)$$

The characteristic polynomial of the SMS (30) is

$$A_{SMS}(s) = \prod_{i=1}^{p}(s^2 + a_i T_i^{-1}s + T_i^{-2}). \quad (31)$$

So, if the steady state of the FMS (26) takes place, then the closed-loop system equations (25) imply the SMS (30). As the result, if a sufficient time-scale separation between the fast and slow modes in the closed-loop system is maintained and exponential convergence of FMS transients to equilibrium is provided, then after the damping of fast transients the behavior of error prescribed by (30) is fulfilled. Note, due to diagonal structure of matrix T in (30), the almost perfect decoupling of control channels is provided as well.

The almost perfect rejection of interaction between loops, nonlinearities, and unknown external disturbances is provided as well due to increase of time-scale separation degree between the fast and slow modes in the closed-loop system (25) via decreasing of μ_i $\forall i$.

3.4 Comments on PID controller representation

From the expressions of the FMS characteristic polynomial (27) and the SMS characteristic polynomial (31) follow that the representation of the controller in the form (20) gives the clear meaning of relationships between parameters of the controller and performances of the fast and slow transients in the discussed closed-loop system.

Obviously, the controller (20) can be represented in the form of the conventional PID controller with an additional first order filter, that is

$$\tilde{u}_i(s) = \frac{1}{\mu_i s + d_i}\left[\frac{1}{\mu_i T_i^2}\frac{1}{s} + \frac{a_i}{\mu_i T_i} + \frac{1}{\mu_i}s\right] e_i(s), \quad (32)$$

and also the transfer function of the controller (20) can be rewritten in the following well-known form:

$$\tilde{u}_i(s) = \left[k_i^P + \frac{k_i^I}{s} + \frac{k_i^D s}{\tau_i s + 1}\right] e_i(s), \quad (33)$$

where

$$k_i^P = \frac{a_i d_i T_i - \mu_i}{\mu_i d_i^2 T_i^2}, \ k_i^I = \frac{1}{\mu_i d_i T_i^2}, \quad (34)$$

$$k_i^D = \frac{\mu_i^2 + d_i^2 T_i^2 - a_i d_i \mu_i T_i}{\mu_i d_i^3 T_i^2}, \ \tau_i = \frac{\mu_i}{d_i}. \quad (35)$$

It is clear to see from (34),(35), the representation of the controller (20) in the form (33) leads to loss of the clarity of the relationships between parameters k_i^P, k_i^I, k_i^D, τ_i of the controller and performances of the fast and slow transients in the discussed closed-loop system. That is the great disadvantage of the widely used representation of the PID controller in the form (33) in contrast to the representation given by (20).

4. TRAJECTORY TRACKING CONTROL OF ROBOT MANIPULATOR

Problem of controller design for tracking of robotic manipulators is treated in numerous research works (Young (1978); Slotine and Sastry (1983); Spong and Vidyasagar (1989); Piltan and Sulaiman (2012); Chanda and Gogoi (2014)). This problem gives an appropriate example of the high nonlinear closely coupled dynamical plant and allows to demonstrate efficiency of the proposed trajectory tracking controller design methodology which is based on the singular perturbation technique.

Let us consider a two-link manipulation robot model as shown on Fig. 2, where the dynamical behavior is described by the following equations (Young (1978)):

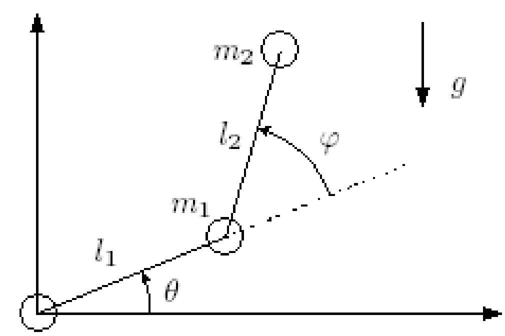

Fig. 2. Two-link robot manipulator model

$$\begin{aligned}
x_1^{(1)} &= x_2,\\
x_2^{(1)} &= [a_{22}/a][\beta_{12}x_2(x_2 + 2x_4) + \gamma_1 g + u_1 + w_1]\\
&\quad -[a_{12}/a][-\beta_{12}x_4^2 + \gamma_2 g + u_2 + w_2],\\
x_3^{(1)} &= x_4, \qquad (36)\\
x_4^{(1)} &= -[a_{12}/a][\beta_{12}x_2(x_2 + 2x_4) + \gamma_1 g + u_1 + w_1]\\
&\quad +[a_{11}/a][-\beta_{12}x_4^2 + \gamma_2 g + u_2 + w_2],
\end{aligned}$$

where

$$a_{11} = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(x_3),$$
$$a_{12} = m_2 l_2^2 + m_2 l_1 l_2 \cos(x_3), \ a_{22} = m_2 l_2^2,$$
$$a = a_{11}a_{22} - a_{12}^2, \ \beta_{12} = m_2 l_1 l_2 \sin(x_3),$$
$$\gamma_1 = -(m_1 + m_2)l_1 \cos(x_1) - m_2 l_2 \cos(x_1 + x_3),$$
$$\gamma_2 = -m_2 l_2 \cos(x_1 + x_3),$$
$$x = [x_1, x_2, x_3, x_4]^T = [\theta, \dot{\theta}, \varphi, \dot{\varphi}]^T,$$
$$y_1 = x_1, \ y_2 = x_3, \ u = [u_1, u_2]^T, \ w = [w_1, w_2]^T,$$

$y = [y_1, y_2]^T$ is the measurable output, u is the vector of joint torques (control variables), w is the vector of external joint torques (disturbances), and g is the gravitational constant.

The model parameters are selected as the following ones:

$$m_1 = m_2 = 1[kg], \ l_1 = l_2 = 1[m], \ g = 9.8[m/s^2].$$

The model equations (36) correspond to the nonlinear system given by (15), where

$$G(x_3) = \frac{1}{a}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{12} & a_{11} \end{bmatrix}$$

and, consequently,

$$G^{-1}(x_3) = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}. \tag{37}$$

Consider the two-input two-output tracking controller given by (17) and (18), where the matching matrix K_0 is selected such that $K_0 = G^{-1}(x_3)$ in accordance with (37). Take

$$T_1 = T_2 = 0.2\ [s],\ \eta = 20,\ \mu_1 = T_1/\eta,\ \mu_2 = T_2/\eta,$$

$$a_1 = a_2 = d_1 = d_2 = 2.$$

The simulation results of the system (36) which is equipped by controller (17), (18) with assigned above parameters are displayed on Figs. 3–6, where the initial conditions are

$$x(0) = [x_1(0), x_2(0), x_3(0), x_4(0)]^T = [-\pi/2, 0, 0, 0]^T,$$

the reference signals are assigned such that

$$r_1(t) = \frac{\pi}{2} + \pi \sin(t - \frac{\pi}{2}),\ r_2(t) = \pi + \pi \sin(2t - \frac{\pi}{2}),$$

whereas the external joint torques (disturbances) are assigned by step-wise functions, that are

$$w_1(t) = 45 \cdot 1(t-1),\ w_2(t) = -45 \cdot 1(t-3).$$

From simulation results displayed on Figs. 3–6 for $\eta = 20$,

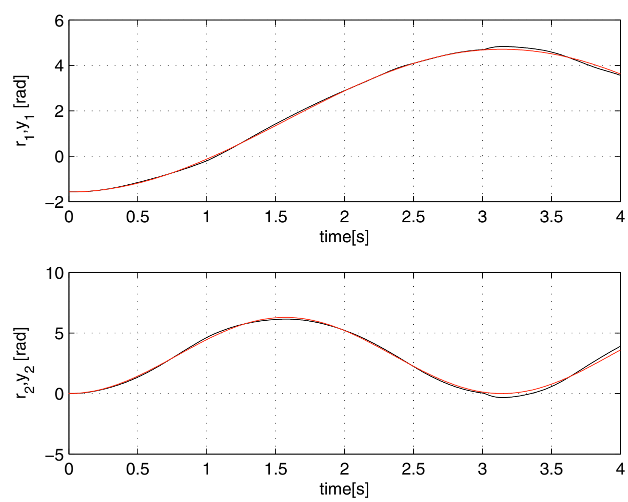

Fig. 3. The sine reference inputs $r_1(t)$, $r_2(t)$ (red lines) and output responses $y_1(t)$, $y_2(t)$ (black lines) of the system (36) with controller (17), (18) for the step disturbances $w_1(t)$, $w_2(t)$ (Fig. 6) when $\eta = 20$

where $\mu_1 = \mu_2 = 0.01$ s., it follows that the almost perfect tracking, disturbance rejection, and decoupling of control channels are provided.

The effect of time-scale separation degree between the fast and slow modes in the system (36) with controller (17), (18) is highlighted by plots for the tracking errors $e_1(t)$, $e_2(t)$ shown on Fig. 7, where simulation results have been done for $\eta = 20$ (blue lines), $\eta = 40$ (red lines), and $\eta = 80$ (black lines). It is clear to see from Fig. 7, the increase time-scale separation degree η leads to reduce of the tracking errors. Hence, the tracking errors approach to zero uniformly as $\mu_i \to 0\ \forall i$.

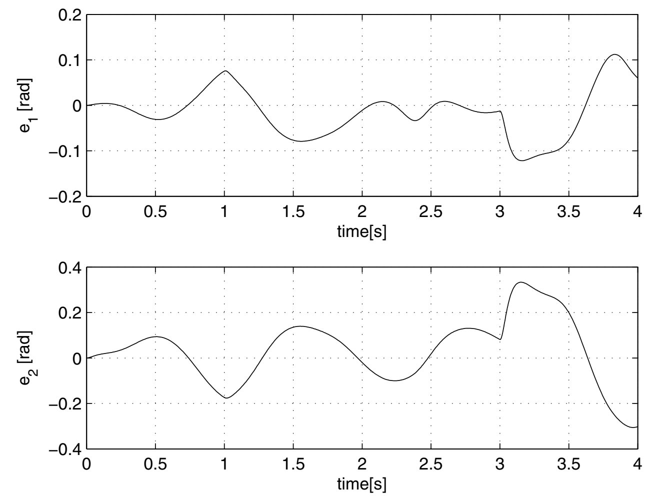

Fig. 4. Tracking errors $e_1(t)$, $e_2(t)$ of the system (36) with controller (17), (18) for the sine reference inputs $r_1(t)$, $r_2(t)$ and the step disturbances $w_1(t)$, $w_2(t)$ when $\eta = 20$

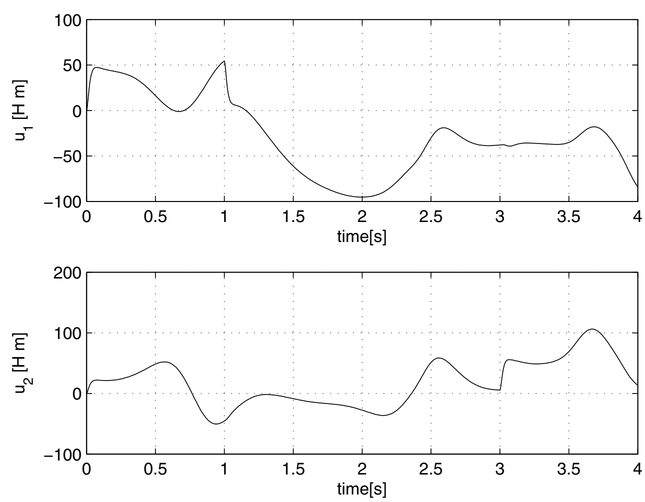

Fig. 5. Control variables $u_1(t)$, $u_2(t)$ of the system (36) with controller (17), (18) for the sine reference inputs $r_1(t)$, $r_2(t)$ and the step disturbances $w_1(t)$, $w_2(t)$ when $\eta = 20$

5. CONCLUSION

The design methodology based on time-scale separation technique allows for design of MIMO tracking PI or PID controllers where the almost perfect decoupling of control channels and unknown external disturbance rejection are provided by increasing the time-scale separation degree between the fast and slow modes that are artificially forced in the closed-loop system. The analytical expressions for calculation of controller parameters have been derived. Moreover, the presented approach gives the clear meaning of relationships between parameters of the controller and performances of the transients in the closed-loop system.

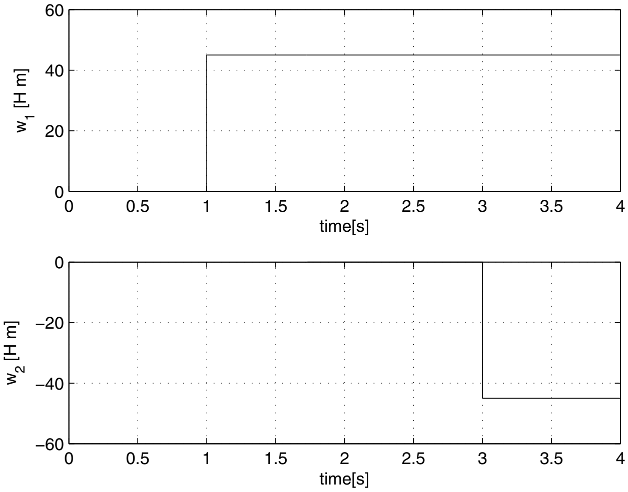

Fig. 6. External disturbances $w_1(t)$, $w_2(t)$ of the system (36) with controller (17), (18) when $\eta = 20$

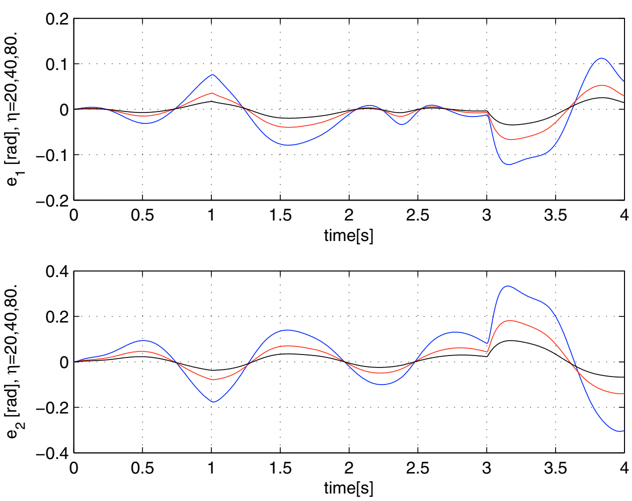

Fig. 7. Tracking errors $e_1(t)$, $e_2(t)$ of the system (36) with controller (17), (18) for the sine reference inputs $r_1(t)$, $r_2(t)$ and the step disturbances $w_1(t)$, $w_2(t)$ when $\eta = 20$ (blue lines), $\eta = 40$ (red lines), and $\eta = 80$ (black lines)

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