Sliding-mode control for uncertain neutral delay systems

Y. Niu, J. Lam and X. Wang

Abstract: The problem of sliding-mode control for a class of neutral delay systems with uncertainties in both the state matrices and the input matrix is considered. By selecting a sliding surface depending on the current state and delayed state, the paper gives a sufficient condition in terms of linear matrix inequalities (LMIs) such that the closed-loop system is guaranteed to be asymptotically stable. When LMIs are feasible, the design of the sliding surface and the sliding-mode control law can be easily obtained by convex optimisation. State trajectories are attracted onto the sliding surface in a finite time and remain there for all subsequent time. A simulation illustrates the application of the method.

1 Introduction

Dynamic systems with time delays are commonly encountered in various areas, such as chemical processes, aircraft stabilisation, manual control, electrical heater and long transmission lines in pneumatic, hydraulic and rolling-mill systems. Over the past decades considerable research effort has been undertaken on time-delay systems, since the existence of a delay in a physical system is often a source of instability or poor performance.

On the other hand, the problem of stability and stabilisation for neutral delay systems has attracted considerable attention. A neutral delay system is one depending not only on state delays, but also on the derivatives of the delayed state [1, 2]. There have recently been a number of developments in the search for a control mechanism for uncertain neutral delay systems [3–7]. Xu *et al.* [4] considered the H_∞ and positive-real control problem for linear neutral delay systems, and presented sufficient conditions for the solvability of these two problems by means of an LMI approach. Wang *et al.* [7] dealt with the problem of robust reliable control for a class of uncertain neutral systems with actuator faults. Xu *et al.* [6] proposed a guaranteed cost controller for uncertain neutral delay systems such that the closed-loop system is not only stable but also guarantees an adequate level of performance for all admissible uncertainties.

Since its early appearance in the 1950 s, sliding-mode control (SMC) has proven to be an effective robust control strategy for incompletely modelled or uncertain systems. An SMC system has various attractive features such as fast response, good transient performance, and robustness with respect to uncertainties and external disturbance. Recently, SMC involving time-delay systems has received increasing attention [8–14]. Niu *et al.* [14] proposed a neural network based sliding-mode control approach to solve the problem of robust control for nonlinear uncertain state-delayed systems. Gouaisbaut *et al.* [13] considered the sliding-mode control of uncertain systems with multiple and time-varying state delays. Yang *et al.* [9] gave an output feedback sliding-mode control law for uncertain systems with unmeasurable states. However, due to the complexity of neutral delay systems, there has been very little work to date on the problem of SMC for uncertain neutral delay systems.

In this paper we deal with the design of SMC for uncertain neutral delay systems. The uncertain system under consideration has mismatched uncertainties in the state matrices, and matching norm-bounded uncertainties in the input matrix. The sliding surface is chosen as a function depending on the current state and delayed state. By utilising a Lyapunov approach a sufficient condition in terms of linear matrix inequalities is derived to guarantee the asymptotic stability of the closed-loop system. Thus when LMIs are feasible, the design of a sliding surface and sliding-mode controller can be constructed. It is also shown that the sliding motion is attained within a region in the state space in finite time.

Throughout, $\boldsymbol{R}^n$ denotes the real n-dimensional linear vector space. $\|\cdot\|$ denotes the Euclidean norm of a vector or the spectral norm of a matrix. For a real symmetric matrix, $\boldsymbol{M} > 0\,(<0)$ means that $\boldsymbol{M}$ is positive definite (negative definite). $\boldsymbol{I}$ is used to represent an identity matrix of appropriate dimensions. Matrices, if their dimensions are not stated, are assumed to have compatible dimensions.

2 Problem formulation

Consider the following uncertain neutral delay system:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) - \boldsymbol{A}_d\dot{\boldsymbol{x}}(t-d) = {} & (\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{x}(t) \\ & + (\boldsymbol{A}_h + \Delta\boldsymbol{A}_h(t))\boldsymbol{x}(t-h) \\ & + (\boldsymbol{B} + \Delta\boldsymbol{B}(t))\boldsymbol{u}(t) \qquad (1)\end{aligned}$$

$$\boldsymbol{x}(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0] \qquad (2)$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the state, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$ is the control input, $\boldsymbol{A}$, $\boldsymbol{A}_h$, $\boldsymbol{A}_d$ and $\boldsymbol{B}$ are known real constant matrices of

IEE Proceedings online no. 20040009

doi: 10.1049/ip-cta:20040009

Paper first received 12th May and in revised form 29th October 2003

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appropriate dimension, $\Delta A(t), \Delta A_h(t)$ and $\Delta B(t)$ are unknown time-varying system parameter uncertainties, and $h > 0$ and $d > 0$ are constant time delays (d may not equal h). Let $\tau = \max(d, h)$. $\varphi(t)$ is a continuous vector-valued initial function. Without loss of generality, matrix $\boldsymbol{B}$ is assumed to have full column rank and $(\boldsymbol{A}, \boldsymbol{B})$ is stabilisable, i.e. there exists $\boldsymbol{K} \in \boldsymbol{R}^{m\times n}$ such that $\boldsymbol{A} - \boldsymbol{BK}$ is stable. Moreover, the uncertainty $\Delta \boldsymbol{B}(t)$ is assumed to be matched, i.e. there exists a matrix $\boldsymbol{\delta}(t) \in \boldsymbol{R}^{m\times m}$ such that $\Delta \boldsymbol{B}(t) = \boldsymbol{B}\boldsymbol{\delta}(t)$ with $\|\boldsymbol{\delta}(t)\| \le \rho_B < 1$, where ρ_B is a positive constant. The admissible parameter uncertainties are of the norm-bounded form

$$\Delta A(t) = \boldsymbol{E}\boldsymbol{F}(t)\boldsymbol{H}, \quad \Delta A_h(t) = \boldsymbol{E}_h \boldsymbol{F}_h(t)\boldsymbol{H}_h \tag{3}$$

where $\boldsymbol{E}$, $\boldsymbol{E}_h$, $\boldsymbol{H}$ and $\boldsymbol{H}_h$ are known constant matrices, and $\boldsymbol{F}(t)$ and $\boldsymbol{F}_h(t)$ are unknown time-varying matrices with Lebesgue measurable elements bounded by

$$\boldsymbol{F}^T(t)\boldsymbol{F}(t) \le I, \quad \boldsymbol{F}_h^T(t)\boldsymbol{F}_h(t) \le \boldsymbol{I}, \quad \forall t$$

The objective is to design an SMC law such that the state trajectories are driven onto the specified sliding surface and remain there in subsequent time. To this end we choose the sliding surface $\boldsymbol{s}(t)$ as

$$\boldsymbol{s}(t) = \boldsymbol{G}[\boldsymbol{x}(t) - \boldsymbol{A}_d \boldsymbol{x}(t-d)] = \Gamma \boldsymbol{B}^T \boldsymbol{X}[\boldsymbol{x}(t) - \boldsymbol{A}_d \dot{\boldsymbol{x}}(t-d)] = 0 \tag{4}$$

where $\boldsymbol{G} \in \boldsymbol{R}^{m\times n}, \boldsymbol{X} \in \boldsymbol{R}^{n\times n}$ is a positive definite matrix to be designed. $\boldsymbol{\Gamma} \in \boldsymbol{R}^{m\times m}$ is some nonsingular matrix: For simplicity, $\boldsymbol{\Gamma}$ is chosen as the identity matrix in this work.

We present an SMC design method and investigate the reachability of the sliding surface and the stability of the closed-loop systems. The following matrix inequalities are useful for the development of our result.

Lemma 1: (i) Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be real matrices of appropriate dimensions, for any scalar $\varepsilon > 0$,

$$\boldsymbol{XY} + \boldsymbol{Y}^T\boldsymbol{X}^T \le \varepsilon^{-1}\boldsymbol{XX}^T + \varepsilon \boldsymbol{Y}^T\boldsymbol{Y}$$

(ii) Let $\boldsymbol{E}$, $\boldsymbol{H}$, and $\boldsymbol{F}(t)$ be real matrices of appropriate dimensions with $\boldsymbol{F}(t)$ satisfying

$$\boldsymbol{F}^T(t)\boldsymbol{F}(t) \le \boldsymbol{I}$$

then, for any $\varepsilon > 0$,

$$\boldsymbol{EF}(t)\boldsymbol{H} + \boldsymbol{H}^T\boldsymbol{F}^T(t)\boldsymbol{E}^T \le \varepsilon^{-1}\boldsymbol{EE}^T + \varepsilon \boldsymbol{H}^T\boldsymbol{H}$$

3 Sliding-Mode control design

Let the SMC strategy be given as follows:

$$\boldsymbol{u}(t) = -\boldsymbol{Kx}(t) + \boldsymbol{u}_r(t) \tag{5}$$

$$\boldsymbol{u}_r(t) = \begin{cases} -\boldsymbol{B}^T\boldsymbol{X}[\boldsymbol{Ax}(t) + \boldsymbol{A}_h\boldsymbol{x}(t-h)] - \rho(\boldsymbol{x},t)\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|}, & \|\boldsymbol{s}(t)\| \ne 0 \\ -\boldsymbol{B}^T\boldsymbol{X}[\boldsymbol{Ax}(t) + \boldsymbol{A}_h\boldsymbol{x}(t-h)], & \|\boldsymbol{s}(t)\| = 0 \end{cases} \tag{6}$$

where $\boldsymbol{K} \in \boldsymbol{R}^{m\times n}$ is chosen such that $\boldsymbol{A} - \boldsymbol{BK}$ is stable, and

$$\begin{aligned} \rho(\boldsymbol{x}, t) = \frac{1}{1-\rho_B}(2(&\|\boldsymbol{B}^T\boldsymbol{XA}\|\ \|\boldsymbol{x}(t)\| \\ &+ \|\boldsymbol{B}^T\boldsymbol{XA}_h\|\ \|\boldsymbol{x}(t-h)\|) + \gamma) \end{aligned} \tag{7}$$

with $\gamma > 0$ constant. Then we obtain the following result.

Theorem 1: For uncertain neutral systems (1), (2), if there exist matrices $\boldsymbol{X} > 0, \boldsymbol{Q}_1 > 0, \boldsymbol{Q}_2 > 0$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ satisfying the following LMIs

$$\begin{bmatrix} \boldsymbol{\Theta} & \boldsymbol{\Pi A}_d & \boldsymbol{XA}_h & \boldsymbol{XB} & \boldsymbol{XE} & \boldsymbol{XE}_h \\ \boldsymbol{A}_d^T\boldsymbol{\Pi}^T & -\boldsymbol{W}_1 & 0 & 0 & 0 & 0 \\ \boldsymbol{A}_h^T\boldsymbol{X} & 0 & -\boldsymbol{W}_2 & 0 & 0 & 0 \\ \boldsymbol{B}^T\boldsymbol{X} & 0 & 0 & -\varepsilon_1\boldsymbol{I} & 0 & 0 \\ \boldsymbol{E}^T\boldsymbol{X} & 0 & 0 & 0 & -\varepsilon_2\boldsymbol{I} & 0 \\ \boldsymbol{E}_h^T\boldsymbol{X} & 0 & 0 & 0 & 0 & -\varepsilon_3\boldsymbol{I} \end{bmatrix} < 0 \tag{8}$$

$$\boldsymbol{W}_1 = \boldsymbol{Q}_1 - \boldsymbol{A}_d^T(\boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \varepsilon_2\boldsymbol{H}^T\boldsymbol{H} + \varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K})\boldsymbol{A}_d > 0 \tag{9}$$

$$\boldsymbol{W}_2 = \boldsymbol{Q}_2 - \varepsilon_3\boldsymbol{H}_h^T\boldsymbol{H}_h > 0 \tag{10}$$

with

$$\begin{aligned} \boldsymbol{\Theta} &= \boldsymbol{X}(\boldsymbol{A} - \boldsymbol{BK}) + (\boldsymbol{A} - \boldsymbol{BK})^T\boldsymbol{X} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \varepsilon_2\boldsymbol{H}^T\boldsymbol{H} \\ &\quad + \varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K} \\ \boldsymbol{\Pi} &= \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \varepsilon_2\boldsymbol{H}^T\boldsymbol{H} + \varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K} + \boldsymbol{X}(\boldsymbol{A} - \boldsymbol{BK}) \end{aligned}$$

then the SMC law (5)–(7) with sliding surface (4) can guarantee that the closed-loop system is globally asymptotically stable.

Proof: Define a difference operator D as

$$D(\phi) = \phi(0) - \boldsymbol{A}_d\phi(-d) \tag{11}$$

From (9) it is easily seen that

$$\boldsymbol{A}_d^T\boldsymbol{Q}_1\boldsymbol{A}_d - \boldsymbol{Q}_1 < 0$$

Thus the operator D is stable. Now choose a Lyapunov functional candidate as follows:

$$\begin{aligned} V(\boldsymbol{x}_t) &= [\boldsymbol{x}(t) - \boldsymbol{A}_d(t-d)]^T\boldsymbol{X}[\boldsymbol{x}(t) - \boldsymbol{A}_d\boldsymbol{x}(t-d)] \\ &\quad + \int_{t-d}^{t}\boldsymbol{x}^T(\tau)\boldsymbol{Q}_1\boldsymbol{x}(\tau)d\tau + \int_{t-h}^{t}\boldsymbol{x}^T(\tau)\boldsymbol{Q}_2\boldsymbol{x}(\tau)d\tau \end{aligned} \tag{12}$$

where $\boldsymbol{x}_t = \boldsymbol{x}(t+\theta),\ \theta \in [-\tau, 0]$. It can be shown that there exist scalars $c_1 > 0$ and $c_2 > 0$ such that the following holds:

$$c_1\|D(\phi)\|^2 \le V(\phi) \le c_2 \sup_{\theta\in[-\tau,0]}\|\phi(\theta)\|^2 \tag{13}$$

Differentiating $V(\boldsymbol{x}_t)$ along the solution of neutral system (1), (2) with (5) yields

$$\begin{aligned} \dot{V}(\boldsymbol{x}_t) &= 2[\boldsymbol{x}(t) - \boldsymbol{A}_d\boldsymbol{x}(t-d)]^T\boldsymbol{X}(\boldsymbol{A} - \boldsymbol{BK} + \Delta\boldsymbol{A}(t) \\ &\quad - \Delta\boldsymbol{B}(t)\boldsymbol{K})\boldsymbol{x}(t) \\ &\quad + 2[\boldsymbol{x}(t) - \boldsymbol{A}_d\boldsymbol{x}(t-d)]^T\boldsymbol{X}(\boldsymbol{A}_h + \Delta\boldsymbol{A}_h(t))\boldsymbol{x}(t-h) \\ &\quad + 2(\boldsymbol{x}(t) - \boldsymbol{A}_d\boldsymbol{x}(t-d))^T\boldsymbol{X}(\boldsymbol{B} + \Delta\boldsymbol{B}(t))\boldsymbol{u}_r(t) \\ &\quad + \boldsymbol{x}^T(t)(\boldsymbol{Q}_1 + \boldsymbol{Q}_2)\boldsymbol{x}(t) - \boldsymbol{x}^T(t-d)\boldsymbol{Q}_1\boldsymbol{x}(t-d) \\ &\quad - \boldsymbol{x}^T(t-h)\boldsymbol{Q}_2\boldsymbol{x}(t-h) \end{aligned} \tag{14}$$

Noting the definitions of the operator D and sliding variable $\boldsymbol{s}(t)$, (14) can be rewritten as

$$
\begin{aligned}
\dot{V}(\boldsymbol{x}_t) = & \boldsymbol{D}^T(\boldsymbol{x}_t)(\boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK})+(\boldsymbol{A}-\boldsymbol{BK})^T\boldsymbol{X}+\boldsymbol{Q}_1+\boldsymbol{Q}_2)\boldsymbol{D}(\boldsymbol{x}_t) \\
& +2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}\Delta\boldsymbol{A}(t)\boldsymbol{x}(t)-2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}\Delta\boldsymbol{B}(t)\boldsymbol{Kx}(t) \\
& +2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}(\boldsymbol{A}_h+\Delta\boldsymbol{A}_h(t))\boldsymbol{x}(t-h) \\
& +2\boldsymbol{s}^T(t)(\boldsymbol{I}+\boldsymbol{\delta}(t))\boldsymbol{u}_r(t) \\
& -\boldsymbol{x}^T(t-d)(\boldsymbol{Q}_1-\boldsymbol{A}_d^T(\boldsymbol{Q}_1+\boldsymbol{Q}_2)\boldsymbol{A}_d)\boldsymbol{x}(t-d) \\
& +2\boldsymbol{D}^T(\boldsymbol{x}_t)(\boldsymbol{Q}_1+\boldsymbol{Q}_2+\boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK}))\boldsymbol{A}_d\boldsymbol{x}(t-d) \\
& -\boldsymbol{x}^T(t-h)\boldsymbol{Q}_2\boldsymbol{x}(t-h)
\end{aligned} \tag{15}
$$

By lemma 1 (i), one can obtain for $\varepsilon_1 > 0$,

$$
\begin{aligned}
-2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}\Delta\boldsymbol{B}(t)\boldsymbol{Kx}(t) \le & \varepsilon_1^{-1}\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{XBB}^T\boldsymbol{XD}(\boldsymbol{x}_t) \\
& + \varepsilon_1\rho_B^2\boldsymbol{x}^T(t)\boldsymbol{K}^T\boldsymbol{Kx}(t)
\end{aligned} \tag{16}
$$

Similarly it follows from (3) and lemma 1 (ii) that for $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$,

$$
\begin{aligned}
2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}\Delta\boldsymbol{A}(t)\boldsymbol{x}(t) \le & \varepsilon_2^{-1}\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{XEE}^T\boldsymbol{XD}(\boldsymbol{x}_t) \\
& + \varepsilon_2\boldsymbol{x}^T(t)\boldsymbol{H}^T\boldsymbol{Hx}(t)
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{X}\Delta\boldsymbol{A}_h(t)\boldsymbol{x}(t-h) \le & \varepsilon_3^{-1}\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{XE}_h\boldsymbol{E}_h^T\boldsymbol{XD}(\boldsymbol{x}_t) \\
& + \varepsilon_3\boldsymbol{x}^T(t-h)\boldsymbol{H}_h^T\boldsymbol{H}_h\boldsymbol{x}(t-h)
\end{aligned} \tag{18}
$$

Utilising (6) and (7), for $\|\boldsymbol{\delta}(t)\| \le \rho_B < 1$ and $\|\boldsymbol{s}(t)\| \ne 0$, one can obtain

$$
\begin{aligned}
& 2\boldsymbol{s}^T(t)(\boldsymbol{I}+\boldsymbol{\delta}(t))\boldsymbol{u}_r(t) \\
& = -2\boldsymbol{s}^T(t)(\boldsymbol{I}+\boldsymbol{\delta}(t))\boldsymbol{B}^T\boldsymbol{X}(\boldsymbol{Ax}(t)+\boldsymbol{A}_h\boldsymbol{x}(t-h)) \\
& \quad -2\rho(\boldsymbol{x},t)\,\|\boldsymbol{s}(t)\| -2\boldsymbol{s}^T(t)\boldsymbol{\delta}(t)\rho(\boldsymbol{x},t)\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|} \\
& \le 2\,\|\boldsymbol{s}(t)\|\,(1+\rho_B)(\|\boldsymbol{B}^T\boldsymbol{XA}\|\|\boldsymbol{x}(t)\| \\
& \quad + \|\boldsymbol{B}^T\boldsymbol{XA}_h\|\|\boldsymbol{x}(t-h)\|) \\
& \quad -2\rho(\boldsymbol{x},t)\,\|\boldsymbol{s}(t)\| + \frac{\rho(\boldsymbol{x},t)}{\|\boldsymbol{s}(t)\|}\left(\boldsymbol{s}^T(t)\boldsymbol{\delta}(t)\boldsymbol{\delta}^T(t)\boldsymbol{s}(t)+\boldsymbol{s}^T(t)\boldsymbol{s}(t)\right) \\
& \le 2\,\|\boldsymbol{s}(t)\|\,(1+\rho_B)(\|\boldsymbol{B}^T\boldsymbol{XA}\|\|\boldsymbol{x}(t)\| \\
& \quad + \|\boldsymbol{B}^T\boldsymbol{XA}_h\|\|\boldsymbol{x}(t-h)\|) - \rho(\boldsymbol{x},t)(1-\rho_B^2)\,\|\boldsymbol{s}(t)\| \\
& \le -\gamma(1+\rho_B)\,\|\boldsymbol{s}(t)\|
\end{aligned} \tag{19}
$$

Thus substituting (16)–(19) into (15) results in

$$
\begin{aligned}
\dot{V}(\boldsymbol{x}_t) \le & \boldsymbol{D}^T(\boldsymbol{x}_t)(\boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK})+(\boldsymbol{A}-\boldsymbol{BK})^T\boldsymbol{X} \\
& + \boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_2^{-1}\boldsymbol{XEE}^T\boldsymbol{X}+\varepsilon_1^{-1}\boldsymbol{XBB}^T\boldsymbol{X} \\
& + \varepsilon_3^{-1}\boldsymbol{XE}_h\boldsymbol{E}_h^T\boldsymbol{X}+\varepsilon_2\boldsymbol{H}^T\boldsymbol{H}+\varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K})\boldsymbol{D}(\boldsymbol{x}_t) \\
& - \boldsymbol{x}^T(t-d)(\boldsymbol{Q}_1-\boldsymbol{A}_d^T(\boldsymbol{Q}_1+\boldsymbol{Q}_2 \\
& + \varepsilon_2\boldsymbol{H}^T\boldsymbol{H}+\varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K})\boldsymbol{A}_d)\boldsymbol{x}(t-d) \\
& + 2\boldsymbol{D}^T(\boldsymbol{x}_t)(\boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_2\boldsymbol{H}^T\boldsymbol{H}+\varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K} \\
& + \boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK}))\boldsymbol{A}_d\boldsymbol{x}(t-d) \\
& + 2\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{XA}_h\boldsymbol{x}(t-h) \\
& - \boldsymbol{x}^T(t-h)(\boldsymbol{Q}_2-\varepsilon_3\boldsymbol{H}_h^T\boldsymbol{H}_h)\boldsymbol{x}(t-h)
\end{aligned} \tag{20}
$$

By considering (9) and (10), it follows from (20) that

$$
\begin{aligned}
\dot{V}(\boldsymbol{x}_t) \le & \boldsymbol{D}^T(\boldsymbol{x}_t)\Sigma\boldsymbol{D}(\boldsymbol{x}_t) - (\boldsymbol{x}^T(t-d)-\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{\Pi}\boldsymbol{A}_d\boldsymbol{W}_1^{-1}) \\
& \times \boldsymbol{W}_1(\boldsymbol{x}(t-d)-\boldsymbol{W}_1^{-1}\boldsymbol{A}_d^T\boldsymbol{\Pi}^T\boldsymbol{D}(\boldsymbol{x}_t)) \\
& - (\boldsymbol{x}^T(t-h)-\boldsymbol{D}^T(\boldsymbol{x}_t)\boldsymbol{XA}_h\boldsymbol{W}_2^{-1})\boldsymbol{W}_2(\boldsymbol{x}(t-h) \\
& - \boldsymbol{W}_2^{-1}\boldsymbol{A}_h^T\boldsymbol{XD}(\boldsymbol{x}_t))
\end{aligned} \tag{21}
$$

with

$$
\begin{aligned}
\Sigma = & \boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK})+(\boldsymbol{A}-\boldsymbol{BK})^T\boldsymbol{X}+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_2^{-1}\boldsymbol{XEE}^T\boldsymbol{X} \\
& + \varepsilon_1^{-1}\boldsymbol{XBB}^T\boldsymbol{X}+\varepsilon_3^{-1}\boldsymbol{XE}_h\boldsymbol{E}_h^T\boldsymbol{X}+\varepsilon_2\boldsymbol{H}^T\boldsymbol{H}+\varepsilon_1\rho_B^2\boldsymbol{K}^T\boldsymbol{K} \\
& + \boldsymbol{\Pi}\boldsymbol{A}_d\boldsymbol{W}_1^{-1}\boldsymbol{A}_d^T\boldsymbol{\Pi}^T+\boldsymbol{XA}_h\boldsymbol{W}_2^{-1}\boldsymbol{A}_h^T\boldsymbol{X}
\end{aligned}
$$

Thus it can be seen from (21) that if $\Sigma < 0$, there exists a scalar $c > 0$ such that

$$
\dot{V}(\boldsymbol{x}_t) \le -c\,\|\boldsymbol{D}(\boldsymbol{x}_t)\|^2
$$

Moreover, noting the stability of the operator D and this inequality and (13), it follows from theorem 7.1 in [1] that the closed-loop system (1), (2) with sliding-mode controller (5)–(7) is globally asymptotically stable.

Finally, by Schur's complement, it is easily shown that the matrix inequality $\Sigma < 0$ is equivalent to LMI (8). □

Corollary 1: Suppose that there exists no uncertainty in the input matrix, that is $\rho_B = 0$. Then, if there exist matrices $\boldsymbol{X} > 0, \boldsymbol{Q}_1 > 0, \boldsymbol{Q}_2 > 0$, and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ satisfying LMIs

$$
\begin{bmatrix}
\hat{\boldsymbol{\Theta}} & \hat{\boldsymbol{\Pi}}\boldsymbol{A}_d & \boldsymbol{XA}_h & \boldsymbol{XE} & \boldsymbol{XE}_h \\
\boldsymbol{A}_d^T\hat{\boldsymbol{\Pi}}^T & -\hat{\boldsymbol{W}}_1 & 0 & 0 & 0 \\
\boldsymbol{A}_h^T\boldsymbol{X} & 0 & -\hat{\boldsymbol{W}}_2 & 0 & 0 \\
\boldsymbol{E}^T\boldsymbol{X} & 0 & 0 & -\varepsilon_1\boldsymbol{I} & 0 \\
\boldsymbol{E}_h^T\boldsymbol{X} & 0 & 0 & 0 & -\varepsilon_2\boldsymbol{I}
\end{bmatrix} < 0 \tag{22}
$$

$$
\hat{\boldsymbol{W}}_1 = \boldsymbol{Q}_1 - \boldsymbol{A}_d^T(\boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_1\boldsymbol{H}^T\boldsymbol{H})\boldsymbol{A}_d > 0 \tag{23}
$$

$$
\hat{\boldsymbol{W}}_2 = \boldsymbol{Q}_2 - \varepsilon_2\boldsymbol{H}_h^T\boldsymbol{H}_h > 0 \tag{24}
$$

with

$$
\begin{aligned}
\hat{\boldsymbol{\Theta}} &= \boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK})+(\boldsymbol{A}-\boldsymbol{BK})^T\boldsymbol{X}+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_1\boldsymbol{H}^T\boldsymbol{H} \\
\hat{\boldsymbol{\Pi}} &= \boldsymbol{Q}_1+\boldsymbol{Q}_2+\varepsilon_1\boldsymbol{H}^T\boldsymbol{H}+\boldsymbol{X}(\boldsymbol{A}-\boldsymbol{BK})
\end{aligned}
$$

the SMC law (5)–(7) with sliding surface (4) guarantees that the closed-loop system is globally asymptotically stable.

Now, consider the special case that both $\Delta A(t)$ and $\Delta A(t)$ are matched, i.e.

$$
\Delta\boldsymbol{A}(t) = \boldsymbol{B}\boldsymbol{\delta}_A(t), \quad \Delta\boldsymbol{A}_h(t) = \boldsymbol{B}\boldsymbol{\delta}_h(t) \tag{25}
$$

where $\|\boldsymbol{\delta}_A(t)\| \le \rho_A$, and $\|\boldsymbol{\delta}_h(t)\| \le \rho_h$ with ρ_A and ρ_h known constants. The following result can easily be obtained from theorem 1.

Corollary 2: Suppose that $\Delta\boldsymbol{A}(t)$ and $\Delta\boldsymbol{A}_h(t)$ are matched such that (25) holds. If there exist matrices $\boldsymbol{X} > 0, \boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$, and scalar $\varepsilon > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \tilde{\Theta} & \tilde{\Pi}A_d & XA_h & XB \\ A_d^T\tilde{\Pi}^T & -\tilde{W} & 0 & 0 \\ A_h^TX & 0 & -Q_2 & 0 \\ B^TX & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{26}$$

$$\tilde{W} = Q_1 - A_d^T(Q_1 + Q_2 + \varepsilon_1\rho_B^2K^TK)A_d > 0 \tag{27}$$

with

$$\tilde{\Theta} = X(A - BK) + (A - BK)^TX + Q_1 + Q_2 + \varepsilon\rho_B^2K^TK$$

then the SMC strategy (5), (6) with

$$\rho(x, t) = \frac{1}{1-\rho_B^2}(2[\rho_A + (1+\rho_B)]\ \|B^TXA\|\ \|x(t)\| + 2[\rho_h + (1+\rho_B)]\ \|B^TXA_h\|\|x(t-h)\| + \gamma) \tag{28}$$

can guarantee that the closed-loop system is globally asymptotically stable.

Proof: By selecting a Lyapunov functional candidate as (12) it is easily shown that

$$\begin{aligned}\dot{V}(x_t) = {} & D^T(x_t)(X(A-BK)+(A-BK)^TX+Q_1+Q_2)D(x_t) \\ & + 2D^T(x_t)XA_h - 2D^T(x_t)X\Delta B(t)Kx(t) \\ & + 2s^T(t)\delta_A(t)x(t) + 2s^T(t)\delta_h(t)x(t-h) \\ & + 2s^T(t)(I+\delta(t))u_r(t) \\ & - x^T(t-d)(Q_1 - A_d^T(Q_1+Q_2)A_d)x(t-d) \\ & + 2D^T(x_t)(Q_1+Q_2+X(A-BK))A_dx(t-d) \\ & - x^T(t-h)Q_2x(t-h)\end{aligned} \tag{29}$$

By (6) and (28) it is seen that

$$\begin{aligned} & 2s^T(t)\delta_A(t)x(t) + 2s^T(t)\delta_h(t)x(t-h) + 2s^T(t)(I+\delta(t))u_r(t) \\ & \le 2\,\|s(t)\|\,(\rho_A\,\|x(t)\| + \rho_h\,\|x(t-h)\| \\ & \quad + (1+\rho_B)\,\|B^TXA\|\|\,x(t)\| \\ & \quad + (1+\rho_B)\,\|B^TXA_h\|\|\,x(t-h)\|) \\ & \quad - \rho(x,t)(1-\rho_B^2)\,\|s(t)\| \\ & \le -\gamma\,\|s(t)\| \end{aligned} \tag{30}$$

Thus, with a similar argument to theorem 1 one can obtain that if there exist matrices $X > 0$, $Q_1 > 0$, $Q_2 > 0$, and scalar $\varepsilon > 0$ satisfying the following LMIs (26) and (27), then it follows from (29) and (30) that

$$\begin{aligned}\dot{V}(x_t) \le {} & D^T(x_t)\tilde{\Sigma}D(x_t) - \left(x^T(t-d) - D^T(x_t)\tilde{\Xi}A_d\tilde{W}^{-1}\right) \\ & \times \tilde{W}\left(x(t-d) - \tilde{W}^{-1}A_d^T\tilde{\Xi}^TD(x_t)\right) - (x^T(t-h) \\ & - D^T(x_t)XA_hQ_2^{-1})Q_2(x(t-h) - Q_2^{-1}A_h^TXD(x_t)) \\ \le {} & -c\|D(x_t)\|^2\end{aligned}$$

where $c > 0$ is some scalar constant, and

$$\begin{aligned}\tilde{\Sigma} = {} & X(A-BK) + (A-BK)^TX + Q_1 + Q_2 + \varepsilon\rho_B^2K^TK \\ & + \varepsilon^{-1}XBB^TX + \tilde{\Xi}A_d\tilde{W}^{-1}A_d\tilde{\Xi}^T + XA_hQ_2^{-1}A_h^TX\end{aligned}$$

This means that the uncertain neutral delay system (1), (2) with SMC law composed of (5), (6) and (28) is globally asymptotically stable. □

At the second step of the sliding-mode control design, in the following, we investigate the reachability of sliding surface.

Theorem 2: Suppose that there exist matrices $X > 0$, $Q_1 > 0, Q_2 > 0$, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and $\varepsilon_3 > 0$ satisfying LMIs (8)–(10), and the sliding surface is given by (4). Then, for neutral delay system (1), (2) with SMC (5)–(7), every state trajectory is attracted towards the sliding surface $s(t) = 0$ in a finite time and once the trajectory hits the sliding surface it remains there for all subsequent time.

Proof: By using (1), (4) and (5), one can obtain for $\|s(t)\| \ne 0$

$$\begin{aligned}\dot{s}(t) = {} & B^TX\{[A - BK + \Delta A(t) - \Delta B(t)K]x(t) \\ & + [A_h + \Delta A_h(t)]x(t-h)\} - B^TXB(I+\delta(t)) \\ & \times \left(B^TX[Ax(t) + A_hx(t-h)] + \rho(x,t)\frac{s(t)}{\|s(t)\|}\right)\end{aligned}$$

which leads to

$$\begin{aligned} & s^T(t)(B^TXB)^{-1}\dot{s}(t) \\ & = s(t)(B^TXB)^{-1}B^TX[A - BK + \Delta A(t) - \Delta B(t)K]x(t) \\ & \quad + s^T(t)(B^TXB)^{-1}B^TX[A_h + \Delta A_h(t)]x(t-h) \\ & \quad - s^T(t)(I+\delta(t))B^TX[Ax(t) + A_hx(t-h)] \\ & \quad - \rho(x,t)\|s(t)\| - \frac{\rho(x,t)}{\|s(t)\|}s^T(t)\delta(t)s(t)\end{aligned} \tag{31}$$

By noting that for $\|\delta(t)\| \le \rho_B$

$$\begin{aligned} & \frac{\rho(x,t)}{\|s(t)\|}s^T(t)\delta(t)s(t) \\ & \quad \le 0.5\frac{\rho(x,t)}{\|s(t)\|}\left(s^T(t)\delta(t)\delta^T(t)s(t) + s^T(t)s(t)\right) \\ & \quad \le 0.5\rho(x,t)\|s(t)\|(1+\rho_B^2)\end{aligned}$$

one can obtain that

$$\begin{aligned} & -\rho(x,t)\|s(t)\| - \frac{\rho(x,t)}{\|s(t)\|}s^T(t)\delta(t)s(t) \\ & \quad \le -0.5\rho(x,t)(1-\rho_B^2)\|s(t)\|\end{aligned} \tag{32}$$

Thus it follows from (7), (31) and (32) and that

$$\begin{aligned} & s^T(t)(B^TXB)^{-1}\dot{s}(t) \\ & \le \|s(t)\|([\|R(A-BK)\| + \|R\Delta A(t)\| + \|RB\delta(t)K\|]\|x(t)\| \\ & \quad + \|s(t)\|(\|RA_h\| + \|R\Delta A_h(t)\|)\|x(t-h)\| \\ & \quad + \|s(t)\|(1+\rho_B)(\|B^TXA\|\|x(t)\| + \|B^TXA_h\|\|x(t-h)\|) \\ & \quad - 0.5\rho(x,t)(1-\rho_B^2)\|s(t)\| \\ & \le \|s(t)\|(\|R(A-BK)\| + \|RE\|\|H\| + \rho_B\|RB\|\|K\|)\|x(t)\| \\ & \quad + \|s(t)\|(\|RA_h\| + \|RE_h\|\|H_h\|)\|x(t-h)\| \\ & \quad - 0.5\gamma(1+\rho_B)\|s(t)\|\end{aligned} \tag{33}$$

where $R = (B^TXB)^{-1}B^TX$.

In the state space, define the following domain:

$$\Omega_\gamma = \{x(t)\colon \zeta\|x(t)\| + \xi\|x(t-h)\| \le 0.5\gamma\}$$

where

$$\zeta = \|\boldsymbol{R}(\boldsymbol{A} - \boldsymbol{BK})\| + \|\boldsymbol{RE}\|\|\boldsymbol{H}\| + \rho_B\|\boldsymbol{RB}\|\|\boldsymbol{K}\|$$
$$\xi = \|\boldsymbol{RA}_h\| + \|\boldsymbol{RE}_h\|\|\boldsymbol{H}_h\|$$

Hence in the domain Ω_γ we have from (33)

$$\begin{aligned} \boldsymbol{s}^T(t)(\boldsymbol{B}^T\boldsymbol{XB})^{-1}\dot{\boldsymbol{s}}(t) \leq & -0.5\gamma\rho_B\|\boldsymbol{s}(t)\| + (\zeta\|\boldsymbol{x}(t)\| \\ & + \xi\|\boldsymbol{x}(t-h)\| - 0.5\gamma)\|\boldsymbol{s}(t)\| \\ \leq & -0.5\gamma\rho_B\|\boldsymbol{s}(t)\| \end{aligned}$$

It is seen that the reachability of sliding mode is satisfied within the domain Ω_γ. Since according to theorem 1 the closed-loop system is globally asymptotically stable, the state trajectories of closed-loop system will enter Ω_γ in a finite time and remain there. □

Remark 1: The sliding motion is attained only when the state trajectories enter the domain Ω_γ. The region in which the sliding motion takes place is usually referred to as the sliding patch [15]. It is seen that the size of the sliding patch depends on the design parameter γ.

4 Numerical simulation

Consider the uncertain neutral delay systems (1), (2) with

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0.3 & 0 \\ -3 & 0.1 & 0 \\ 0.1 & 0 & -2 \end{bmatrix}, \quad \boldsymbol{A}_d = \begin{bmatrix} -0.1 & 0 & 0.1 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.02 \end{bmatrix},$$

$$\boldsymbol{A}_h = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0 & 0 \\ 0.1 & -0.1 & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 5 & -7 \\ -9 & 8 \\ 3 & 5 \end{bmatrix}, \boldsymbol{\delta}(t) = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \boldsymbol{E} = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix},$$

$$\boldsymbol{E}_h = \begin{bmatrix} -0.2 \\ 0 \\ 0.1 \end{bmatrix}$$

$$\boldsymbol{H} = [-0.02 \quad 0.02 \quad 0.10], \quad \boldsymbol{H}_h = [0.1 \quad 0.01 \quad 0.02]$$

and $d = 2$, $h = 1$. It is easily seen that the matrix $\mathbf{A}$ has eigenvalues $0.55 \pm 0.8352i$ and -2, and $\|\boldsymbol{\delta}(t)\| \leq \rho_B = 0.1$. For this uncertain neutral delay system, it is required to construct the SMC law as (5)–(7) such that the closed-loop system is asymptotically stable. To this end, select the feedback matrix $\mathbf{K}$ as

$$\boldsymbol{K} = \begin{bmatrix} 0.2 & -0.2 & -0.05 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$

such that $\boldsymbol{A} - \boldsymbol{BK}$ is stable. By solving LMIs (8)–(10)

$$\boldsymbol{X} = \begin{bmatrix} 0.1407 & 0.0796 & 0.0317 \\ 0.0796 & 0.1085 & -0.0279 \\ 0.0317 & -0.0279 & 0.2686 \end{bmatrix}$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} 0.0384 & 0.0059 & 0.0232 \\ 0.0059 & 0.0818 & -0.0225 \\ 0.0232 & -0.0225 & 0.2343 \end{bmatrix}$$
$$\boldsymbol{Q}_2 = \begin{bmatrix} 0.0589 & 0.0145 & 0.0340 \\ 0.0145 & 0.0347 & -0.0230 \\ 0.0340 & -0.0230 & 0.2317 \end{bmatrix}$$

and $\varepsilon_1 = 5.5257$, $\varepsilon_2 = 0.8804$, $\varepsilon_3 = 0.8496$. Hence, by theorem 1 the sliding surface is given as

$$\begin{aligned} \boldsymbol{s}(t) = & \begin{pmatrix} 0.0822 & -0.6623 & 1.2158 \\ -0.1897 & 0.1711 & 0.8979 \end{pmatrix}\boldsymbol{x}(t) \\ & - \begin{pmatrix} -0.0082 & -0.3312 & 0.0325 \\ 0.0190 & 0.0855 & -0.0010 \end{pmatrix}\boldsymbol{x}(t-d) \end{aligned}$$

and the desired SMC law is given as

$$\boldsymbol{u}(t) = -\begin{bmatrix} 0.2 & -0.2 & -0.05 \\ 0 & 0.1 & 0.1 \end{bmatrix}\boldsymbol{x}(t) + \boldsymbol{u}_r(t)$$

where

$$\begin{aligned} \boldsymbol{u}_r(t) = & -\begin{bmatrix} 2.1908 & -0.0416 & -2.4317 \\ -0.6132 & -0.0398 & -1.7957 \end{bmatrix}\boldsymbol{x}(t) \\ & - \begin{bmatrix} 0.0718 & -0.1134 & 0.0082 \\ 0.0690 & -0.1088 & -0.0190 \end{bmatrix}\boldsymbol{x}(t-h) \\ & - \frac{10}{9}(6.8780\|\boldsymbol{x}(t)\| + 0.3722\|\boldsymbol{x}(t-h)\| + 5)\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|} \end{aligned}$$

First, consider the case with initial function $\varphi(\theta) = (1 \quad 0 \quad -1)^T$, $\theta \in [-2, 0]$, $\boldsymbol{x}(0) = (1 \quad 0 \quad -1)^T$. This implies that $\varphi(\theta) = \boldsymbol{x}(0)$ for $\theta \in [-2, 0]$. The simulation results are given in Figs. 1–4. Furthermore, the case with $\boldsymbol{x}(0) \neq \varphi(\theta)$ for $\theta \in [-2, 0]$ is considered. Let $\varphi(\theta) = (2 \quad 2 \quad 2)^T$, $\theta \in [-2, 0]$, $\boldsymbol{x}(0) = (0 \quad -1 \quad 1)^T$. The simulation results are given in Figs. 5–7. The Figures demonstrate that the SMC law effectively eliminates the effect of uncertainties. The state trajectories are attracted towards the designed sliding surface and the closed-loop neutral delay system is asymptotically stable. Moreover, it

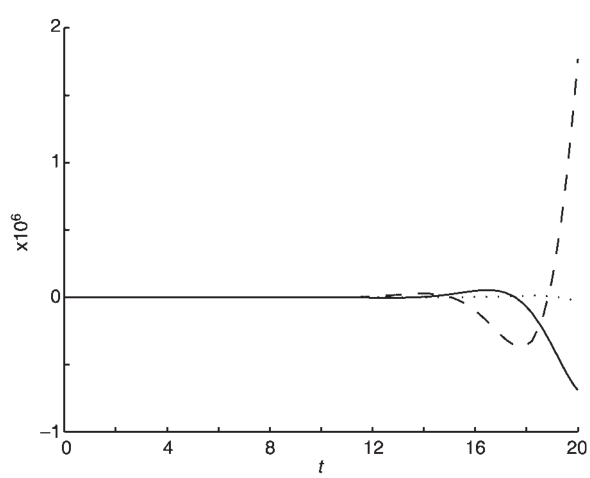

Fig. 1 *Trajectories of state with u(t) = 0*

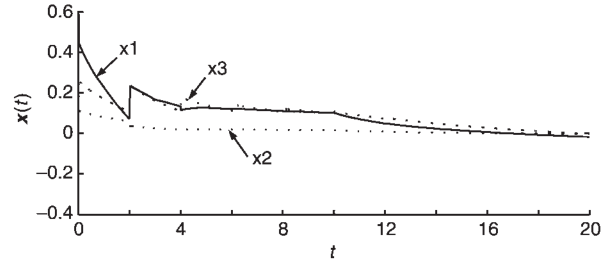

Fig. 2 *Trajectories of state variable x(t)*

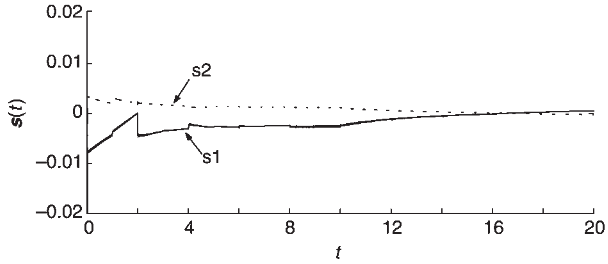

Fig. 3 *Trajectories of sliding variable s(t)*

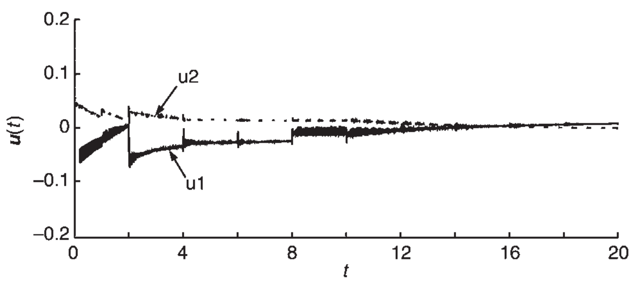

Fig. 4 *Control signals u(t)*

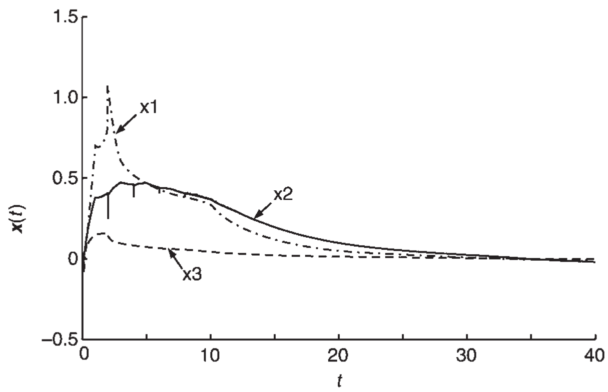

Fig. 5 *Trajectories of state variable x(t)*

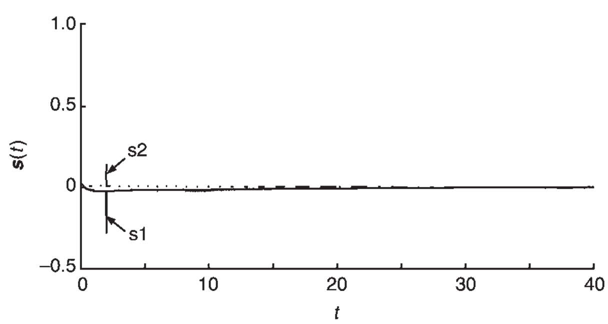

Fig. 6 *Trajectories of sliding variable s(t)*

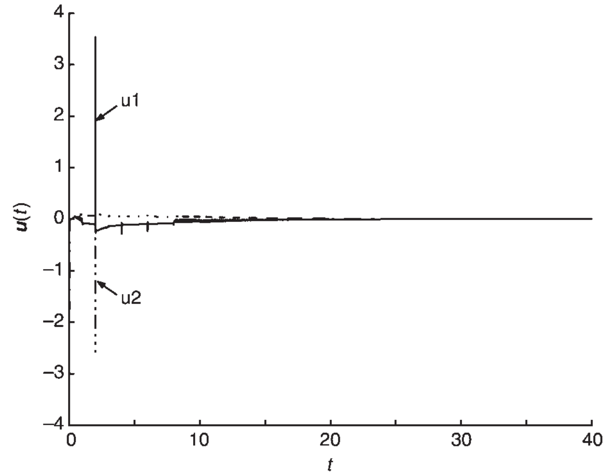

Fig. 7 *Control signals u(t)*

can be seen from Figs. 2, 3, 5 and 6 that there are notable variations in the curves of state variables and sliding-mode variables around $t = 2$, caused by the effect of the time delay $d = 2$ in the derivative term $\dot{x}(t - d)$. Nevertheless, these undesirable effects are effectively attenuated by the present method.

5 Conclusions

We have investigated the problem of sliding-mode control for a class of uncertain neutral delay systems. The neutral delay systems under consideration may have parameter uncertainties in the state matrices as well as in the input matrix. A sufficient condition in terms of LMIs is derived to guarantee the asymptotic stability of the closed-loop systems. It was shown that the sliding motion is expected to happen in the *sliding patch* which is a subset of the sliding surface.

6 Acknowledgment

The research was partially supported by RGC grant HKU 7103/01P.

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