



## Dynamic economic dispatch of a microgrid: Mathematical models and solution algorithm



Hongbin Wu\*, Xingyue Liu, Ming Ding

School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China

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### ABSTRACT

Dynamic economic dispatch of a microgrid is better suited to the requirements of a system in actual operation because it not only considers the lowest cost in a scheduling cycle but also coordinates between different distribution generations (DGs) over many periods. So it is very significant to research the dynamic economic dispatch of a microgrid. Since wind energy and solar energy are subject to random variations and intervals, there is great difficulty in solving the dynamic economic dispatch. In this paper, we establish a combined heat and power (CHP) microgrid system which includes wind turbines (WT), photovoltaic arrays (PV), diesel engines (DE), a micro-turbine (MT), a fuel cell (FC) and a battery (BS). Comprehensively considering the operation cost and the pollutant treatment cost of the microgrid system, we choose the maximum comprehensive benefits as the objective function for the dynamic economic dispatch. At the same time, we establish the spinning reserve probability constraints of the microgrid considering the influence of uncertainty factors such as the fluctuation of the renewable energy, load fluctuation error, and fault shutdown of the unit. Also researched are four different operation scheduling strategies under grid-connected mode and island mode of the microgrid. An improved particle swarm optimization (PSO) algorithm combined with Monte Carlo simulation is used to solve the objective function. With the example system, the proposed models and improved algorithm are verified. When the microgrid is running under the grid-connected mode, we discuss the influence of different scheduling strategies, optimization goals and reliability indexes on the dynamic economic dispatch. And when the microgrid is running under the island mode, we discuss the influence of the uncertainty factors and the capacity of the battery on the dynamic economic dispatch. The presented research can provide some reference for dynamic economic dispatch of microgrid on making full use of renewable energy and improving the microgrid system reliability.

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### Introduction

In recent years, the whole world has been paying more and more attention to developing renewable energy sources such as wind energy and solar energy owing to the serious global depletion of energy and environmental problems. With the development of distribution technology, the microgrid [1–6] provides an effective way for the comprehensive use of renewable energy.

The economic dispatch of power system can be divided into static dispatch and dynamic dispatch [7–10]. Static economic dispatch determines the priority and operation mode of the power generating equipment based on the operating conditions of the system in each independent period.

The dynamic economic dispatch is better suited to the requirements of a system in actual operation because it not only considers the lowest cost in a scheduling cycle but also coordinates between the different distribution generations (DGs) over several periods. So it is very significant to research the dynamic economic dispatch. Renewable energy sources [11] are subject to randomness and interruptions, which makes it very difficult to solve the dynamic economic dispatch.

The power system which includes wind energy and solar energy have been developed so far in terms of dynamic economic load dispatch problem. An optimal economical dispatch model was established in [12], it developed a method to estimate the risk and to manage conventional power systems with wind power systems for the short-term operation. [13] proposed a stochastic model and a solution technique for optimal scheduling of the generators in a wind integrated power system considering the demand and wind generators uncertainties. The research in [14]

\* Corresponding author. Tel.: +86 13856013658; fax: +86 05512903929.

E-mail address: [hfwuhongbin@163.com](mailto:hfwuhongbin@163.com) (H. Wu).

proposed a new real-time dynamic economic dispatch method to meet the requirements of power system based on investigation of future circumstance, the research in [15] proposed dynamic economic dispatch based on the market price of power system, considering the uncertainties in deregulated energy and reserve markets. Recently, some studies about the dynamic economic dispatch of a microgrid have been published. Online optimization method developed in [16] used particle swarm optimization (PSO), however some issues need to be further investigated regarding the optimal operation for a number of DGs such as the WT, FC and BS. A dynamic economic dispatch model was proposed in [17], it compared the dynamic dispatch results with those of static dispatch, and reached the conclusion that dynamic economic dispatch for a microgrid could reduce the operation costs, however gas turbines and the randomness of renewable resources were not taken into account. On the other hand, they did not take into account that a microgrid has distinct operation modes, namely the grid-connected mode and the island mode. Generally, without the support of the grid, the effect of uncertainty factors on the operation of the system under island mode is more obvious than that under grid-connected mode, and the generating units participating in the economic scheduling are different. Therefore the study of dynamic economic dispatch should take into account the two different operation modes.

In fact, there are many kinds of DGs in a microgrid. The DGs will show different features in the dynamic economic dispatch under different operation modes and scheduling strategies. Randomness and interruptions will increase the difficulty of the economic dispatch. In this paper, mathematical models and an algorithmic solution of dynamic economic dispatch on a microgrid are presented. After formulating a combined heat and power (CHP) microgrid system which including wind turbines (WT), photovoltaic arrays (PV), diesel engines (DE), a micro turbine (MT), a fuel cell (FC) and a battery (BS), we choose the maximum comprehensive benefits as the objective function for dynamic economic dispatch. We also establish the spinning reserve probability constraints on the microgrid taking uncertainty into account. An improved particle swarm optimization (PSO) algorithm combined with Monte Carlo simulation is used to solve the objective function. Using an example, we discuss the various influences on the dynamic economic dispatch of different scheduling strategies, optimization goals, reliability indexes, uncertainty factors and the capacity of the battery.

### The mathematical model of DGs

#### The model of an MT

The exhaust emissions of  $NO_x$  and  $CO_2$  of MT is much lower than traditional technologies used in centralized power plants [18]. The mathematical model of an MT can be shown as follows:

$$Q_{MT} = P_{GT}(1 - \eta_e - \eta_l) / \eta_e \quad (1)$$

$$Q_{he} = Q_{MT} K_{he} \quad (2)$$

$$C_{MT} = C_{nl} \times (\sum P_{GT} \Delta t / \eta_e L) \quad (3)$$

where  $Q_{MT}$  is the residual heat of the exhaust,  $\eta_e$  is the generating efficiency of the MT,  $\eta_l$  is the heat loss coefficient,  $P_{GT}$  is the output power of the MT during the calculation period  $\Delta t$  in kW,  $Q_{he}$  is the heat provided by the MT,  $K_{he}$  is the heat coefficient of the cooler,  $C_{MT}$  is the gas consumption cost of the MT,  $L$  is the net thermal value of the gas, 9.7 g/kW, and  $C_{nl}$  is the price of the gas, 2.05 ¥/m<sup>3</sup>.

#### The model of the FC

The mathematical model of the FC can be shown as follows:

$$C_{FC} = C_{nl} \times (\sum P_{FC} \Delta t / \eta_f L) \quad (4)$$

where  $C_{FC}$  is the gas consumption cost of the FC,  $P_{FC}$  is the output power of the FC during the calculation period  $\Delta t$  in kW,  $\eta_f$  is the efficiency of the FC,  $L$  is the net thermal value of gas, 9.7 g/kW, and  $C_{nl}$  is the price of gas, 2.05 ¥/m<sup>3</sup>.

#### The model of the BS

The state of charge (SOC) of the battery refers to the ratio of the residual energy to the rated energy. It is very important to predict the SOC of the battery accurately for controlling the charging/discharging process and the system economic dispatching.

The charging formula of the battery is described as follows:

$$SOC(t) = (1 - \delta)SOC(t - 1) - P_c \Delta t \eta_c / E_c \quad (5)$$

where  $P_c$  is negative, it represents the charging power,  $\eta_c$  is the charging efficiency,  $E_c$  is the total capacity of the BS during the calculation period  $\Delta t$  in kW,  $SOC(t)$  is the SOC of the BS in period  $t$ , and  $SOC(t - 1)$  is the SOC of the BS in period  $t - 1$ .

The discharging formula of the battery is described as follows:

$$SOC(t) = (1 - \delta)SOC(t - 1) - P_d \Delta t / (E_c \eta_d) \quad (6)$$

where  $P_d$  is positive, it represents the discharging power,  $\eta_d$  is the discharging efficiency, and  $\delta$  is the self-discharge rate of storage in%/h.

### The mathematical model for the dynamic economic dispatch of a microgrid

#### The objective function for the dynamic economic dispatch of a microgrid

##### The operating cost of the microgrid system

For the microgrid, the operating cost  $C_1$  of the system can be described as follows:

$$C_1 = C_{Fuel} + C_{OM} + C_{DC} + M(\sum \lambda_t^i r_t^i + C_{GRID}) \quad (7)$$

where  $C_{Fuel}$  is the fuel consumption cost of the DGs,  $C_{OM}$  is the operation and management cost of the DGs,  $C_{DC}$  is the depreciation cost of the DGs,  $M$  indicates whether the microgrid is connected with the grid or not: when the microgrid is connected with the grid,  $M = 1$ , when the microgrid is in island mode,  $M = 0$ ,  $C_{GRID}$  is the cost of interaction between the microgrid and the grid: it is being positive, represents that the microgrid is purchasing power from the grid, when it is negative, that represents that the microgrid is selling power to the grid. And because there are some uncertain factors, the spinning reserve capacity of the microgrid is limited by the DGs, so the microgrid needs to purchase some spinning reserve power from the grid,  $\lambda_t^i$  is the price of the spinning reserve, and  $r_t^i$  is the purchasing power of the spinning reserve.

$C_{Fuel}$ ,  $C_{OM}$  and  $C_{DC}$  can be described as follows:

$$\begin{cases} C_{Fuel} = K_{fc} * P \\ C_{OM} = K_{om} * P \\ C_{DC} = \frac{ADCC}{P_{max} \times 8760 \times cf} \times P \\ ADCC = InCost \times d(1 + d)^l / [(1 + d)^l - 1] \end{cases} \quad (8)$$

where  $P$  is the output power of the DGs,  $K_{fc}$  is the coefficient of fuel consumption,  $K_{om}$  is the coefficient of operation and management.  $P_{max}$  is the maximum power of the DGs,  $cf$  is a capacity factor,  $ADCC$  is the depreciation cost per kilowatt-hour of the DGs,  $InCost$  is the

installation cost per capacity of the DGs,  $d$  is the interest rate, set at 8%,  $l$  is the lifetime of the DGs.

#### The pollutant treatment cost of the microgrid system

For the microgrid, the pollutant treatment cost  $C_2$  of the system can be described as follows:

$$C_2 = \sum_{i=1}^N \sum_k (C_k \gamma_{ik}) P_i + \sum_k (C_k \gamma_{gridk}) P_{grid} \quad (9)$$

where  $i$  is the number of DGs,  $N$  is the total number of the DGs in the microgrid,  $k$  is the type of pollutant emission ( $\text{CO}_2$ ,  $\text{SO}_2$ ,  $\text{NO}_x$ ),  $C_k$  is the treatment cost of the  $k^{\text{th}}$  class of pollutants per kilogram,  $\gamma_{ik}$  is the coefficient of pollutant emissions in g/kW,  $P_i$  is the output power of DG  $i$  in kW,  $\gamma_{gridk}$  is the coefficient of pollutant emissions of the grid in g/kW,  $P_{grid}$  is the output power of the grid in kW.

#### The objective function for the dynamic economic dispatch of a microgrid

After comprehensively considering  $C_1$  and  $C_2$ , we choose the maximum comprehensive benefits  $C$  (the minimum total cost) as the objective function for the dynamic economic dispatch of the microgrid:

$$\min C = C_1 + C_2 \quad (10)$$

#### The constraints of the system

##### (1) Power balance of the microgrid system

$$\sum_{i=1}^N P_i + P_{grid} + P_B = P_{Load} \quad (11)$$

where  $P_{Load}$  is the system load,  $P_i$  is the output power of DG  $i$ , and  $P_{grid}$  is the output power of the grid—if  $P_{grid}$  is positive, the grid transmits power to the microgrid, if  $P_{grid}$  is negative, the grid absorbs power from the microgrid.  $P_B$  is the output of the battery: when  $P_B$  is positive, the battery is discharging, if  $P_B$  is negative, the battery is charging.

##### (2) Power limits of the DGs

$$P_{i\min} \leq P_i \leq P_{i\max} \quad (12)$$

where  $P_{i\min}$  is the lower limit of DG  $i$  and  $P_{i\max}$  is the upper limit of DG  $i$ .

##### (3) Ramp rate limits of DE

$$r_{downi} \times \Delta t \leq P_{Gi}(t) - P_{Gi}(t-1) \leq r_{upi} \times \Delta t \quad (13)$$

where  $r_{downi}$  is the lower limit of DE  $i$  ramp rate and  $r_{upi}$  is the upper limit of DE  $i$  ramp rate in kW/min,  $\Delta t$  is the calculation period,  $P_{Gi}(t)$  is the output of DE  $i$  in period  $t$ , and  $P_{Gi}(t-1)$  is the output of DE  $i$  in period  $t-1$ .

##### (4) Operation constraints of the battery

$$SOC_{\min} \leq SOC(t) \leq SOC_{\max} \quad (14)$$

$$-P_{B\max} \leq P_B \leq P_{B\max} \quad (15)$$

$$SOC_{end} = SOC_{start} + \sum_{t=0}^{n-1} P_B \Delta t = SOC_{start} \quad (16)$$

where we set  $SOC_{\min}$  to be 0.1, which represents the lower limit of the SOC, and we set  $SOC_{\max}$  to be 0.9, which represents the upper limit of the SOC,  $P_{B\max}$  is the maximum power of charging and discharging the battery. Considering that the dynamic economic dispatch scheme for the microgrid is executed in cycles, it may be assumed that the final SOC value  $SOC_{end}$  of the battery is equal to the starting SOC value  $SOC_{start}$  of the battery, as shown in formula (16),  $n$  is the total number of calculation periods in the whole day.

(5) Constraints of the line transmission capacity between the microgrid and the grid

$$-P_L^{\max} \leq P_{grid} \leq P_L^{\max} \quad (17)$$

where  $P_{grid}$  is the transmission power between the microgrid and the grid and  $P_L^{\max}$  is the upper limit of the transmission power.

## Spinning reserve under uncertain conditions

### Spinning reserve constraints

It is necessary to schedule a spinning reserve to maintain the system's reliability, due to power fluctuations of the renewable energy, load fluctuations, and unit outage or failure. But in fact the cost will be very high if we consider all the uncertainty factors. Based on various kinds of uncertain factors, we adopt probability constraints for the spinning reserve for a given confidence level  $\alpha$  to meet the system's requirements so as to achieve a balance between reliability and economy.

$$0 \leq r_{it} \leq r_{it\max} \quad (18)$$

$$P_{Git} + r_{it} \leq P_{Gi\max} \quad (19)$$

$$r_{it} \leq r_{upi} \times \Delta t \quad (20)$$

$$P \left\{ \sum_{i=1}^{N_w} u_{ti}^w (P_{wti} + \sigma_{wti}) + \sum_{i=1}^{N_s} u_{ti}^s (P_{sti} + \sigma_{sti}) + P_{FC} + P_{GT} + \sum_{i=1}^{N_d} u_{ti} (P_{Gti} + r_{ti}) + P_{gridt} + r_t^L + P_{Bt} \geq P_{Lt} + \sigma_{Lt} \right\} \geq \alpha \quad (21)$$

where  $\sigma_{wti}$ ,  $\sigma_{sti}$ ,  $\sigma_{Lt}$  are, respectively, the fluctuation of the wind engine  $i$ , the photovoltaic array  $i$  and the load in period  $t$ .  $u_{ti}^w$  and  $u_{ti}^s$  are, respectively, the state of startup or shutdown of wind engine  $i$  and of the photovoltaic array  $i$ ; and  $u_{ti}$  is the state of startup or shutdown of the diesel engine;  $P_{wti}$ ,  $P_{sti}$ ,  $P_{Gti}$  are, respectively, the output of wind turbine  $i$ , photovoltaic array  $i$ , and diesel engine DE  $i$  in period  $t$ .  $P_{Lt}$  is the load in period  $t$ ,  $r_t^L$  is the spinning reserve power provided by the grid,  $r_{ti}$  is the spinning reserve power provided by diesel engine DE  $i$  in period  $t$ , it is less than the maximum of spinning reserve  $r_{ti\max}$ ,  $r_{ti}$  is determined by the surplus capacity and ramp rate of diesel engine DE  $i$ ,  $N_d$  is the number of DEs,  $\alpha$  is the confidence level, and  $1 - \alpha$  can be thought of as the upper limit of the probability of load loss.

### The simulation of the uncertainty factors

#### The simulation of uncertainty in the load and renewable resources

Research has shown that the load fluctuation of a power system obeys the normal distribution. The probability density function is shown as follows:

$$f(P_{La}) = (1/\sqrt{2\pi} * \delta_L) e^{-(P_{La} - P_L)/2\delta_L^2} \quad (22)$$

where  $\delta_L$  is the standard deviation of load forecasting,  $P_L$  is the value of load forecasting, and  $P_{La}$  is the value of the actual load.

Similarly, the fluctuations of renewable energy follow the normal distribution model,  $\delta_L/P_L$  is 0.1,  $\delta_W/P_W$ ,  $\delta_S/P_S$  both are 0.05.

#### The simulation of unit fault shutdown

We set the forced outage rate of DG  $i$  as  $f_i$ . According to the 0–1 uniformly distribution, if a number  $\xi$  which randomly generated is less than the forced outage rate  $f_i$ ,  $u_{it}$  equals 0 and the DG exits the operation. Otherwise  $u_{it}$  equals 1, the DG is in normal operation.

### An improved PSO algorithm combined with Monte Carlo simulation

#### The standard PSO algorithm

Dynamic economic dispatching of a microgrid is an optimization problem with nonlinear, high dimensionality, multi-index constraints, which is also a typical optimization problem in a power system. There are many modern heuristics stochastic optimization techniques in recent years, such as Hopfield neural networks [19,20], evolutionary program [21–23], genetic algorithm [24,25], differential evolution algorithm [26,27]. Compared with other algorithms, the PSO algorithm [28,29] has easier implementation, fewer parameter settings, and stronger optimization capability, but it may easily plunge into a local optimum and it finds it hard to deal with equality constraint conditions.

The traditional PSO algorithm forms a particle community by a random initialization [30]. The position of each particle is expressed by  $X_i = (x_{i1}, x_{i2}, \dots, x_{id})^T$ , and the speed is expressed by  $V_i = (v_{i1}, v_{i2}, \dots, v_{id})^T$ , where  $i = 1, 2, \dots, n$ , and  $n$  is the size of the population. Through the analysis and statistics of the individual optimal value  $pbest$  and the optimal value of group  $gbest$ , it adjusts the particle's position and speed until it meets the termination conditions, according to the following formula.

$$\begin{cases} x_{i,d}^{k+1} = x_{i,d}^k + v_{i,d}^k \\ v_{i,d}^k = \omega v_{i,d}^k + c_1 \cdot rand_1^k \cdot (pbest_{i,d}^k - x_{i,d}^k) + c_2 \cdot rand_2^k \cdot (gbest_{i,d}^k - x_{i,d}^k) \end{cases} \quad (23)$$

where  $v_{i,d}^k$  is the speed along dimension  $d$  of particle  $i$  in iteration  $k$ ,  $x_{i,d}^k$  is the position along dimension  $d$  of particle  $i$  in iteration  $k$ ,  $\omega$  is the inertia weight factor,  $c_1$  and  $c_2$  are learning factors,  $pbest_{i,d}^k$  is the local best value of coordinate  $d$  of particle  $i$  in iteration  $k$ ,  $gbest_{i,d}^k$  is the global best value of coordinate  $d$  in iteration  $k$ , and  $rand_1^k$  and  $rand_2^k$  are random numbers uniformly distributed in (0,1).

The updated value  $x_{i,d}^k$  takes its boundary value, such as formula (24) when it is beyond its scope of the particle position.

$$x_{i,d}^k = \begin{cases} x_{i,d}^k & x_{d \min} \leq x_{i,d}^k \leq x_{d \max} \\ x_{d \min} & x_{i,d}^k < x_{d \min} \\ x_{d \max} & x_{i,d}^k > x_{d \max} \end{cases} \quad (24)$$

where  $x_{d \min}$  is the minimum value in dimension  $d$  and  $x_{d \max}$  is the maximum value of dimension  $d$ .

The present paper implements some improvements on the standard PSO algorithm to make it more suitable for solving the mathematical model for dynamic economic dispatch of the microgrid model in this paper. In addition, due to the introduction of the random variables and probability constraints, the present paper adopts the Monte Carlo method to simulate the variables, and transform it into a deterministic optimization model. And then we adopt the improved PSO algorithm combined with Monte Carlo simulation to solve the objective function.

#### The improved PSO algorithm

To make PSO more suitable for solving the model, this paper improved it in two aspects.

#### Variable weighting factor and learning factors

At first, this paper assumes that the weighting factor and learning factor are not fixed, but change along with the number of iterations for getting better results. The formula are shown as follows:

$$\begin{cases} \omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) / Iter_{\max} \cdot Iter \\ c_1 = (c_{1f} - c_{1i}) Iter / Iter_{\max} + c_{1i} \\ c_2 = (c_{2f} - c_{2i}) Iter / Iter_{\max} + c_{2i} \end{cases} \quad (25)$$

where  $Iter$  is the number of iterations,  $Iter_{\max}$  is the total number of iterations,  $c_{1f}$  and  $c_{2f}$  are the stop values of  $c_1$  and  $c_2$ , set to 2.5 and 0.5.  $c_{1i}$  and  $c_{2i}$  are set to 2.5 and 0.5: they represent the initial values of  $c_1$  and  $c_2$ .  $\omega_{\max}$  is 0.8 and  $\omega_{\min}$  is 0.2.

#### Dynamic processing strategy for the equality constraints

In addition, how to deal with various equality constraints is a challenge for traditional PSO. Now, many scholars have introduced a penalty function into the objective function to render obsolete infeasible solutions and get the optimal solution [31], but the proportion of the feasible part in the solution space is small for dynamic economic dispatch, and the value of the punishment factor can only been determined by experience: but there is no definitive theory to prove whether it is optimal or not. Using the penalty function method alone will result in a slow calculation rate, and the optimal solution cannot be found and equality constraint cannot be satisfied, which leads to a lot of power deficiency or excess.

In order to solve this problem, this paper adopts a dynamic strategy which can deal with equality constraints in the process of initialization and updating of the particle swarm. The particles always satisfy the equality constraint conditions in the process of optimization. The processes of particle initialization and updating combined with this strategy are as follows:

- (1) Dealing with equality constraints in the process of initialization

Set the dimension of the particles to  $N$ . This indicates the number of generator units in the system.

Step 1: Produce the position and velocity of the first  $N - 1$  dimensions of the particle randomly, and satisfy their inequality constraints.

Step 2: The final dimension is determined by the equality constraint (11). If this value is within its range, it then goes to Step 3, otherwise proceeding to Step 1.

Step 3: The end of initialization. (2) Dealing with equality constraints in the process of updating

Step 1: Employing formulas (23)–(25), update the position and velocity of the first  $N - 1$  dimensions of each particle.

Step 2: The final dimension is determined by the equality constraint (11). If this value is within its range, then go to Step 5. If it is above the boundary value, we make its value equal to the boundary value, as in formula (24), and proceed to Step 3.

Step 3: Set  $l = 1$ .

Step 4: The value of dimension  $l$  is determined by the equality constraint (11). If it is within its range, we turn to Step 5. If it is above boundary value, we make its value equal to the boundary value, as in formula (24), set  $l = l + 1$ , then proceed to Step 4.

Step 5: Stop the update process.

#### Monte Carlo simulation

For a given set of decision variables, the Monte Carlo stochastic simulation method is applied to test whether the spinning reserve probability constraint satisfies the inequality (21), then we use penalty function to deal with it if it is unsatisfied. The steps are as follows:

- Step 1: Set the spinning reserve probability constrained counter  $K' = 0$ .

Step 2: Produce load fluctuation parameters which obey the normal distribution  $N(0, \delta_t^2)$ , wind turbine output power fluctuation parameters using  $N(0, \delta_w^2)$ , and PV array output power fluctuation parameters using  $N(0, \delta_s^2)$ .

Produce a random number which obeys the 0–1 distribution, if it is less than the forced outage rate,  $u_{it}$  equals 0 and the unit exits the operation; otherwise  $u_{it}$  equals 1 and the unit is under normal operation.

Step 3: Put the random value and decision variables into the probability constraint inequalities for the spinning reserve. If the inequality can be satisfied, set  $K' = K' + 1$ .

Step 4: Repeat  $K$  times. If  $K$  is big enough and  $K'/K \geq \alpha$ , we consider the inequality (21) can be satisfied, otherwise the inequality (21) cannot be satisfied. We will use a penalty function to deal with this inequality.

#### The process of the algorithmic solution

The improved PSO algorithm combined with the Monte Carlo simulation can solve the dynamic economic dispatch model very well in this paper. The algorithm process is shown as Fig. 1.

The specific steps are as follows:

Step 1: Initialize the particle swarm, dealing with the equality constraints as in the process of initialization in 5.2.

Step 2: Simulate the uncertainty values of the microgrid with a Monte Carlo simulation.

Step 3: Calculate the objective function for the fitness of the current particle. Through putting the uncertainty values into the spinning reserve probability constraints, judge whether the system meets the inequality constraints or not. If the system does not meet the inequality constraints, deal with it in the form of a penalty function.

Step 4: Update the local optimum value and the global optimum value of particle swarm.

Step 5: Update the particle position and velocity, dealing with the equality constraints as in the process of updating in 5.2.

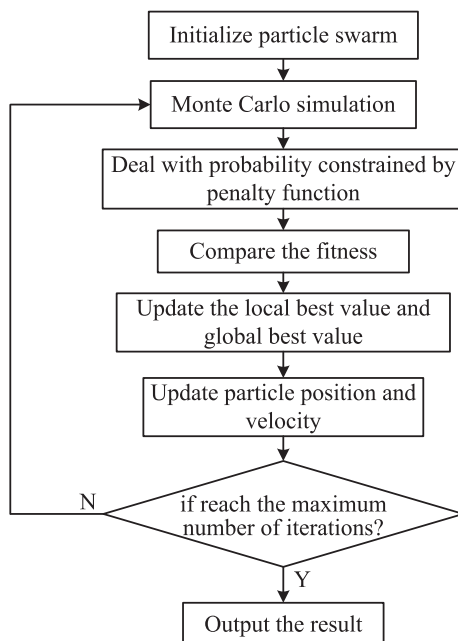


Fig. 1. Flow chart of the algorithm.

Step 6: If the maximum number of iterations has been reached, stop the search and output the result. Otherwise return to step 2 and continue the iterations.

#### Operation scheduling strategies for the microgrid system

In this paper, the microgrid includes WT, PV, MT, FC, DE and battery.

When the microgrid is running under the grid-connected mode, the battery neither charges nor discharges, owing to the support from the grid. When the microgrid is running under the island mode, the battery will participate in the operation dispatch.

And because the wind and solar energy are clean and renewable energy, they will be utilized the maximum scheduling in this paper. At the same time, the MT has been ordered to give priority to supplying the heat power.

So considering the two different operation modes, the four different operation scheduling strategies for dynamic economic dispatch are researched.

#### Running under the grid-connected mode

According to whether the DGs will use priority scheduling and the interactive mode between the grid and the microgrid, the operation scheduling strategies of dynamic economic dispatch on the grid-connected mode can be divided into three kinds.

##### Scheduling strategy 1:

The microgrid cannot output power to the grid.

The DGs are priority scheduled. When they cannot meet the load demand, the microgrid system will purchase power from the grid.

##### Scheduling strategy 2:

The microgrid still cannot output power to the grid.

The grid and the microgrid both participate in economic operation. If the cost of power generation of the DGs is cheaper, the DGs will receive priority scheduling. If the cost of power generation of the DGs is more expensive, the microgrid will absorb power from the grid.

##### Scheduling strategy 3:

The grid and the microgrid both participate in economic operation. And the grid and the microgrid can freely exchange power. If the cost of power generation of the DGs is higher than the purchasing cost, the DGs will be given priority. If the cost of power generation of the DGs is less than the purchasing cost, the grid will be given priority in the scheduling. And the microgrid can also sell redundant power to the grid in order to obtain profits.

#### Running under the island mode

When the microgrid is in the island mode, we design the fourth scheduling strategy.

##### Scheduling strategy 4:

The DGs will contribute according to optimal scheduling. If the output power of the DGs is greater than the load demand, the battery will be charged within its range. If the output power of the DGs is less than the load demand, the battery will be discharged within its range, and if a power shortage still exists in the

microgrid, the system have to interrupt part of the unimportant load to ensure the power supply.

**Numerical examples**

*The example system*

Visual C++ is adopted for the simulation calculation in this paper. In order to suit the actual power grid scheduling and reflect the dynamic scheduling of microgrid better, this paper set the calculation cycle as 1 d, setting 5 min as a calculation period, then the whole day could be divided into 288 periods. The related parameters about PSO were set as follows: particle population size was 60, and the largest number of iteration was 100. The output power of renewable energy, heat load, power load, real-time price, spinning reserve price are presented with the system data in Fig. 2. The cost parameters of the DGs are shown as Table 1. And the parameters of the pollutant discharge coefficient and the treatment cost of the pollutant are shown as Table 2.

The capacity of the battery was 150 kW/h, and the initial state of SOC was 50%. When it was in island mode, without the support of a large power grid, the DE3 was increased to ensure meeting the load demand. The compensation cost of load loss is 1.458 ¥/kW h. The parameter table of the DGs is shown in Table 3, and the exchange power limit value between the grid and microgrid is 80 kW.

*Comparative analysis of the PSO algorithm*

In this paper, we adopt the dynamic processing strategy of equality constraint under the scheduling strategy 1 and 2. The

**Table 1**  
Cost parameters of DGs.

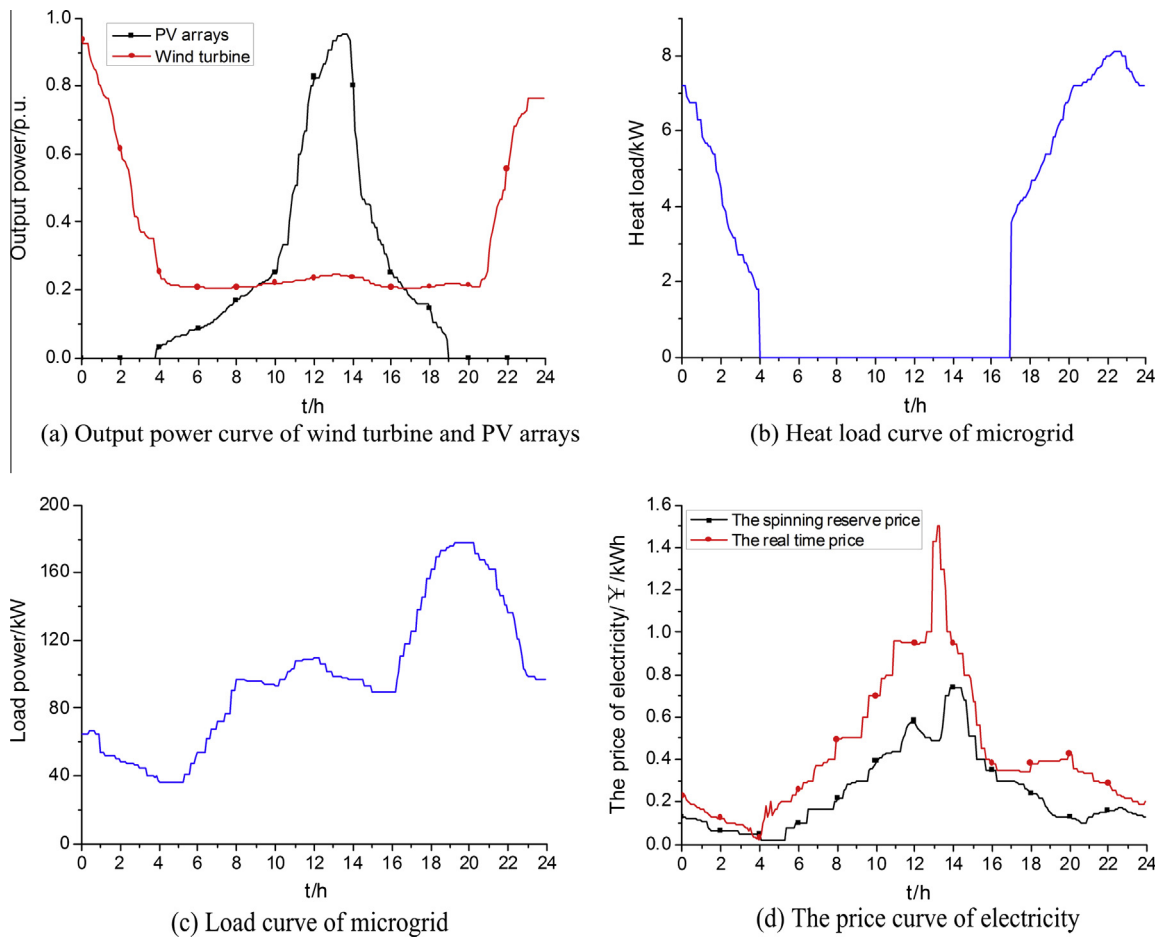
Type	PV	WT	GT	DE1, DE2	DE3	FC
<i>l</i> (year)	20	10	10	10	10	10
<i>InCost</i> (¥/kW)	66.50	22.35	13.06	16.00	18.21	42.75
<i>K<sub>om</sub></i> (¥/kW h)	0.0096	0.0296	0.0648	0.088	0.09	0.0293
<i>K<sub>fc</sub></i> (¥/kW h)	0	0	0.695	0.396	0.396	0.206

**Table 2**  
Parameters of pollutant discharge.

Pollutant type		CO <sub>2</sub>	SO <sub>2</sub>	NO <sub>x</sub>
Treatment cost (¥/kg)		0.21	14.842	62.964
Pollutant discharge coefficient (g/KW)	PV	0	0	0
	WT	0	0	0
	GT	724	0.0036	0.2
	DE1, DE2	649	0.206	9.89
	DE3	680	0.306	10.09
	FC	489	0.003	0.01
	GRID	889	1.8	1.6

results of adopting the traditional PSO algorithm and the improved PSO algorithm to calculate the model of scheduling strategy 1 when connected to grid are shown in Table 4, and the convergence curves are shown in Fig. 3.

Table 4 shows that the calculation time of the improved PSO is longer than the traditional PSO, since the improved PSO adopts the equality constraint strategy in the process of particle swarm initialization and update. This strategy adjusts each particle



**Fig. 2.** The microgrid system data.

**Table 3**  
DGs in microgrid.

Type	Power (kW)		Ramp rate (kW/min)	Forced outage rate	Number
	Lower limit	Upper limit			
DE1	0	30	1	0.009	1
DE2	0	30	1	0.008	1
DE3	0	60	2	0.002	1
FC	0	30	–	–	1
PV	0	6	–	0.003	1
WT	0	30	–	0.004	1
BS	–30	30	–	–	1

**Table 4**  
Results comparison between the traditional PSO and the improved PSO.

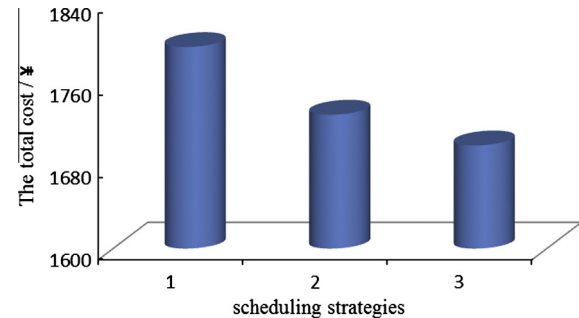
Algorithm type	The traditional PSO	The improved PSO
Calculation time (min)	12.564	30.784
Convergence iteration	80	24
Convergence value	1838.43	1795.23
Average power shortage (kW)	8.78	0
Average power excess (kW)	10.12	0

repeatedly to make it satisfy the equality constraint so the calculation time is longer than that of the traditional PSO. But the equality constraints also can be completely satisfied. It can also be seen from Fig. 3 that the convergence rate of the improved PSO is faster than the traditional PSO, it converges at about the 20th generation, but the traditional PSO converges at about the 80th generation. And the improved PSO obtained better results than the traditional PSO. Considering the improved PSO algorithm deals with the equality constraint effectively and the convergence value is better, we select the improved PSO as the dynamic economic dispatch algorithm in this paper.

#### The influence of the scheduling strategy

When different scheduling strategies are adopted, the optimization results are shown in Fig. 4.

We can see from Fig. 4, when we adopt scheduling strategy 2, the total cost is less than scheduling strategy 1, this is because both the grid and the microgrid participate in scheduling in scheduling strategy 2, so the load can rely more on the generating units that have lesser generating costs. And because power can be freely exchanged between the grid and the microgrid in scheduling strategy 3, when the purchase cost is higher than the generating cost of

**Fig. 4.** Optimization results under different control strategies.

the other DGs, the redundant output power can be sold to the grid to obtain economic benefits, so the total cost is lower than the first two scheduling strategies.

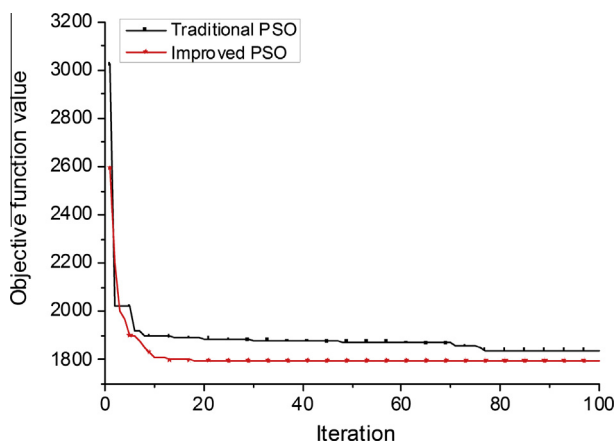
When adopting the three different scheduling strategies, the economic dispatch results are shown in Fig. 5.

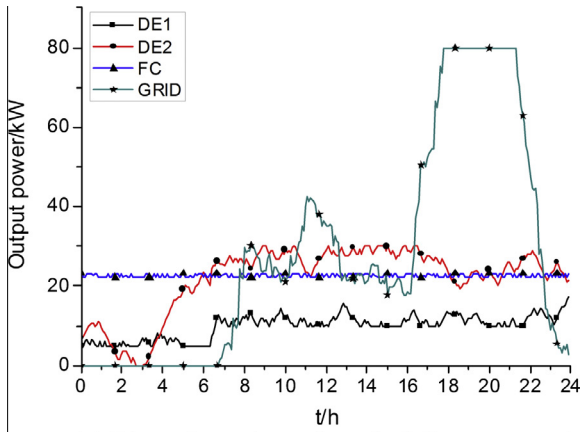
From Fig. 5, when choosing scheduling strategy 1, the DGs can be prior scheduled, and because the generating cost of an FC is the least, it is used first and the output is steady. After 6 o'clock in the morning, if the DGs cannot satisfy the load demanded, the microgrid absorbs power from the grid. When choosing scheduling strategy 2, the grid and the DGs both participate in the scheduling. We can see that the FC which has the lowest cost is prior scheduled, and before 8 o'clock in the morning, the purchase cost is less than the cost of the DE, so the output of the grid is bigger than the DE. When choosing scheduling strategy 3, the FC is still prior scheduled. Before 6 o'clock in the morning, according to the scheduling results, the output of the DGs is more than the load met, then spare energy can be sold to the grid to earn profits. And in periods of peak load, when the power from the DGs cannot meet the need of the load, the microgrid will absorb power from the grid.

#### The influence of the objective functions

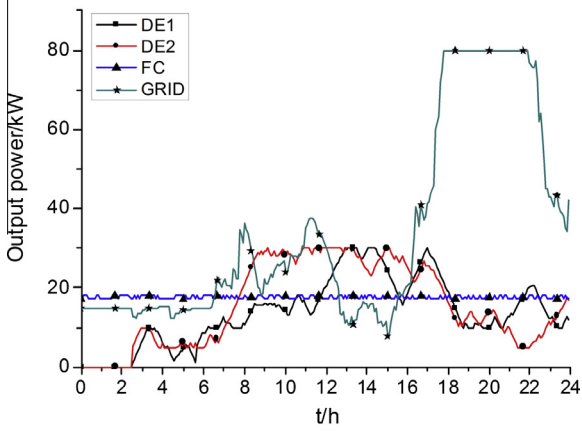
To analyze the impact of the optimization goals on the scheduling results, we used the operation cost  $C_1$ , the pollutant treatment cost  $C_2$  and the maximum comprehensive benefits  $C$  respectively as objection functions under scheduling strategy 1. The scheduling results are shown in Fig. 6.

From Fig. 6, we find the fuel cell is prior scheduled no matter which objective function is chosen, and its output is steady because its operation costs and pollutant treatment costs are the best. When choosing  $C_1$  for optimization, the output of the diesel engine is greater than the fuel cell during some periods, because there is so little difference in the operating costs between the fuel cell and the diesel engine; But when choosing  $C_2$  as the objection function, the output of the two diesel engines is always less than the fuel cell, and the output of the fuel cell is almost at its maximum. This is because the pollution discharge of the diesel engine is far greater than that of the fuel cell. When choosing  $C$  as the objective function, the difference of total costs is less than the

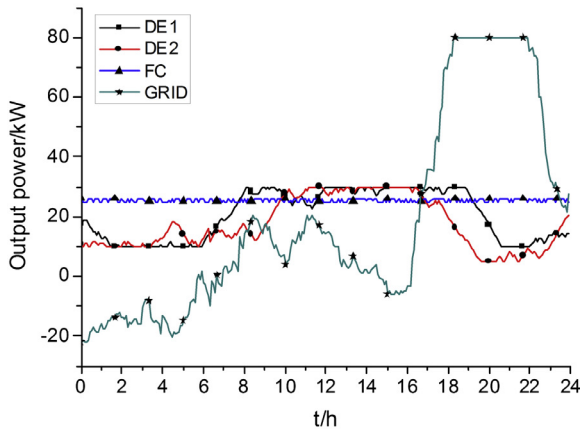
**Fig. 3.** Convergence curve based on the algorithm.



(a) Dispatch results under scheduling strategy 1



(b) Dispatch results under scheduling strategy 2



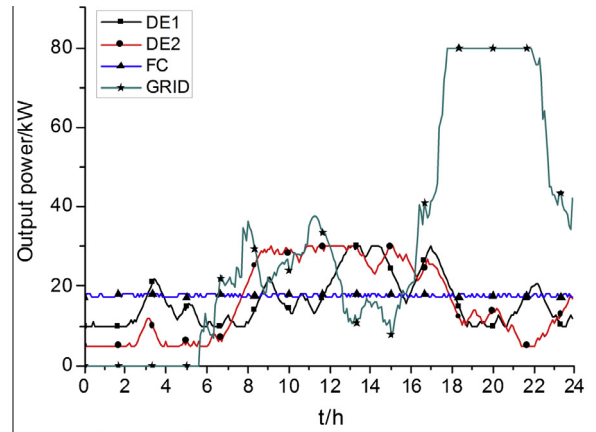
(c) Dispatch results under scheduling strategy 3

Fig. 5. Dispatch results under the different scheduling strategies.

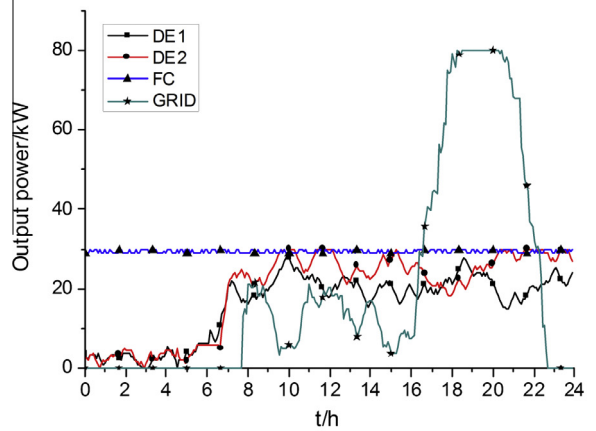
difference of pollutant treatment costs between the diesel engine and the fuel cell due to a comprehensive consideration of the operating costs. So during some periods, the output of the diesel engine is greater than the fuel cell. The grid will supply power to the microgrid when their output cannot meet the load owing to the impact of the scheduling strategy,

*The influence of the reliability indexes*

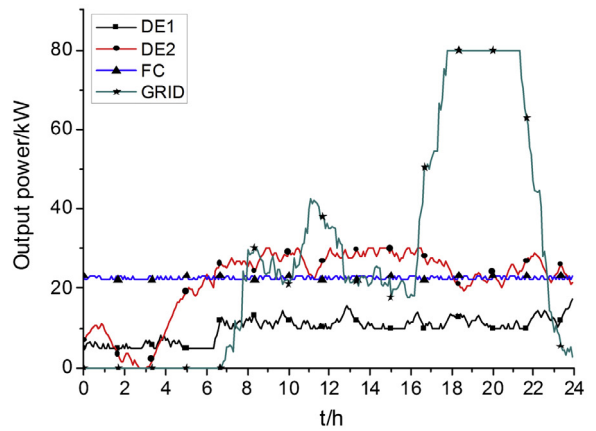
The confidence level  $\alpha$  in the model, the selection of the unit failure rate and the uncertainty of renewable resources output, will all affect the optimization results.



(a) The objective function-the operation cost



(b) The objective function-the pollutant treatment cost



(c) The objective function-the total cost

Fig. 6. Dispatch results under different objective functions.

In order to analyze the influence of the reliability indexes on the microgrid operation, this paper discusses the changes of total cost corresponding to different scheduling strategies and different confidence levels. The results are shown in Fig. 7.

From the analysis of Section ‘The influence of the scheduling strategy’, the maximum comprehensive benefits of strategy 1 are always the highest, followed by those of strategy 2 and strategy 3, regardless of the value of the confidence level. From Fig. 7, we can see that the maximum comprehensive benefits of strategy 1 are also always the highest, followed by those of strategy 2 and strategy 3 at the confidence level. We can also see that the



maximum comprehensive benefits always gradually increase as  $\alpha$  increases. And when  $\alpha$  is close to 1, the maximum comprehensive benefits  $C$  sharply increase. This is because the higher is the system reliability level, the more spinning reserve the system needs, and the more spinning reserve power has to be purchased from the grid. Therefore the improvement of reliability is at the expense of higher total costs.

*The analysis of the operating results under the island mode*

When the microgrid is in island mode, the system load is provided by the DGs, so the scheduling results are quite different from those when grid-connected. And without the support of the grid, the effect of uncertain factors on the operation of the system is more obvious than that under grid-connected mode. We set the island operating conditions as an example to analyze its scheduling results. The load fluctuation and uncertainty of the renewable resources were also set as examples to analyze the influence of the uncertainty on the microgrid and the characteristics of the battery.

*Scheduling results analysis*

Fig. 8 shows the optimization scheduling results under the operation condition of the island mode.

We can see from Fig. 8 that the fuel cell FC is prior scheduled, because the generating cost of the fuel cell is optimal. Following are the diesel engines DE1 and DE2. Engine DE3 is scheduled last because its generating cost is greater than that of the others. But during periods of peak load, the output of DE3 is at its maximum. This is because its capacity is greater than that of the others. From midnight to 3 o'clock in the morning, we can see battery BS charges within its operation period, because the output of the renewable energy and other micro sources is greater than the load demand. But it no longer charges after 3 o'clock in the morning owing to the limitation of the battery capacity and the SOC. From 6 o'clock in the evening to 9 o'clock at night during the peak load periods, when the output of other micro sources no longer meets the load demand, battery BS discharges in its scope. And from 7 o'clock to 8 o'clock at night, when micro sources still cannot meet the load demand, some unimportant parts of the load LS are interrupted.

*The influence of uncertainty under the island mode*

(1) The influence of load fluctuation under the island mode

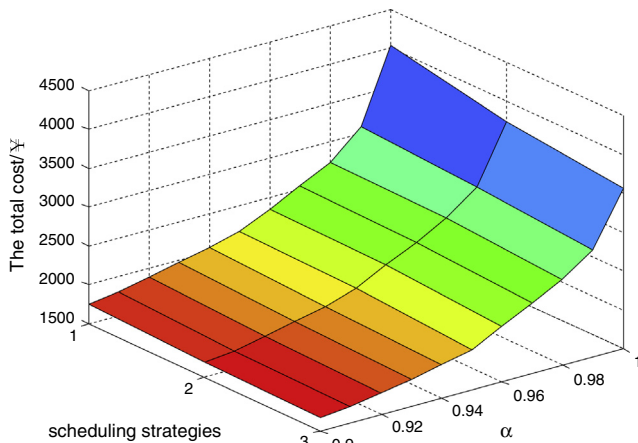


Fig. 7. Optimization results under the different scheduling strategies and confidence levels.

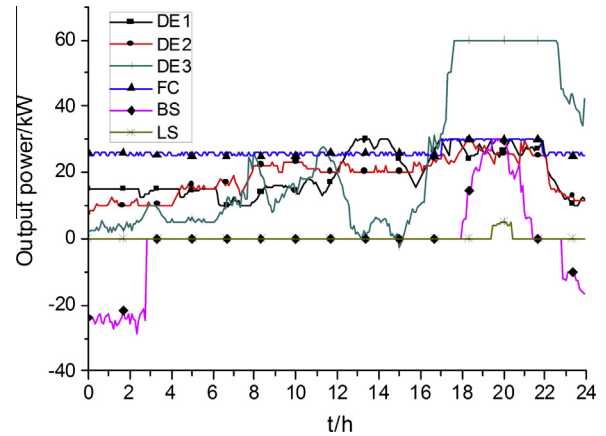


Fig. 8. Dispatch results in island mode.

The load fluctuation parameter  $\delta_L/P_L$  reflects the accuracy of load forecasting. In order to study the influence of load fluctuation on island operation, this paper assumes that there is no fluctuation in the renewable energy. The influences on the optimization results with different  $\delta_L/P_L$  values are shown in Fig. 9.

Fig. 9 shows that the total cost increases as the load fluctuation worsens. For example, when  $\delta_L/P_L = 0.2$ , the cost increases by 2.5% more than when  $\delta_L/P_L = 0.18$ . And when  $\delta_L/P_L = 0.18$ , the cost increases by 1.8% more than when  $\delta_L/P_L = 0.16$ . This is because the system needs more spinning reserves from the DE, which may lead to more compensation costs for load interruption.

(2) The influence of renewable energy fluctuations under the island mode

In order to study the influence of the fluctuation renewable resources under island mode, this paper assumed that the load fluctuation parameter  $\delta_L/P_L$  is 0. Fig. 10 shows the influence of renewable resource fluctuation parameters  $\delta_W/P_W$  and  $\delta_S/P_S$  on the total cost of the microgrid.

When  $\delta_W/P_W$  is set to a certain value, the total cost increases as  $\delta_S/P_S$  increases. Similarly, when  $\delta_S/P_S$  is set to a certain value, the total cost increases as  $\delta_W/P_W$  increases. And the increased generating cost increases as the renewable energy fluctuation worsens. This is because the system needs more spinning reserve from DE. Also, there may be more compensation costs for interruption of

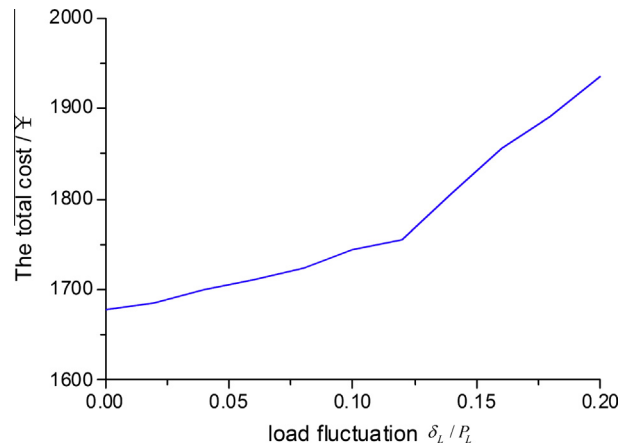


Fig. 9. Dispatch results under different load fluctuations.

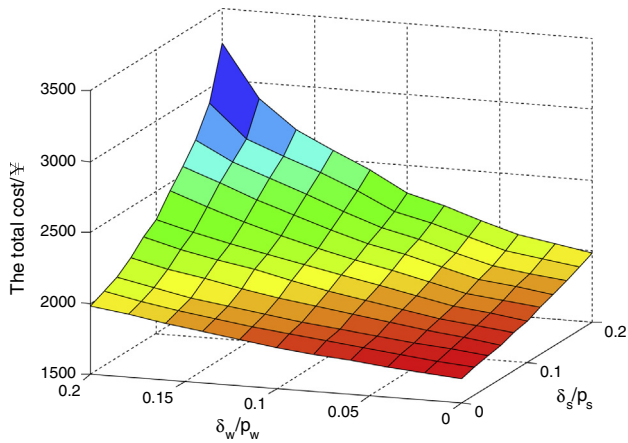


Fig. 10. The total cost curves with wind turbine  $\delta_w/P_w$  and PV arrays fluctuation  $\delta_s/P_s$ .

the load due to fluctuations in the renewable energy, leading to an increase in the generation costs.

(3) Analysis of battery characteristics

(i) The analysis of battery impact

Uncertainty factors in the microgrid will affect the operation of the storage battery. This paper has only studied the effect of wind fluctuations on the battery when the fluctuation parameter  $\delta_w/P_w$  was set to different values. The impact on the performance of battery BS is shown in Fig. 11.

We can see from Fig. 11 that while the microgrid is running in island mode, the battery BS charges within its operating range during low load periods, from midnight to 3 o'clock in the morning when the output of the renewable energy units and other micro sources in the micro grid is greater than the load demand. And during the periods of peak load, from 6 o'clock in the evening until 9 o'clock at night when the output of other micro sources cannot meet the load demand, the battery BS discharges within its operating range. Because the SOC of the battery needs to return to the same value after the dynamic scheduling cycle, namely, the discharge power needs to be equal to the charge power of the battery, the battery BS no longer discharges after a certain period. And it does not start charging until the period when the output of the other micro sources is greater than the load at night. Therefore, the battery plays the role of peak load shifting in the operation

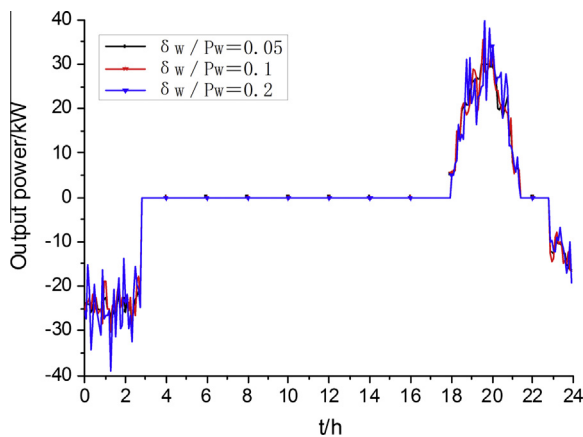


Fig. 11. The output power curve of the battery.

Table 5  
The influence of battery capacity on outage index.

Battery capacity (kW h)	$T_{LS}$ (h)	$\bar{L}S$ (kW)
80	2	18.86
120	1.5	10.32
150	1	4.03
200	0.5	1.23

of the microgrid. We also can see the fluctuation of the battery BS worsens as  $\delta_w/P_w$  increases. This is because the feature of battery energy storing makes the battery smooth the power fluctuations in the microgrid.

(ii) Analysis of battery capacity

As a matter of fact, the microgrid may not be able to meet the load demand when it runs in island mode owing to the existence of the uncertainty factors. Some unimportant load may be disrupted in the periods of peak load, causing some power loss for the users. The characteristic of battery energy storing makes the battery able to discharge power during peak load, reducing the extent of any interrupted loads. This paper sets the program to run  $N$  times repeatedly, and took the average outage time  $T_{LS}$ , average outage power  $\bar{L}S$  as outage indexes, studying the influence of the battery capacity on the outage indexes. The results are shown in Table 5.

Table 5 shows that in this example, the load interruption occurs from 7 o'clock to 8 o'clock at night, the second peak load. The average outage time  $T_{LS}$  lasts an hour. The average outage power  $\bar{L}S$  is 4.03 KW. When the capacity of battery is reduced, both the average outage time  $T_{LS}$  and average outage power  $\bar{L}S$  increase. And when the capacity of the battery is increased,  $T_{LS}$  and  $\bar{L}S$  both decrease. This shows that increasing the capacity of the battery can reduce the outage power and the power loss of users.

Conclusions

Through establishing a combined heat and power (CHP) microgrid system which includes wind turbines, photovoltaic arrays, diesel engines, a micro turbine, a fuel cell, and a battery, mathematical models and an algorithmic solution of dynamic economic dispatch for the microgrid were presented in this paper. We choose the maximum comprehensive benefits as the objective function for dynamic economic dispatch. At the same time, it establishes the spinning reserve probability constraints of a microgrid considering the influence of the uncertainty factors of renewable energy and load fluctuations. An improved particle swarm optimization algorithm combined with Monte Carlo simulation is used to solve the objective function. And we research four different operation scheduling strategies under the grid-connected mode and the island mode of the microgrid. The proposed models and algorithm are verified by case studies based on an example system. When the microgrid was running under the grid-connected mode, we discussed the influence of different scheduling strategies, optimization goals, and reliability indexes on the dynamic economic dispatch. And when the microgrid was running under the island mode, we discussed the influence of the uncertainty factors and the capacity of the battery on the dynamic economic dispatch. Through a study of the dynamic economic dispatch of the microgrid, it can be concluded that an improvement of the reliability of the microgrid carries an economic cost, the battery fulfills the role of peak load shifting and stabilizing power fluctuations, and increasing the capacity of the battery can reduce system power

loss. The presented research can provide a reference for the dynamic economic dispatch of a microgrid in making full use of renewable energy and improving its reliability.

### Acknowledgements

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