

Stochastic approach to represent distributed energy resources in the form of a virtual power plant in energy and reserve markets

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Abstract: Today, traditional networks are changing to active grids due to the burgeoning growth of distributed energy resources (DER), which demands scrupulous attention to technical infrastructures, as well as economic aspects. In this study, from economic point of view, the aggregation of DERs in a distribution network to participate in joint energy and reserve markets is investigated. This approach, which is predicated upon price-based unit commitment method, has considered virtually all the technical data in the proposed model. It is worth to mention that uncertainties of loads and market prices, as an inherent characteristic of the electricity markets, are treated in this study, and their effect on the operation of virtual power plants in energy and reserve markets has been thoroughly discussed. To this end, for both uncertain parameters, a good number of scenarios are generated and using the backward reduction method the number of these scenarios is reduced. The problem is formulated as a MINLP model and is implemented in GAMS software, while its authenticity is validated using particle swarm optimisation method.

 $P_{\operatorname{ch}_i^t}; P_{\operatorname{Dch}_i^t}$

N	l٥	m	Δn	ام	atı	ıre

Indices

i,j index for buses t index for hours N total number of buses

Sets

 S_{DG} set of distributed generation (DG) units S_{IL} set of interruptible load (IL)

 $S_{\rm EES}$ set of electrical energy storage (EES)

 S_{int} set of tie-lines S_{b} set of buses

Variables

$P_{\mathrm{DG}_{i}^{t}};Q_{\mathrm{DG}_{i}^{t}}$	amount of active and reactive power generated
1 1	by <i>i</i> th DG unit at hour <i>t</i> for the energy market
$R_{\mathrm{DG}_i^t}$	amount of active power generated by ith DG
	unit at hour t for reserve market
$P_{\mathrm{IL}^t}; Q_{\mathrm{IL}^t}$	amount of curtailed load of <i>i</i> th IL at hour <i>t</i>
$P_{\mathrm{IL}_i^t};Q_{\mathrm{IL}_i^t}$	amount of curtailed load of ith IL allotted to
	spinning reserve market at hour t
$P_{\mathrm{DG}_{i}}^{\mathrm{min}}; P_{\mathrm{DG}_{i}}^{\mathrm{max}}$	minimum and maximum active power
-1 -1	generation of <i>i</i> th DG unit
$P_{\mathrm{int}_i^t};Q_{\mathrm{int}_i^t}$	amount of active and reactive exchanged
ı ı	power between virtual power plant (VPP) and
	the <i>i</i> th neighbouring grid (positive sign for
	purchasing from the neighbouring grid and
	negative sign for selling to it)
$P_{E_t}; Q_{E_t}$	amount of active and reactive exchanged
E_{t} , $\sum E_{t}$	power between VPP and upstream network
	(positive sign for purchasing from the
	neighbouring grid and negative sign for selling
	to it)
R_t	sum of curtailed load and DG generation
ι	allotted to spinning reserve market at hour t
$P_{\text{Load}_t}; Q_{\text{Load}_t}$	supplied active and reactive load by VPP to
¹ Load _t , ∠Load _t	supplied delive and reactive load by vii to

end customers

	state and negative sign for discharging state)
$P_{\mathrm{Str}}^{\mathrm{min}}; P_{\mathrm{Str}}^{\mathrm{max}}$	minimum and maximum capacity of <i>i</i> th EES in kWh
$R_{\mathrm{ch}_i}; R_{\mathrm{Dch}_i}$	maximum charge/discharge rate of <i>i</i> th EES in kW
Cap_i^t	state of charge of the ith EES at hour t
$C(P_{\mathrm{DG}_{i}^{t}})$	generation cost function of ith DG unit
$C(P_{\operatorname{Str}_i^t})$	operation cost function of ith EES
$C(P_{\mathrm{IL}_i^t})$	contracted cost function of IL to curtail its load in specified hours
$I_t; L_t; J_{i,t}; K_{i,t}; I_i^t$	binary variables
$V_{i,t} \angle \delta_{i,t}$	voltage phasor at bus i at hour t
$Y_{ij} \angle \theta_{ij}$	polar form of <i>ij</i> th element of admittance matrix
$Y_{ij} \angle \theta_{ij}$ $T_{i,t-1}^{\text{off}}; T_{i,t-1}^{\text{on}}$	number of hours that <i>i</i> th DG unit has been on $(+)$ or off $(-)$ at the end of hour $t-1$
$V_{i,t}^{\min}; V_{i,t}^{\max};$	minimum and maximum voltage magnitude at bus <i>i</i> at hour <i>t</i>
S_{ij}^t	apparent power flow in feeder i – j at hour t
$S_{ij,t}^{max}$	maximum apparent power flow in feeder i – j at hour t
$P_{\mathrm{IL}_{:}^{t}}^{\mathrm{max}}$	maximum load curtailment on <i>i</i> th IL at hour <i>t</i>
$P_{\mathrm{sub}_{\mathrm{t}}}^{'}$	capacity of upstream substation transformer
$E_{\mathrm{Ex},t}^{\mathrm{Max}}$	maximum exchanged power with upstream grid
AR_t	adequacy reserve maintained by VPP

EES at hour t

amount of power charged/discharged into ith

amount of charged/discharged capacity of ith

EES at hour t in kW (positive sign for charging

Constants

$ ho_{\mathrm{E},t}$	energy market price
$ ho_{\mathrm{R},t}$	spinning reserve market price
$oldsymbol{ ho}_{ ext{int}_i^t}$	price of power exchange with neighbouring grid
$ ho_{L,t}$	retail energy price
r_t	probability of reserve delivery
$\eta_{ m Str}$	efficiency of EES

 $SUC_{DG_i^t}$; $SDC_{DG_i^t}$ R_i^{UP} ; R_i^{DOWN} MUT_i ; MDT_i start up and shut down costs of *i*th DG unit ramp-up and ramp-down rate of *i*th DG unit minimum up time and minimum down time of *i*th DG unit

1 Introduction

The electricity industry restructuring and deregulation, the energy crisis and limited fossil fuels, environmental concerns ensued from the overconsumption of fossil fuels, burgeoning technological developments in the field of distributed energy resources (DERs), and the competitive drive of the electricity market over the past two decades have instigated the spread of DERs and utilisation of renewable energy resources in industrial, residential, and territory sector, as Denmark already secures 40% of its electricity demand through mentioned resources and Germany, as it is anticipated, will provide 80% of its electricity demand through renewable resources by 2050 [1, 2]. Although employing new technologies like photovoltaic (PV) cells, wind turbines, combined heat and power generation (CHP), and electric vehicles are costly, governments are inclined to utilise them as alternative energy resources in the future.

On the other hand, while utility companies are striving to keep up with the pace of the progressive development of new technologies, like smart grids, regulatory authorities are putting extra pressure on them to reduce the amount of detrimental derivatives of carbon and increase their flexibility regarding consumer requests [3]. To this end, these companies should adopt economical approaches in order to equip themselves with the newest technologies, provide the consumers with reasonable electricity prices, and satisfy their shareholders as well. The question posed is: how to accomplish all these objectives all together? The following solutions can be proposed:

- Utilising DERs to decrease harmful carbon derivatives.
- Using situation-based prices for emergency conditions, and demand response techniques, in order to alleviate the peak load through load shedding.
- Using DERs to increase the security and reliability of the power grid.
- Adjusting energy and reserve prices through load shedding, local generation, and ancillary services provided by DERs.
- Offering incentives to customers for decreasing electricity consumption within peak load hours.

This study, since achieving all the aforementioned objectives is highly challenging and requires special infrastructures, focuses on economical energy procurement with consideration of power system reliability and optimal use of DERs, which are currently included as a small share of the electricity market and generally serve as emergency or optional resources to postpone the power system need for reinforcement. The main challenge is making preparations for these resources to participate in the market and maximising the benefits of their integration [4–6].

Considering growing penetration of DERs, the old passive structures of the grid have been superseded by new active ones. These resources, which have caused economical and technical effects in restructured environment of electricity industry, are expected to play an important role in electricity market scheduling and operating through offering energy and reserve services, two key features of electricity markets [7, 8]. Although in the past, the main objective was generally to receive maximum power from DERs, however, currently the ultimate goal is integrating these resources into the grid in order to attain maximum benefits. Broadly speaking, regarding some newly introduced issues like high penetration rate of DERs, demand response, competing over energy procurement, and further need to more reliable power systems and more secure power supplies, a control infrastructure to coordinate numerous elements of such an elaborate system is inevitable and virtual power plants (VPPs) are the radical solutions [9]. As the PricewaterhouseCoopers has claimed, VPPs are

indispensable for transition to alternative energy resources. These software-based systems are in charge of dispatching and optimising output power of distributed generations (DGs) and storage devices, as well as managing demand side operation and exchanged power with upper or adjacent networks.

Having an intermittent nature, limited storage capabilities, and lower inertia in comparison with the traditional resources, increasing penetration of DERs have engendered worries about the reliability of power systems among system operators. Furthermore, since individual participation of these resources in the market, for their notoriously different and inflexible operation profiles, would not be economically justifiable, aggregation of them in order to have a more efficacious role in energy and reserve markets is necessary. To this end, VPPs provide these resources with economically favourable conditions through aggregation and integration, in order to demonstrate a more cohesive and flexible profile suitable enough for energy and reserve markets [10].

The methods used for aggregation and integration need to be precisely delineated, as well as operation strategies. VPPs are categorised as technical VPPs (TVPPs) and commercial VPPs [11]. The former integrates DER located in a specific geographic region and distribution system operator, owing to the fact that TVPPs are highly dependent on detailed data of distribution network, is typically the operator of this kind of VPPs. On the other hand, the latter is not confined to a particular region and provides a variety of services, such as transactions in the wholesale market, portfolio balancing, and services for transmission system operator, in various points of the grid [12, 13].

For DGs installed in the distribution networks to participate in energy and reserve markets, several references have addressed a variety of strategies [14]. In [15], an energy acquisition model is presented for distribution companies (DISCOs) to participate in day-ahead market with the presence of wholesale suppliers, independent DGs, and DGs which belong to DISCOs which is based on the bilateral contracts. In [16], in order to determine the optimal contract prices for dispatchable DGs in distribution grids, a bi-level programming method has been adopted. In [17], a practical decision-making approach is proposed for a competitive market with DISCOs, DGs, and interruptible loads (ILs) as market players. In [18], a short-term hierarchical performance framework is proposed for DISCOs, where the first level is related to DGs participation and making purchases from market considering uncertainty, and the second level evaluates real-time performance of DISCO. In [19], an energy acquisition model, for those DISCOs which aim to participate in day-ahead market and possess both ILs and DGs, is presented. In [20], the integrated approach in the competitive market for DGs in a pool-based market is discussed. In [21], based on a decentralised control strategy, optimal performance of a VPP operator which owns a number of CHPs is investigated. In [22], a new demand response scheme has been proposed that can be applied in VPP context to participate in the electricity market. In [23], an approach is proposed to aggregate wind power generation and electric vehicles in the form of VPP for participation in the energy market in order to maximise VPP profit. In [24], optimal bidding strategies for VPPs are discussed, and in [25-28], using price-based unit commitment (PBUC), DER integration is carried out and optimal bidding in energy and ancillary service markets is achieved. In [29], in order to make the dispatch more feasible in the real world, the above-mentioned method has been modified and applied to geographically dispersed DERs.

In this study, using the PBUC approach as in aforementioned works, DER integration in a TVPP framework and an optimal bidding strategy in day-ahead energy and spinning reserve markets are elaborated, with consideration to uncertainty in load and market prices, and regarding all the constraints related to suppliers, consumers, and loads.

The rest of the paper is organised as follows. Section 2 is dedicated to VPP modelling; market interactions and problem formulation are addressed. In Section 3, the effect of uncertain parameters is discussed, which is followed by simulation and result analysis in Section 4 and conclusion in Section 5.

2 Methodology

In this study, a part of an active distribution network which consists of DGs, storages, and ILs (demand side resources) has been considered as a VPP. In other words, the VPP operator aggregates, in a specified geographical region of a distribution network, the DGs and loads in order to meet the internal demand with the minimum cost through utilising internal generation, exchanging energy with upstream networks, or demand side management.

2.1 Modelling VPP elements

In this section, conventional models for VPP elements are proposed.

(i) DG: These elements are instrumental parts of VPPs, and the cost function of dispatchable types can be modelled as a polynomial function [19]

$$C(P_{\mathrm{DG}_{i}^{t}}) = \alpha \times P_{\mathrm{DG}_{i}^{t}}^{2} + \beta \times P_{\mathrm{DG}_{i}^{t}} + \gamma \tag{1}$$

(ii) Electrical energy storage (EES): Another indispensable element of a VPP is EES.

In [26], the following equations are proposed as mathematical models for ${\sf EES}$

$$-\left(P_{\operatorname{Str}_{i}^{t}}^{\min}-\operatorname{Cap}_{i}^{t-1}\right) \leq P_{\operatorname{Str}_{i}^{t}} \leq P_{\operatorname{Str}_{i}^{t}}^{\max}-\operatorname{Cap}_{i}^{t-1} \tag{2}$$

$$\operatorname{Cap}_{i}^{t-1} - \operatorname{Cap}_{i}^{t} \le R_{\operatorname{Dch}} \tag{3}$$

$$\operatorname{Cap}_{i}^{t} - \operatorname{Cap}_{i}^{t-1} \le R_{\operatorname{ch}} \tag{4}$$

$$-\left(\operatorname{Cap}_{i}^{t-1} - P_{\operatorname{Str}_{i}^{t}}^{\min}\right) \le \sum_{k=1}^{24} P_{\operatorname{Str}_{i}^{t}} \le P_{\operatorname{Str}_{i}^{t}}^{\max} - \operatorname{Cap}_{i}^{t-1} \tag{5}$$

$$P_{\operatorname{Str}_{i}^{t}} \leq R_{\operatorname{ch}} \tag{6}$$

$$P_{\text{Str}_{i}^{t}} \le R_{\text{Dch}} \tag{7}$$

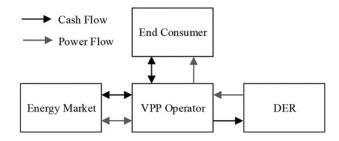


Fig. 1 Schematic representation of power and cash flow

Table 1 Probability distribution attributed to the load uncertainty

Scenario	Load factor	Probability
high-high	1.1	0.1
medium-high	1.05	0.2
medium	1	0.4
medium-low	0.95	0.2
low-low	0.9	0.1

In this study, the behaviour of storages is simulated using (2)–(7) and the following cost function is deployed

$$C(P_{Str_i'}) = A \times \left| P_{Str_i'} \right| + B \tag{8}$$

(iii) *ILs*: A polynomial function represents the cost function of ILs [15]

$$C(P_{\mathrm{IL}_{i}^{t}}) = \alpha_{\mathrm{IL}} \times P_{\mathrm{IL}_{i}^{t}}^{2} + \beta_{\mathrm{IL}} \times P_{\mathrm{IL}_{i}^{t}}$$

$$\tag{9}$$

2.2 Proposed method

On the basis of energy price, retail price, and the price of spinning reserve and regarding to the operating costs of DGs, ILs, and

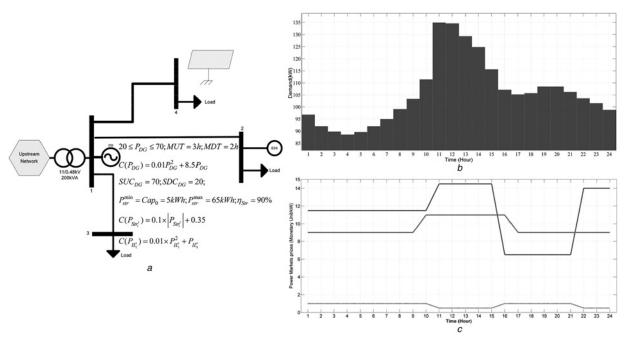


Fig. 2 Characteristics of studied VPP

- a VPP configuration
- b Forecasted day-ahead load curve
- \boldsymbol{c} Day-ahead energy, spinning reserve, and retail market prices

energy storages, in day-ahead market VPPs propose bids for the value of unit generations, charge and discharge of storages, the state of ILs, and the amount of power exchange in energy and reserve markets. Of course grid structure and limitations are considered in bidding strategy of VPPs, while the objective of bidding problem is to maximise the benefit of their transactions in both energy and reserve markets, as well as direct transactions with end users. Fig. 1 demonstrates the schematic representation of power and cash flow in the electricity market regarding VPP participation. Since a VPP can connect to several adjacent or upstream networks, and since it can have different market interactions with each, separate financial/power exchanges have been defined for each network.

2.3 Objective function

The objective function of this problem, which is to be maximised, is the benefit of VPP in day-ahead market. It is defined as the aggregated value of costs subtracted from the aggregated value of incomes as follows

$$\begin{aligned} \text{Benefit} &= -\sum_{t=1}^{24} \rho_{E,t} \times P_{E_t} + \sum_{i=1}^{\text{No}_{\text{TicLine}}} \sum_{t=1}^{24} \rho_{\text{int}_i^t} \times P_{\text{int}_i^t} \\ &+ \sum_{t=1}^{24} \left[r_t \cdot (\rho_{E,t} + \rho_{R,t}) + (1 - r_t) \cdot \rho_{R,t} \right] R_t + \sum_{t=1}^{24} \rho_{L,t} \times P_{\text{Load}_t} \\ &- \sum_{i=1}^{\text{No}_{\text{DG}}} \sum_{t=1}^{24} \left[r_t \cdot C(P_{\text{DG}_i^t} + R_{\text{DG}_i^t}) + (1 - r_t) \cdot C(P_{\text{DG}_i^t}) \cdot I_t \right] \\ &- \sum_{i=1}^{\text{No}_{\text{DG}}} \sum_{t=1}^{24} \left[\text{SUC}_{\text{DG}_i^t} + \text{SDC}_{\text{DG}_i^t} \right] - \sum_{i=1}^{\text{No}_{\text{Str}}} \sum_{t=1}^{24} C(P_{\text{Str}_i^t}) \\ &- \sum_{i=1}^{\text{No}_{\text{Curt}}} \sum_{t=1}^{24} r_t \cdot C(P_{\text{IL}_i^t} + R_{\text{IL}_i^t}) + (1 - r_t) \cdot C(P_{\text{IL}_i^t}) \cdot L_t \end{aligned}$$

It is worth to mention that if the system operator opts to utilise reserve capacity, VPP should be paid for both energy and reserve services [30].

2.4 Constraints

The constraints of VPP scheduling problem are as follows

· Power balance constraint

$$P_{E_{t}} + R_{t} + P_{\mathrm{DG}_{i}^{t}} + P_{\mathrm{int}_{i}^{t}} + P_{\mathrm{IL}_{i}^{t}} + (P_{\mathrm{ch}_{i}^{t}} - \eta P_{\mathrm{Dch}_{i}^{t}}) - P_{\mathrm{Load}_{t}}$$

$$= \sum_{j=1}^{N} |V_{i,t}| |V_{j,t}| |Y_{jj}| \cos(\delta_{j,t} - \delta_{i,t} + \theta_{ij})$$
(11)

$$i \in S_{b}$$
; $t = 1, 2, ..., 24$

$$Q_{E_{t}} + Q_{\mathrm{DG}_{i}^{t}} + Q_{\mathrm{int}_{i}^{t}} + Q_{\mathrm{IL}_{i}^{t}} - Q_{\mathrm{Load}_{t}}$$

$$= -\sum_{j=1}^{N} |V_{i,t}| |V_{j,t}| |Y_{ij}| \sin(\delta_{j,t} - \delta_{i,t} + \theta_{ij})$$

$$i \in S_{\mathrm{b}}; t = 1, 2, \dots, 24$$
(12)

DG capacity

$$P_{\mathrm{DG}_i}^{\mathrm{min}} \cdot I_i \le (P_{\mathrm{DG}_i} + R_{\mathrm{DG}_i}) \le P_{\mathrm{DG}_i}^{\mathrm{max}} \cdot I_i \tag{13}$$

• Ramp-up for generation units

$$P_{\mathrm{DG}_{i}^{t+1}} - P_{\mathrm{DG}_{i}^{t}} \le R_{i}^{\mathrm{UP}}, \quad i \in S_{\mathrm{b}}, \quad t = 1, 2, \dots, 24$$
 (14)

• Ramp-down for generation units

$$P_{\mathrm{DG}_{i}^{t}} - P_{\mathrm{DG}_{i}^{t+1}} \le R_{i}^{\mathrm{DOWN}}, \quad i \in S_{\mathrm{b}}, \quad t = 1, 2, \dots, 24$$
 (15)

• Minimum up time for generation units

$$[T_{i,t-1}^{\text{on}} - \text{MUT}_i][I_i^t - I_i^{t-1}] \ge 0, \quad i \in S_b,$$

 $t = 1, 2, \dots, 24$ (16)

Table 2 Results of VPP elements behaviour in scenario 1

Hour	Total DG generation, kW	DG generation for energy market, kW	DG generation for reserve market, kW	Charging and discharging of EES, kWh	Interrupted load, kW
1	100	70	30	5	0
2	100	70	30	17	0
3	100	70	30	29	0
4	100	70	30	41	0
5	100	70	30	53	0
6	100	70	30	65	0
7	100	70	30	65	25
8	100	70	30	65	25
9	100	70	30	65	0
10	100	70	30	65	0
11	100	70	30	53	25
12	100	70	30	41	25
13	100	70	30	29	25
14	100	70	30	17	25
15	100	70	30	5	25
16	0	0	0	5	0
17	0	0	0	17	0
18	0	0	0	29	0
19	0	0	0	41	0
20	0	0	0	41	0
21	0	0	0	41	0
22	100	70	30	29	0
23	100	70	30	17	0
24	100	70	30	5	0

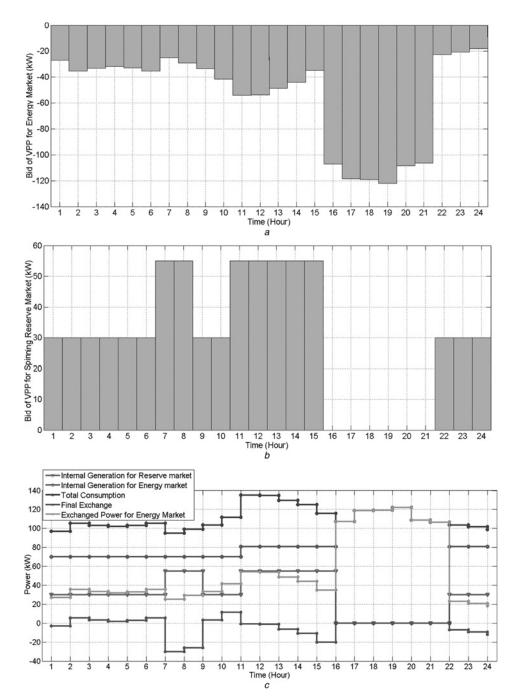


Fig. 3 Results for VPP decision making in scenario 1

- a Bids of VPP for energy market
- b Bids of VPP for spinning reserve market
- c Summary of VPP behaviour
- Minimum down time for generation units

$$[T_{i,t-1}^{\text{off}} - \text{MDT}_i][I_i^t - I_i^{t-1}] \ge 0, \quad i \in S_b,$$

$$t = 1, 2, \dots, 24$$
(17)

• Inhibition of decision interference

These constraints ensure the logical order of binary variables. The first two constraints approve the validity of turning on or shutting down decisions, while the third one inhibits the decision interference

$$\begin{cases} I_{i,t} - I_{i,t-1} \leq J_{i,t} \\ I_{i,t-1} - I_{i,t} \leq K_{i,t} \\ I_{i,t} - I_{i,t-1} \leq J_{i,t} - K_{i,t} \end{cases}$$
 (18)

• IL constraint

This constraint limits the amount of load shed between two extremes

$$0 \le P_{IL_i^t} + R_{IL_i^t} \le P_{IL_i^t}^{Max} \tag{19}$$

• Technical constraint for the flow of power in feeders

This constraint limits the amount of power carried by VPP's feeders

$$S_{ij}^{t}(\theta_{t}, V_{t}) \le S_{ij,t}^{\text{max}}$$
(20)

Table 3 Hourly benefits in scenario 2

Hour	Benefit	Hour	Benefit	Hour	Benefit			
1	-82.0375	9	-28.3327	17	174.8			
2	-155.09	10	174.259	18	176.059			
3	-149.484	11	266.75	19	182.921			
4	-146.43	12	267.81	20	271.266			
5	-148.975	13	286.35	21	265.654			
6	-155.087	14	302.036	22	27.085			
7	48.799	15	334.121	23	107.27			
8	38.612	16	462.407	24	121.02			
Net ber	Net benefit of VPP in scenario 1: 2641.785 monetary unit							

Voltage limitations

$$V_{i,t}^{\min} \le V_{i,t} \le V_{i,t}^{\max}, \quad i \in S_{b}, \quad t = 1, 2, \dots, 24$$
 (21)

Power exchange constraint

$$\left| P_{\text{Sub}_t} \right| \le P_{\text{Sub}_t}^{\text{Max}} \tag{22}$$

System adequacy constraints

VPP's central controller is in charge of scheduling excess capacity of dispatchable DGs, storages, ILs, and exchanged power with the main network to maintain a reserve margin for VPP during each hour

$$\sum_{i=1}^{\text{No}_{\text{DG}}} (P_{\text{DG}_{i,\text{max}}^{t}} - P_{\text{DG}_{i}^{t}} - R_{\text{DG}_{i}^{t}})$$

$$+ \sum_{i=1}^{\text{No}_{\text{IL}}} (P_{\text{IL}_{i,\text{max}}^{t}} - P_{\text{IL}_{i}^{t}} - R_{\text{IL}_{i}^{t}})$$

$$+ \eta \cdot (\text{Cap}_{0} + \sum_{i=1}^{\text{No}_{\text{curt}}} P_{\text{Str}_{i}^{t}}) + E_{\text{Ex},t}^{\text{Max}} - E_{t} - R_{t} \ge AR_{t}$$
(23)

$$P_{\mathrm{DG}_{i,\max}^{l}}^{\mathrm{available}} = \mathrm{Min}(R_{i,t}^{\mathrm{up}} + P_{\mathrm{DG}_{i}^{l-1}}, P_{\mathrm{DG}_{i}^{l}}^{\mathrm{Max}})$$
 (24)

• EES constraints

These constraints are already presented in Section 2, i.e. (2)–(7) are used as EES constraints and (8) is used as EES cost function.

3 Uncertainty modelling

In this study, the energy and reserve prices are assumed to be uncertain parameters, as well as load values. To generate stochastic data, log-normal probability distribution function (PDF) is deployed for price values [31], while load values are generated using normal PDF.

3.1 Scenario generation for price value

The price data which is extracted from [25] comprises both anticipated price values and corresponding standard deviations resulted from price forecast procedure. Having generated 2000 random price scenarios, a 2000 branch fan-type scenario tree is built and reduced to 30 scenarios using backward reduction method afterwards. Each scenario represents three strings for energy, reserve, and retail prices for a 24 h time horizon. Every member of this string is generated using log-normal PDF with a mean value equal to forecasted price and standard deviation as mentioned before.

3.2 Scenario generation for load value

To include load uncertainty, load curve is uniformly scaled with a five step load factor as shown in Table 1.

The probability values assigned to load factors represent a stepwise estimation of normal PDF with a mean value equal to 1 and a standard deviation equal to 0.1.

3.3 Uncertainty augmented objective function

To observe the effect of uncertain parameters on benefit, the expected value of the benefit is maximised

Benefit_{expected} =
$$\sum_{S=1}^{s} P_S \times \text{Benefit}_S$$
 (25)

In case both price and load are deemed as stochastic parameters, the probability of each scenario is the product of corresponding probabilities associated to price and load scenarios. To solve the mixed integer nonlinear programming (MINLP) problem GAMS

 Table 4
 Results of VPP elements behaviour in scenario 2

Hour	Total DG generation, kW	DG generation for energy market, kW	DG generation for reserve market, kW	Charging and discharging of EES, kWh	Interrupted load, kW
1	96.776	66.776	30	5	0
2	96.962	66.962	30	10.314	0
3	96.152	66.152	30	14.36	0
4	98.514	68.514	30	17.549	0
5	98.481	68.481	30	21.714	0
6	98.671	68.671	30	28.206	0
7	99.2	69.2	30	32.597	23.75
8	97.486	67.486	30	37.274	25
9	95.181	65.181	30	42.261	0
10	98.096	68.096	30	44.009	0
11	100	70	30	35.342	23.393
12	100	70	30	27.743	24.238
13	100	70	30	19.154	24.5
14	100	70	30	9.676	24.286
15	100	70	30	5	24.345
16	0	0	0	11.511	0
17	0	0	0	18.408	0
18	0	0	0	24.861	0
19	0	0	0	32.308	0
20	0	0	0	39.223	0
21	0	0	0	41	0
22	100	70	30	29	0
23	100	70	30	17	0
24	100	70	30	5	0

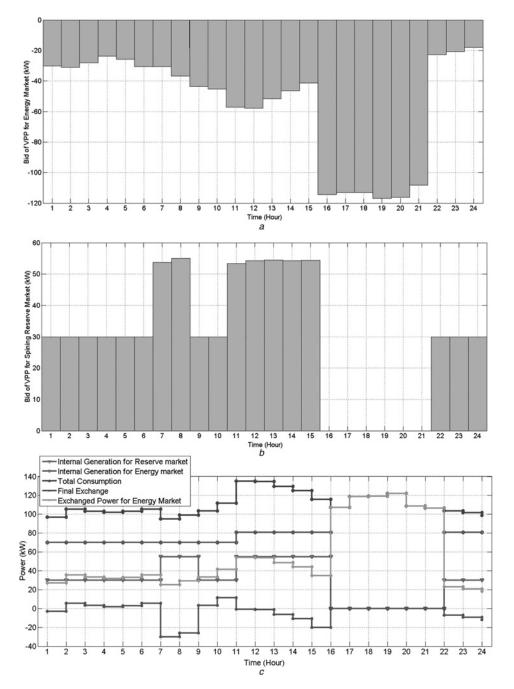


Fig. 4 Results of VPP decision making in scenario 2

- a Bids of VPP for energy market
- b Bids of VPP for spinning reserve market c Summary of VPP behaviour

optimisation software was deployed. The run time, with a 2.5 GHz dual-core computer, was 3 s and particle swarm optimisation heuristic optimisation method was used to validate the results.

Table 5 Hourly benefit in scenario 2

Hour	Benefit	Hour	Benefit	Hour	Benefit
1	-77.36	9	-71.936	17	225.26
2	-86.58	10	186.47	18	210.55
3	-61.29	11	233.95	19	229.44
4	-28.30	12	182.94	20	242.06
5	-51.58	13	252.95	21	261.77
6	-76.30	14	274.45	22	28.81
7	-2.45	15	238.93	23	121.36
8	-16.78	16	432.72	24	136.72
Net bene	efit of VPP in so	enario 2: 27	85.691 monetar	y unit	

Simulation result and discussion

Fig. 2a shows the topology of the network used for this study, which is obtained from [27] and augmented with a PV unit for scenario 4. Figs. 2b and c show the forecasted day-ahead load and energy, reserve, and retail prices, respectively. In this study, we intend to investigate the performance of VPP in energy and reserve markets simultaneously. First, we appraise the VPP's performance in energy and spinning reserve markets, regardless of uncertainties in price and load values. In the second scenario, solely the price values are deemed as uncertain parameters and the load values are assumed to be deterministic. The third scenario considers both price and load values as uncertain parameters. Finally, in the fourth scenario, the PV unit is added to scenario 1. It is worth to mention that, while for scenario 4 the PV unit is incorporated, for scenarios 1, 2, and 3 the mentioned element has been neglected.

4.1 Scenario 1: deterministic prices and loads

The first scenario discusses VPP's behaviour in energy and reserve markets regarding price and load values as deterministic parameters. Table 2 indicates DG unit's performance in energy and reserve markets in scenario 1. The second column shows total DG generation. The portion of DG generation allotted to the energy market and reserve market are shown in third and fourth columns of this table, respectively.

Here, we assume that the probability of reserve deployment equals to 1. Also the price of both energy and reserve services are paid in case a generation unit's reserve capacity is requested. Accordingly, DG unit prefers to offer maximum available reserve capacity to attain maximum benefit. Since the reserve capacity should be available in 10 min and the ramp rate of DG is 3 kW/min, this unit is capable of offering 30 kW in reserve market.

The fifth column of Table 2 shows the state of EES. Considering the capacity and charge rate of EES, it is charged with maximum charge rate within hours 1–6, when the price of energy in energy market is low enough (11.5 monetary unit/kWh) and providing energy from upstream network is justifiable. However, DG unit still have a good share in providing energy for EES because of its low cost (9.5 monetary unit/kWh). It is worth to mention that, to examine the effect of charge rate in VPP's benefit, a 10% increase in the value of charge and discharge rates was exerted which resulted in more benefit and a similar charge and discharge pattern. In the sixth column of Table 2, the ILs states are shown. Within hours 7-8 and 11-15, the ILs are shed by 25 kW. The reason behind is that load shedding allows the VPP to release the capacity of DG in order to offer in energy market, which results in more benefit. Figs. 3a and b show the VPP's offers in energy and reserve markets, respectively. It is obvious that VPP purchases energy from energy market all the time. Since the probability of reserve request is assumed to be 1, during hours 1-6, 9-10, and 22-24, when it is beneficial for DG to participate in reserve market, this unit allots as much capacity as possible to reserve service and fails to meet the internal loads requirements. As a result, the VPP is obliged to purchase energy from energy market to supply its load. Fig. 3c briefly reports the VPP's performance in energy and reserve markets, and shows how the VPP exchanges energy with upstream network and how it supplies its internal consumers. Finally, in Table 3 VPP's benefits for a 24 h time horizon is presented which shows that to acquire maximum benefit the VPP does not necessarily need to have positive income values for all time intervals, and sometimes it attains maximum benefit in expense of negative benefit values in some hours.

4.2 Scenario 2: stochastic prices and deterministic loads

In this part, we aim to investigate price uncertainties in VPP's performance in energy and reserve markets. As the main aspects of results are illustrated in base scenario (scenario 1), only the deviations of these results, engendered by price uncertainty, are considered in the following part. Table 4 indicates DG unit's performance in energy and reserve markets in scenario 2.

The second, third, and fourth columns of table show DG's performance in energy and reserve markets. As it can be seen, notwithstanding the price uncertainty, the value of reserve capacity remains unchanged at its highest possible value, because of its high potential benefit. On the other hand, the value of capacity allotted to energy market undergoes some changes, and during hours 1-11 experiences a lower value in comparison with scenario 1. The state of EES, affected by price uncertainty, is shown in fifth column. In comparison with Table 2, main changes are observable. Facing uncertainty in price values, the EES is charged with less charge rate and reaches a peak value equal to 44 kWh which is less than its maximum charge capacity reached in scenario 1. The key point to notice is that charge and discharge intervals are the same in both scenarios. Furthermore, less peak charge value has repercussions on DG generation and exchanged power with upstream network.

The sixth column shows the state of ILs. In comparison with Table 2, there are slight changes in the amount of shed load, which occurs exactly within hours it occurred in scenario 1. In Figs. 4a and b capacity offered by VPP in energy and reserve markets are shown, respectively. As it can be seen, the VPP purchases energy from the upstream network all the time; however, the amount of the purchased power demonstrates a slight decrease as compared with scenario 1, the underlying reason for which is the decrease in the charges and discharges of the EES. In comparison with Fig. 3a, Fig.4b demonstrates slight changes in capacity offered by VPP in reserve market, which is the result of variations in shed load. Fig. 4c briefly reports the VPP's performance in energy and reserve markets in this scenario, and shows how the VPP exchanges energy with upstream network and how it supplies its internal consumers. Finally, Table 5 represents the hourly benefits of VPP in the present scenario.

4.3 Scenario 3: stochastic loads and prices

In this scenario, both loads and prices are considered to be stochastic parameters. Table 6 indicates DG unit's performance in energy and

 Table 6
 Results of VPP elements behaviour in scenario 3

Hour	Total DG generation, kW	DG generation for energy market, kW	DG generation for reserve market, kW	Charging and discharging of EES, kWh	Interrupted load, kW
1	96.790	66.790	30	5	0
2	97.004	67.004	30	10.3143	0
3	96.152	66.152	30	14.360	0
4	98.514	68.514	30	17.549	0
5	98.482	68.482	30	21.787	0
6	98.673	68.673	30	28.272	0
7	99.202	69.202	30	32.665	23.725
8	97.708	67.708	30	37.402	24.999
9	95.390	65.406	29.984	42.449	0
10	98.208	68.208	30	44.220	0
11	100	70.409	29.591	35.560	23.393
12	100	70.620	29.380	27.956	24.188
13	100	70.004	29.996	19.294	24.499
14	100	70	30	9.791	24.248
15	100	70	30	5	24.328
16	2	2	0	11.418	0
17	2.0004	2.0004	0	18.533	0
18	2	2	0	25.154	0
19	2.16	2.059	0.102	32.334	0
20	2	2	0	39.093	0
21	2	2	0	40.999	0
22	100	70	30	29	0
23	100	70	30	17	0
24	100	70	30	5	0

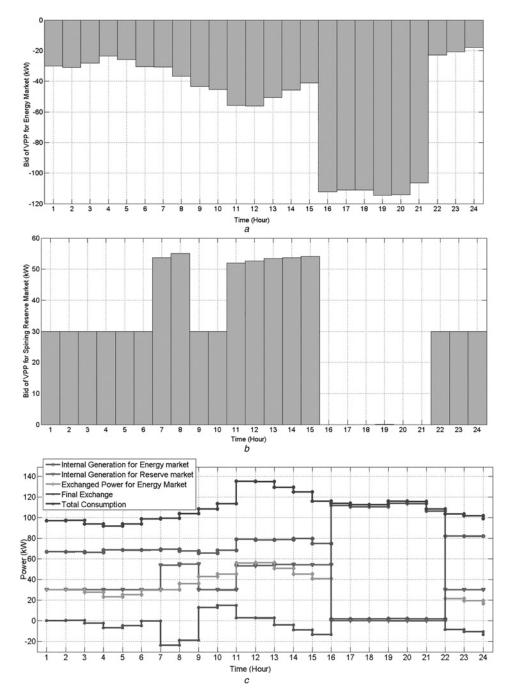


Fig. 5 Results of VPP decision making in scenario 3

- a Bids of VPP for energy market
- b Bids of VPP for spinning reserve market c Summary of VPP behaviour

reserve markets in this scenario. The second, third, and fourth columns depict DG's generation for energy and reserve markets.

Table 7 Hourly benefits in scenario 3

Hour	Benefit	Hour	Benefit	Hour	Benefit			
1	-77.358	9	-72.750	17	219.097			
2	-86.585	10	186.164	18	204.998			
3	-61.289	11	233.181	19	226.712			
4	-28.330	12	182.132	20	238.324			
5	-52.589	13	253.370	21	256.330			
6	-76.219	14	274.436	22	35.798			
7	-2.470	15	240.289	23	121.356			
8	-17.590	16	430.477	24	136.720			
Net ben	Net benefit of VPP in scenario 3: 2764.207 monetary unit							

In the present scenario, in comparison with scenario 1 (Table 2), DG's generation for reserve market demonstrates slight variations which can be construed as the result of coincidence of uncertain loads and uncertain market prices culminated in decrease in reserve capacity.

The fifth column shows the storage state of charge, which shows remarkable changes in comparison with Table 2, analogous to scenario 2, resulted from uncertain loads and prices. The sixth column shows the state of ILs. It can be inferred from discrepancies between Tables 6 and 4 that in case which both loads and prices are stochastic parameters, results are more volatile comparing to deterministic prices and stochastic loads. However, in aggregate, comparing Tables 6 and 2 shows that the ILs are not highly sensitive to these uncertain parameters. In Figs. 5a and b, VPP's bidding is presented for energy and reserve markets,

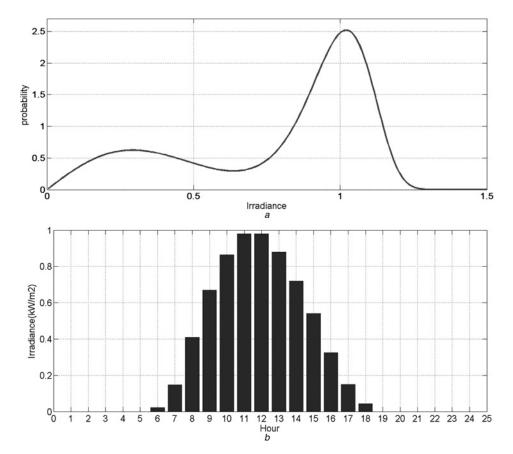


Fig. 6 *PV unit data a* PDF of sunlight irradiance *b* Hourly sunlight irradiance

respectively. In comparison with Fig. 4a, Fig.5a shows a slight decrease in exchanged energy with upstream network, ensued from uncertain loads and prices. This issue leads to changes in VPP elements behaviour, and changes in exchanged power, consequently. Fig. 5c briefly describes the VPP's condition in scenario 3, and shows how the VPP exchanges the energy with upstream network, and how it supplies internal consumers, considering both energy and reserve markets. In Table 7, the hourly benefits of VPP corresponding to scenario 3 for a 24 h time horizon are presented. The results show the benefit for scenario 3 is higher than that for scenario 1, while it is lower than the benefit of scenario 2, which insinuates that the results are more realistic in case both prices and loads are considered to be stochastic parameters.

4.4 Scenario 4: VPP's strategy considering PV integration

In the present scenario, a PV unit has been integrated into bus 4. The hourly distribution of sunlight irradiance, since the output power of the PV unit depends on the value of this parameter, has been modelled using a bimodal distribution, a combination of two Weibull distributions as follows

$$f(G) = \omega \left(\frac{k_1}{C_1}\right) \left(\frac{G}{C_1}\right)^{(k_1 - 1)} e^{-(G/C_1)^{K_1}} + (1 - \omega) \left(\frac{k_2}{C_2}\right) \left(\frac{G}{C_2}\right)^{(k_2 - 1)} e^{-(G/C_2)^{K_2}}$$
(26)

The corresponding values for the parameters of bimodal distribution are as follows

$$\omega = 0.3;$$
 $k_1 = 2;$ $k_2 = 10;$ $C_1 = 0.4C_2;$ $C_2 = \frac{G_{\text{mean}}}{\Gamma(1 + 1/K_2)}$

Fig. 6a shows the PDF of sunlight irradiance (G), while Fig. 6b delineates the value of sunlight irradiance for a 24 h time horizon.

The output power of PV unit is obtained substituting the value of sunlight irradiance in the following equation

$$P_{\text{out}} = \eta^{\text{PV}} S^{\text{PV}} G \tag{27}$$

To begin with, the bidding problem of the VPP has been investigated under the assumption that the output power of the PV unit is deterministic. However, because of the intermittent nature of the sunlight irradiance, the next step posits the output power of the PV unit to be uncertain and a stochastic model has been proposed. In order to include the PV's intermittency effect on VPP's bidding strategy, the Monte Carlo method has been deployed.

Simulation results show that DG generation, EES behaviour and ILs behaviour are all the same as scenario 1. The reason that the DG generation, as well as EES and ILs behaviour have remained unchanged is, since the operational cost of the PV unit is considered to be zero, the power generated by this unit is consumed by the internal consumers, which results in the aforementioned condition. In other words, during the hours that the PV unit delivers the power to the grid, it can be treated as a negative load for the VPP.

Fig. 7a shows the VPP's bids in the energy market for a 24 h time period, which confirms the notion of the PV unit being considered as a negative load. In fact the amount of decrease in

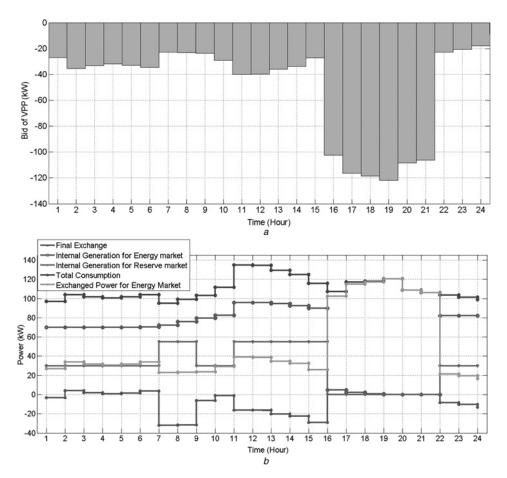


Fig. 7 Results of VPP decision making in scenario 4 a Bids of VPP for energy market

b Summary of VPP behaviour

Table 8 Hourly benefits in scenario 4

Hour	Benefit	Hour	Benefit	Hour	Benefit
1	-82.037	9	-4.212	17	169.435
2	-155.087	10	180.488	18	174.475
3	-149.484	11	316.139	19	182.921
4	-146.429	12	317.206	20	271.266
5	-148.974	13	330.702	21	265.654
6	-153.215	14	338.325	22	27.085
7	54.091	15	361.338	23	107.27
8	53.372	16	441.347	24	121.02
Net ben	efit of VPP in sce	nario 4: 287	2.695 monetar	y unit	

the present bids, in comparison with the scenario 1, is exactly equal to the PV unit's output power. Fig. 7b shows the power interchange for the VPP in the presence of PV unit. The first curve represents the total consumption of the VPP, which equals to the summation of the consumers' demand and the power

supplied to charge the EES. The second one shows the power offered by the VPP in energy market which comprises a share of the power generated by dispatchable units, the PV unit, and the storages. The third curve corresponds to the capacity allotted to the reserve market procured by the DG unit and the ILs. The fourth curve, which can be obtained from subtracting the second curve from the first one, and the fifth one, which similarly is obtainable through subtracting the third curve from the fourth one, are the power exchange for the energy market and the final exchange, respectively. It should be noted that the final exchange represents the resultant power exchanges of the VPP with the main grid with both energy and spinning reserve markets taken into account.

In Table 8, the hourly benefits of VPP corresponding to scenario 4 for a 24 h time horizon is presented. Comparing Table 3 and the results presented in Table 8, it can be noticed that the revenue of the VPP has increased during the hours the PV unit generates power, and the total revenue shows 8.75% increase in comparison with the corresponding scenario without PV.

Table 9 Bids of VPP for power market

Hour	Mean	Standard deviation	Hour	Mean	Standard deviation	Hour	Mean	Standard deviation
1	815.26	0	9	23.685	3.038	17	116.457	0.499
2	35.361	0	10	29.025	3.940	18	118.478	0.0267
3	33.120	0	11	39.995	4.495	19	121.861	0
4	31.898	0	12	39.969	4.495	20	108.528	0
5	32.916	0	13	35.834	4.036	21	106.287	0
6	34.612	0.058	14	33.656	3.280	22	22.737	0
7	22.864	0.488	15	27.081	2.422	23	20.700	0
8	23.152	1.780	16	102.524	1.388	24	17.95	0

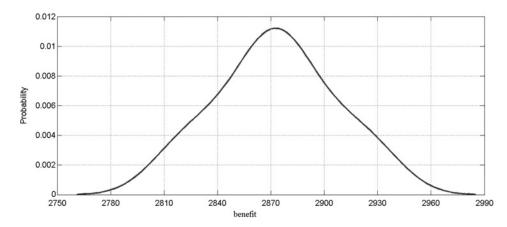


Fig. 8 PDF of the VPP benefit

In Table 9, the hourly bids of VPP for energy market and associated standard deviation when PV unit assumed as an uncertain source of power generation are presented.

As it is manifest, for every hour, there is a negligible difference between the average value of the offered power, and the power offered using the anticipated sunlight irradiance. On the other hand, during the hours that the PV unit is capable of generating power, the offered power demonstrates high values of standard deviation, which is depicted in Table 9.

Finally, Fig. 8 shows the PDF of the VPP, with the average value and the standard deviation equal to 2872.95 and 31.966, respectively. Obviously the effect of the uncertainty in PV unit's output is far less than that for the uncertain loads and prices, which can be attributed to limited generating hours and smaller share of power.

5 Conclusion

In this study, the aggregation of DER in the form of a VPP and the bidding strategy of this participant in the joint energy and reserve markets is proposed, considering almost all the technical constraints and uncertain behaviour of the prices, loads, and a non-dispatchable DG unit. The results show that the VPP has a dual role and regarding to the market circumstances and uncertainty effects, it may have bilateral transactions with the upstream network. VPP may bid a part of its capacity in reserve market to be purchased by the upstream network meanwhile it is purchasing energy from it, because trading in reserve market is more beneficial. Also, as the results demonstrated, uncertain parameters are very important in VPP's bidding strategy and neglecting them would lead to deviation from optimal scheduling. Furthermore, as it can be inferred from results, the bidding strategy of VPP is more sensitive to price uncertainties than it is to load uncertainties, even though the standard deviation of the prices is considered to be less than that for load values. Hence, considering this issue in VPP scheduling is highly recommended in order to achieve more realistic results. Finally, adding a PV unit to the portfolio of the VPP, the important effects of this agent on VPP's bidding and benefit are observable.

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