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Time-varying sliding-coefficient-based decoupled terminal sliding-mode control for a class of fourth-order systems

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ABSTRACT

A time-varying sliding-coefficient-based decoupled terminal sliding mode control strategy is presented for a class of fourth-order systems. First, the fourth-order system is decoupled into two second-order subsystems. The sliding surface of each subsystem was designed by utilizing time-varying coefficients. Then, the control target of one subsystem to another subsystem was embedded. Thereafter, a terminal sliding mode control method was utilized to make both subsystems converge to their equilibrium points in finite time. The simulation results on the inverted pendulum system demonstrate that the proposed method exhibits a considerable improvement in terms of a faster dynamic response and lower IAE and ITAE values as compared with the existing decoupled control methods.

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1. Introduction

Sliding mode control (SMC) has received much attention due to its major advantages such as guaranteed stability, robustness against parameter variations, fast dynamic response and simplicity in implementation and therefore has been widely applied to the control of a class of nonlinear systems [1,2]. The design of a SMC consists of two steps: design of a sliding surface and design of a control law. Once a suitable sliding surface function and a suitable control law are designed, the system states can be forced to move towards the sliding surface and slide on the surface until the equilibrium (origin) point is reached. In most SMC schemes, the most commonly used sliding surface is the linear sliding surface which is based on linear combination of the system states by using an appropriate time-invariant coefficient (commonly termed as sliding coefficient). Although this coefficient can be adjusted such that the convergence rate is arbitrarily fast, the system states cannot reach the equilibrium point in finite time [3].

In order to achieve finite time convergence of the system states, a terminal sliding mode control (TSMC) approach has been firstly proposed [4]. The TSMC approach has been developed further for the control of a simple second-order nonlinear system and *n*th-order nonlinear rigid robotic manipulator system which results zero tracking error in finite time [5,6]. Thereafter, the TSMC method has been widely applied to many practical areas [3,7,8]. However, the TSMC approaches proposed in all of these studies are only suitable for second-order systems. The performance of these TSMC approaches for a class of fourth-order system is questionable. For example, in a cart–pole system controlled by an existing TSMC method, either pole or cart can be controlled successfully, but not both. A remedy to this problem is to decouple the system states and apply a suitable control law to stabilize the whole system. Recently, a decoupled sliding mode control (DSMC) has been developed to cope with this issue [9]. It provides a simple way to decouple a class of fourth-order nonlinear systems into two second-order subsystems such that each subsystem has a separate control objective expressed in terms of a sliding surface. An important consequence of using the DSMC method is that the second subsystem is successfully embedded into the first one via a two-level decoupling strategy. An alternative decoupling approach termed as single-input decoupled fuzzy-logic control (SIDFLC) is also proposed as an improved version of the DSMC method [10]. The SIDFLC method embeds the signed distance concept [11] together with the single-input fuzzy-logic control (SIFLC) and the DSMC methods. The results have shown a considerable improvement of the SIDFLC method in terms of a reduction in the number of fuzzy rules and a faster dynamic response as compared with the DSMC method. However, the time-invariant coefficient based sliding surface still suffers from the slow convergence speed. A self-tuning signed-distance fuzzy sliding mode control (DSSFSMC) and a decoupled sliding-mode control based on neural network methods are proposed to improve the convergence speed [12,13]. However, these methods are based on complicated algorithms which increase the complexity of the controller design. Recently,

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a new DSMC method based on time-varying sliding surface slope (TVSSS) has been developed [14]. The time-varying sliding surface function of each subsystem is composed of the corresponding subsystem states and a time-varying sliding coefficient which is computed by a linear function derived from the input–output mapping of the one-dimensional fuzzy rule base. Despite the faster convergence rate obtained by this method, the system states still cannot converge to the equilibrium point in finite time. Although the DSMC method presented in [15] makes use of a multi-objective particle swarm optimization (MOPSO) algorithm for optimizing the controller parameters in order to achieve a better performance, again the states cannot converge to the equilibrium point in finite time.

In this paper, a time-varying sliding-coefficient-based decoupled terminal sliding mode (DTSMC) control is proposed for a class of fourth-order systems ensuring finite time convergence of the system states. The system under consideration was firstly decoupled into two second-order subsystems. Then a separate sliding surface function was defined for each subsystem using the idea in [9]. Afterwards, the idea of TSMC has been applied to each subsystem separately to ensure that the states of both subsystems convergence to their equilibrium points in finite time. Since each subsystem has its own equilibrium point, then their convergence times are different. When the finite time expressions of these subsystems are solved analytically, it is seen that each expression is a function of an initial value of the corresponding state and a sliding coefficient appearing in the denominator of each expression. This means that the dynamic characteristics of the system can be changed by adjusting these sliding coefficients. Therefore, we propose to tune these coefficients based on time-varying basis so as to minimize the convergence time of each subsystem. The proposed DTSMC strategy is applied to control a cart–pole system. Simulations are carried out and the results are compared with the TVSSS and DSMC methods.

The rest of this paper is divided into six sections. In Section 2, the problem formulation is described for fourth-order systems. In Section 3, review of decoupled sliding mode control strategy is given. In Section 4, the conventional terminal sliding mode control was described for a second-order system. Then, it has been extended to a class of fourth-order system. In Section 5, the stability analysis of the proposed method is derived. In Section 6, the proposed method is used to control a cart–pole system and the computer simulation results are presented and compared with the existing decoupled methods. Finally, the conclusions are addressed in Section 7.

2. Problem formulation

Consider a second-order nonlinear system represented by the following canonical state-space form

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= f(\mathbf{x},t)+b(\mathbf{x},t)u(t)+d(t)\end{aligned} \tag{1}$$

where $\mathbf{x}=[x_1,x_2]^T$ is the state vector, $f(\mathbf{x},t)$ and $b(\mathbf{x},t)$ are nonlinear functions representing system dynamics, $u(t)$ is the control input, and $d(t)$ represents the external disturbance. The control of such systems can be easily achieved by using well known SMC [1] or any other nonlinear control method. However, the dynamic representation of such systems is generally not in a canonical form exactly. For example, the dynamic representation of a cart–pole (single-inverted pendulum) system appears in the following form [9]:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= f_1(\mathbf{x},t)+b_1(\mathbf{x},t)u_1(t)+d_1(t)\\ \dot{x}_3(t) &= x_4(t)\\ \dot{x}_4(t) &= f_2(\mathbf{x},t)+b_2(\mathbf{x},t)u_2(t)+d_2(t)\end{aligned} \tag{2}$$

where $\mathbf{x}=[x_1,x_2,x_3,x_4]^T$ is the state vector, $f_1(\mathbf{x},t)$, $b_1(\mathbf{x},t)$, $f_2(\mathbf{x},t)$, and $b_2(\mathbf{x},t)$ are nonlinear functions representing system dynamics, $u_1(t)$, $u_2(t)$ are the control inputs, and $d_1(t)$ and $d_2(t)$ represent external disturbances. The disturbances are assumed to be bounded as $|d_1(t)| \le D_1(t)$ and $|d_2(t)| \le D_2(t)$. The control objective is to design a control strategy that would force the states (x_1, x_2, x_3 and x_4) to the origin of the state-space ($\mathbf{x}=[0,0,0,0]^T$). However, if $u_1(t)$ is used to control the system in (2), only state x_1 and x_2 will tend to zero. On the other hand, if $u_2(t)$ is used to control the same system, only state x_3 and x_4 will move to zero. This means that with these control inputs either the pole or the cart can be controlled, but not both. The main reason behind this control failure is due to the coupled nature of the system. Therefore, in order to deal with the coupling problem, a control strategy that decouples the system states and controls the whole system using a single control input ($u(t)=u_1(t)=u_2(t)$) is needed.

3. Review of decoupled sliding mode control

A decoupled control method based on SMC (DSMC) was firstly proposed to solve the decoupling problem in the fourth-order nonlinear systems [9]. It provides a simple way to decouple a class of fourth-order nonlinear systems into two second-order subsystems such that each subsystem has a separate control objective expressed in terms of a sliding surface. The main idea behind the decoupled strategy is to decouple a nonlinear system appearing in the form of (2) into two subsystems as A and B in the form of (1). The subsystem A is chosen as a primary target while the subsystem B is used as a secondary target. However, the selection of the primary and the secondary subsystems is problem dependent. Here, the control objective is to devise a control strategy that would move the states of both subsystems towards their sliding surfaces $S_1=0$ and $S_2=0$, and eventually, converge to the points $[x_1,x_2]^T=[0,0]^T$ and $[x_3,x_4]^T=[0,0]^T$, respectively. The subsystem A involves knowledge from subsystem B if the sliding surface function S_1 is defined as

$$S_1=\lambda_1(x_1-z)+x_2, \quad \lambda_1>0 \tag{3}$$

where λ_1 is the time-invariant coefficient of the sliding surface function. In phase plane, $S_1=0$ represents a line (commonly called sliding line) passing through the points $x_1=z$ and $x_2=0$ with a slope equal to $-\lambda_1$. The intermediate signal represented by z is defined as

$$z=sat\left(\frac{S_2}{\Phi_Z}\right)z_U, \quad 0<z_U<1 \tag{4}$$

In (4), Φ_Z is the boundary layer of S_2 used to smooth z and S_2 is the sliding surface function for subsystem B defined as

$$S_2=\lambda_2x_3+x_4, \quad \lambda_2>0 \tag{5}$$

It is important to note here that the control objective for the subsystem A is changed from $[x_1,x_2]^T=[0,0]^T$ to $[x_1,x_2]^T=[z,0]^T$. It is clear from (4) that z is a function of S_2 transforming S_2 to the proper range of x_1. This means that the sliding-mode condition $S_2=0$ for the subsystem B is embedded into S_1 through the intermediate signal z. Hence, the control input that controls both subsystems simultaneously can be obtained easily (see Section 5).

Remark 1. Since z is a function of S_2, and $0<z_U<1$, z can be considered as a bounded oscillatory signal decaying to zero [14].

In order to verify this, we start our analysis by taking the time derivative of (3) and equating it to zero as follows:

$$\dot{S}_1=\lambda_1(\dot{x}_1-\dot{z})+\dot{x}_2=0 \tag{6}$$

Obviously, (6) is a first-order linear differential equation which can be rewritten as

$$\lambda_1x_2+\dot{x}_2=\lambda_1\dot{z} \tag{7}$$

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The general solution of $x_2(t)$ is given by

$$x_2(t) = x_2(0)e^{-\lambda_1 t} + \lambda_1 \int_0^t e^{-\lambda_1(t-\tau)}\dot{z}(\tau)d\tau \tag{8}$$

where $x_2(0)$ is the initial value of $x_2(t)$. It is clear that if $\dot{z}(t)$ is known, then $x_2(t)$ can be determined easily. As mentioned before, the control objective is to achieve $x_1 = z(t)$ and $x_2 = 0$ in the steady-state. In order to achieve this objective, the second term in (8) must converge to zero $(\int_0^t e^{-\lambda_1(t-\tau)}\dot{z}(\tau)d\tau \to 0)$ which is possible if and only if $z(t)$ decays to zero.

4. Terminal sliding mode based control strategy

4.1. Conventional TSMC for a second-order system

Let a nonlinear sliding surface function for the second-order system given (1) be defined as

$$S = \lambda x_1^{\gamma} + \dot{x}_1 \tag{9}$$

where $\lambda > 0$, and $0 < (\gamma = q/p) < 1$ where p and q are positive odd integers satisfying $p > q$. When the system is in the terminal sliding mode $(S = 0)$, its dynamics can be determined by the following nonlinear differential equation:

$$\frac{dx_1}{dt} = \dot{x}_1 = -\lambda x_1^{\gamma} \tag{10}$$

It has been shown that $x_1 = 0$ is the terminal attractor [4] of the system defined in (1). The equation in (10) can also be written as

$$dt = -\frac{dx_1}{\lambda x_1^{\gamma}} \tag{11}$$

Taking integral of both sides of (11) and evaluating the resulting equation on the closed interval $x_1(0) \neq 0, x_1(t_s) = 0)$ gives the following equation [6]:

$$t_s = \frac{|x_1(0)|^{1-\gamma}}{\lambda(1-\gamma)} \tag{12}$$

Eq. (12) means that when the system enters to the terminal sliding mode at $t = t_r$ with initial condition $x_1(0) \neq 0$, the system state x_1 converges to $x_1(t_s) = 0$ in finite time and stay there for $t \geq t_s$. In other words, when the state trajectory hits the sliding surface at time t_r, the system state cannot leave the sliding line meaning that the state trajectory will belong to the sliding line for $t \geq t_r$. However, for a class of fourth-order systems with strong coupling, when some of the states reach zero in a finite time, they will not have any effect on other states.

4.2. Decoupled TSMC for a class of fourth-order system

The idea of conventional second-order TSMC can be extended to the control problem of a class of fourth-order system shown in (2). Now, let us define two nonlinear sliding surfaces in the following form:

$$S_1 = \lambda_1(x_1 - z)^{\gamma_1} + \dot{x}_1 \tag{13}$$

$$S_2 = \lambda_2 x_3^{\gamma_2} + \dot{x}_3 \tag{14}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$, $0 < (\gamma_1 = q_1/p_1) < 1$, $0 < (\gamma_2 = q_2/p_2) < 1$, p_1, p_2, q_1 and q_2 are positive odd integers satisfying $p_1 > q_1$ and $p_2 > q_2$. When the system is in the terminal sliding mode ($S_1 = 0$ and $S_2 = 0$), its dynamics can be determined by the following nonlinear differential equations

$$\dot{x}_1 = -\lambda_1(x_1 - z)^{\gamma_1} \tag{15}$$

$$\dot{x}_3 = -\lambda_2 x_3^{\gamma_2} \tag{16}$$

It should be noted the solution of x_2 given in (8) is not valid here. It is quite difficult to ascertain an analytical solution for x_2 as $\dot{S}_1 = \lambda_1\gamma_1(x_1 - z)^{\gamma_1 - 1}(\dot{x}_1 - \dot{z}) + \dot{x}_2 = 0$ is complicated. Nevertheless, it is clear from (15) that the steady-state solution of x_2 can be written as $x_2 = -\lambda_1(x_1 - z)^{\gamma_1}$. Hence, it is obvious that $x_2 \to 0$ if and only if $(x_1 - z) \to 0$. Since, $x_1 = 0$ in the steady-state, then it can be concluded that $z = 0$ in the steady-state. On the other hand, it should be noted that $x_1 = z$ and $x_3 = 0$ are the equilibrium points of (15) and (16), respectively. Eqs. (15) and (16) can also be written as

$$dt = -\frac{dx_1}{\lambda_1(x_1 - z)^{\gamma_1}} \tag{17}$$

$$dt = -\frac{dx_3}{\lambda_2 x_3^{\gamma_2}} \tag{18}$$

Now, the total sliding time of subsystem A from initial point $xz(0) \neq 0$ to equilibrium point $xz(t_{s1}) = 0$ can be obtained by taking integral of both sides of (17) as follows:

$$t_{s1} = -\frac{1}{\lambda_1}\int_{xz(0)}^{0} \frac{dx_1}{(x_1 - z)^{\gamma_1}} = -\frac{1}{\lambda_1}\int_{xz(0)}^{0} (x_1 - z)^{-\gamma_1} dx_1 = -\left.\frac{(x_1 - z)^{1-\gamma_1}}{\lambda_1(1-\gamma_1)}\right|_{xz(0)}^{0} \tag{19}$$

where $xz(0)$ is the initial value of $x_1 - z$ at the beginning of the sliding mode. Evaluating (19) gives

$$t_{s1} = 0 - \left[-\frac{xz(0)^{1-\gamma_1}}{\lambda_1(1-\gamma_1)}\right] = \frac{|xz(0)|^{1-\gamma_1}}{\lambda_1(1-\gamma_1)} \tag{20}$$

In (20), absolute value of $xz(0)$ is included intentionally in order to avoid a negative t_{s1} when $xz(0)$ is negative. Similarly, the total sliding time of subsystem B from initial point $x_3(0) \neq 0$ to equilibrium point $x_3(t_{s3}) = 0$ can be obtained as

$$t_{s3} = \frac{|x_3(0)|^{1-\gamma_2}}{\lambda_2(1-\gamma_2)} \tag{21}$$

where $x_3(0)$ is the initial value of x_3 at the beginning of the sliding mode.

The stability of (16) can easily be proved by using a Lyapunov function $V = (1/2)x_3^2$. Taking time derivative of V and making use of (16) yields

$$\dot{V} = x_3\dot{x}_3 = -\lambda_2 x_3^{(p_2+q_2)/p_2} \tag{22}$$

Clearly, since $(p_2 + q_2)$ is even and $\lambda_2 > 0$, then $\dot{V}$ is always negative definite meaning that $x_3 = 0$ is terminally stable. On the other hand, since (15) is a function of z, it is quite difficult to find a Lyapunov function of (15) and ascertain the stability of (15). Therefore, the stability of (15) and (16) can be proved by using the Jacobian evaluated around the equilibrium points $x_1 = z$ and $x_3 = 0$, respectively.

The Jacobian of (15) around $x_1 = z$ can be written as

$$J_1 = \frac{\partial \dot{x}_1}{\partial x_1} = -\lambda_1\gamma_1(x_1 - z)^{\gamma_1 - 1} = -\frac{\lambda_1\gamma_1}{(x_1 - z)^{1-\gamma_1}} \tag{23}$$

where J_1 is the eigenvalue of the first order approximation matrix for scalar x_1. One can easily see that $J_1 \to -\infty$ as $x_1 \to z$ which shows that at the equilibrium point the eigenvalue tends to negative infinity. The trajectory of subsystem with infinitely negative eigenvalue will converge to the equilibrium point $x_1 = z$ with an infinitely large speed in finite time.

Similarly, the Jacobian of (16) around $x_3 = 0$ can be written as

$$J_3 = \frac{\partial \dot{x}_3}{\partial x_3} = -\lambda_2\gamma_2 x_3^{\gamma_2 - 1} = -\frac{\lambda_2\gamma_2}{x_3^{1-\gamma_2}} \tag{24}$$

where J_3 is the eigenvalue of the first order approximation matrix for scalar x_3. Again, one can easily see that $J_3 \to -\infty$ as $x_3 \to 0$ which shows that the eigenvalue tends to negative infinity.

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The trajectory of subsystem with infinitely negative eigenvalue will converge to the equilibrium point $x_1 = z$ with an infinitely large speed in finite time. Therefore, it can be concluded that both (15) and (16) are stable.

It is obvious from (20) and (21) that the time to reach the equilibrium is related to the coefficients λ_1, λ_2, γ_1 and γ_2. Since $\gamma_1 = q_1/p_1$ and $\gamma_2 = q_2/p_2$ should have predefined fixed values satisfying the fraction of two odd integers, then λ_1 and λ_2 can be adjusted in such a way that the time taken to reach the equilibrium is minimized for both states. In order to investigate the influence of λ_1 on the convergence rate of state x_1, a sample study has been carried out by computing t_{s1} using (20). In each computation step, the values of $xz(0)$ and γ_1 were kept constant, and the value of λ_1 was gradually changed. Fig. 1 shows the computed convergence time (t_{s1}) for different λ_1 values. It is evident that the convergence time of state x_1 becomes shorter for large values of λ_1. Therefore, it is most desirable if the sliding surface coefficients (λ_1 and λ_2) could be tuned so as to shorten the convergence time of the system states.

Inspired from the TVSSS method presented in [14], a decoupled TSMC strategy using time-varying coefficients is proposed for a class of fourth-order system. The time-varying coefficient computations are performed by the following first-order linear functions derived from the input–output mapping of the one-dimensional fuzzy rule bases [14]

$$\overline{\lambda}_1^L(t) = -0.9X_D^A(t)+1 \tag{25}$$

$$\overline{\lambda}_1^L(t) = -0.9X_D^A(t)+1 \tag{26}$$

where $X_D^A(t) = |X_1| - |X_2|$ and $X_D^B(t) = |X_3| - |X_4|$ are single-input variables of the subsystems A and B, respectively. The variables $X_1 = G_3x_1$, $X_2 = G_4x_2$, $X_3 = G_6x_3$, and $X_4 = G_7x_4$ denote the scaled versions of x_1, x_2, x_3, and x_4, respectively. The block diagram of the proposed approach is shown in Fig. 2. Clearly, the dynamic performance of the proposed controller depends on the values of K, q_1, q_2, p_1, and p_2. The optimum values of these parameters resulting in low integral absolute error (IAE) value for x_1 and x_3 can be determined by the help of a program written in Matlab. This program which contains five nested loops for these parameters works interactively with the Simulink model of the system. In this program, an initial and a final value have been defined for each parameter in each loop. The Simulink model gets the value of each parameter from the program for each run and computes an IAE value. This operation is repeated until all values defined in the program are entered into the Simulink model. Thereafter, the best set of parameter values satisfying the minimum IAE values are chosen.

5. Stability analysis

In this section, the stability analysis of the proposed decoupling method is investigated. The most crucial parameters for the stability of a fourth-order system are λ_1 and λ_2 which must be always positive. Since the proposed decoupled control strategy always ensures $\lambda_1^L > 0$ and $\lambda_2^L > 0$, then the use of DSMC method will be sufficient to demonstrate the stability of the proposed method.

Remark 2. Since the state knowledge in S_2 is transferred into S_1, then it is sufficient to consider only S_1 in the following stability analysis. It is pointed out by many researchers [9,10,12,13,16] that the stabilization of subsystem A is possible if the knowledge of subsystem B is made available in the subsystem A.

Taking the derivative of (13) and equating the resulting equation to zero gives

$$\dot{S}_1 = \lambda_1\gamma_1(\dot{x}_1 - \dot{z})(x_1 - z)^{\gamma_1 - 1} + \dot{x}_2 = 0 \tag{27}$$

Substituting $\dot{x}_2(t) = f_1(\mathbf{x}, t) + b_1(\mathbf{x}, t)u_1(t)$ into (27) and solving for $u_1(t)$ gives

$$u_{eq1}(t) = \frac{\lambda_1\gamma_1(\dot{z} - \dot{x}_1)(x_1 - z)^{\gamma_1 - 1} - f_1(\mathbf{x}, t)}{b_1(\mathbf{x}, t)} \tag{28}$$

Eq. (28) is the equivalent control that is necessary to keep the states on the sliding surface sliding towards the equilibrium point. However, it is well known that a switching control action is also needed to move the system states towards the sliding surface [17]. Therefore, the control input for subsystem A in terms of equivalent

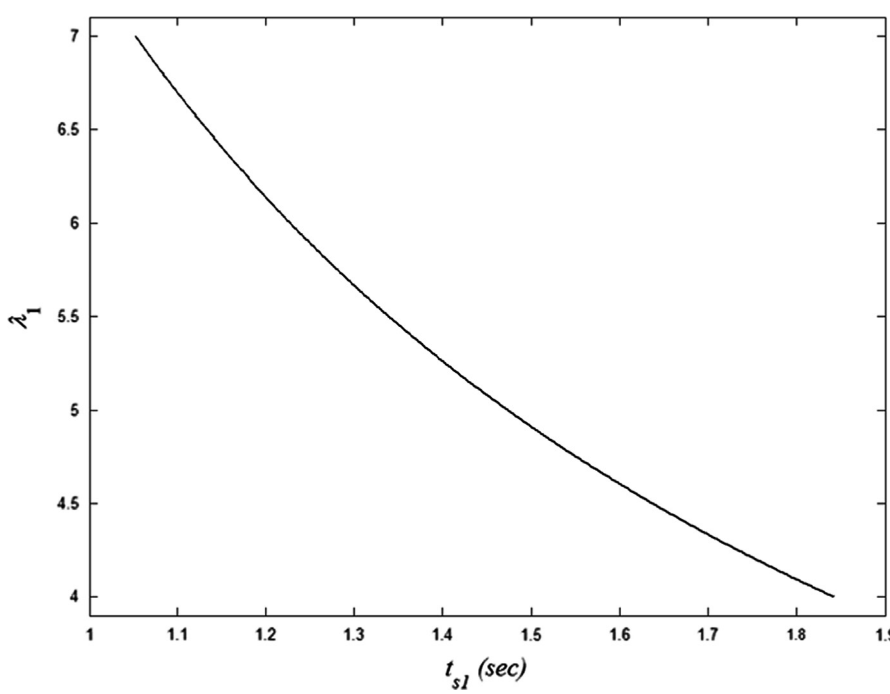

Fig. 1. Computed convergence time (t_{s1}) for different λ_1 values.

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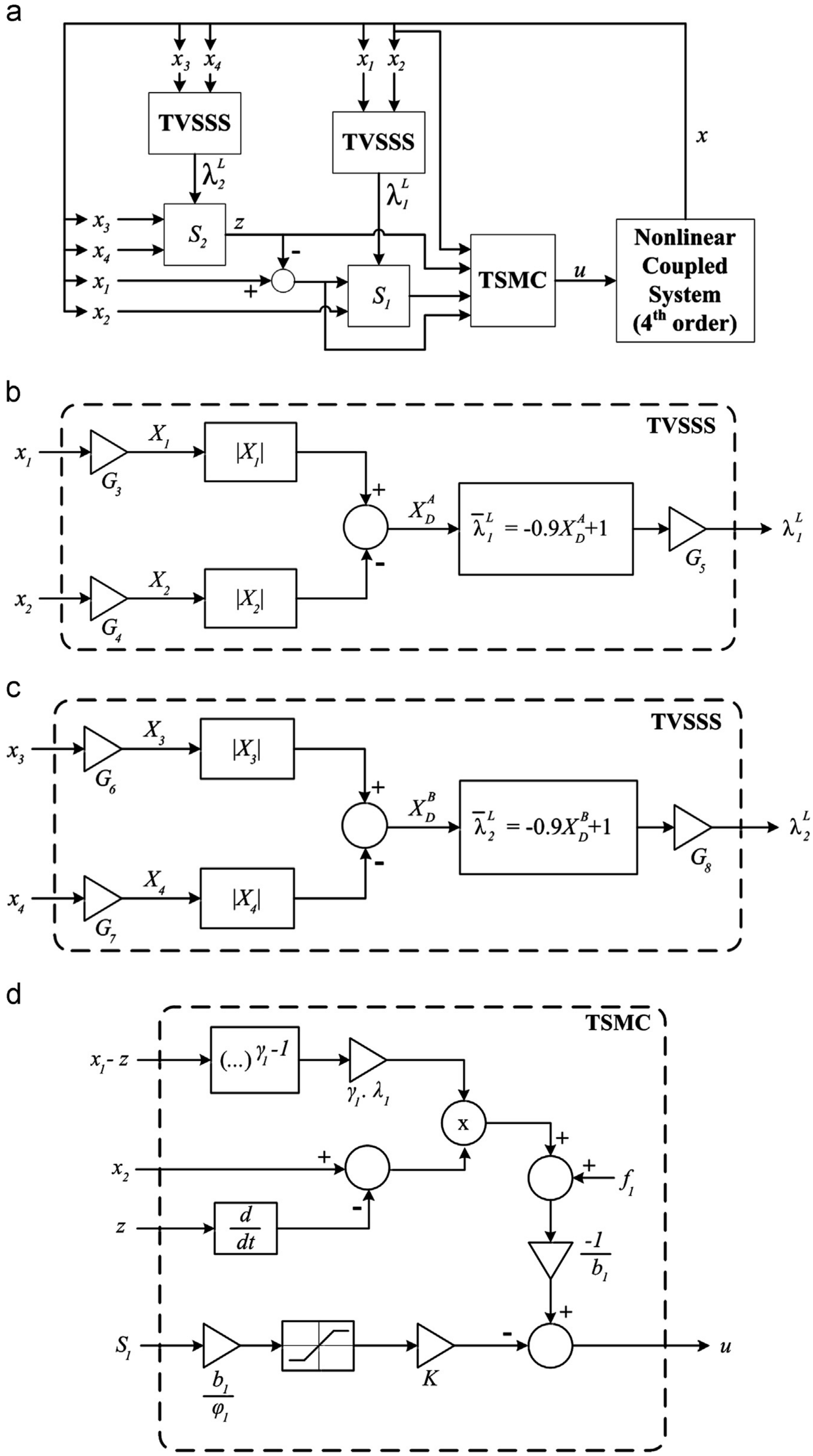

Fig. 2. (a) Block diagram of the proposed method, (b) internal representation of the TVSSS block used to compute λ_1^L, (c) internal representation of the TVSSS block used to compute λ_2^L and (d) internal representation of the TSMC block used compute u.

control and switching control can be written as

$$u_1(t) = \frac{\lambda_1\gamma_1(\dot{z}-\dot{x}_1)(x_1-z)^{\gamma_1-1} - f_1(\mathbf{x},t)}{b_1(\mathbf{x},t)} - K\,\mathrm{sat}\left(\frac{S_1 b_1(\mathbf{x},t)}{\Phi_1}\right) \tag{29}$$

where $K > (D_1(t))/|b_1(\mathbf{x},t)|$ and $\Phi_1 > 0$.

According to the TSMC theory, a terminal sliding-mode will exist if the following sufficient condition is satisfied:

$$S_1\dot{S}_1 \le 0 \tag{30}$$

That is, if a control law can be written which ensures (30), then the system will be forced towards the sliding surface and remains

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on it until origin is reached asymptotically. Now, let

$$V(t) = \tfrac{1}{2} S_1^2 \tag{31}$$

be a Lyapunov function candidate for the fourth-order system described in (2). The time derivative of this Lyapunov function can be written as

$$\dot{V}(t) = S_1 \dot{S}_1 \le 0 \tag{32}$$

Substitution of (27) and $\dot{x}_2(t) = f_1(\mathbf{x}, t) + b_1(\mathbf{x}, t)u_1(t) + d_1(t)$ into (32) gives

$$\dot{V}(t) = S_1\dot{S}_1 = S_1[(\lambda_1\gamma_1\dot{x}_1 - \lambda_1\gamma_1\dot{z})(x_1 - z)^{\gamma_1 - 1} + f_1(\mathbf{x}, t) + b_1(\mathbf{x}, t)u_1(t) + d_1(t)] \le 0 \tag{33}$$

Now, substituting the control input derived in (29) into (33) yields

$$\dot{V}(t) = S_1\dot{S}_1 = \left[-S_1 b_1(\mathbf{x}, t) K \ \mathrm{sat}\left(\frac{S_1 b_1(\mathbf{x}, t)}{\Phi_1}\right) + S_1 d_1(t)\right] \le 0 \tag{34}$$

Since, the switching control gain must satisfy $K > (D_1(t)) / |b_1(\mathbf{x}, t)|$, then regardless of sign of S_1, $\dot{V}(t)$ is always negative for all values of the system states. The proposed control method forces S_1 to converge to $x_1 = z$ and $x_2 = 0$ in the steady-state. It is clear from (8) that $x_2 \to 0$ if and only if $z \to 0$ which implies that $S_2 \to 0$. Therefore, it can be concluded that the stabilization of both subsystems can be achieved.

6. Simulation results and discussion

In order to verify the theoretical considerations and show the correct operation of the proposed control method, a cart–pole system is simulated and comparisons between the proposed method and the existing decoupled methods (DSMC and TVSSS) are demonstrated. All simulations were carried out by Matlab/Simulink.

The dynamic behavior of the cart–pole system shown in Fig. 3 can be described by the following nonlinear equations:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f_1(\mathbf{x}, t) + b_1(\mathbf{x}, t)u(t) + d_1(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= f_2(\mathbf{x}, t) + b_2(\mathbf{x}, t)u(t) + d_2(t)
\end{aligned} \tag{35}$$

where

$$f_1(\mathbf{x}, t) = \frac{m_t g \ \sin(x_1) - m_p L \ \sin(x_1)\cos(x_1) x_2^2}{L((4/3)m_t - m_p \cos^2(x_1))}$$

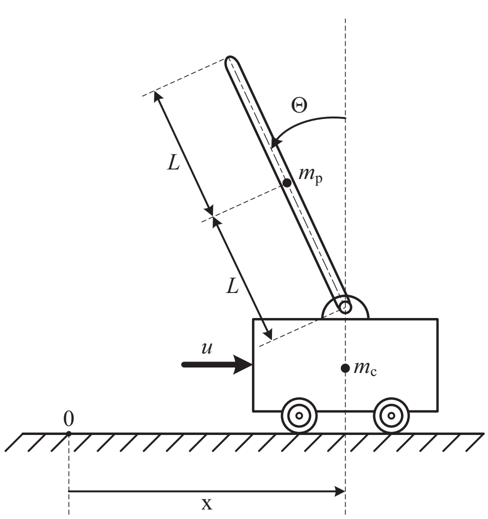

Fig. 3. Cart–pole system.

$$b_1(\mathbf{x}, t) = \frac{\cos(x_1)}{L((4/3)m_t - m_p \cos^2(x_1))}$$

$$f_2(\mathbf{x}, t) = \frac{-(4/3)m_p L x_2^2 \ \sin(x_1) + m_p g \ \sin(x_1)\cos(x_1)}{(4/3)m_t - m_p \cos^2(x_1)}$$

$$b_2(\mathbf{x}, t) = \frac{4}{3((4/3)m_t - m_p \cos^2(x_1))} \tag{36}$$

where $x_1(t)$ is the angular position of the pole from the vertical axis, $x_2(t)$ is the angular velocity of the pole with respect to the vertical axis, $x_3(t)$ is the position of the cart, $x_4(t)$ is the velocity of the cart, m_t is the total mass of the system (which includes the mass of the pole, m_p, and the mass of the cart, m_c), and L is the half-length of the pole. The state variables x_1 and x_2 are chosen to form the subsystem A (primary target) and the state variables x_3 and x_4 are used to form the subsystem B (secondary target). In the simulation study, the system state is assumed to be $\mathbf{x}(0) = [-60^\circ, 0, 0, 0]^T$ and the following parameters were used for the cart–pole system [9,10]:

$$m_p = 0.05 \text{ kg}, \quad m_c = 1 \text{ kg}, \quad L = 0.5 \text{ m}, \quad \text{and} \quad g = 9.8 \ m/s^2$$

The integral of absolute error (IAE) and the integral of time multiplied by absolute error (ITAE) defined below are used to make a quantitative comparison

$$\mathrm{IAE} = \int |e(t)| dt \tag{37}$$

$$\mathrm{ITAE} = \int t|e(t)| dt \tag{38}$$

where $e(t)$ represents the error signal between the actual and the desired state. The controller parameters used in the simulations are shown in Table 1.

Fig. 4 shows the response of the angular position of the pole obtained by the proposed method, DSMC and TVSSS methods. Clearly, the proposed method exhibits a faster response than the other methods. This fact can also be seen in the response of the intermediate signal z presented in Fig. 5. Fig. 6 shows the position evolution of the cart for the same case. Again, it is obvious that the proposed control strategy is capable of keeping the cart in a shorter distance which verifies that it is faster than the other methods. Fig. 7 shows the control input of the proposed method, DSMC, and TVSSS. Clearly, the proposed method generates a faster control input with more undershoot. The time-varying coefficients obtained from the proposed method are presented in Fig. 8. It is interesting to note that the change in the coefficients has similar characteristics to the back and forth action exerted on the cart to keep both the pole and the cart back to the zero steady-state. The simulation results of the proposed method are compared with that of the existing decoupled methods based on the quantitative measures such as IAE and ITAE. Table 2 gives the IAE and ITAE values calculated using (37) and (38). As can be seen, the proposed

Table 1
The controller parameters used for the cart–pole system.

	DSMC	TVSSS	Proposed method
λ_1/λ_2	5/0.5	–	–
K	10	–	64
Φ_1/Φ_z	5/15	–/15	5/15
z_U	0.9425	0.9425	0.9425
G_1/G_2	–	1/40	–
$G_3/G_4/G_5$	–	0.05/0.05/5	0.05/0.05/5
$G_6/G_7/G_8$	–	0.05/0.05/0.5	0.05/0.05/0.5
$\|d_1\| = \|d_2\|$	≤ 0.0873	≤ 0.0873	≤ 0.0873
q_1/p_1	–	–	19/21
q_2/p_2	–	–	17/21

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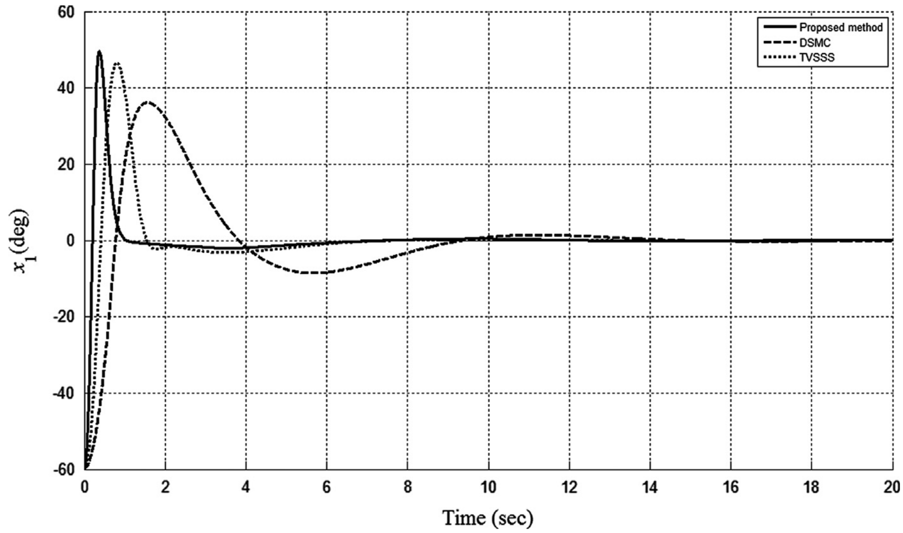

Fig. 4. Angular position of the pole.

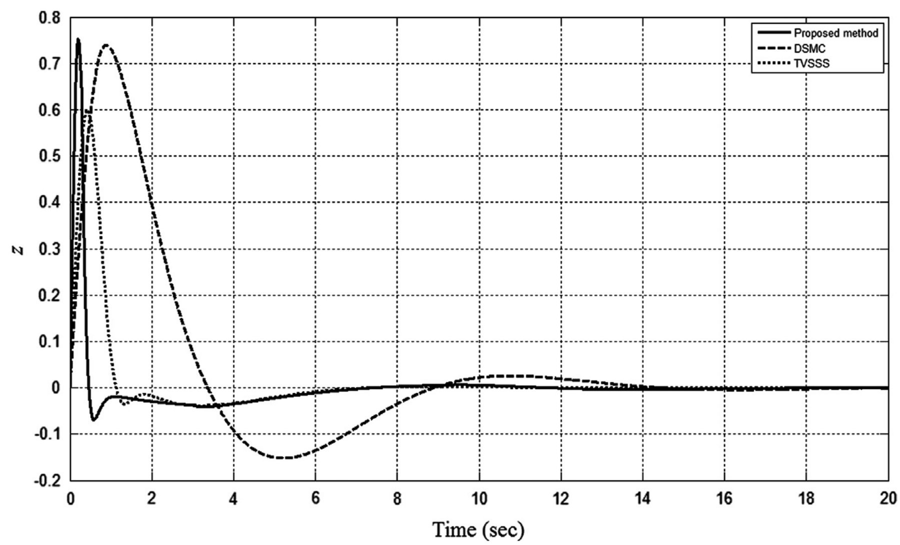

Fig. 5. Intermediate signal.

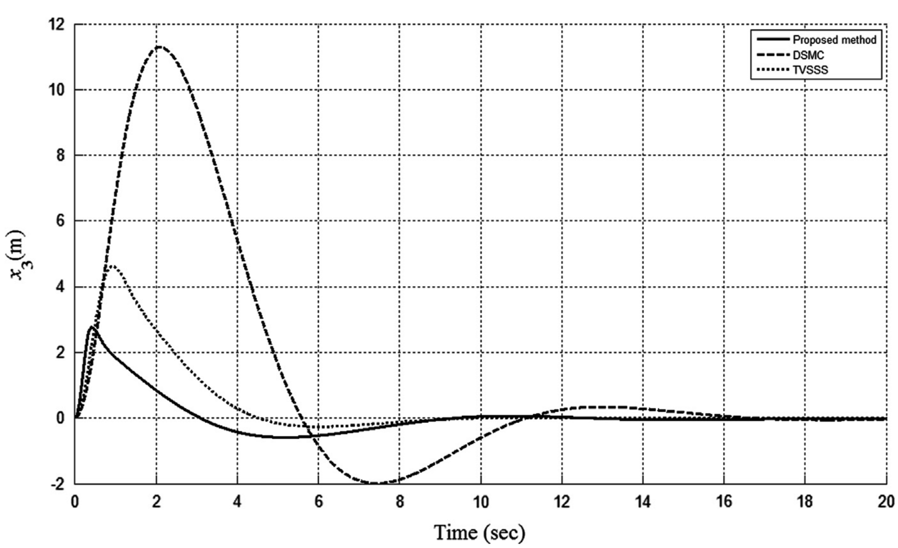

Fig. 6. Position of the cart.

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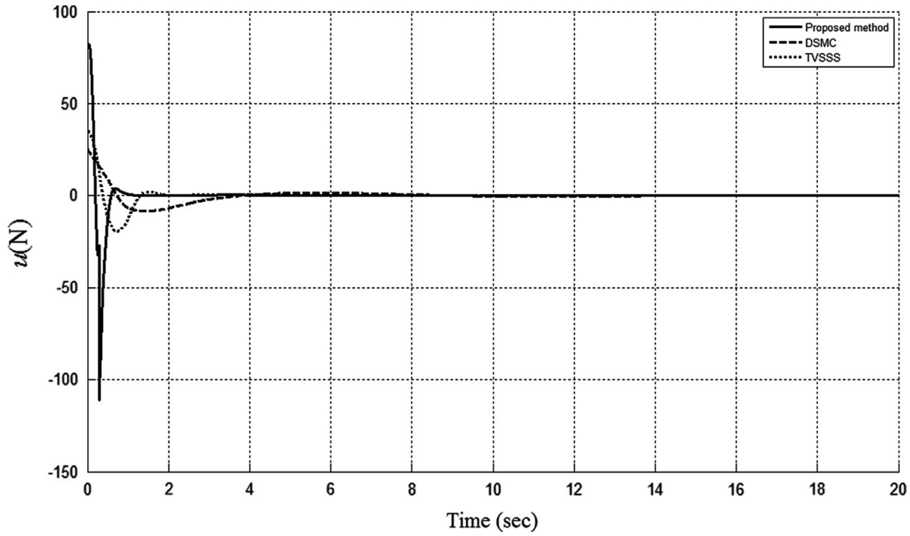

Fig. 7. Control input.

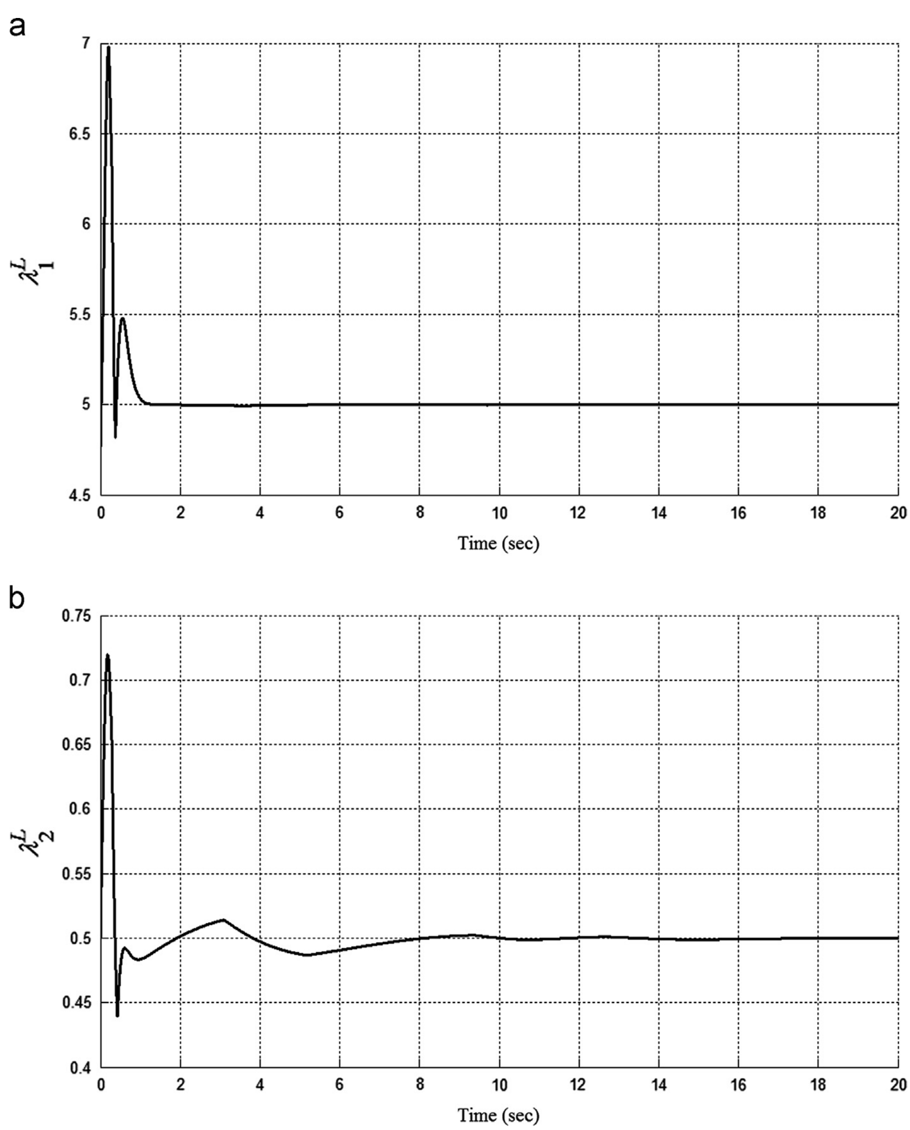

Fig. 8. Time-varying coefficients: (a) λ_1^L and (b) λ_2^L.

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control method provides lesser error values in terms of both the angular position of the pole and the position of the cart.

The performance of the proposed method is also compared with that of a recently published control method introduced in [15]. In order to have a fair comparison of the two methods, the cart–pole system parameters, the initial condition of the state vector, and other common parameters should be equal to each other. In this simulation study the cart–pole parameters which are used in [15] are chosen as $m_p = 0.5$ kg, $m_c = 2$ kg, $L = 0.5$ m. The initial condition of the state vector is assumed to be $\mathbf{x}(0) = [30^\circ, 0, 0, 0]^T$. Other common parameters which exist in both methods are chosen as $\Phi_1 = 1/9.949$, $\Phi_z = 1/0.18574$, $z_U = 0.9966$. On the other hand, the control method in [15] utilizes time-invariant coefficients for the sliding surface functions which are chosen as $\lambda_1 = 1.436$ and $\lambda_2 = 0.26172$, respectively. It should be noted that the time-varying coefficients (λ_1^L and λ_2^L) of the sliding surface functions of the proposed control method converge to 1.436 and 0.26172 in the steady-state. The gains of proposed controller were set to $K = 563$, $q_1 = q_2 = 99$, and $p_1 = p_2 = 101$.

Table 2
Computed IAE and ITAE values for the cart–pole system.

		DSMC	TVSSS	Proposed method
Angle	IAE	125.69	57.48	34.10
	ITAE	370.46	75.79	54.80
Position	IAE	42.29	9.84	6.28
	ITAE	156.24	20.41	21.64

Fig. 9 shows the responses of the angular position of the pole (x_1) obtained by the proposed method and the method in [15]. It is clear that the proposed method exhibits a slightly faster response than that obtained by the method in [15]. The IAE of x_1 in the proposed method was computed as 18.55, which is smaller than IAE = 22.47 obtained from the method in [15]. Fig. 10 shows the position evolution of the cart (x_3) obtained by the two methods. Although the responses of x_3 are very similar to each other, the IAE of x_3 in the proposed method was computed as 13.47, which is smaller than IAE = 13.84 obtained from the method in [15].

7. Conclusions

A time-varying sliding-coefficient-based decoupled sliding-mode control method has been proposed for a class of fourth-order systems. The proposed method decouples the nonlinear

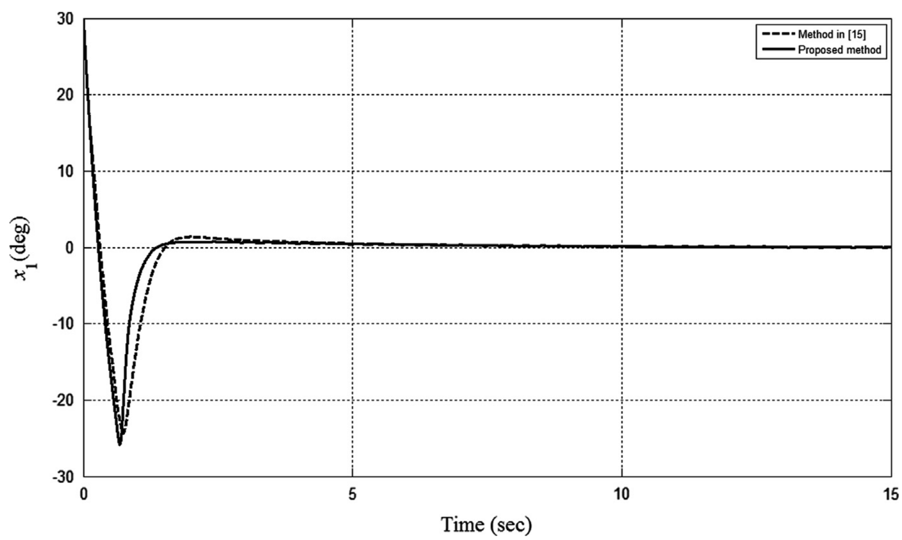

Fig. 9. Angular positions of the pole obtained by the two methods.

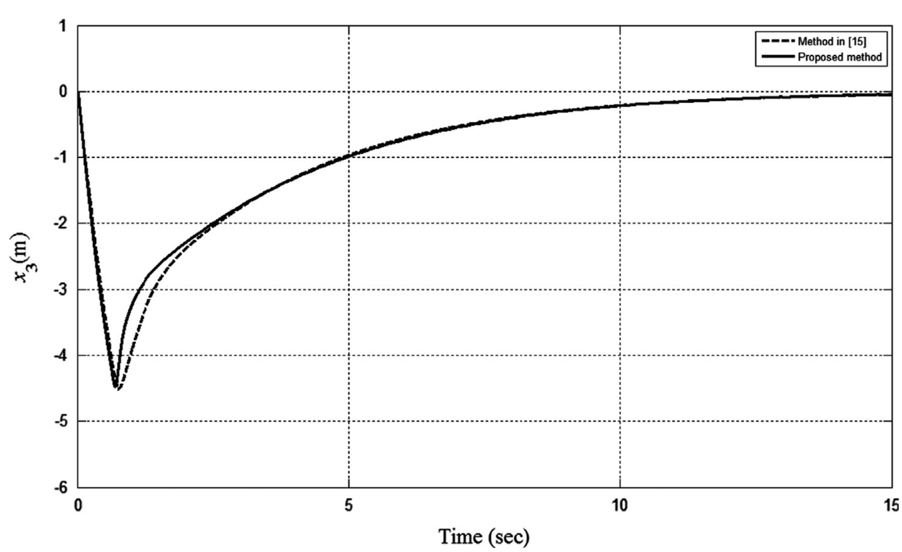

Fig. 10. Positions of the cart obtained by the two methods.

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fourth-order system into two second-order subsystems. The sliding surface of each subsystem was designed by utilizing time-varying sliding coefficients which are generated with the help of linear functions derived from the input–output mapping of the one-dimensional fuzzy rule bases. The terminal sliding mode control (TSMC) method was utilized to make both subsystems converge to the equilibrium points in finite time. The simulation results on the inverted pendulum system were given to show the effectiveness of the proposed method. Simulation results of a cart–pole system with the proposed control method demonstrate that the dynamic response is much faster than that of obtained with the decoupled control methods in literature. In addition, the proposed method exhibits lower IAE and ITAE values compared with the existing decoupled control methods.

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