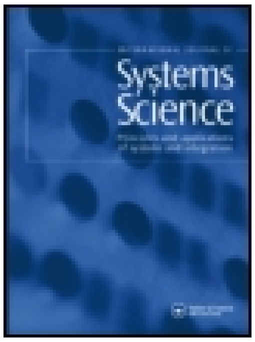

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Networked iterative learning control approach for nonlinear systems with random communication delay

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ABSTRACT

This paper constructs a proportional-type networked iterative learning control (NILC) scheme for a class of discrete-time nonlinear systems with the stochastic data communication delay within one operation duration and being subject to Bernoulli-type distribution. In the scheme, the communication delayed data is replaced by successfully captured one at the concurrent sampling moment of the latest iteration. The tracking performance of the addressed NILC algorithm is analysed by statistic technique in virtue of mathematical expectation. The analysis shows that, under certain conditions, the expectation of the tracking error measured in the form of 1-norm is asymptotically convergent to zero. Numerical experiments are carried out to illustrate the validity and effectiveness.

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Iterative learning control; nonlinear systems; networked control systems; stochastic communication delay

1. Introduction

As one of the intelligent control strategies for a system operating repetitively over a fixed time interval, iterative learning control (ILC) was firstly proposed in 1984 for a robotic manipulator to track a given trajectory (Arimoto, Kawamura, & Miyazaki, 1984). Its schematic structure is to iteratively generate a sequence of control inputs in a recursive mode by compensating for the control input at the current operation with its and/or the historical tracking discrepancy to compose an upgraded control input for the next operation. The aim is to ensure that the sequential control inputs may drive the system to successively track a predetermined desired trajectory more and more precise as the iteration index goes on. Since the ILC algorithmic construction requires only the system tracking information rather than the precise system model, the efficacy of tracking performance of various ILCs has been acknowledged in the aspects of theoretical developments and practical applications over the past three decades (Bristow, Tharayil, & Alleyne, 2006; Chen, Moore, Yu, & Zhang, 2008; Mi, Lin, & Zhang, 2005). The concerned issues include ILC strategies handling initial states shifts or parameter interval uncertainties (Ahn, Moore, & Chen, 2007; Meng, Jia, Du, & Yu, 2010; Park, 2005; Yin, Xu, & Hou, 2010), ILC modes for stochastic, discrete or continuous systems (Chen, Wen, & Sun, 1997; Chien, 1998; Ouyang, Zhang, & Gupta, 2006; Ruan, & Bien, 2011; Saab, 2001), ILC algorithms dealing with operation-varying lengths (Li, Xu, & Huang, 2014; Seel, Schauer, & Raisch, 2011) as well as ILC schemes involving time delay (Chen & Zhang, 2010; Ma, Li, & Huang, 2011) and so on.

With the progressive development of information technology, the Internet has been becoming a most popular service for its efficient convenience and lower cost, by which the traditional ILC schemes can be real-timely networked. The utility of the communication network for executing the traditional iterative learning control schemes constitutes a networked iterative learning control (NILC) profile. It is evident that, while implementing the NILC scheme, the sampled big data needs to be communicated via the Internet with limited bandwidth and thus the data possibly suffers from not only packet dropout but also communication delay. This implies that the dropped and/or delayed data must be properly estimated for satisfactory tracking performance. Thus the algorithmic construction and the convergence analysis of the NILC scheme become complicated in treating the data packet dropout or communication delay as mentioned in Ahn, Moore, and Chen (2008), Bu and Hou (2011), Bu, Hou, and Yu (2011). Usually, in the networked control systems, the communication delay stochastically occurs in two communication channels (Yang, Wang, Huang, & Gani, 2006). The one is the input communication channel which transmits the control inputs from the ILC unit to the executor for stimulating the system, whilst another is the output channel that communicates the system output to the ILC unit for composing the control signal of the next operation. Regarding the Internet communication delay and dropout issue, up to date, a few of NILCs have been made which mainly focus on the estimating formulation of the delayed and/or dropped data. In specific, literature (Liu, Xu, & Wu, 2012) has constructed a P-type NILC updating rule for a linear time-invariant (LTI) system by replacing the one-step communication delayed output with the captured one-step ahead at the concurrent iteration and zeroing the dropped output. Bu, Yu, Hou, and Wang (2013) have formulated a P-type NILC law for a class of discrete-time nonlinear system by replacing the delayed data with the latest one captured at the concurrent iteration for the case when the one-step data communication delay is stochastic and subject to Bernoulli-type distribution. The convergence analysis in terms

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of expectation presents that the tracking error is asymptotically upper-bounded but non-zero. This means that it is inspiring to develop a more effective NILC for nonlinear systems with random communication delay for further refinement of tracking performance.

For the NILC construction, it is reminded that the key theme of the ILC strategy is its learning capability which exhibits that the learning effect at each instant is mainly from the compensation for the control input along the iteration axis rather than the time axis. This implies that the substitution of the delayed data with the synchronous one captured at the concurrent instant of the previous iteration possibly owns better learning advantage over the data received at the latest sampling instant of the current iteration. This motivates the paper to investigate a P-type NILC strategy which replaces the delayed data at the current operation with the one captured at the corresponding sampling instant of the previous operation for a class of nonlinear systems with the stochastic Bernoulli-type data communication delay within one iteration duration. The learning gain is designed properly and the tracking performance is derived on behalf of expectation involving of the probability of the data transmission success.

The rest of the paper is organised as follows. Some necessary definitions and properties are given in Section 2. In Section 3, a proportional-type NILC algorithm is constructed and some notations are exhibited. Section 4 presents the convergence analysis and Section 5 displays numerical simulations. Conclusion is given in Section 6.

2. Preliminaries

In this section, we introduce some necessary definitions and properties which will be used later.

Definition 2.1. Let $x = (x_1, \ldots, x_n)^\top \in R^n$ and $H = (h_{ij})_{m\times n} \in R^{m\times n}$. Then $||x||_1 = \sum_{i=1}^{n} |x_i|$ denotes the 1-norm with respect to x and $||H||_1 = \max\limits_{1\le j\le n}\{\sum_{i=1}^{m} |h_{ij}|\}$ denotes the induced 1-norm of matrix H. Besides, denote $|x| = (|x_1|, \ldots, |x_n|)^\top$ and $|H| = (|h_{ij}|)_{m\times n}$.

Definition 2.2. Let $x = (x_1, \ldots, x_n)^\top \in R^n$ and $y = (y_1, \ldots, y_n)^\top \in R^n$ be any two n-dimensional real vectors. Their partial order relation $\prec$ is defined as $x \prec y$ if and only if $x_i \le y_i$ for all $i = 1, 2, \ldots, n$.

What follows are basic properties of the above definitions.

(P1) Let $x, y, z \in R^n$. If $|x| \prec |y|$ and $|y| \prec |z|$, then $|x| \prec |z|$.
(P2) Let $x \in R^n$, $y \in R^m$ and $H \in R^{m\times n}$. If $y = Hx$, then $|y| \prec |H||x|$.
(P3) Let $x, y \in R^n$. If $|x| \prec |y|$, then $||x||_1 \le ||y||_1$.
(P4) Let $x, y, z \in R^n$. If $z = x + y$, then $|z| \prec |x| + |y|$.
(P5) Let $x, y \in R^n$. Then $|||x| + |y|||_1 \le |||x|||_1 + |||y|||_1$.
(P6) Let $H_1, H_2 \in R^{m\times n}$. If $H_2 = |H_1|$, then $||H_2||_1 = ||H_1||_1$.
(P7) Let $x, y \in R^n$ be stochastic vectors. If $|x| \prec |y|$, then $E\{|x|\} \prec E\{|y|\}$.
(P8) Let $x \in R^n$ be a stochastic vector. Then $||E\{|x|\}||_1 = E\{||x||_1\}$.

3. NILC algorithm and notations

Consider a class of repetitive single-input-single-output (SISO) nonlinear systems as follows.

$$\begin{cases} x_k(t+1) = f(x_k(t)) + b(t)u_k(t), \\ y_k(t) = g(x_k(t)) + d(t)u_k(t), \\ x_k(0) = x_0, \end{cases} \tag{1}$$

where k denotes the iteration index and t refers to the discrete sampling variable with $t \in S = \{0, 1, 2, \ldots, T-1\}$. Meanwhile, $x_k(t) \in R^n$, $u_k(t) \in R$ and $y_k(t) \in R$ are n-dimensional state, scalar input and output at the kth iteration, respectively. The nonlinear functions $f(z)$ and $g(z)$ are uniformly globally Lipschitz with respect to z, i.e. for any $z_1, z_2 \in R^n$, there exist positive constants K_f and K_g, such that

$$\begin{aligned} &\left\| f(z_1) - f(z_2) \right\|_1 \le K_f \|z_1 - z_2\|_1, \\ &\left| g(z_1) - g(z_2) \right| \le K_g \|z_1 - z_2\|_1. \end{aligned}$$

Here, $\|f(z)\|_1$ refers to 1-norm of the vector function $f(z)$ as expressed in Definition 2.1 and $|g(z)|$ denotes the absolute value of a scalar function $g(z)$. Additionally, $b(t)$ is a vector function of the sampling time variable $t \in S$ and $d(t) \neq 0$, $(t \in S)$.

Under the assumption that $d(t) \neq 0$ for $t \in S$, for a given desired output $y_d(t)$, $t \in S$, there exist such a desired input $u_d(t)$ and a desired state $x_d(t)$ that they obey the dynamics of system (1) as follows.

$$\begin{cases} x_d(t+1) = f(x_d(t)) + b(t)u_d(t), \\ y_d(t) = g(x_d(t)) + d(t)u_d(t), \\ x_d(0) = x_0, \end{cases} \tag{2}$$

where $t \in S$.

In system (1), when the control input $u_k(t)$ is iteratively generated in learning mode and is transmitted through the Internet to the actuator for the system stimulation, and simultaneously the system output $y_k(t)$ is transferred via the Internet to the ILC unit, the mode is regarded as a NILC scheme. The diagram of the NILC scheme is illustrated in Figure 1, where $u_k(t)$ is the system input whilst $y_k(t)$ refers to the system output. For the case when the output data $y_k(t)$ is transmitted from the sensor to the ILC unit via output communication channel, it may be delayed due

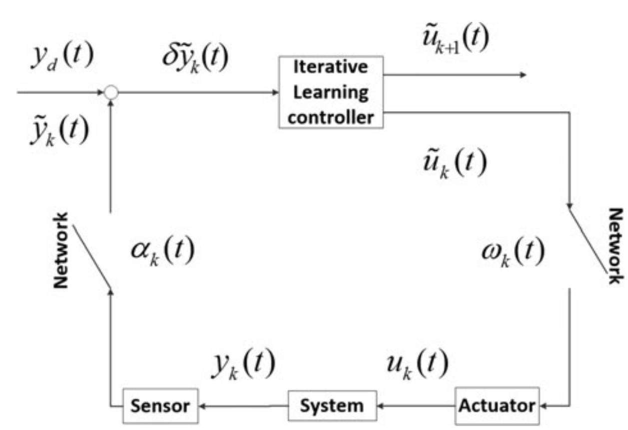

Figure 1. Schematic diagram of NILC.

to Internet bandwidth limitation. In this scenario, the delayed data must be estimated for the ILC algorithmic construction. Let $\tilde{y}_k(t)$ be the estimated output with respect to the system output $y_k(t)$ for the ILC formulation and $\tilde{u}_{k+1}(t)$ be the corresponding generated control input for the next operation. When $\tilde{u}_{k+1}(t)$ is transmitted through input communication channel to the executor, it is possibly delayed and thus is approximated as $u_{k+1}(t)$ for the system stimulation.

The estimated formulae are given as follows.

$$\begin{cases} \tilde{y}_1(t) = y_1(t), u_1(t) = \tilde{u}_1(t), \\ \tilde{y}_k(t) = \alpha_k(t)y_k(t) + (1-\alpha_k(t))\, y_{k-1}(t), k = 2, 3, \ldots, \\ u_k(t) = \omega_k(t)\tilde{u}_k(t) + (1-\omega_k(t))\, \tilde{u}_{k-1}(t), k = 2, 3, \ldots, \end{cases} \tag{3}$$

where $t \in S$, $\alpha_k(t)$ and $\omega_k(t)$ are stochastic variables subject to 0-1 Bernoulli distributions. Specifically, $\alpha_k(t) = 1$ means that the data $y_k(t)$ in the current sampling instant t is successfully captured and then is estimated as is, whilst $\alpha_k(t) = 0$ implies that the transmission of the current data $y_k(t)$ is failed and thus is estimated by the synchronous system output $y_{k-1}(t)$ at the previous iteration. Meanwhile, $\omega_k(t) = 1$ represents that the transmission of the data $\tilde{u}_k(t)$ is in success and then is utilised for system stimulation in a direct manner, whilst $\omega_k(t) = 0$ means that the transmission of the data $\tilde{u}_k(t)$ is stuck but the previous-iteration signal $\tilde{u}_{k-1}(t)$ is borrowed for the use of $u_k(t)$. Moreover, assume that the test input $\tilde{u}_1(t)$ and output $y_1(t)$ at the first iteration are available in full. It should be pointed out that both the captured and the delayed data need to be saved in a memory for its possible use at the next iteration. This does not mean that the NILC scheme uses too much memory to save the data of all operations but extra memory for saving the data of the latest previous operation.

Assume that the probabilities of successful transmission are as follows.

$$\begin{aligned} &\mathrm{Prob}\{\alpha_k(t) = 1\} = \bar{\alpha}_k(t), 0 \le \bar{\alpha}_k(t) \le 1, \text{ for } t \in S, k = 2, 3, \ldots, \\ &\mathrm{Prob}\{\omega_k(t) = 1\} = \bar{\omega}_k(t), 0 \le \bar{\omega}_k(t) \le 1, \text{ for } t \in S, k = 2, 3, \ldots. \end{aligned}$$

By considering the 0-1 Bernoulli distribution assumption, it is easy to calculate the expectations of those stochastic variables as follows.

$$\begin{aligned} E\{\alpha_k(t)\} &= \mathrm{Prob}\{\alpha_k(t) = 1\} = \bar{\alpha}_k(t), 0 \le \bar{\alpha}_k(t) \le 1, \text{ for} \\ &\qquad t \in S, k = 2, 3, \ldots, \\ E\{\omega_k(t)\} &= \mathrm{Prob}\{\omega_k(t) = 1\} = \bar{\omega}_k(t), 0 \le \bar{\omega}_k(t) \le 1, \text{ for} \\ &\qquad t \in S, k = 2, 3, \ldots. \end{aligned}$$

Here $t \in S$, $\bar{\alpha}_k(t)$ and $\bar{\omega}_k(t)$ are known constants for the given iteration index k and time t. $E\{\cdot\}$ represents the expectation operator.

Remark 3.1. It is recalled that in Bu et al. (2013) the estimated formulae are as follows.

$$\begin{cases} \tilde{y}_k(t) = \alpha_k(t)y_k(t) + (1-\alpha_k(t))\, y_k(t-1), \\ u_k(t) = \omega_k(t)\tilde{u}_k(t) + (1-\omega_k(t))\, \tilde{u}_k(t-1). \end{cases} \tag{3*}$$

Comparing the estimation formulae (3) with the existing estimation expression (3*), it is observed that the proposed scheme (3) borrows the corresponding sampled data at the previous iteration rather than that the existing estimation expression (3*) uses the one-step ahead data at the current iteration. On behalf of handling the communication delayed data, the proposed estimation scheme (3) may own a wider application scope as it relaxes the one-step delay to the one-iteration delay.

In general, it is known that the occurrences of the communication delays at different operations are mutually independent. It is characterised as the assumptions (I), (II) and (III) as follows.

(I) $\alpha_k(t)$ is independent on $\alpha_l(s)$ for the case when $k \neq l$.
(II) $\omega_k(t)$ is independent on $\omega_l(s)$ for the case when $k \neq l$.
(III) $\omega_k(t)$ is independent on $\alpha_l(s)$ for any k, l, t, s.

It is usual that the data transmission quality relies on the Internet quality. Thus, it is reasonable to assume that the expectations $\bar{\alpha}_k(t)$ of $\alpha_k(t)$ and $\bar{\omega}_k(t)$ of $\omega_k(t)$ are constant in short-term implementation.

Let $\bar{\alpha}_k(t) = \bar{\alpha}$ and $\bar{\omega}_k(t) = \bar{\omega}$.

A proportional-type (P-type) NILC algorithm is constructed as follows.

$$\tilde{u}_{k+1}(t) = \tilde{u}_k(t) + \Gamma(t)\delta\tilde{y}_k(t), \tag{4}$$

where $t \in S$ and $\delta\tilde{y}_k(t) = y_d(t) - \tilde{y}_k(t)$. In particular, $\Gamma(t)$ is assigned as the proportional learning gain.

For the sake of simplifying the statements and the analysis, a set of denotations are introduced as follows.

$$\begin{aligned} x^0 &= \left[(x_0)^\top, 0, \ldots, 0\right]^\top \in R^{nT}, \\ x_k &= \left[(x_k(0))^\top, (x_k(1))^\top, \ldots, (x_k(T-1))^\top\right]^\top \in R^{nT}, \\ x_d &= \left[(x_d(0))^\top, (x_d(1))^\top, \ldots, (x_d(T-1))^\top\right]^\top \in R^{nT}, \\ y_k &= \left[y_k(0), y_k(1), \ldots, y_k(T-1)\right]^\top \in R^T, \\ y_d &= \left[y_d(0), y_d(1), \ldots, y_d(T-1)\right]^\top \in R^T, \\ u_k &= \left[u_k(0), u_k(1), \ldots, u_k(T-1)\right]^\top \in R^T, \\ u_d &= \left[u_d(0), u_d(1), \ldots, u_d(T-1)\right]^\top \in R^T, \\ f(x_k) &= \left[\left(f(x_k(0))\right)^\top, \left(f(x_k(1))\right)^\top, \ldots, \left(f(x_k(T-1))\right)^\top\right]^\top \in R^{nT}, \\ f(x_d) &= \left[\left(f(x_d(0))\right)^\top, \left(f(x_d(1))\right)^\top, \ldots, \left(f(x_d(T-1))\right)^\top\right]^\top \in R^{nT}, \\ g(x_k) &= \left[g(x_k(0)), g(x_k(1)), \ldots, g(x_k(T-1))\right]^\top \in R^T, \\ g(x_d) &= \left[g(x_d(0)), g(x_d(1)), \ldots, g(x_d(T-1))\right]^\top \in R^T. \end{aligned}$$

Thus, the dynamic description (1) is rewritten as follows.

$$\begin{cases} x_k = Af(x_k) + ABu_k + x^0, \\ y_k = g(x_k) + Du_k. \end{cases} \tag{5}$$

Likewise, the dynamics (2) is reformulated as follows.

$$\begin{cases} x_d = Af(x_d) + ABu_d + x^0, \\ y_d = g(x_d) + Du_d, \end{cases} \tag{6}$$

where

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 \\ I_{n(T-1)\times n(T-1)} & 0 \end{pmatrix} \in R^{nT\times nT} \text{ with } I_{n(T-1)\times n(T-1)} \\ &= \mathrm{diag}\,(1, 1, \ldots, 1) \in R^{n(T-1)\times n(T-1)}, \end{aligned}$$

$$B=\begin{pmatrix} b(0) & 0 & \cdots & 0 \\ 0 & b(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b(T-1) \end{pmatrix} \in R^{nT\times T},$$

$$D=\begin{pmatrix} d(0) & 0 & \cdots & 0 \\ 0 & d(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d(T-1) \end{pmatrix} \in R^{T\times T}.$$

In addition, set

$$\begin{aligned}
\tilde{u}_k &= \left[\tilde{u}_k(0), \tilde{u}_k(1), \ldots, \tilde{u}_k(T-1)\right]^\top \in R^T,\\
\tilde{y}_k &= \left[\tilde{y}_k(0), \tilde{y}_k(1), \ldots, \tilde{y}_k(T-1)\right]^\top \in R^T,\\
\Lambda_k &= \operatorname{diag}\left(\alpha_k(0), \ldots, \alpha_k(T-1)\right) \in R^{T\times T},\\
\Omega_k &= \operatorname{diag}\left(\omega_k(0), \ldots, \omega_k(T-1)\right) \in R^{T\times T},\\
\Lambda &= \operatorname{diag}\left(\bar{\alpha}, \ldots, \bar{\alpha}\right) \in R^{T\times T},\\
\Omega &= \operatorname{diag}\left(\bar{\omega}, \ldots, \bar{\omega}\right) \in R^{T\times T},\\
\Gamma &= \operatorname{diag}\left(\Gamma(0), \Gamma(1), \ldots, \Gamma(T-1)\right) \in R^{T\times T}.
\end{aligned}$$

Hence the expression (3) is compacted as follows.

$$\begin{cases} \tilde{y}_1 = y_1,\ u_1 = \tilde{u}_1, \\ \tilde{y}_k = \Lambda_k y_k + (I-\Lambda_k)\, y_{k-1},\ k=2,3,\ldots, \\ u_k = \Omega_k \tilde{u}_k + (I-\Omega_k)\, \tilde{u}_{k-1},\ k=2,3,\ldots, \end{cases} \tag{7}$$

where I is an identity matrix with appropriate dimension.

The P-type NILC algorithm (4) is compacted as

$$\tilde{u}_{k+1} = \tilde{u}_k + \Gamma\delta\tilde{y}_k, \tag{8}$$

where $\delta\tilde{y}_k = y_d - \tilde{y}_k$.

4. Convergence analysis

Lemma 4.1. Suppose that a real nonnegative series $\{e_n\}_{n=1}^{\infty}$ satisfies

$$e_{n+3} \le \rho_1 e_{n+2} + \rho_2 e_{n+1} + \rho_3 e_n,\ \left(n \in N^*\right), \tag{9}$$

where N^* is the set of positive integers, $\rho_i \ge 0$ $(i=1,2,3)$ and $\rho = \rho_1+\rho_2+\rho_3 < 1$. Then we have

$$\max\{e_{n+5}, e_{n+4}, e_{n+3}\} \le \rho \max\{e_{n+2}, e_{n+1}, e_n\},\ \left(\forall n \in N^*\right), \tag{10}$$

$$e_{3k+i} \le \rho^k \max\{e_1, e_2, e_3\},\ \left(\forall k \in N^*, i=1,2,3\right), \tag{11}$$

and

$$\lim_{n\to\infty} e_n = 0. \tag{12}$$

Proof. By (9), we obtain

$$\begin{aligned}
e_{n+3} &\le \rho \max\{e_{n+2}, e_{n+1}, e_n\},\\
e_{n+4} &\le \rho \max\{e_{n+2}, e_{n+1}, e_n\},\\
e_{n+5} &\le \rho \max\{e_{n+2}, e_{n+1}, e_n\}.
\end{aligned} \tag{13}$$

This implies that (10) is true.

By recursive formula (10), it is easy to check that (11) is true.

From (11), it follows

$$\lim_{n\to\infty} e_n = 0. \tag{14}$$

This completes the proof. □

Theorem 4.1. Assume that the proposed updating NILC law (4) with estimation formulation (3) is applied to the nonlinear system (1). Then the expectation of tracking error $E\{\|\delta y_k\|_1\}$ asymptotically converges to zero along the iteration axis if the following conditions (C1) $\rho = \rho_1+\rho_2+\rho_3 < 1$ and (C2) $K_f < 1$ are satisfied. Here

$$\begin{aligned}
\rho_1 &= \|E\{|I-\Gamma\Lambda_k D\Omega_k|\}\|_1 + \|\Gamma\|_1 \bar{\alpha}\bar{\omega}\frac{K_g\|AB\|_1}{1-K_f},\\
\rho_2 &= \|\Gamma\|_1\left[\bar{\alpha}(1-\bar{\omega}) + (1-\bar{\alpha})\bar{\omega}\right]\Phi,\\
\rho_3 &= \|\Gamma\|_1(1-\bar{\alpha})(1-\bar{\omega})\Phi,\ \Phi = \frac{K_g\|AB\|_1}{1-K_f} + \|D\|_1.
\end{aligned}$$

Proof: The formulae (7) and (8) give rise to

$$\begin{aligned}
\tilde{u}_{k+1} &= \tilde{u}_k + \Gamma\left(y_d - \tilde{y}_k\right)\\
&= \tilde{u}_k + \Gamma\left(y_d - \left[\Lambda_k y_k + (I-\Lambda_k) y_{k-1}\right]\right)\\
&= \tilde{u}_k + \Gamma\Lambda_k\delta y_k + \Gamma\left(I-\Lambda_k\right)\delta y_{k-1},
\end{aligned} \tag{15}$$

where $\delta y_k = y_d - y_k$.

Notice that

$$y_d = g(x_d) + Du_d, \tag{16}$$

and

$$y_k = g(x_k) + Du_k. \tag{17}$$

Equations (16) and (17) lead to

$$\delta y_k = \left[g(x_d) - g(x_k)\right] + D\left[u_d - u_k\right]. \tag{18}$$

Furthermore, considering formula (7) reduces

$$u_d - u_k = \Omega_k\delta\tilde{u}_k + \left(I-\Omega_k\right)\delta\tilde{u}_{k-1}, \tag{19}$$

where $\delta\tilde{u}_k = u_d - \tilde{u}_k$.

Substituting (19) into the right-hand side of (18), we have

$$\delta y_k = \left[g(x_d) - g(x_k)\right] + D\Omega_k\delta\tilde{u}_k + D\left(I-\Omega_k\right)\delta\tilde{u}_{k-1}. \tag{20}$$

Substituting (20) into (15) yields

$$\begin{aligned}
\tilde{u}_{k+1} &= \tilde{u}_k + \Gamma\Lambda_k\left(\left[g(x_d) - g(x_k)\right] + D\Omega_k\delta\tilde{u}_k + D\left(I-\Omega_k\right)\delta\tilde{u}_{k-1}\right)\\
&\quad + \Gamma(I-\Lambda_k)([g(x_d) - g(x_{k-1})] + D\Omega_{k-1}\delta\tilde{u}_{k-1}\\
&\quad + D(I-\Omega_{k-1})\delta\tilde{u}_{k-2})\\
&= \tilde{u}_k + \Gamma\Lambda_k D\Omega_k\delta\tilde{u}_k + \Gamma\Lambda_k[g(x_d) - g(x_k)]\\
&\quad + \Gamma(I-\Lambda_k)[g(x_d) - g(x_{k-1})]\\
&\quad + \Gamma[\Lambda_k D(I-\Omega_k) + (I-\Lambda_k)D\Omega_{k-1}]\delta\tilde{u}_{k-1}\\
&\quad + \Gamma(I-\Lambda_k)D(I-\Omega_{k-1})\delta\tilde{u}_{k-2}.
\end{aligned} \tag{21}$$

By (21) and $\delta\tilde{u}_k = u_d - \tilde{u}_k$, it follows that

$$\begin{aligned}\delta\tilde{u}_{k+1} = &\left(I - \Gamma\Lambda_k D\Omega_k\right)\delta\tilde{u}_k - \Gamma\Lambda_k\left[g(x_d) - g(x_k)\right]\\ &- \Gamma\left(I - \Lambda_k\right)\left[g(x_d) - g(x_{k-1})\right]\\ &- \Gamma\left[\Lambda_k D\left(I - \Omega_k\right) + \left(I - \Lambda_k\right)D\Omega_{k-1}\right]\delta\tilde{u}_{k-1}\\ &- \Gamma\left(I - \Lambda_k\right)D\left(I - \Omega_{k-1}\right)\delta\tilde{u}_{k-2}.\end{aligned} \tag{22}$$

By Definition 2.2 and the properties (P1), (P2) and (P4), we have

$$\begin{aligned}|\delta\tilde{u}_{k+1}| \prec\; &|I - \Gamma\Lambda_k D\Omega_k||\delta\tilde{u}_k| + |\Gamma||\Lambda_k D(I - \Omega_k)\\ &+ (I - \Lambda_k)D\Omega_{k-1}||\delta\tilde{u}_{k-1}|\\ &+ |\Gamma||\Lambda_k||g(x_d) - g(x_k)|\\ &+ |\Gamma||I - \Lambda_k||g(x_d) - g(x_{k-1})|\\ &+ |\Gamma||I - \Lambda_k||D||I - \Omega_{k-1}||\delta\tilde{u}_{k-2}|.\end{aligned} \tag{23}$$

Notice that the stochastic variable $|I - \Gamma\Lambda_k D\Omega_k|$ is independent on $|\delta\tilde{u}_k|$, $|\Lambda_k D(I - \Omega_k) + (I - \Lambda_k)D\Omega_{k-1}|$ is independent on $|\delta\tilde{u}_{k-1}|$, $|\Lambda_k|$ is independent on $|g(x_d) - g(x_k)|$ as well as $|I - \Lambda_k|$ is independent on $|g(x_d) - g(x_{k-1})|$ and $|I - \Lambda_k|$ is independent on $|\delta\tilde{u}_{k-2}|$. Calculating the expectation on both sides of (23) and taking the property (P7) into consideration give rise to

$$\begin{aligned}E\left\{\left|\delta\tilde{u}_{k+1}\right|\right\} \prec\; &E\left\{|I - \Gamma\Lambda_k D\Omega_k|\right\}E\left\{\left|\delta\tilde{u}_k\right|\right\}\\ &+ |\Gamma|\,E\left\{|\Lambda_k|\right\}E\left\{\left|g(x_d) - g(x_k)\right|\right\}\\ &+ |\Gamma|\left(E\left\{|\Lambda_k|\right\}|D|\,E\left\{|I - \Omega_k|\right\}\right.\\ &\left.+ E\left\{|I - \Lambda_k|\right\}|D|\,E\left\{|\Omega_{k-1}|\right\}\right)E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\\ &+ |\Gamma|\,E\left\{|I - \Lambda_k|\right\}E\left\{\left|g(x_d) - g(x_{k-1})\right|\right\}\\ &+ |\Gamma|\,E\left\{|I - \Lambda_k|\right\}|D|\,E\left\{|I - \Omega_{k-1}|\right\}E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}.\end{aligned} \tag{24}$$

Computing 1-norm on both sides of (24) and considering the properties (P3), (P5) and (P6) as well as the facts that $\|E\{|I - \Lambda_k|\}\|_1 = 1 - \bar{\alpha}$, $\|E\{|\Lambda_k|\}\|_1 = \bar{\alpha}$ and $\|E\{|I - \Omega_k|\}\|_1 = 1 - \bar{\omega}$ achieve

$$\begin{aligned}\left\|E\left\{\left|\delta\tilde{u}_{k+1}\right|\right\}\right\|_1 \le\; &\|E\left\{|I - \Gamma\Lambda_k D\Omega_k|\right\}\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\\ &+ \bar{\alpha}\|\Gamma\|_1\left\|E\left\{\left|g(x_d) - g(x_k)\right|\right\}\right\|_1\\ &+ \|\Gamma\|_1\left(\bar{\alpha}(1 - \bar{\omega}) + (1 - \bar{\alpha})\bar{\omega}\right)\\ &\times \|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1\\ &+ \|\Gamma\|_1(1 - \bar{\alpha})(1 - \bar{\omega})\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1\\ &+ \|\Gamma\|_1(1 - \bar{\alpha})\|E\{|g(x_d)\\ &- g(x_{k-1})|\}\|_1.\end{aligned} \tag{25}$$

From that $\|E\{|g(x_d) - g(x_k)|\}\|_1 = E\{\|g(x_d) - g(x_k)\|_1\}$ and the function g is Lipschitz continuous, we get

$$\begin{aligned}\left\|E\left\{\left|g(x_d) - g(x_k)\right|\right\}\right\|_1 &\le K_g E\left\{\|x_d - x_k\|_1\right\}\\ &= K_g\|E\left\{|x_d - x_k|\right\}\|_1.\end{aligned} \tag{26}$$

Substituting (26) into (25) results in

$$\begin{aligned}\left\|E\left\{\left|\delta\tilde{u}_{k+1}\right|\right\}\right\|_1 \le\; &\|E\left\{|I - \Gamma\Lambda_k D\Omega_k|\right\}\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\\ &+ \bar{\alpha}K_g\|\Gamma\|_1\|E\left\{|x_d - x_k|\right\}\|_1\\ &+ \|\Gamma\|_1\left(\bar{\alpha}(1-\bar{\omega}) + (1-\bar{\alpha})\bar{\omega}\right)\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1\\ &+ \|\Gamma\|_1(1 - \bar{\alpha})(1 - \bar{\omega})\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1\\ &+ K_g\|\Gamma\|_1(1 - \bar{\alpha})\|E\left\{|x_d - x_{k-1}|\right\}\|_1.\end{aligned} \tag{27}$$

By making use of (5), (6) and (19), we reach

$$\begin{aligned}x_d - x_k &= A\left[f(x_d) - f(x_k)\right] + AB\left[u_d - u_k\right]\\ &= A\left[f(x_d) - f(x_k)\right] + AB\Omega_k\delta\tilde{u}_k + AB\left(I - \Omega_k\right)\delta\tilde{u}_{k-1}.\end{aligned} \tag{28}$$

By the above equality (28), Definition 2.2 and taking properties (P1), (P2) as well as (P4) into account, we obtain

$$\begin{aligned}|x_d - x_k| \prec\; &|A|\left|f(x_d) - f(x_k)\right| + |AB|\,|\Omega_k|\left|\delta\tilde{u}_k\right|\\ &+ |AB|\,|I - \Omega_k|\left|\delta\tilde{u}_{k-1}\right|.\end{aligned} \tag{29}$$

Taking expectation on both sides of (29) and considering the independence of the stochastic variable $|\Omega_k|$ on $|\delta\tilde{u}_k|$ and $|I - \Omega_k|$ on $|\delta\tilde{u}_{k-1}|$ yield

$$\begin{aligned}E\left\{|x_d - x_k|\right\} \prec\; &|A|\,E\left\{\left|f(x_d) - f(x_k)\right|\right\} + |AB|\,E\left\{|\Omega_k|\right\}\\ &\times E\left\{\left|\delta\tilde{u}_k\right|\right\} + |AB|\,E\left\{|I - \Omega_k|\right\}E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}.\end{aligned} \tag{30}$$

Taking 1-norm on both sides of (30) and benefiting from properties (P3), (P5) and (P6) as well as the expressions $\|E\{|\Omega_k|\}\|_1 = \bar{\omega}$, $\|E\{|I - \Omega_k|\}\|_1 = 1 - \bar{\omega}$ and $\|A\|_1 = 1$, we get

$$\begin{aligned}&\|E\left\{|x_d - x_k|\right\}\|_1\\ &\quad\le \left\|E\left\{\left|f(x_d) - f(x_k)\right|\right\}\right\|_1 + \bar{\omega}\|AB\|_1\\ &\quad\times \left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1 + (1 - \bar{\omega})\|AB\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1.\end{aligned} \tag{31}$$

Considering the fact that $\|E\{|f(x_d) - f(x_k)|\}\|_1 = E\{\|f(x_d) - f(x_k)\|_1\}$ and the function f is Lipschitz continuous, we have

$$\left\|E\left\{\left|f(x_d) - f(x_k)\right|\right\}\right\|_1 \le K_f\|E\left\{|x_d - x_k|\right\}\|_1. \tag{32}$$

Substituting (32) into (31) yields

$$\begin{aligned}\|E\left\{|x_d - x_k|\right\}\|_1 \le\; &K_f\|E\left\{|x_d - x_k|\right\}\|_1 + \bar{\omega}\|AB\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\\ &+ (1 - \bar{\omega})\|AB\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1.\end{aligned} \tag{33}$$

From condition (C2) and (33), it achieves

$$\begin{aligned}\|E\left\{|x_d - x_k|\right\}\|_1 \le\; &\bar{\omega}\frac{\|AB\|_1}{1 - K_f}\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\\ &+ (1 - \bar{\omega})\frac{\|AB\|_1}{1 - K_f}\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1.\end{aligned} \tag{34}$$

Furthermore, substituting (34) into (27) leads to

$$\left\|E\left\{\left|\delta\tilde{u}_{k+1}\right|\right\}\right\|_1 \le \|E\left\{|I - \Gamma\Lambda_k D\Omega_k|\right\}\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1$$

$$+\|\Gamma\|_1(1-\bar{\alpha})(1-\bar{\omega})\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1$$
$$+\|\Gamma\|_1\left(\bar{\alpha}(1-\bar{\omega})+(1-\bar{\alpha})\bar{\omega}\right)\|D\|_1$$
$$\times\|E\{|\delta\tilde{u}_{k-1}|\}\|_1+\bar{\alpha}K_g\|\Gamma\|_1$$
$$\times\left(\bar{\omega}\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\right.$$
$$\left.+(1-\bar{\omega})\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1\right)$$
$$+K_g\|\Gamma\|_1(1-\bar{\alpha})\left(\bar{\omega}\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1\right.$$
$$\left.+(1-\bar{\omega})\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1\right)$$
$$=\left(\|E\{|I-\Gamma\Lambda_k D\Omega_k|\}\|_1\right.$$
$$\left.+\|\Gamma\|_1\bar{\alpha}\bar{\omega}\frac{K_g\|AB\|_1}{1-K_f}\right)\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1$$
$$+\|\Gamma\|_1\left(\bar{\alpha}(1-\bar{\omega})+(1-\bar{\alpha})\bar{\omega}\right)\Phi\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1$$
$$+\|\Gamma\|_1(1-\bar{\alpha})(1-\bar{\omega})\Phi\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1$$
$$=\rho_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1+\rho_2\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1$$
$$+\rho_3\left\|E\left\{\left|\delta\tilde{u}_{k-2}\right|\right\}\right\|_1, \tag{35}$$

where $\Phi=\frac{K_g||AB||_1}{1-K_f}+||D||_1$.

Then, according to condition (C1) in Theorem 4.1 and Lemma 4.1, the recursive relation (35) leads to

$$\lim_{k\to\infty}\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1=0. \tag{36}$$

From (20), Definition 2.2, and the properties (P1), (P2) and (P4), it follows

$$\left|\delta y_k\right|\prec\left|g(x_d)-g(x_k)\right|+|D|\,\Omega_k\left|\delta\tilde{u}_k\right|+|D|\left(I-\Omega_k\right)\left|\tilde{u}_{k-1}\right|. \tag{37}$$

Taking the expectation on both sides of (37) and taking the independence of the stochastic variables into consideration, we find

$$E\left\{\left|\delta y_k\right|\right\}\prec E\left\{\left|g(x_d)-g(x_k)\right|\right\}+|D|\,E\{|\Omega_k|\}\,E\left\{\left|\delta\tilde{u}_k\right|\right\}$$
$$+|D|\,E\{|I-\Omega_k|\}\,E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}. \tag{38}$$

Taking 1-norm on both sides of (38) and taking $\|\{|I-\Omega_k|\}\|_1=1-\bar{\omega}$, $\|E\{|\Omega_k|\}\|_1=\bar{\omega}$ and (26) into consideration, it induces

$$\left\|E\left\{\left|\delta y_k\right|\right\}\right\|_1\le K_g\|E\{|x_d-x_k|\}\|_1+\bar{\omega}\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1$$
$$+(1-\bar{\omega})\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1. \tag{39}$$

Substituting (34) into (39) produces

$$\left\|E\left\{\left|\delta y_k\right|\right\}\right\|_1\le K_g\left(\bar{\omega}\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1\right.$$
$$\left.+(1-\bar{\omega})\frac{\|AB\|_1}{1-K_f}\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1\right)$$
$$+\bar{\omega}\|D\|_1\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1+(1-\bar{\omega})\|D\|_1$$
$$\times\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1$$
$$=\bar{\omega}\Phi\left\|E\left\{\left|\delta\tilde{u}_k\right|\right\}\right\|_1+(1-\bar{\omega})\Phi\left\|E\left\{\left|\delta\tilde{u}_{k-1}\right|\right\}\right\|_1. \tag{40}$$

Hence, by (36) and (40), it follows

$$\lim_{k\to\infty}\left\|E\left\{\left|\delta y_k\right|\right\}\right\|_1=0, \tag{41}$$

that is,

$$\lim_{k\to\infty}E\left\{\left\|\delta y_k\right\|_1\right\}=0. \tag{42}$$

This completes the proof. □

Remark 4.1. From Lemma 4.1 and the inequality (35), it is seen that

$$E\left\{\left\|\delta\tilde{u}_{3k+i}\right\|_1\right\}\le\rho^k\max\left\{E\left\{\left\|\delta\tilde{u}_1\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_2\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_3\right\|_1\right\}\right\},$$
$$\left(\forall k\in N^*,\ i=1,2,3\right)$$

and

$$\max\left\{E\left\{\left\|\delta\tilde{u}_{n+5}\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_{n+4}\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_{n+3}\right\|_1\right\}\right\}$$
$$\le\rho\max\left\{E\left\{\left\|\delta\tilde{u}_{n+2}\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_{n+1}\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_n\right\|_1\right\}\right\},$$
$$\left(\forall n\in N^*\right).$$

The above relationship among the expectations of the system control errors conveys that $E\{\|\delta\tilde{u}_k\|_1\}$ is 3-iteration-wise-monotonously convergent. Meanwhile, there hold the following estimation formulae.

$$E\left\{\left\|\delta y_{3k+1}\right\|_1\right\}\le\rho^{k-1}\Phi\max\left\{E\left\{\left\|\delta\tilde{u}_1\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_2\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_3\right\|_1\right\}\right\},$$
$$E\left\{\left\|\delta y_{3k+2}\right\|_1\right\}\le\rho^k\Phi\max\left\{E\left\{\left\|\delta\tilde{u}_1\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_2\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_3\right\|_1\right\}\right\},$$
$$E\left\{\left\|\delta y_{3k+3}\right\|_1\right\}\le\rho^k\Phi\max\left\{E\left\{\left\|\delta\tilde{u}_1\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_2\right\|_1\right\},E\left\{\left\|\delta\tilde{u}_3\right\|_1\right\}\right\}.$$

Remark 4.2. As the proposed NILC scheme (4) adopts the estimation formulation (3) which replaces the delayed data by the captured data at the concurrent sampling instant of the previous iteration, the influence of the random communication delay on the tracking performance may be completely swept away. This benefits from the time point-point mapping of NILC mechanism. But, for the scheme in Bu et al. (2013) which approximates the delayed data by its latest reached data at the concurrent iteration, the influence of the random communication delay may not be wiped away completely. Further, it is observed that in the case when the communication delay vanishes, the proposed iterative learning process is strictly monotonically convergent in the sense of 1-norm, that is to say, $\|\delta\tilde{u}_k\|_1=\sum_{t=0}^{T-1}|u_d(t)-\tilde{u}_k(t)|$ converges monotonically to zero.

Remark 4.3. It should be pointed out that the convergent condition (C1) does not always hold for any successful transmission probability. In other words, (C1) is the constraints on successful transmission probability.

Remark 4.4. It is noticed that condition (C2) $K_f<1$ is sufficient to guarantee the convergence. This does not mean that the condition is necessary though some NILC schemes are effective. In authors' simulation experience, the proposed NILC may be effective for some systems where condition (C2) does not hold.

This implies that condition (C2) for guaranteeing the convergence is rigorous. It is inspiring to seek a weaker condition to ensure the convergence.

5. Numerical simulations

Consider the following nonlinear system

$$\begin{cases} x_k(t+1) = \frac{1}{6}\sin(x_k(t)) + u_k(t), \\ y_k(t) = \frac{1}{6}\cos(x_k(t)) + u_k(t). \end{cases}$$

The operation time interval is set as $S = \{0, 1, 2, \ldots, 60\}$, the initial state is set as $x_k(0) = 0$ and the desired trajectory is given as $y_d(t) = \cos(\frac{\pi t}{30})$, $t \in S$. The control profile of the beginning iteration is $u_1(t) = 0$ for $t \in S$. The learning gain of the NILC scheme (4) is restricted to $\Gamma \in (0, 1]$. In addition, it is easy to compute that the maximal absolute values of the derivatives of functions $\frac{1}{6}\sin(x)$ and $\frac{1}{6}\cos(x)$ are $\frac{1}{6}$. This implies that $K_f = K_g = \frac{1}{6}$ which guarantees that the convergent condition (C2) holds. Notice that $B = D = I$, which implies $\|B\|_1 = \|D\|_1 = 1$, the convergent factor in Theorem 4.1 is $\rho = \rho_1 + \rho_2 + \rho_3 = 1 - 2\Gamma[\bar{\alpha}\bar{\omega} - \frac{3}{5}]$ and the convergent factor in Bu et al. (2013) is $\tilde{\rho} = 1 - \Gamma\bar{\alpha}\bar{\omega}$. By definition, the tracking error is $\|\delta y_k\|_1 = \sum_{t=0}^{60} |y_d(t) - y_k(t)|$. For numerical simulation, we evaluate the expectation of the tracking errors as the average of tracking errors expressed as $\frac{1}{100}\sum_{i=1}^{100}(\sum_{t=0}^{60} |y_d(t) - y_k^i(t)|)$ for the NILC scheme to be executed for 100 runs, where $y_k^i(t)$ is the system output at the kth iteration of the ith run and the terminology 'one run' means that the proposed NILC algorithm (4) with (3) or (4) with (3*) is executed till a perfect tracking is achieved. For the purpose of simplifying the comparative statements of the simulation results performed by the developed scheme in this paper with that done in Bu et al. (2013) , the following context abbreviates the NILC scheme (4) with the proposed estimation formula (3) as scheme I, whilst the scheme (4) with the existing formula (3*) is abbreviated as scheme II.

5.1 Comparative tracking performance of scheme I with scheme II

Case 1. Set $\Gamma = \frac{1}{3}, \bar{\alpha} = \bar{\omega} = 1$.

The case means that the probabilities of the communication delays are zero. It is testified that both the convergent conditions (C1) $\rho = \frac{11}{15} < 1$ in Theorem 4.1 and $\tilde{\rho} = \frac{2}{3} < 1$ in Bu et al. (2013) hold. The tendencies of the tracking errors $\|\delta y_k\|_1$ driven by the NILC schemes I and II along the iteration axis are shown in Figure 2, where the solid curve refers to the tracking error driven by scheme I whilst the dashed curve stands for the tracking error driven by scheme II. Figure 3 displays the system outputs driven by schemes I and II at the sixth iteration, where the circle-solid curve is the desired trajectory, the solid one presents the system output driven by scheme I and the dashed one refers to the system output driven by scheme II. From Figures 2 and 3, it is seen that both schemes I and II are equivalently efficient.

Case 2. Set $\Gamma = \frac{1}{3}, \bar{\alpha} = \bar{\omega} = 0.9$.

The case represents that the probabilities of the random communication delays are 0.1. It is testified that the convergent conditions $\rho = 0.86 < 1$ and $\tilde{\rho} = 0.73 < 1$ are guaranteed. The tracking errors driven by schemes I and II are shown in Figure 4, where the solid curve stands for the tracking error driven by scheme I whilst the dashed curve refers to the tracking error driven by scheme II, which exhibits that the tracking error produced by scheme I is convergent to zero whilst the tracking error made by scheme II is bounded but non-zero as the iteration goes on.

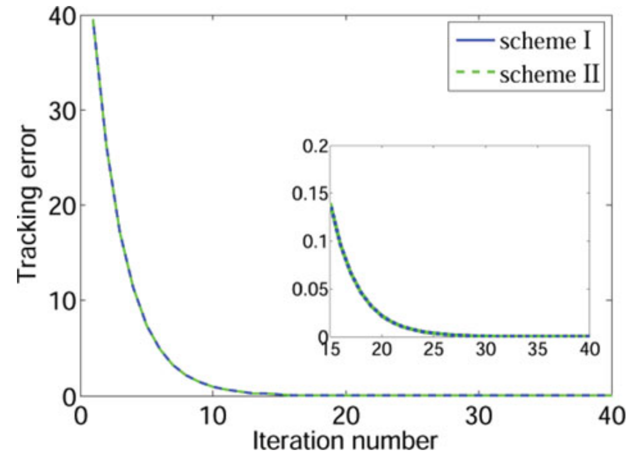

Figure 2. Tracking errors with null communication delay probabilities.

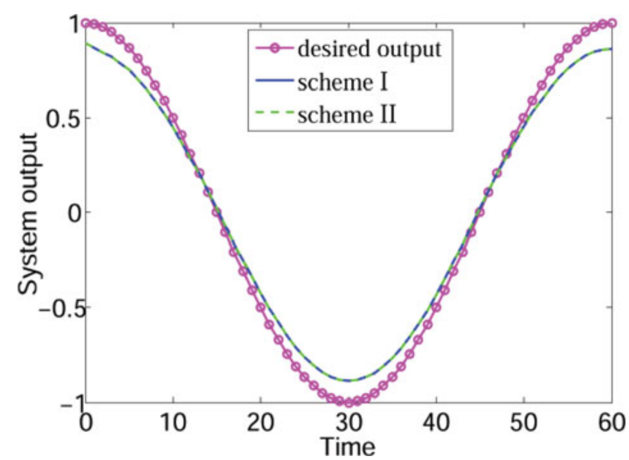

Figure 3. System outputs at the sixth iteration with null communication delay probabilities.

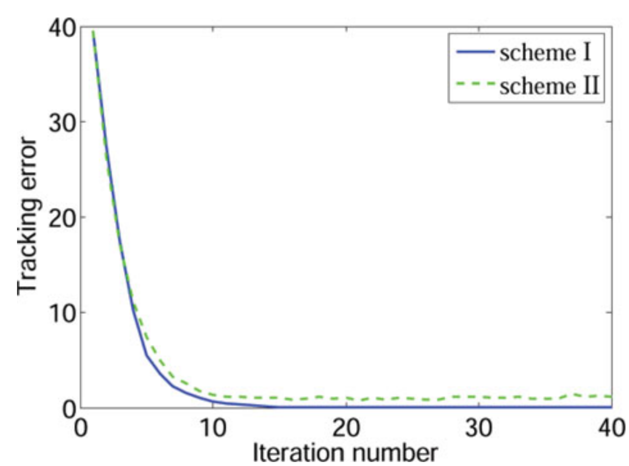

Figure 4. Tracking errors with 0.1 communication delay probabilities.

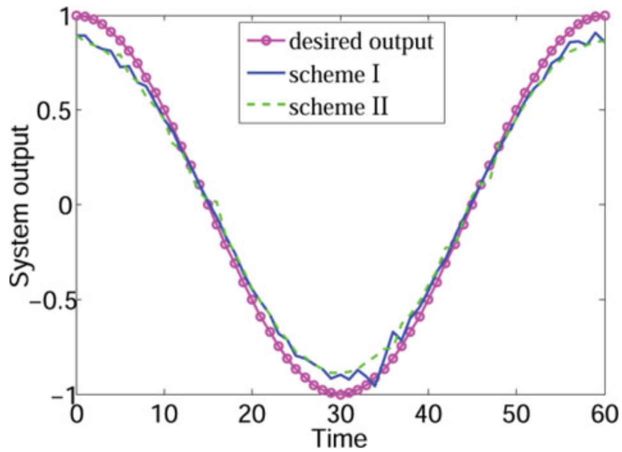

Figure 5. System outputs at the sixth iteration with 0.1 communication delay probabilities

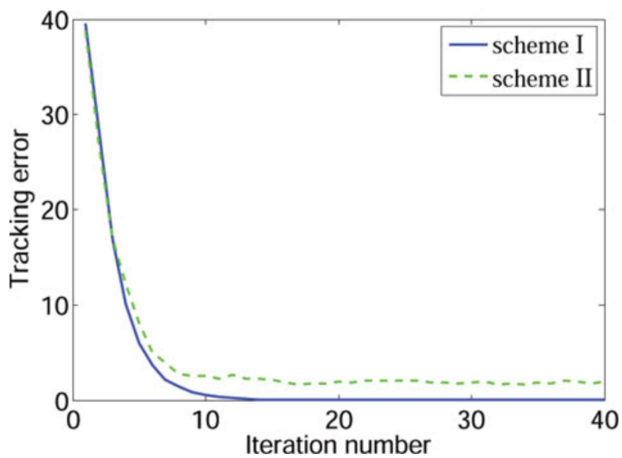

Figure 6. Tracking errors with 0.2 communication delay probabilities.

The system outputs driven by the two schemes at the sixth iteration are displayed in Figure 5, where the circle-solid curve is the desired output, the solid one refers to the system output driven by scheme I and the dashed one stands for the system output driven by scheme II, from which it is seen that both schemes track the desired trajectory well and scheme I performs slightly better.

Case 3. Set $\Gamma = \frac{1}{3}, \bar{\alpha} = \bar{\omega} = 0.8$.

The case tells that the probabilities of the random communication delays are 0.2. By simple computing, it is easy to verify that the convergent conditions $\rho = \frac{73}{75} < 1$ and $\tilde{\rho} = \frac{59}{75} < 1$ are ensured. The tracking errors driven by the two schemes are depicted in Figure 6, where the solid curve is the tracking error driven by scheme I whilst the dashed curve refers to tracking error driven by scheme II. Analogously, Figure 6 shows that the tracking error of scheme I vanishes whilst that of scheme II lies at a nonzero level as the iteration increases. Figure 7 shows the system outputs driven by the two schemes at the sixth iteration, where the circle-solid curve is the desired output, the solid curve stands for the system output driven by scheme I and the dashed one presents the system output driven by scheme II. It is worth to observe from Figure 7 that scheme I tracks the desired trajectory closer than the done by scheme II.

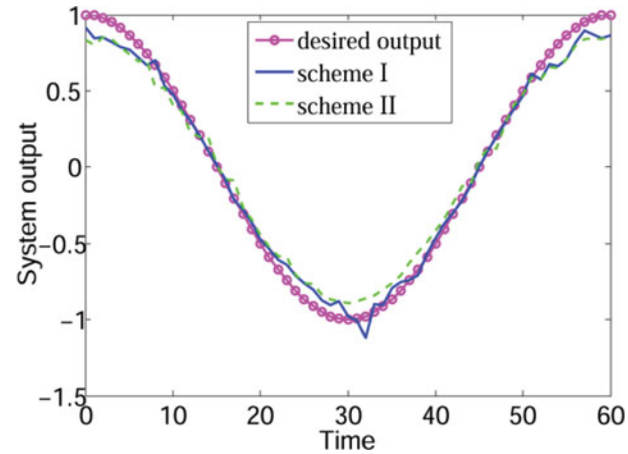

Figure 7. System outputs at the sixth iteration with 0.2 communication delay probabilities.

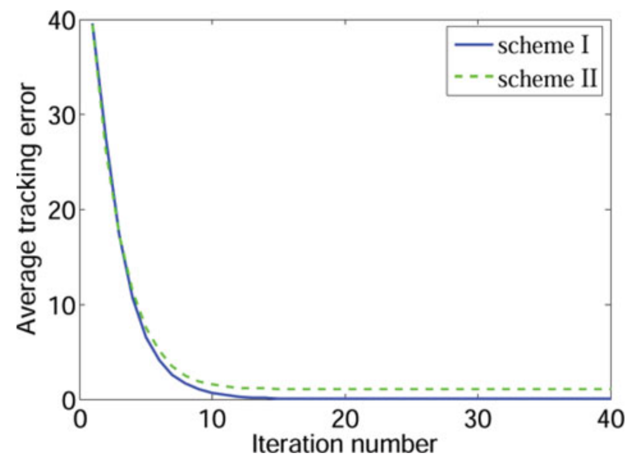

Figure 8. 100-run averages of the tracking errors with 0.1 communication delay probabilities.

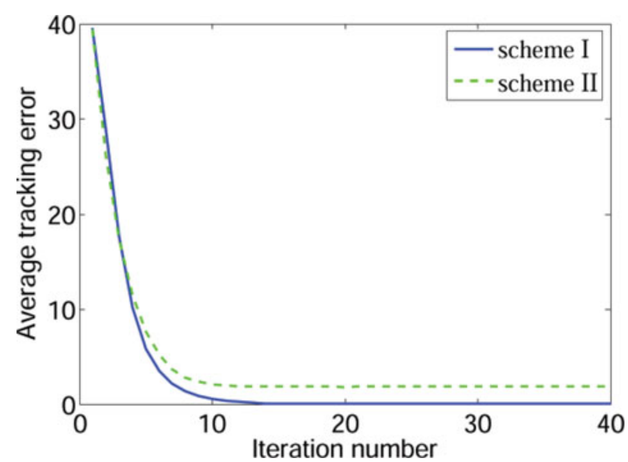

Figure 9. 100-run averages of the tracking errors with 0.2 communication delay probabilities.

Figure 8 exhibits the 100-run average of tracking errors driven by schemes I and II with 0.1 communication delay probabilities and Figure 9 shows the 100-run average of the tracking errors driven by the two schemes with communication delay

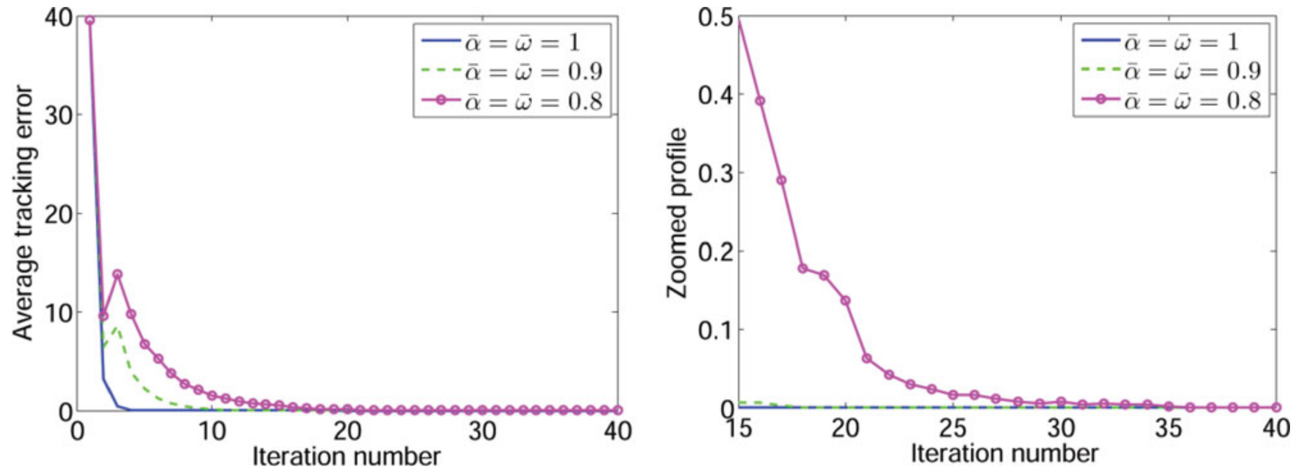

Figure 10. 100-run average of the tracking error with $\Gamma = 1$ and different communication delay probabilities.

probabilities being 0.2, where the solid curve refers to the average of tracking errors driven by scheme I whilst the dashed one stands for the average of tracking errors driven by scheme II.

It is observed from Figures 4, 6, 8 and 9 that the tracking errors driven by scheme I converge to zero whilst the tracking errors driven by scheme II remain in a nonzero level as the iteration evolves. This means that scheme I owns a better tracking performance compared with scheme II.

5.2 Tracking performance of scheme I with diverse communication delay probabilities

Case 1. Set $\Gamma = 1, \bar{\alpha} = \bar{\omega} = 1, \bar{\alpha} = \bar{\omega} = 0.9$ and $\bar{\alpha} = \bar{\omega} = 0.8$.

It is testified that the convergent conditions in Theorem 4.1 of those three groups of probabilities are $\rho = 0.2$, $\rho = 0.58$ and $\rho = 0.92$. In Figure 10, the solid curve is the average of tracking error of the scheme with zero communication delay probabilities, whilst the dashed and circle-solid ones mark the average of tracking errors with communication delay probabilities 0.1 and 0.2, respectively, which shows that scheme I with larger delay probabilities renders bigger tracking error.

Case 2. Set $\Gamma = \frac{1}{3}, \bar{\alpha} = \bar{\omega} = 1, \bar{\alpha} = \bar{\omega} = 0.9$ and $\bar{\alpha} = \bar{\omega} = 0.8$.

It is calculated that the convergent conditions in Theorem 4.1 are $\rho = \frac{11}{15}$, $\rho = 0.86$ and $\rho = \frac{73}{75}$. Figure 11 depicts the average of tracking errors stimulated by scheme I with communication delay probabilities being 0, 0.1 and 0.2, which presents that the scheme with larger delay probabilities performs with smaller tracking error after the third iteration.

Case 3. Set $\Gamma = 0.5$, $\bar{\alpha} = \bar{\omega} = 1$, $\bar{\alpha} = \bar{\omega} = 0.9$ and $\bar{\alpha} = \bar{\omega} = 0.8$.

It is computed that the convergent conditions in Theorem 4.1 are $\rho = 0.6$, $\rho = 0.79$ and $\rho = 0.96$. Figure 12 shows the average tracking errors driven by the proposed NILC scheme I with communication delay probabilities being 0, 0.1 and 0.2. It is observed from Figure 12 that the average of tracking error of scheme I with larger communication delay probabilities 0.2 is still larger than that of the scheme with delay probabilities nullity and 0.1, but the error of the scheme with the probabilities 0.1 is smaller than the one with nullity probabilities after the 10th iteration. This implies that the scheme with relatively larger delay probabilities may incur either larger or smaller tracking error.

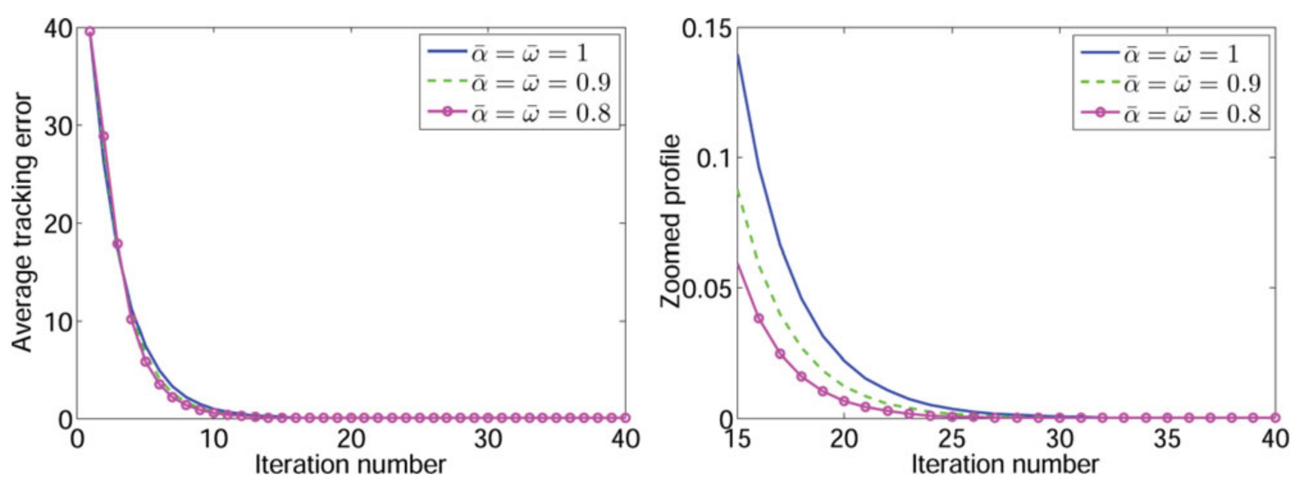

Figure 11. 100-run average of the tracking error with $\Gamma = 1/3$ and different communication delay probabilities.

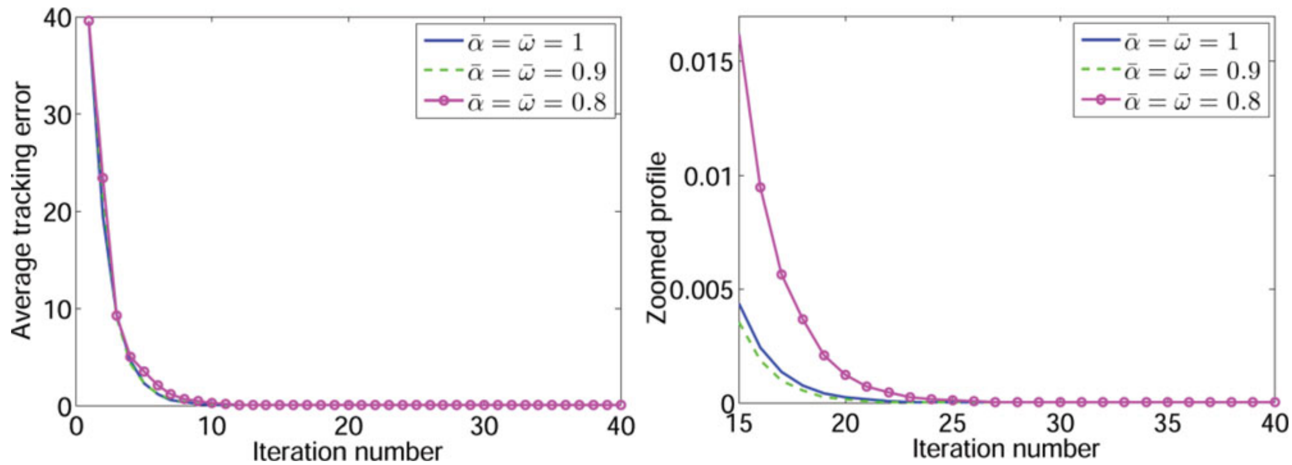

Figure 12. 100-run average of the tracking error with $\Gamma = 0.5$ and different communication delay probabilities.

Overall, from Figures 10–12 we may not assert that for a chosen learning gain the scheme with larger communication delay probabilities delivers larger tracking error.

6. Conclusion

In this paper, a P-type NILC scheme for a class of nonlinear systems with Bernoulli-type communication delay is constructed. By means of evaluating expectation of the tracking error in the sense of 1-norm, the convergence is analysed for the general circumstance. The influence of the communication delay can eventually be wiped off and perfect tracking performance can be achieved. The analysis conveys that, under appropriate assumptions with respect to the systems dynamics and the learning gain, the tracking error is asymptotically convergent to zero. However, it is still a challenging to investigate the tracking performance of the NILC scheme for MIMO systems.

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