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Optimal Multi-objective Number, Locating, and Sizing of Distributed Generations and Distributed Static Compensators Considering Loadability using the Genetic Algorithm

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1. INTRODUCTION

Recently, distribution systems (DSs) have increasingly found more importance as they can play essential roles in power system quality and planning [1, 2]. Due to the inherent high R/X ratio and significant voltage drop of DSs, these systems have high losses. Indeed, the major part of losses in a power system belongs to DSs, which are approximated in previous literature at about 13% [3]. In addition, DSs usually have radial feeders. Therefore, the electrical demand increment and DS development will result in more voltage drop and power losses [4]. Complete utilization of the line capacity in these networks is not possible for several reasons [5, 6]. Generally, one of the factors that limit the capacity of a DS is voltage limits [7]. The voltage limits determine the system loadability. The load increment of a DS is allowable until the voltages of the systems maintain acceptable voltage limits [8]. To improve these limitations, it is necessary to use advanced
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{DG-inv}$</td>
<td>distributed generation investment cost ($/kW$)</td>
</tr>
<tr>
<td>$C_{DG-O&amp;M}$</td>
<td>distributed generation operation and maintenance cost ($/kWh$)</td>
</tr>
<tr>
<td>$C_{DSTATCOM-inv}$</td>
<td>distributed static compensator investment cost ($/kVAR$)</td>
</tr>
<tr>
<td>$C_{loss}$</td>
<td>cost of losses ($$/kWh$)</td>
</tr>
<tr>
<td>$i$</td>
<td>index of bus number</td>
</tr>
<tr>
<td>$I$</td>
<td>branch current (p.u.)</td>
</tr>
<tr>
<td>$I_{DSTATCOM}$</td>
<td>distributed static compensator current (p.u.)</td>
</tr>
<tr>
<td>$OF_{cost}$</td>
<td>objective function of costs ($)</td>
</tr>
<tr>
<td>$P$</td>
<td>local active load of bus (p.u.)</td>
</tr>
<tr>
<td>$P_0$</td>
<td>nominal active load (kW)</td>
</tr>
<tr>
<td>$P_{loss}$</td>
<td>reactive power loss (kW)</td>
</tr>
<tr>
<td>$Q$</td>
<td>local reactive load of bus (p.u.)</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>nominal reactive load (kVAR)</td>
</tr>
<tr>
<td>$Q_{DSTATCOM}$</td>
<td>distributed static compensator injected reactive power (p.u.)</td>
</tr>
<tr>
<td>$Q_{max}$</td>
<td>maximum distributed static compensator capacity (kVAR)</td>
</tr>
<tr>
<td>$Q_{min}$</td>
<td>minimum distributed static compensator capacity (kVAR)</td>
</tr>
<tr>
<td>$R$</td>
<td>branch resistance (p.u.)</td>
</tr>
<tr>
<td>$S_{DG}$</td>
<td>distributed generation capacity (kVA)</td>
</tr>
<tr>
<td>$S_{max}$</td>
<td>maximum distributed generation capacity (kVA)</td>
</tr>
<tr>
<td>$S_{min}$</td>
<td>minimum distributed generation capacity (kVA)</td>
</tr>
<tr>
<td>$U$</td>
<td>bus voltage (p.u.)</td>
</tr>
<tr>
<td>$U'$</td>
<td>bus voltage in presence of distributed static compensator (p.u.)</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>maximum acceptable voltage (p.u.)</td>
</tr>
<tr>
<td>$U_{min}$</td>
<td>minimum acceptable voltage (p.u.)</td>
</tr>
<tr>
<td>$X$</td>
<td>branch reactance (p.u.)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>angle of branch current (rad)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angle of bus voltage (rad)</td>
</tr>
<tr>
<td>$\theta'$</td>
<td>angle of bus voltage in presence of distributed static compensator (rad)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>nominal system loadability</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>maximum loadability</td>
</tr>
</tbody>
</table>

Equipment. Power electronic devices and modern techniques, such as flexible AC transmission systems (FACTS) and distributed generation (DG), have improved considerably in these issues. The development of such networks requires active and reactive power provisions by appropriate sources. These are elements that are used for loss reduction, power quality improvement, and loadability improvement in DSs. Therefore, it is necessary to identify the size, location, and amount of required equipment for installation in an electrical distribution network in order to improve the performance of the network. It has been a challenging issue for power system researchers to find the proper location and size of DGs and distributed static compensator (DSTATCOMs) in a DS. Following is a review of some categorized research.

#### 1.1. Optimal Dstatcom Locating and Sizing without DG

The DSTATCOM as a shunt-connected voltage source converter (VSC) is frequently employed to improve power quality concerns. During overloading and voltage sag, the load voltage of the bus in which the DSTATCOM is connected can be regulated by compensating current injection into the system [9]. The injected current is evaluated using bus voltage and reactive power required for balancing/regulating voltage in the desired value. The design of a prototype DSTATCOM for voltage sag mitigation in an unbalanced DS was investigated in [10]. In [11], the optimal sizing and placement of a DSTATCOM using an immune algorithm was studied. In [12], DSTATCOM performance using a control strategy based on the improved instantaneous active and reactive current component theory to generate reference currents was evaluated under various source and load conditions. The proposed control strategy performance was evaluated in terms of load balancing, reactive power compensation, compensator rating, and harmonic mitigation. A practical example of the use of STATCOMs to reduce voltage fluctuations was given in [13]. In [14], a new sensitivity index was developed for the placement of STATCOMs to ensure fast voltage recovery at all buses of interest. A number of works have been carried out on the optimal location of a STATCOM using optimization techniques, such as particle swarm optimization (PSO) or genetic algorithm (GA) [15-16], and on optimal allocation and sizing of a STATCOM using PSO [17].

#### 2.2. Optimal DG Locating and Sizing without DSTATCOM

In [18], an analytical approach was been proposed by Wang and Nehrir to find the optimal DG location in DSs considering different load patterns in order to reduce the real power loss. In [19], a sensitivity analysis method on the basis of power flow equations was used to find the optimal places and sizes...
of DGs; in that study, two indices were considered at each bus of the network to sit the DGs. In [20], the primal–dual interior point (IP) method was applied for optimal DG placement considering the both voltage profile improvement and line loss reduction indices. However, the accurate results of the proposed approach depended on the initial location of DGs at all of the buses; hence, this approach would not be applicable in large power networks. In [21], only the DG sizing problem using the generalized reduced-gradient method was treated. The main objective of this study has been loss minimization.

In [22], to optimize the location of DG, the GA method was used to maximize a predefined index of DG’s benefit–cost ratio considering the both voltage drop and feeder transfer capacity. In [23], DG sizing and placement were been studied using a hybrid technique. Voltage profile, loss reduction, and voltage stability were considered as objective functions to find optimal DG places using a hybrid GA and PSO. In the research, shunt capacitance has been neglected in the overhead lines studies.

Some studies have been conducted on the impact of DG on loadability and its improvement [24, 25]. It has been shown that the system loadability is increased in the presence of DGs. In [24], an eigenvalue-based method was proposed to find the optimum place and size of DG to improve the loadability. The continuation power flow (CPF) approach was utilized in [25] to sit the DG on the bus that has the weakest voltage.

1.3. Optimal DG and DSTATCOM Simultaneous Locating and Sizing

Due to development of DSs, the limitation of active and reactive powers of DG sources, increments in demands and loads, and high installation cost, the usage of DG sources is limited. Therefore, it is essential to use an alternative element, such as a DSTATCOM for which the implementation cost is lower than DGs, to improve voltage profile, losses, and power factor in DSs. Placement and sizing of DGs and DSTATCOMs should be studied simultaneously in the optimization procedure to reach the optimal operation of a power system.

In [26], the optimal placement and sizing of DGs and a DSTATCOM with the aim of reducing total power loss was studied using PSO. In [27], simultaneous reconfiguration and optimal placement of a DSTATCOM and DGs was presented using fuzzy and ant colony optimization (ACO) to reduce losses and voltage profile improvement. Both of the aforementioned studies were conducted as single-objective optimization and loadability, and Pareto analysis was been considered.

It should be noted that the optimal number of DGs and DSTATCOMs is an important issue in the investment cost of installed devices in the system, which has not been considered in the previously mentioned literature. In those studies, the researchers considered a determined number of these two devices in their simultaneous locating and sizing, for example, two DGs and two DSTATCOMs or three of each; this issue is considered herein as well. The investment costs are inserted in the objective function to determine the optimal number of devices for system cost minimization. If too many devices are installed, the system costs will be increased, and if a few devices are installed, the other aims of the article, such as loadability improvement or loss reduction, cannot be supported. Therefore, the number of each device that should be installed is very important and is considered in this article in the objective function.

There are several optimization algorithms that have been introduced and adapted to engineering problems [28–33]; however, having no need for setting up normalizing factors is one of the most important advantages of the Pareto front approach [34]. Different objective functions are orthogonal with each other, and the non-dominated solutions have no dependency on the units associated with each individual objective. The Pareto front makes a decision more flexible and allows better adaptation to the best policies for each electrical distribution company. Thus, a set of solutions is available that ranges between one that minimizes total loss and device costs and another that improves the loadability, each satisfying the operational constraints.

In this article, the GA and Pareto optimization are used to evaluate the number, placement, and sizing of DSTATCOMs and DGs in DSs, simultaneously, according to system loadability and total cost, including the installation and operation and maintenance (O&M) costs of DGs and DSTATCOMs and the cost of losses. In this approach, the optimal solution has been investigated in two categories: continuous (DSTATCOMs and DGs size) and discrete (location and number of devices). The proposed multi-objective function is implemented on the IEEE 33-bus and 69-bus test systems, and the results are presented.

It can be concluded that the advantages of the proposed method herein are as follows:

- simultaneous locating and sizing of DGs and DSTATCOMs;
- consideration of operating, maintenance, and investment costs of devices;
- finding the optimal number of devices, which is very important from the system cost point of view;
- consideration of total loss of the system;
- consideration of loadability of the system;
consideration of operation of a 30-year period for the system;
adaption of a multi-objective function to the objectives and system; and
investigation of the proposed multi-objective function on IEEE 33- and 69-bus test systems.

The rest of the article is organized as follows. In Section 2, the implementation of the DSTATCOM in distribution load flow is explained. In Section 3, the problem formulation and constraints are introduced. The simulation results are presented in Section 4, and finally, the conclusion is presented in Section 5.

2. DSTATCOM IN DISTRIBUTION LOAD FLOW
A DSTATCOM is a shunt device that absorbs or injects both the active and reactive current at a point of common coupling (PCC) connection. The DSTATCOM is also a DC/AC converter consisting of a DC-link capacitor or a DC energy storage device that provides constant DC voltage and a three-phase pulse-width modulation (PWM) VSC bridge. All these are usually connected to the DS via a coupling transformer [35]. A DSTATCOM can work as a synchronous voltage source with a variable phase angle and magnitude. Hence, it is capable of controlling its voltage bus and correcting the power factor. Figure 1 shows a bus in a DS equipped with a proposed DSTATCOM.

In some short-circuit events or steady-state operation with heavy loading, a DSTATCOM typically injects an appropriate compensating current to the PCC connection, and thus, voltage at the load bus regulated by the DSTATCOM will be lifted close to a given value (for example, a normal value) [35, 36]. Generally, a DSTATCOM has the ability to exchange active and reactive power simultaneously. The amount of exchanging active power depends on the energy source capacity. In this article, only a DSTATCOM application for reactive power exchange is considered and active power exchange with the network is neglected.

Backward/forward sweep load flow calculations are used in this work, in combination with a proper steady-state model for DSTATCOMs as presented in [37]. Many DSs have radial structures that only feed from one side. A section of a sample DS is shown in Figure 2 [38] that assumes that a three-phase radial DS is in balance. The phasor diagram for Figure 2 is presented in Figure 3. The Kirchhoff’s Voltage Law (KVL) equation can be stated as

$$U_i + jX_i = U_{i+1} - (R_i + jX_i)I_i \angle \delta.$$  \hspace{1cm} (1)

The values of the variables are derived from the load flow. In traditional networks, the bus voltage is usually less than 1 p.u., in which case it is assumed that voltage of bus $i+1$ is also less than 1 p.u. A DSTATCOM device is installed in this study to compensate the voltage of bus $i+1$ to a desired value. The DSTATCOM is used for voltage regulation and loss reduction in the steady-state condition, as noted earlier, and can inject only reactive power to the system in this article. Consequently, $I_{DSTATCOM}$ must be kept in quadrature with respect to the system voltage. As shown in Figures 4 and 5, by installing a DSTATCOM in bus $i+1$, $I_i$ and $I_{DSTATCOM}$ flow simultaneously in the branch;

$$\angle I_{DSTATCOM} = \frac{\pi}{2} - \theta_{i+1},$$  \hspace{1cm} (2)
As seen from Eq. (4), there are two roots for variable $x$, and so two values are calculated for $\angle I_{\text{DSTATCOM}}$ and $|I_{\text{DSTATCOM}}|$. To determine the correct answer, the boundary conditions are considered in these roots:

$$U_{i+1}' = U_{i+1} \Rightarrow I_{\text{DSTATCOM}} = 0 \text{ and } \theta_{i+1}' = \theta_{i+1}. \quad (12)$$

Results show that $x = -\frac{\theta_{i+1}' + \sqrt{\Delta}}{2A}$ is the answer of Eq. (4); therefore, $\angle I_{\text{DSTATCOM}}$ can be determined as follows:

$$\angle I_{\text{DSTATCOM}} = \frac{\pi}{2} - \theta_{i+1}' = \frac{\pi}{2} - \sin^{-1} x. \quad (13)$$

The magnitude of the DSTATCOM current can be calculated as follows:

$$|I_{\text{DSTATCOM}}| = \frac{U_{i+1}' - \angle \theta_{i+1}' - a_1 - a_4 \sin \theta_{i+1}' - a_3 \cos \theta_{i+1}'}{-a_4 \sin \theta_{i+1}' - a_3 \cos \theta_{i+1}'} \quad (14)$$

Thus, $I_{\text{DSTATCOM}}$ and the PCC bus voltage are calculated using Eqs. (13), (14), and (3). Finally, the injected reactive power to the power system by the DSTATCOM, to correct the voltage of the connected bus up to $U_{i+1}'$, can be expressed as follows:

$$jQ_{\text{DSTATCOM}} = (U_{i+1}' - \angle \theta_{i+1}'). (I_{\text{DSTATCOM}} \angle \left(\frac{\pi}{2} - \theta_{i+1}'\right))^*. \quad (15)$$

such that the symbol $^*$ denotes a complex conjugate.

To evaluate load flow considering a DSTATCOM in a DS with any iteration in forward sweep, the magnitude of the compensated node voltage can be assumed to be any desired value (for example, 1 p.u.). The angle of the compensated node phase, reactive power, and also DSTATCOM size can now be calculated using the above equations. The updated voltage and injected reactive power by the DSTATCOM are used for continuing the forward sweep to determine load currents in the next backward sweep load flow. This procedure is repeated until load flow converges [39].

### 3. PROBLEM FORMULATION

#### 3.1. Objective Function

The number, place, and size of the DGs and DSTATCOMs are optimized using the non-linear multi-objective optimization procedure. The proposed multi-objective function has two parts—total costs and system loadability—which have conflicting behaviors. Improvement of loadability leads to increasing total cost, and minimization of the total costs causes loadability to decline. The multi-objective function is formulated taking into account the following objectives considering a period of 30 years.
3.1.1. Total Cost
The first part of the multi-objective function is all costs of the system. This objective function has three parts as shown in the following equation:

\[ OF_{\text{costs}} = OF_1 + OF_2 + OF_3, \]  

where each part is calculated as follows:

3.1.1.1. Total Power Losses. These two elements in the power system considering the equal and unequal constraints will cause loss reduction in the DS. To calculate the network real power loss,

\[ P_{\text{loss}} = \sum_{i=1}^n R_i |I_i|^2. \]  

Therefore, the cost of the power loss for a period of 30 years [11] can be obtained as the following equation:

\[ OF_1 = P_{\text{loss}} \times C_{\text{loss}} \times 8760 \times 30. \]  

3.1.1.2. DG O&M and Investment Cost. In this study, the power factors of all DGs are assumed to be 0.85 [4]. Therefore, the investment and O&M costs of DGs are calculated as follows:

\[ OF_2 = S_{DG} \times 0.85 \times C_{DG-\text{inv}} + S_{DG} \times 0.85 \times C_{DG-O&\text{M}} \times 8760 \times 30. \]  

3.1.1.3. DSTATCOM Investment Cost. To consider the investment cost of a DSTATCOM, the following equation should be calculated:

\[ OF_3 = Q_{\text{DSTATCOM}} \times C_{\text{DSTATCOM-\text{inv}}}. \]  

3.1.1.4. Loadability. To find the maximum loadability of the system, all the active and reactive loads are increased in the power flow problem in the 4th step as follows:

\[ P_i = \lambda_i \times P_0, \quad Q_i = \lambda_i \times Q_0, \]  

\[ \lambda_i = \lambda_0 + (i \times 0.01) \quad i = 1, 2, \ldots \]  

In this article, \( \lambda_0 \) is assumed to be equal to 1. \( \lambda_i \) is increased until the voltage collapse occurs and the divergence of the load flow calculation is observed. The latest acceptable \( \lambda \) is the maximum loadability (\( \lambda_{\text{max}} \)), which is shown in Figure 6. To improve the loadability (maximize it), the following objective function is defined to minimize:

\[ OF_{\text{Loadability}} = \frac{1}{\lambda_{\text{max}}}. \]  

3.2. Constraints
The voltage constraints of the buses and the allowable size of DGs and DSTATCOMs should be considered as follows:

\[ U_{\text{min}} \leq U_i \leq U_{\text{max}}, \]  

\[ S_{\text{min}} \leq S_{DG} \leq S_{\text{max}}, \]  

\[ Q_{\text{min}} \leq Q_{\text{DSTATCOM}} \leq Q_{\text{max}}, \]  

where \( U_{\text{min}} = 0.95 \text{ p.u.}, \ U_{\text{max}} = 1.05 \text{ p.u.}, \ S_{\text{min}} = 20 \text{ kVA}, \ S_{\text{max}} = 200 \text{ kVA}, \ Q_{\text{min}} = 20 \text{ kVAR}, \) and \( Q_{\text{max}} = 200 \text{ kVAR}. \)  

4. GA-BASED MULTI-OBJECTIVE OPTIMIZATION APPROACH
The crossover and mutation operators in a GA-based multi-objective are the most important parts of the algorithm and are introduced in what follows.

4.1. The Crossover Operator
The crossover is formulated as follows:

\[ c_{1,k} = \frac{1}{2} [(1 - \beta_k)p_{1,k} + (1 + \beta_k)p_{2,k}], \]  

\[ c_{2,k} = \frac{1}{2} [(1 + \beta_k)p_{1,k} + (1 - \beta_k)p_{2,k}], \]  

where \( c_{i,k} \) is the \( i \)th child with the \( k \)th component, \( p_{i,k} \) is the selected parent, and \( \beta_k \) is a sample from a random number generated having the following density:

\[ p(\beta) = \begin{cases} \frac{1}{2}(\eta_c + 1)\beta^{\eta_c}, & \text{if } 0 \leq \beta \leq 1, \\ \frac{1}{2}\eta_c^{\frac{1}{\eta_c + 1}}, & \text{if } \beta > 1. \end{cases} \]  

This distribution can be obtained from a uniformly sampled random number \( u \) between \((0, 1)\). \( \eta_c \) is the distribution index for crossover, and that is \( \beta(u) = (2u)^{1/\eta_c} \).
\[ \beta(u) = \frac{1}{[2(1-u)]^{1/\eta_m}}. \] (32)

### 4.2. The Mutation Operator

The mutation can be calculated as follows:

\[ c_k = p_k + (p_u^k - p_l^k) \delta_k, \] (33)

where the child is \( c_k \) and the parent is \( p_k \). \( p_u^k \) is the upper bound on the parent component, \( p_l^k \) is the lower bound, and \( \delta_k \) is a small variation calculated from a polynomial distribution as follows:

\[ \delta_k = \begin{cases} 2 r_k^{1/\eta_m} - 1, & \text{if } r_k < 0.5, \\ 1 - 2(1 - r_k)^{1/\eta_m}, & \text{if } r_k \geq 0.5. \end{cases} \] (34, 35)

where \( r_k \) is an uniformly sampled random number that is between \((0, 1)\), and \( \eta_m \) is the mutation distribution index.

### 5. SIMULATION RESULTS

As mentioned before, this article aims to optimize the number, place, and size of the DGs and DSTATCOMs using the multi-objective functions introduced in Section 3. The case studies are the IEEE 33-bus and 69-bus test systems, as shown in Figures 7 and 8. For the 33-bus test system, there are 32 candidates (buses) to site the DSTATCOMs and DGs. In addition, the size of each device should be determined. Therefore, there are 64 variables for DSTATCOMs, 32 variables (\( \text{var}_1 \) to \( \text{var}_{32} \)) for sizing, and 32 discrete variables (\( \text{var}_{33} \) to \( \text{var}_{64} \) indicate buses of the system except slack bus) for sitting. \( \text{var}_1 \) to \( \text{var}_{32} \) are selected from \( Q_{\text{min}} \) to \( Q_{\text{max}} \); \( \text{var}_{33} \) to \( \text{var}_{64} \) are set to 1 if the device is placed and 0 if not in the optimization procedure. Similar to the DSTATCOM, there are 64 variables for sizing (\( \text{var}_{65} \) to \( \text{var}_{96} \) selected between \( S_{\text{min}} \) and \( S_{\text{max}} \)) and sitting (\( \text{var}_{97} \) to \( \text{var}_{128} \) set to 0 or 1) of DGs. All 128 variables for each individual of the GA are shown in Figure 9.

Therefore, an individual shows a situation of places and sizes of devices in the system. In each iteration, for each individual in the optimization procedure, the total cost of the system is calculated according to Eq. (16), and then \( \lambda_i \) is increased to determine the maximum loadability of this situation of the system according to Eq. (23). The constant parameters of the objective functions are shown in Table 1.

![Figure 7. IEEE 33-bus test system.](image7)

![Figure 8. IEEE 69-bus test system.](image8)

![Figure 9. Variables of each individual in the GA for 33-bus case.](image9)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG investment cost</td>
<td>318</td>
<td>DSTATCOM cost</td>
<td>50</td>
</tr>
<tr>
<td>( C_{\text{DG-inv}} ) ($/kW)</td>
<td></td>
<td>( C_{\text{DSTATCOM-inv}} ) ($/kVAR)</td>
<td></td>
</tr>
<tr>
<td>DG O&amp;M cost</td>
<td>0.036</td>
<td>Loss cost ( C_{\text{loss}} ) ($/kVAR)</td>
<td>0.06</td>
</tr>
<tr>
<td>( C_{\text{DG-op}} ) ($/kWh)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years</td>
<td>30</td>
<td>Population size (N)</td>
<td>300</td>
</tr>
</tbody>
</table>

**Table 1. Objective functions parameters [4, 11]**
loadability axis should be selected because these points lead to minimum values of total cost.

For example, according to the importance of each objective functions that is shown in Eq. (36), an optimal point is selected considering the minimum distance from the origin point, or (0, 0). To do this, the values of both objective functions should first be normalized using the maximum values that have been evaluated for each of them. The distance of the points from the origin can be calculated as follows (as shown in Figure 12; \( \sqrt{2/\lambda_{\text{max}}} \) versus total cost):

\[
D_i = \sqrt{2 \times \text{loadability}_i^2 + \text{Total Cost}_i^2},
\]

(36)

where \( D_i \) is the distance of the \( i \)th points from the origin. Thereafter, the point that has the minimum distance from the origin can be selected as an optimal point.

The aforementioned procedure is done for all evaluated Pareto front points. For the 33-bus test system, three sample points of Pareto front curves are given in Table 2 and the number of DGs and DSTATCOMs, their optimal sizes, and the optimal locations corresponding to these points are listed in Table 3. Similar results are provided for the 69-bus test systems in Tables 4 and 5. The values of Tables 2 and 4 are normalized for each objective function according to the maximum of loadability and total cost. In these tables, the optimal point according to (36) is shown as bold text. The results of each system can be achieved by combination of Tables 2 and 4, or Tables 3 and 5. For example, in the first Pareto point of the 33-bus system in Table 3, from \( var_97 \) to \( var_{128} \), only \( var_{105} \), \( var_{107} \), and \( var_{124} \) are set to 1, and all else is zero. It means that only three DGs should be installed in buses 10, 12, and 29. The sizes of these DGs are defined by \( var_{65} \) to \( var_{96} \). Therefore, 64 variables are needed for the number, size, and place of the DGs and 64 variables for DSTATCOMs; in sum, 128 variables are needed.

In Tables 2 and 4, the second point has the minimum distance from the origin among all Pareto front points. The first point is the point that has the lower value of maximum loadability (higher value of \( 1/\lambda_{\text{max}} \)) but lower total cost. This is because of the lower number of DGs and DSTATCOMs and small sizes. On the other hand, point number 3 is the point that has a higher value of maximum loadability (lower value of \( 1/\lambda_{\text{max}} \)) but a higher total cost. This is because of the higher number of DGs and DSTATCOMs and large sizes. All Pareto front points may be selected as an optimal point depending on the priority of maximum loadability or total cost.

In the simulations, the maximum available number of DSTATCOMs has been assumed equal to 10. According to Tables 2 and 4, it is shown that the optimization procedure has placed all ten DSTATCOMs in each Pareto front point. First, it is because of the low investment cost of DSTATCOMs. Compared to DGs, DSTATCOMs have no operation cost. Second, DSTATCOMs have been placed on the buses that usually are at
The results of Tables 3 and 5 clearly prove this fact.

6. CONCLUSION

In this article, the Pareto analysis of optimal number, placement, and sizing of DGs and DSTATCOMs in a radial DS has been presented. The maximum loadability and total cost of loss and installed units including DSTATCOM investment cost, DG investment, and O&M costs are considered as two parts of the multi-objective functions. The proposed multi-objective function is applied to the IEEE 33-bus and 69-bus test systems using a multi-objective GA. The simulation results show that the total cost will increase when loadability improves and vice versa. This is because of the number of DGs and DSTATCOMs and their sizes. Therefore, all Pareto front points can be selected as an optimal point depending on the priority of loadability or total cost.

REFERENCES


**BIographies**

**Hossein Karami** was born in 1988 in Iran. He received his B.Sc. and M.Sc. from the Electrical Engineering Department of AUT, Tehran, Iran, in 2010 and 2012, respectively. His B.Sc. thesis was about the feasibility of increasing the operating voltage of current transmission lines, and his M.Sc. thesis was on the detection and localization of partial discharge in a power transformer using simulations in CST software. Since 2012, he has been a Ph.D. student at AUT and working with Prof. G. B. Gharehpetian; his Ph.D. thesis is about the feasibility of a combination of partial discharge localization and synthetic aperture radar (SAR) imaging of power transformer windings using electromagnetic antennas. His research interests include power transformer monitoring, partial discharge, short-term load forecasting, optimization algorithms and artificial intelligence, power quality, relays coordination, and smart homes.

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