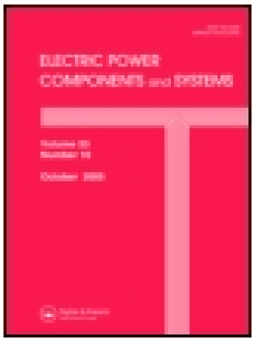

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CONTENTS

Abstract—Due to the development of distribution systems and an increase in electricity demand, and to overcome the shortcomings of distribution systems, such as high loss and low loadability, the use of distributed generations and flexible AC transmission systems devices, such as distributed static compensators, has been increased. The number, place, and capacity of these devices are significant factors affecting network loss reduction, influencing investment costs, and improving network performance. In this article, a new multi-objective function is proposed and optimized based on the genetic algorithm. The multi-objective function includes system loadability, and total costs include investment costs of distributed generations and distributed static compensators and network loss. The proposed method is applied to the IEEE 33-bus and 69-bus test systems. According to the simulation results, the total costs and system loadability have conflict with each other, and one can select each Pareto front point as an optimal point depending on the priority of system loadability or total cost.

1. INTRODUCTION

Recently, distribution systems (DSs) have increasingly found more importance as they can play essential roles in power system quality and planning [1, 2]. Due to the inherent high R/X ratio and significant voltage drop of DSs, these systems have high losses. Indeed, the major part of losses in a power system belongs to DSs, which are approximated in previous literature at about 13% [3]. In addition, DSs usually have radial feeders. Therefore, the electrical demand increment and DS development will result in more voltage drop and power losses [4]. Complete utilization of the line capacity in these networks is not possible for several reasons [5, 6]. Generally, one of the factors that limit the capacity of a DS is voltage limits [7]. The voltage limits determine the system loadability. The load increment of a DS is allowable until the voltages of the systems maintain acceptable voltage limits [8]. To improve these limitations, it is necessary to use advanced

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/uemp.

Nomenclature

$C_{DG\text{-}inv}$	distributed generation investment cost ($/kW)
$C_{DG\text{-}O\&M}$	distributed generation operation and maintenance cost ($/kWh)
$C_{DSTATCOM\text{-}inv}$	distributed static compensator investment cost ($/kVAR)
C_{loss}	cost of losses ($/kWh)
i	index of bus number
I	branch current (p.u.)
$I_{DSTATCOM}$	distributed static compensator current (p.u.)
OF_{cost}	objective function of costs ($)
P	local active load of bus (p.u.)
P_0	nominal active load (kW)
P_{loss}	reactive power loss (kW)
Q	local reactive load of bus (p.u.)
Q_0	nominal reactive load (kVAR)
$Q_{DSTATCOM}$	distributed static compensator injected reactive power (p.u.)
$Q_{\max}$	maximum distributed static compensator capacity (kVAR)
$Q_{\min}$	minimum distributed static compensator capacity (kVAR)
R	branch resistance (p.u.)
S_{DG}	distributed generation capacity (kVA)
$S_{\max}$	maximum distributed generation capacity (kVA)
$S_{\min}$	minimum distributed generation capacity (kVA)
U	bus voltage (p.u.)
U'	bus voltage in presence of distributed static compensator (p.u.)
$U_{\max}$	maximum acceptable voltage (p.u.)
$U_{\min}$	minimum acceptable voltage (p.u.)
X	branch reactance (p.u.)
δ	angle of branch current (rad)
θ	angle of bus voltage (rad)
θ'	angle of bus voltage in presence of distributed static compensator (rad)
λ_0	nominal system loadability
$\lambda_{\max}$	maximum loadability

equipment. Power electronic devices and modern techniques, such as flexible AC transmission systems (FACTS) and distributed generation (DG), have improved considerably in these issues. The development of such networks requires active and reactive power provisions by appropriate sources. These are elements that are used for loss reduction, power quality improvement, and loadability improvement in DSs. Therefore, it is necessary to identify the size, location, and amount of required equipment for installation in an electrical distribution network in order to improve the performance of the network. It has been a challenging issue for power system researchers to find the proper location and size of DGs and distributed static compensator (DSTATCOMs) in a DS. Following is a review of some categorized research.

1.1. Optimal Dstatcom Locating and Sizing without DG

The DSTATCOM as a shunt-connected voltage source converter (VSC) is frequently employed to improve power quality concerns. During overloading and voltage sag, the load voltage of the bus in which the DSTATCOM is connected can be regulated by compensating current injection into the system [9]. The injected current is evaluated using bus voltage and reactive power required for balancing/regulating voltage in the desired value. The design of a prototype DSTATCOM for voltage sag mitigation in an unbalanced DS was investigated in [10]. In [11], the optimal sizing and placement of a DSTATCOM using an immune algorithm was studied. In [12], DSTATCOM performance using a control strategy based on the improved instantaneous active and reactive current component theory to generate reference currents was evaluated under various source and load conditions. The proposed control strategy performance was evaluated in terms of load balancing, reactive power compensation, compensator rating, and harmonic mitigation. A practical example of the use of STATCOMs to reduce voltage fluctuations was given in [13]. In [14], a new sensitivity index was developed for the placement of STATCOMs to ensure fast voltage recovery at all buses of interest. A number of works have been carried out on the optimal location of a STATCOM using optimization techniques, such as particle swarm optimization (PSO) or genetic algorithm (GA) [15-16], and on optimal allocation and sizing of a STATCOM using PSO [17].

2.2. Optimal DG Locating and Sizing without DSTATCOM

In [18], an analytical approach was been proposed by Wang and Nehrir to find the optimal DG location in DSs considering different load patterns in order to reduce the real power loss. In [19], a sensitivity analysis method on the basis of power flow equations was used to find the optimal places and sizes of DGs; in that study, two indices were considered at each bus of the network to sit the DGs. In [20], the primal–dual interior point (IP) method was applied for optimal DG placement considering the both voltage profile improvement and line loss reduction indices. However, the accurate results of the proposed approach depended on the initial location of DGs at all of the buses; hence, this approach would not be applicable in large power networks. In [21], only the DG sizing problem using the generalized reduced-gradient method was treated. The main objective of this study has been loss minimization.

In [22], to optimize the location of DG, the GA method was used to maximize a predefined index of DG's benefit–cost ratio considering the both voltage drop and feeder transfer capacity. In [23], DG sizing and placement were been studied using a hybrid technique. Voltage profile, loss reduction, and voltage stability were considered as objective functions to find optimal DG places using a hybrid GA and PSO. In the research, shunt capacitance has been neglected in the overhead lines studies.

Some studies have been conducted on the impact of DG on loadability and its improvement [24, 25]. It has been shown that the system loadability is increased in the presence of DGs. In [24], an eigenvalue-based method was proposed to find the optimum place and size of DG to improve the loadability. The continuation power flow (CPF) approach was utilized in [25] to sit the DG on the bus that has the weakest voltage.

1.3. Optimal DG and DSTATCOM Simultaneous Locating and Sizing

Due to development of DSs, the limitation of active and reactive powers of DG sources, increments in demands and loads, and high installation cost, the usage of DG sources is limited. Therefore, it is essential to use an alternative element, such as a DSTATCOM for which the implementation cost is lower than DGs, to improve voltage profile, losses, and power factor in DSs. Placement and sizing of DGs and DSTATCOMs should be studied simultaneously in the optimization procedure to reach the optimal operation of a power system.

In [26], the optimal placement and sizing of DGs and a DSTATCOM with the aim of reducing total power loss was studied using PSO. In [27], simultaneous reconfiguration and optimal placement of a DSTATCOM and DGs was presented using fuzzy and ant colony optimization (ACO) to reduce losses and voltage profile improvement. Both of the aforementioned studies were conducted as single-objective optimization and loadability, and Pareto analysis was been considered.

It should be noted that the optimal number of DGs and DSTATCOMs is an important issue in the investment cost of installed devices in the system, which has not been considered in the previously mentioned literature. In those studies, the researchers considered a determined number of these two devices in their simultaneous locating and sizing, for example, two DGs and two DSTATCOMs or three of each; this issue is considered herein as well. The investment costs are inserted in the objective function to determine the optimal number of devices for system cost minimization. If too many devices are installed, the system costs will be increased, and if a few devices are installed, the other aims of the article, such as loadability improvement or loss reduction, cannot be supported. Therefore, the number of each device that should be installed is very important and is considered in this article in the objective function.

There are several optimization algorithms that have been introduced and adapted to engineering problems [28–33]; however, having no need for setting up normalizing factors is one of the most important advantages of the Pareto front approach [34]. Different objective functions are orthogonal with each other, and the non-dominated solutions have no dependency on the units associated with each individual objective. The Pareto front makes a decision more flexible and allows better adaptation to the best policies for each electrical distribution company. Thus, a set of solutions is available that ranges between one that minimizes total loss and device costs and another that improves the loadability, each satisfying the operational constraints.

In this article, the GA and Pareto optimization are used to evaluate the number, placement, and sizing of DSTATCOMs and DGs in DSs, simultaneously, according to system loadability and total cost, including the installation and operation and maintenance (O&M) costs of DGs and DSTATCOMs and the cost of losses. In this approach, the optimal solution has been investigated in two categories: continuous (DSTATCOMs and DGs size) and discrete (location and number of devices). The proposed multi-objective function is implemented on the IEEE 33-bus and 69-bus test systems, and the results are presented.

It can be concluded that the advantages of the proposed method herein are as follows:

- simultaneous locating and sizing of DGs and DSTATCOMs;
- consideration of operating, maintenance, and investment costs of devices;
- finding the optimal number of devices, which is very important from the system cost point of view;
- consideration of total loss of the system;
- consideration of loadability of the system;

- consideration of operation of a 30-year period for the system;
- adaption of a multi-objective function to the objectives and system; and
- investigation of the proposed multi-objective function on IEEE 33- and 69-bus test systems.

The rest of the article is organized as follows. In Section 2, the implementation of the DSTATCOM in distribution load flow is explained. In Section 3, the problem formulation and constraints are introduced. The simulation results are presented in Section 4, and finally, the conclusion is presented in Section 5.

2. DSTATCOM IN DISTRIBUTION LOAD FLOW

A DSTATCOM is a shunt device that absorbs or injects both the active and reactive current at a point of common coupling (PCC) connection. The DSTATCOM is also a DC/AC converter consisting of a DC-link capacitor or a DC energy storage device that provides constant DC voltage and a three-phase pulse-width modulation (PWM) VSC bridge. All these are usually connected to the DS via a coupling transformer [35]. A DSTATCOM can work as a synchronous voltage source with a variable phase angle and magnitude. Hence, it is capable of controlling its voltage bus and correcting the power factor. Figure 1 shows a bus in a DS equipped with a proposed DSTATCOM.

In some short-circuit events or steady-state operation with heavy loading, a DSTATCOM typically injects an appropriate compensating current to the PCC connection, and thus, voltage at the load bus regulated by the DSTATCOM will be lifted close to a given value (for example, a normal value) [35, 36]. Generally, a DSTATCOM has the ability to exchange active and reactive power simultaneously. The amount of

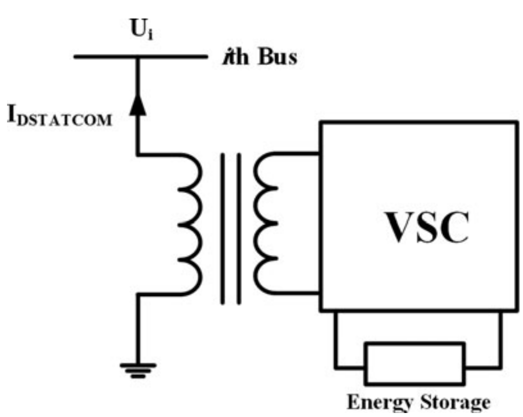

FIGURE 1. Typical DSTATCOM.

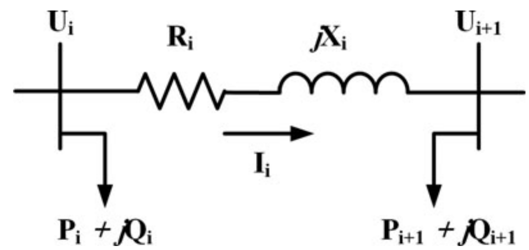

FIGURE 2. Single-line diagram of two buses of a DS.

exchanging active power depends on the energy source capacity. In this article, only a DSTATCOM application for reactive power exchange is considered and active power exchange with the network is neglected.

Backward/forward sweep load flow calculations are used in this work, in combination with a proper steady-state model for DSTATCOMs as presented in [37]. Many DSs have radial structures that only feed from one side. A section of a sample DS is shown in Figure 2 [38] that assumes that a -hree phase radial DS is in balance. The phasor diagram for Figure 2 is presented in Figure 3. The Kirchhoff's voltage law (KVL) equation can be stated as

$$U_{i+1} \angle \theta_{i+1} = U_i \angle \theta_i - (R_i + jX_i) I_i \angle \delta. \quad (1)$$

The values of the variables are derived from the load flow. In traditional networks, the bus voltage is usually less than 1 p.u., in which case it is assumed that voltage of bus $i + 1$ is also less than 1 p.u. A DSTATCOM device is installed in this study to compensate the voltage of bus $i + 1$ to a desired value. The DSTATCOM is used for voltage regulation and loss reduction in the steady-state condition, as noted earlier, and can inject only reactive power to the system in this article. Consequently, $I_{DSTATCOM}$ must be kept in quadrature with respect to the system voltage. As shown in Figures 4 and 5, by installing a DSTATCOM in bus $i + 1$, I_i and $I_{DSTATCOM}$ flow simultaneously in the branch;

$$\angle I_{DSTATCOM} = \frac{\pi}{2} - \theta'_{i+1}, \quad (2)$$

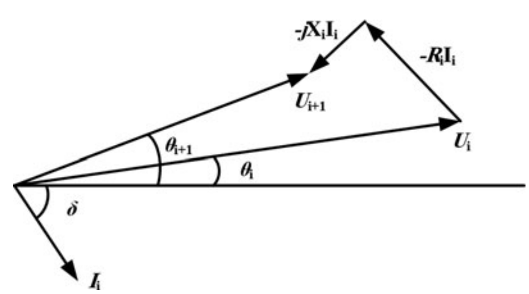

FIGURE 3. Phasor diagram of voltage and current of the system shown in Figure 2.

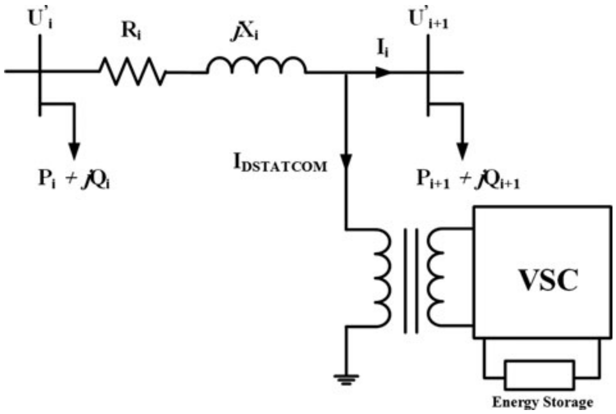

FIGURE 4. DSTATCOM installation in bus $i+1$ of a DS.

$$U'_{i+1}\angle\theta'_{i+1} = U'_i\angle\theta'_i - (R_i + jX_i)(I_i\angle\delta + I_{DSTATCOM}\angle\left(\frac{\pi}{2} - \theta'_{i+1}\right)). \quad (3)$$

Separating the imaginary and real parts of Eq. (3) and some computation gives

$$x = \frac{-B \pm \sqrt{\Delta}}{2A}; \quad (4)$$

that

$$x = \sin\theta'_{i+1}, \quad (5)$$

$$A = (a_1a_3 - a_2a_4)^2 + (a_1a_4 + a_2a_3)^2, \quad (6)$$

$$B = 2(a_1a_3 - a_2a_4)\,.\,(U'_{i+1})(-R_i), \quad (7)$$

$$C = (U'_{i+1}\,.\,R)^2 - (a_1a_4 + a_2a_3)^2, \quad (8)$$

$$a_1 = \text{Real}\left(U'_i\angle\theta'_i\right) - \text{Real}\left((R_i + jX_i)\,.\,(I_i\angle\delta)\right), \quad (9)$$

$$a_1 = \text{Imag}\left(U'_i\angle\theta'_i\right) - \text{Imag}\left((R_i + jX_i)\,.\,(I_i\angle\delta)\right), \quad (10)$$

$$a_3 = -X_i\,,\quad a_4 = -R_i. \quad (11)$$

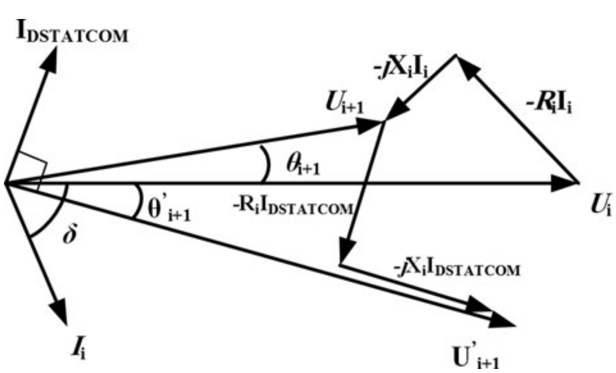

FIGURE 5. Phasor diagram of voltage and current of system shown in Figure 4.

As seen from Eq. (4), there are two roots for variable x, and so two values are calculated for $\angle I_{DSTATCOM}$ and $|I_{DSTATCOM}|$. To determine the correct answer, the boundary conditions are considered in these roots:

$$U'_{i+1} = U_{i+1} \;\Rightarrow\; I_{DSTATCOM} = 0 \;\; and \;\; \theta'_{i+1} = \theta_{i+1}. \quad (12)$$

Results show that $x = \frac{-B+\sqrt{\Delta}}{2A}$ is the answer of Eq. (4); therefore, $\angle I_{DSTATCOM}$ can be determined as follows:

$$\angle I_{DSTATCOM} = \frac{\pi}{2} - \theta'_{i+1} = \frac{\pi}{2} - \sin^{-1}x. \quad (13)$$

The magnitude of the DSTATCOM current can be calculated as follows:

$$|I_{DSTATCOM}| = \frac{U'_{i+1}\angle\theta'_{i+1} - a_1}{-a_4\sin\theta'_{i+1} - a_3\cos\theta'_{i+1}}. \quad (14)$$

Thus, $I_{DSTATCOM}$ and the PCC bus voltage are calculated using Eqs. (13), (14), and (3). Finally, the injected reactive power to the power system by the DSTATCOM, to correct the voltage of the connected bus up to U'_{i+1}, can be expressed as follows:

$$jQ_{DSTATCOM} = (U'_{i+1}\angle\theta'_{i+1})\,.\,\left(I_{DSTATCOM}\angle\left(\frac{\pi}{2} - \theta'_{i+1}\right)\right)^*, \quad (15)$$

such that the symbol * denotes a complex conjugate.

To evaluate load flow considering a DSTATCOM in a DS with any iteration in forward sweep, the magnitude of the compensated node voltage can be assumed to be any desired value (for example, 1 p.u.). The angle of the compensated node phase, reactive power, and also DSTATCOM size can now be calculated using the above equations. The updated voltage and injected reactive power by the DSTATCOM are used for continuing the forward sweep to determine load currents in the next backward sweep load flow. This procedure is repeated until load flow converges [39].

3. PROBLEM FORMULATION

3.1. Objective Function

The number, place, and size of the DGs and DSTATCOMs are optimized using the non-linear multi-objective optimization procedure. The proposed multi-objective function has two parts—total costs and system loadability—which have conflicting behaviors. Improvement of loadability leads to increasing total cost, and minimization of the total costs causes loadability to decline. The multi-objective function is formulated taking into account the following objectives considering a period of 30 years.

3.1.1. Total Cost

The first part of the multi-objective function is all costs of the system. This objective function has three parts as shown in the following equation:

$$OF_{\text{costs}} = OF_1 + OF_2 + OF_3, \tag{16}$$

where each part is calculated as follows:

3.1.1.1. Total Power Losses. These two elements in the power system considering the equal and unequal constraints will cause loss reduction in the DS. To calculate the network real power loss,

$$P_{loss} = \sum_{i=1}^{n} R_i \, |I_i|^2. \tag{17}$$

Therefore, the cost of the power loss for a period of 30 years [11] can be obtained as the following equation:

$$OF_1 = P_{loss} \times C_{loss} \times 8760 \times 30. \tag{18}$$

3.1.1.2. DG O&M and Investment Cost. In this study, the power factors of all DGs are assumed to be 0.85 [4]. Therefore, the investment and O&M costs of DGs are calculated as follows:

$$OF_2 = S_{DG} \times 0.85 \times C_{DG-inv} + S_{DG} \times 0.85 \times C_{DG-O\&M} \times 8760 \times 30. \tag{19}$$

3.1.1.3. DSTATCOM Investment Cost. To consider the investment cost of a DSTATCOM, the following equation should be calculated:

$$OF_3 = Q_{DSTATCOM} \times C_{DSTATCOM-inv}. \tag{20}$$

3.1.1.4. Loadability. To find the maximum loadability of the system, all the active and reactive loads are increased in the power flow problem in the ith step as follows:

$$P_i = \lambda_i \times P_0\,, \;\; Q_i = \lambda_i \times Q_0, \tag{21}$$

$$\lambda_i = \lambda_0 + (i \times 0.01) \quad i = 1, 2, \ldots. \tag{22}$$

In this article, λ_0 is assumed to be equal to 1. λ_i is increased until the voltage collapse occurs and the divergence of the load flow calculation is observed. The latest acceptable λ is the maximum loadability ($\lambda_{\max}$), which is shown in Figure 6. To improve the loadability (maximize it), the following objective function is defined to minimize:

$$OF_{Loadability} = \frac{1}{\lambda_{\max}}. \tag{23}$$

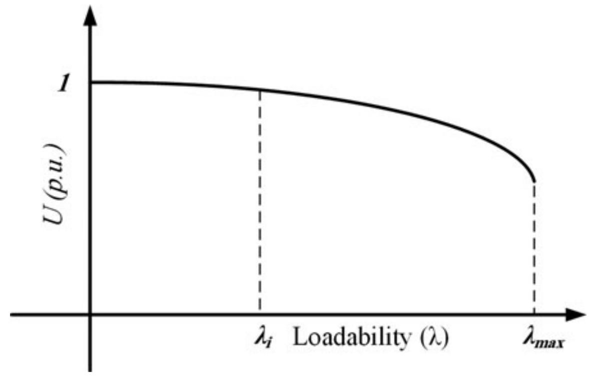

FIGURE 6. Voltage loadability curve.

3.2. Constraints

The voltage constraints of the buses and the allowable size of DGs and DSTATCOMs should be considered as follows:

$$U_{\min} \le U_i \le U_{\max}, \tag{24}$$

$$S_{\min} \le S_{DG} \le S_{\max}, \tag{25}$$

$$Q_{\min} \le Q_{DSTATCOM} \le Q_{\max}, \tag{26}$$

where $U_{\min} = 0.95$ p.u., $U_{\max} = 1.05$ p.u., $S_{\min} = 20$ kVA, $S_{\max} = 200$ kVA, $Q_{\min} = 20$ kVAR, and $Q_{\max} = 200$ kVAR.

4. GA-BASED MULTI-OBJECTIVE OPTIMIZATION APPROACH

The crossover and mutation operators in a GA-based multi-objective are the most important parts of the algorithm and are introduced in what follows.

4.1. The Crossover Operator

The crossover is formulated as follows:

$$c_{1,k} = \frac{1}{2}[(1 - \beta_k)p_{1,k} + (1 + \beta_k)p_{2,k}], \tag{27}$$

$$c_{2,k} = \frac{1}{2}[(1 + \beta_k)p_{1,k} + (1 - \beta_k)p_{2,k}], \tag{28}$$

where $c_{i,k}$ is the ith child with the kth component, $p_{i,k}$ is the selected parent, and β_k is a sample from a random number generated having the following density:

$$p(\beta) = \frac{1}{2}(\eta_c + 1)\beta^{\eta_c}, \quad if \;\; 0 \le \beta \le 1, \tag{29}$$

$$p(\beta) = \frac{1}{2}(\eta_c + 1)\frac{1}{\beta^{\eta_c + 2}}, \quad if \;\; \beta > 1. \tag{30}$$

This distribution can be obtained from a uniformly sampled random number u between (0, 1). η_c is the distribution index for crossover, and that is

$$\beta(u) = (2u)^{\frac{1}{(\eta+1)}}, \tag{31}$$

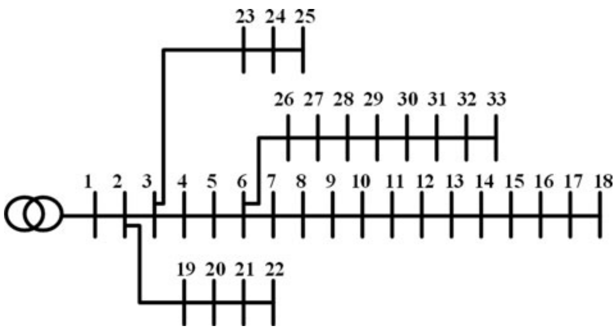

FIGURE 7. IEEE 33-bus test system.

$$\beta(u) = \frac{1}{[2(1-u)]^{\frac{1}{(\eta+1)}}}. \tag{32}$$

4.2. The Mutation Operator

The mutation can be calculated as follows:

$$c_k = p_k + \left(p_k^u - p_k^l\right)\delta_k, \tag{33}$$

where the child is c_k and the parent is p_k. p_k^u is the upper bound on the parent component, p_k^l is the lower bound, and δ_k is a small variation calculated from a polynomial distribution as follows:

$$\delta_k = (2\mathrm{r}_k)^{\frac{1}{\eta_m+1}} - 1, \quad if \;\; \mathrm{r}_k < 0.5, \tag{34}$$

$$\delta_k = 1 - [2(1-\mathrm{r}_k)]^{\frac{1}{\eta_m+1}}, \quad if \;\; \mathrm{r}_k \geq 0.5, \tag{35}$$

where r_k is an uniformly sampled random number that is between (0, 1), and η_m is the mutation distribution index.

5. SIMULATION RESULTS

As mentioned before, this article aims to optimize the number, place, and size of the DGs and DSTATCOMs using the multi-objective functions introduced in Section 3. The case studies are the IEEE 33-bus and 69-bus test systems, as shown in Figures 7 and 8. For the 33-bus test system, there are 32 candidates (buses) to site the DSTATCOMs and DGs. In addition, the size of each device should be determined. Therefore, there are 64 variables for DSTATCOMs, 32 variables (var_1 to var_{32}) for sizing, and 32 discrete variables (var_{33} to var_{64} indicate buses of the system except slack bus) for sitting. var_1 to var_{32} are selected from $Q_{\min}$ to $Q_{\max}$; var_{33} to var_{64} are set to 1 if the device is placed and 0 if not in the optimization procedure. Similar to the DSTATCOM, there are 64 variables for sizing (var_{65} to var_{96} selected between $S_{\min}$ and $S_{\max}$) and sitting (var_{97} to var_{128} set to 0 or 1) of DGs. All 128 variables for each individual of the GA are shown in Figure 9.

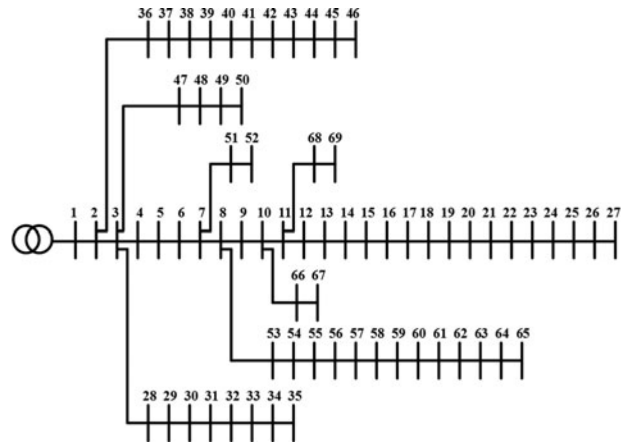

FIGURE 8. IEEE 69-bus test system.

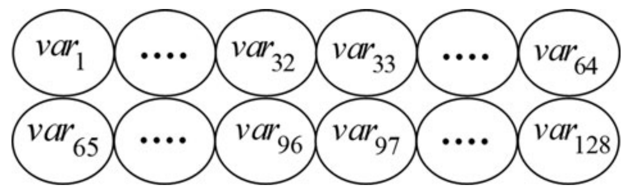

FIGURE 9. Variables of each individual in the GA for 33-bus case.

Therefore, an individual shows a situation of places and sizes of devices in the system. In each iteration, for each individual in the optimization procedure, the total cost of the system is calculated according to Eq. (16), and then λ_i is increased to determine the maximum loadability of this situation of the system according to Eq. (23). The constant parameters of the objective functions are shown in Table 1.

Figures 10 and 11 present Pareto analysis results that have been run using GA. According to these figures, one can choose any point as an optimal point depending on the importance of the two objective functions. If the maximum loadability is more important, the points that are located near the cost axis should be selected because these points lead to minimum values of maximum loadability. On the other hand, if the total cost is more important, the points that are located near the maximum

Parameter	Value	Parameter	Value
DG investment cost C_{DG-inv} (\$/kW)	318	DSTATCOM cost $C_{DSTATCOM-inv}$ (\$/kVAR)	50
DG O&M cost C_{DG-op} (\$/kWh)	0.036	Loss cost C_{loss} (\$/kWh)	0.06
Years	30	Population size (N)	300

TABLE 1. Objective functions parameters [4, 11]

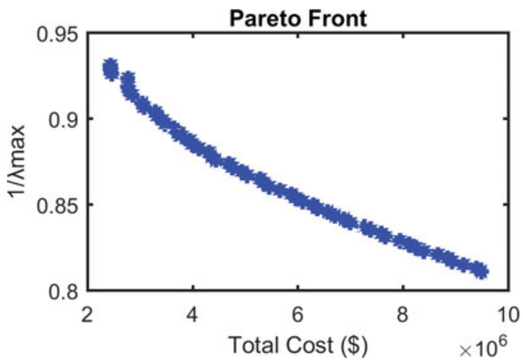

FIGURE 10. Pareto front for IEEE 33-bus test system.

loadability axis should be selected because these points lead to minimum values of total cost.

For example, according to the importance of each objective functions that is shown in Eq. (36), an optimal point is selected considering the minimum distance from the origin point, or (0, 0). To do this, the values of both objective functions should first be normalized using the maximum values that have been evaluated for each of them. The distance of the points from the origin can be calculated as follows (as shown in Figure 12; $\sqrt{2}/\lambda_{\max}$ versus total cost):

$$D_i = \sqrt{2 \times \text{loadability}_i^2 + \text{Total Cost}_i^2}, \qquad (36)$$

where D_i is the distance of the ith points from the origin. Thereafter, the point that has the minimum distance from the origin can be selected as an optimal point.

The aforementioned procedure is done for all evaluated Pareto front points. For the 33-bus test system, three sample points of Pareto front curves are given in Table 2 and the number of DGs and DSTATCOMs, their optimal sizes, and the optimal locations corresponding to these points are listed in Table 3. Similar results are provided for the 69-bus test systems in Tables 4 and 5. The values of Tables 2 and 4 are normalized for each objective function according to the maximum

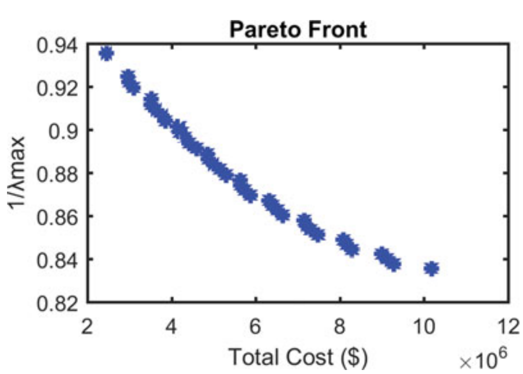

FIGURE 11. 1. Pareto front for IEEE 69-bus test system.

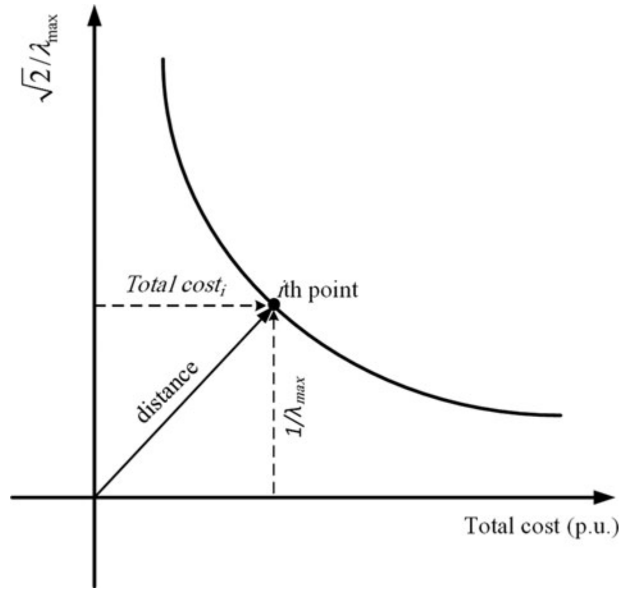

FIGURE 12. Distance from the origin.

of loadability and total cost. In these tables, the optimal point according to (36) is shown as bold text. The results of each system can be acheived by combination of Tables 2 and 4, or Tables 3 and 5. For example, in the first Pareto point of the 33-bus system in Table 3, from var_{97} to var_{128}, only var_{105}, var_{107}, and var_{124} are set to 1, and all else is zero. It means that only three DGs should be installed in buses 10, 12, and 29. The sizes of these DGs are defined by var_{65} to var_{96}. Therefore, 64 variables are needed for the number, size, and place of the DGs and 64 variables for DSTATCOMs; in sum, 128 variables are needed.

In Tables 2 and 4, the second point has the minimum distance from the origin among all Pareto front points. The first point is the point that has the lower value of maximum loadability (higher value of $1/\lambda_{\max}$) but lower total cost. This is because of the lower number of DGs and DSTATCOMs and small sizes. On the other hand, point number 3 is the point that has a higher value of maximum loadability (lower value of $1/\lambda_{\max}$) but a higher total cost. This is because of the higher number of DGs and DSTATCOMs and large sizes. All Pareto front points may be selected as an optimal point depending on the priority of maximum loadability or total cost.

In the simulations, the maximum available number of DSTATCOMs has been assumed equal to 10. According to Tables 2 and 4, it is shown that the optimization procedure has placed all ten DSTATCOMs in each Pareto front point. First, it is because of the low investment cost of DSTATCOMs. Compared to DGs, DSTATCOMs have no operation cost. Second, DSTATCOMs have been placed on the buses that usually are at

Pareto points	$1/\lambda_{max}$ (p.u.)	Cost ($)	Cost (p.u.)	Distance	Loss (kW)	Voltage profile index	Number of DGs	Number of DSs
1	0.981	$3.09E+06$	0.34	1.43	142.3	0.0704	1	10
2	**0.950**	**$4.05E+06$**	**0.45**	**1.42**	**129.1**	**0.0525**	**2**	**10**
3	0.938	$4.74E+06$	0.52	1.43	119.6	0.0450	4	10

TABLE 2. Sample points of Pareto front curve for 33-bus test system

Pareto points	DG buses				DG capacities (kVA)				DS buses					DS capacities (kVAR)				
1	18	—	—	—	96.3	—	—	—	7	8	10	12	17	123.0	110.1	109.5	96.9	113.0
									26	28	29	30	32	98.2	95.7	99.8	65.0	98.2
2	**17**	**18**	—	—	**137.5**	**98.1**	—	—	**7**	**8**	**12**	**17**	**18**	**123.4**	**109.6**	**97.2**	**113.0**	**156.7**
									26	**28**	**29**	**30**	**32**	**97.7**	**94.9**	**99.3**	**65.3**	**98.1**
3	13	15	16	17	82.3	55.4	62.3	137.5	7	8	12	17	18	123.1	109.5	98.0	112.9	157.6
									26	28	29	30	32	98.9	94.6	98.8	66.2	98.8

TABLE 3. Optimal locations and sizes of DGs and DSTATCOMs for 33-bus test system

Pareto points	$1/\lambda_{max}$ (p.u.)	Cost ($)	Cost (p.u.)	Distance	Loss (kW)	VPI	Number of DGs	Number of DSs
1	0.991	$3.01E$+06	0.33	2.01	139.7	0.0608	1	10
2	**0.937**	**$5.39E$+06**	**0.58**	**1.96**	**97.1**	**0.0424**	**4**	**10**
3	0.910	$7.41E$+06	0.80	1.99	74.5	0.0318	7	10

TABLE 4. Sample points of Pareto front curve for 69-bus test system

Pareto points	DG buses				DG capacities (kVA)				DS buses					DS capacities (kVAR)				
1	61	—	—	—	88.9	—	—	—	17	19	57	58	60	135.0	138.3	112.5	119.2	119.1
									61	62	63	64	65	120.9	97.4	138.8	155.0	121.9
2	**62**	**63**	**64**	**65**	**122.7**	**89.8**	**112.4**	**131.4**	**19**	**57**	**58**	**59**	**60**	**137.8**	**112.8**	**121.5**	**111.6**	**118.7**
									61	**62**	**63**	**64**	**65**	**121.6**	**98.2**	**141.0**	**152.5**	**122.2**
3	59	60	61	62	99.8	121.8	89.1	120.6	19	57	58	59	60	138.1	112.4	121.3	114.1	118.1
	63	64	65	—	88.4	106.7	116.0	—	61	62	63	64	65	122.8	98.9	139.8	152.9	120.5

TABLE 5. Optimal locations and sizes of DGs and DSTATCOMs for 69-bus test system

the end of the feeders of both the 33- and 69-bus test systems. The results of Tables 3 and 5 clearly prove this fact.

6. CONCLUSION

In this article, the Pareto analysis of optimal number, placement, and sizing of DGs and DSTATCOMs in a radial DS has been presented. The maximum loadability and total cost of loss and installed units including DSTATCOM investment cost, DG investment, and O&M costs are considered as two parts of the multi-objective functions. The proposed multi-objective function is applied to the IEEE 33-bus and 69-bus test systems using a multi-objective GA. The simulation results show that the total cost will increase when loadability improves and *vice versa*. This is because of the number of DGs and DSTATCOMs and their sizes. Therefore, all Pareto front points can be selected as an optimal point depending on the priority of loadability or total cost.

REFERENCES

[1] Khan, N. A., Ghoshal, S. P., and Ghosh, S., "Optimal allocation of distributed generation and shunt capacitors for the reduction of total voltage deviation and total line loss in radial distribution systems using binary collective animal behavior optimization algorithm," *Electr. Power Compon. Syst.*, Vol. 43, No. 2, pp. 119–133, 2015.

[2] Karami, H., Vahidi, B., Naderi, M. S., Gharehpetian, G., and Vatani, M. R., "A novel and practical solution to mitigate volt-

age unbalance in LV networks by connection and on-line control of a single phase DG," *Proceedings of IEEE 17th Conference on Electrical Power Distribution Networks (EPDC)*, 2012.

[3] Ng, H. N., Salama, M. A., and Chikhani, A. Y., "Classification of capacitor allocation techniques," *IEEE Trans. Power Del.*, Vol. 5, No. 1, pp. 387–392, 2000.

[4] Moradi, M. H., Zeinalzadeh, A., Mohammadi, Y., and Abedini, M., "An efficient hybrid method for solving the optimal sitting and sizing problem of DG and shunt capacitor banks simultaneously based on imperialist competitive algorithm and genetic algorithm," *Electr. Power Energy Syst.*, Vol. 54, pp. 101–111, 2014.

[5] Wang, J. C., Chiang, H. D., Cookings, O., and Shin, H. D., "Optimal capacitor placements in distribution systems: A new formulation of the overall problem," *IEEE Trans. Power Del.*, Vol. 5, No. 2, pp. 634–642, 1990.

[6] Chen, C. S., Hsu, C. T., and Yan, Y. H., "Optimal distribution feeder capacitor placement considering mutual coupling effect of conductors," *IEEE Trans. Power Del.*, Vol. 10, No. 2, pp. 987–994, 1995.

[7] Biswas, S., Kumar Goswami, S., and Chatterjee, A., "Optimum distributed generation placement with voltage sag effect minimization," *Energy Convers. Manag.*, Vol. 53, No. 1, pp. 163–174, 2012.

[8] Prada, R. B., and Souza, L. J., "Voltage stability and thermal limit: constraints on the maximum loading of electrical energy distribution feeders," *IEE Proc. Gener. Transm. Distrib.*, Vol. 145, pp. 573–577, 1998.

[9] Sensarma, P. S., Padiyar, K. R., and Ramanarayanan, V., "Analysis and performance evaluation of a distribution STATCOM for compensating voltage fluctuations," *IEEE Trans. Power Del.*, Vol. 16, No. 2, pp. 259–264, 2001.

[10] Masdi, H., Mariun, N., Mahmud, S., Mohamed, A., and Yusuf, S., "Design of a prototype DSTATCOM for voltage sag mitigation," *IEEE Conference on Power and Energy*, pp. 61–66, Kuala Lumpur, Malaysia, November 2004.

[11] Taher, S. A., and Afsari, S. A., "Optimal location and sizing of DSTATCOM in distribution systems by immune algorithm original research article," *Int. J. Electr. Power Energy Syst.*, Vol. 60, pp. 34–44, 2014.

[12] Zaveri, T., Bhalja, B. R., and Zaveri, N., "A novel approach of reference current generation for power quality improvement in three-phase, three-wire distribution system using DSTATCOM," *Int. J. Electr. Power Energy Syst.*, Vol. 33, No. 10, pp. 1702–1710, 2011.

[13] Hill, J. E., "A practical example of the use of distribution static compensator (D-STATCOM) to reduce voltage fluctuations," *IEE Colloquium on Power Electronics for Renewable Energy (Digest No. 1997/170)*, pp. 5–12, 16 June 1997.

[14] Aziz, T., Mhaskar, U. P., Saha, T. K., and Mithulananthan, N., "An index for STATCOM placement to facilitate grid integration of DER," *IEEE Trans. Sustain. Energy*, Vol. 4, No. 2, pp. 451–460, 2013.

[15] Azadani, E. N., Hosseinian, S. H., and Hasanpor, P., "Optimal placement of multiple STATCOM for voltage stability margin enhancement using particle swarm optimization," *Electr. Eng.*, Vol. 90, No. 7, pp. 503–510, 2008.

[16] Gerbex, S., Cherkaouti, R., and Germond, A., "Optimal location of multi-type FACTS devices in a power system by means of genetic algorithms," *IEEE Trans. Power Syst.*, Vol. 16, No. 3, pp. 537–544, 2001.

[17] Del Valle, Y., Hernandez, J. C., Venayagamoorthy, G. K., and Harley, R. G., "Multiple STATCOM allocation and sizing using particle swarm optimization," *IEEE Conference on Power Systems and Exposition*, pp. 1884–1891, GA, USA, 29 October–November 1 2006.

[18] Wang, C., and Nehrir, M. H., "Analytical approaches for optimal placement of DG sources in power systems," *IEEE Trans. Power Syst.*, Vol. 19, No. 4, pp. 2068–2076, 2004.

[19] Popovic, D. H., Greatbanks, J. A., and Pregelj, A., "Placement of distributed generators and reclosers for distribution network security and reliability," *Int. J. Electr. Power Energy Syst.*, Vol. 27, No. 5, pp. 398–408, 2005.

[20] Iyer, H., Ray, S., and Ramakumar, R., "Assessment of distributed generation based on voltage profile improvement and line loss reduction," *IEEE/PES Transmission and Distribution Conference and Exposition*, pp. 1171–1176, Dallas, TX, May 2006.

[21] Rau, N. S., and Yih-Heui, W., "Optimum location of resources in distributed planning," *IEEE Trans. Power Syst.*, Vol. 9, No. 4, pp. 2014–2020, 2004.

[22] Teng, J. H., Lour, T. S., and Liu, Y. H., "Strategic distributed generator placements for service reliability improvements," *IEEE Power Eng. Soc. Gen. Meet.*, Vol. 2, pp. 719–724, 2002.

[23] Moradi, M. H., and Abedini, M., "A combination of genetic algorithm and particle swarm optimization for optimal DG location and sizing in distribution systems," *Int. J. Electr. Power Energy Syst.*, Vol. 34, No. 1, pp. 66–74, 2012.

[24] Mithulananthan, N., and Oo, T., "Distributed generator placement to maximize the loadability of a distribution system," *Int. J. Electr. Eng. Educ.*, Vol. 43, No. 2, pp. 107–118, 2006.

[25] Hedayati, H., Nabaviniaki, S., and Akbarimajd, A., "A method for placement of DG units in distribution networks," *IEEE Trans. Power Del.*, Vol. 23, No. 3, pp. 1620–1628, 2008.

[26] Devi, S., and Geetanjali, M., "Optimal location and sizing determination of distributed generation and DSTATCOM using particle swarm optimization algorithm," *Electr. Power Energy Syst.*, Vol. 62, pp. 562–570, 2014.

[27] Bagheri Tolabi, H., Ali, M. H., and Rizwan, M., "Simultaneous reconfiguration, optimal placement of DSTATCOM, and photovoltaic array in a distribution system based on fuzzy-ACO approach," *IEEE Trans. Sustain. Energy*, Vol. 6, No. 1, pp. 210–218, 2015.

[28] Karami, H., Sanjari, M. J., and Gharehpetian, G. B., "Hyper-spherical search (HSS) algorithm: A novel meta-heuristic algorithm to optimize nonlinear functions," *Neural Comput. Appl.*, Vol. 25, No. 6, pp. 1455–1465, 2014.

[29] Karami, H., Sanjari, M. J., Hosseinian, S. H., and Gharehpetian, G. B., "Stochastic load effect on home energy system scheduling optimization," *Int. Trans. Electr. Energy Syst.*, Vol. 25, No. 10, pp. 2412–2426, 2015.

[30] Karami, H., Sanjari, M. J., Hosseinian, S. H., and Gharehpetian, G. B., "An optimal dispatch algorithm for managing residential distributed energy resources," *IEEE Trans. Smart Grid*, Vol. 5, No. 5, pp. 2360–2367, 2014.

[31] Karami, H., Sanjari, M. J., Tavakoli, A., and Gharehpetian, G. B., "Optimal scheduling of residential energy system including combined heat and power system and storage device," *Electr. Power Compon. Syst.*, Vol. 41, No. 8, pp. 765–781, 2013.

[32] Karami, H., Sanjari, M. J., Tavakoli, A., and Gharehpetian, G. B., "HSA-based optimal allocation of DGs considering cable aging constraint," *Third Iranian Conference on Renewable Energies and Distributed Generation (ICREDG 2013)*, Isfahan, Iran, February 2013.

[33] Sanjari, M. J., Karami, H., Yatim, A. H., and Gharehpetian, G. B., "Application of hyper-spherical search algorithm for optimal energy resources dispatch in residential microgrids," *Appl. Soft Comput.*, Vol. 37, pp. 15–23, 2015.

[34] Zhao, B., Zhang, X., Chen, J., Wang, C., and Guo, L., "Operation optimization of standalone microgrids considering lifetime characteristics of battery energy storage system," *IEEE Trans. Sustain. Energy*, Vol. 4, No. 4, pp. 934–943, 2013.

[35] Blazic, B., and Papic, I., "A new mathematical model and control of DSTATCOM for operation under unbalanced conditions," *Electr. Power Syst. Res.*, Vol. 72, No. 3, pp. 279–287, 2004.

[36] Hingorani, N., "Introducing custom power," *IEEE Spectrum*, Vol. 32, No. 6, pp. 41–48, 1995.

[37] Ghosh, S., and Das, D., "Method for load-flow solution of radial distribution networks," *IEE Proc. Gener. Transm. Distrib.*, Vol. 146, No. 6, pp. 641–648, 1999.

[38] Hosseini, M., Shayanfar, H. A., and Fotuhi, M., "Modeling of series and shunt distribution facts devices in distribution load flow," *J. Electr. Syst.*, pp. 1–12, 2008.

[39] Baran, M. E., and Wu, F. F., "Optimal capacitor placement on radial distribution systems," *IEEE Trans. Power Del.*, Vol. 4, No. 1, pp. 725–732, 1989.

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