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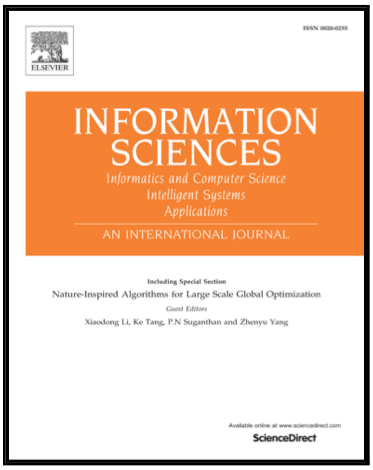

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Output-Based Event-Triggered Schemes on Leader-Following Consensus of A Class of Multi-Agent Systems with Lipschitz-Type Dynamics

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Abstract

This paper addresses the problem of leader-follower consensus for a class of nonlinear multi-agent systems with Lipschitz-type dynamics and a directed communication topology. First, in order to reduce communication frequency among followers, an output-based event-triggered control strategy is proposed for leader-following consensus. Second, neural-network-based observers are designed to reconstruct those immeasurable information of follower agents. Third, instead of continuously monitoring follower agents' measurements, a distributed self-triggered control strategy is put forward, which is based only on each agent's local estimation and its previous triggered instants. Furthermore, it is proved theoretically that the consensus tracking errors are ultimately bounded and the Zeno behavior is definitely excluded. Finally, simulation results are given to show the effectiveness and advantage of the proposed results.

Keywords: Multi-agent systems, Neural-network-based observers, Event-triggered control, Consensus

1. Introduction

Consensus control of multi-agent systems (MASs) has received increasing attention from control community due to its extensive applications, such as teaming of robots, satellite formation, and wireless sensor networks, to name just a few [6, 14, 15, 26, 31]. Traditionally, design of suitable consensus control schemes are accomplished in a periodic manner. Up to date, fruitful results on multi-agent consensus control have been reported under the periodic sampling framework and have provided a mature theoretical background as well as numerous practical examples [2, 30, 37]. However, it should be noted that periodic sampling paradigm disregards communication resource allocation and may become inefficient or infeasible for implementations when agents are deployed within a shared communication network environment and communication resource budget is a major concern. As an admissible alternative, aperiodic control schemes are preferable for networked connected systems under limited communication and energy resources [17, 21, 43]. Specifically, the event-triggered paradigm provides a promising way to aperiodically update control commands but only when needed [38–42]. This is because an event-triggered control scheme is dependent closely on a prescribed event triggering condition and control commands update only when the triggering condition is violated [7, 11, 28]. From this perspective, an event-triggered control scheme is superior to periodic counterparts with reduced sampling or control updates, thus having more potential in alleviating limited communication resources [23, 44].

Note that event-triggered schemes have become a hot topics in the field of multi-agent consensus during recent years, see, e.g., the recent survey papers [6, 12]. In [5], event-triggered consensus control mechanisms were introduced for a group of single-integrators, and then were extended to suit for both first-integrators and double-integrators under undirected communication graphs in [27]. It was reported that event-triggered control tasks were designed for multiple integrators with a fixed time interval in [25], and the extension was developed in [16]. Event-based broadcasting algorithms were proposed to achieve consensus for linear MASs with time-delay inputs by [45]. In [46], two

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triggered conditions were presented for linear MASs with/without continuous communication between neighboring agents. However, the complicated triggering function involves agents' own state information and their neighbors' information as well as control signals. The event-based broadcasting consensus algorithms were discussed for leadless general linear MASs with a fixed undirected topology [10] and for leader-following general linear MASs with different topologies [35]. Nevertheless, the dynamics of MASs are limited to be linear [5, 10, 16], and the designed control approaches are not feasible in certain practical applications [25, 27, 35]. Event-triggered controllers were designed in both [1] and [22] for nonlinear MASs under switching networks. However, these controllers require continuous communication for regeneration of control commands. Self-triggered control strategies [8, 9] were employed for next updating instants via a proactively predictive triggered mechanism using current measurement information, and thus the amount of interaction among agents was significantly reduced. In [19], both the event-triggered and self-triggered consensus problems were studied for linear MASs with an undirected communication graph. Furthermore, both event-triggered and self-triggered output consensus protocols were established for heterogeneous general linear MASs [20]. A nonsmooth analysis technique, including differential inclusion and Filippov solution, was employed in [34] to avoid continuous communication between neighboring linear agents for the leader-following and leaderless consensus cases. Whereas, a common feature of the results above mentioned lies in that only state feedback scenarios were considered for MASs. A few results concerning observer-based event-triggered consensus control methods were developed, such as consensus control of general linear MASs under an undirected graph [40] and time-varying network topology [3], and consensus tracking control of directed connected double-integrator MASs in [18]. To the best of the authors' knowledge, the problem of event-triggered observer-based leader-following consensus for nonlinear MASs under a directed communication topology has been rarely investigated, which motivates this study.

In this paper, the observer-based leader-following consensus is considered for a class of nonlinear MASs under a directed communication topology by developing an event-triggered mechanism. The main contributions of this paper are summarized as follows.

- *An output-based event-triggered leader-following consensus control strategy* is proposed for a class of nonlinear MASs with Lipschitz-type dynamics connected through a directed topology by an event-triggered mechanism. Compared with some existing methods in [4, 19, 29, 36, 45], the newly proposed strategy does not need the assumption that all states of neighboring agents should be measurable at any time. Moreover, with the proposed strategy, the measurement equipment such as speed-sensor devices is not necessary, which is of great significance from the industrial application points of view;
- *Neural-network-based state observers are introduced* to reconstruct immeasurable information of the nonlinear follower agents. Specifically, the universal approximation theory is employed to deal with those unknown functions of individual following agents. It should be noted that the observers designed in [18] and [40] for linear agents are not valid in the scenario under consideration. Furthermore, such an assumption that communication topologies should be undirected [19, 24, 29] is removed;
- *A self-triggered leader-following consensus control algorithm is presented*, where the triggered instants of an individual agent depend only on the previous triggered instants from the agent itself. Hence, additional sampling can be avoided, and it is not required to continuously monitor agents' measurements as is done in [33].

The rest of this paper is organized as follows. In Section 2, we present the problem formulation and give some preliminaries. The design of the output-based distributed event-triggered control approach and the main results are elaborated in Section 3. Section 4 presents a self-triggered control strategy to avoid the continuous measurement information monitoring, and Section 5 contains illustrative examples. Finally, we end this paper with some further directions in Section 6.

2. Problem Formulation and Preliminaries

We consider a group of agents, including one leader and N followers connected through a directed communication topology, and the dynamics of the ith follower agent are

$$\begin{cases} \dot{x}_i = Ax_i + f(x_i) + Bu_i, \\ y_i = Cx_i, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the inner state variable vector, $i = 1, \cdots, N$, $u_i \in \mathbb{R}^p$ and $y_i \in \mathbb{R}^q$ are the control input and output vector, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $f(x_i) = \left[f_1(x_i), \ldots, f_n(x_i)\right]^{\mathrm{T}} \in \mathbb{R}^n$ is a function vector standing for nonlinear dynamics of the ith agent satisfying Assumption 1, and $C \in \mathbb{R}^{q\times n}$ is the output matrix. It is assumed that the inner state vector x_i is not be directly measurable and the ith follower agent is observable. Denote $x_l \in \mathbb{R}^n$ as a trajectory vector of the leader, and the dynamics of the leader are

$$\dot{x}_l = Ax_l + f(x_l), \tag{2}$$

where x_l is measurable.

Assumption 1. *There exists a positive constant ρ satisfying the following inequality*

$$\|f(y) - f(z)\| \leq \rho\|y - z\|, \tag{3}$$

where $\forall y, z \in \mathbb{R}^n$.

Remark 1. *Assumption 1 is the so-called Lipschitz condition, and this assumption is normal and mild. The more details can be referred to [32].*

The interaction topology for the $N + 1$ agents, including one leader agent and N follower agents, is described by a directed network. $\bar{\mathcal{V}}$ and $\bar{\mathcal{E}}$ is defined as a node set and an edge set, respectively. A graph $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$, where $\bar{\mathcal{V}} = \{\upsilon_0, \upsilon_1, \cdots, \upsilon_N\}$, $\bar{\mathcal{E}} = \{(\upsilon_i, \upsilon_j) \in \bar{\mathcal{V}}\times\bar{\mathcal{V}}\}$ with (υ_i, υ_j) denoted that Node i can obtain information from Node j. The node named 0 is viewed as the leader, and other nodes, constituting a set $\mathcal{V} = \{\upsilon_1, \cdots, \upsilon_N\}$, are assigned to represent followers. $\mathcal{E} = \{(\upsilon_i, \upsilon_j) \in \mathcal{V} \times \mathcal{V}\}$, and the neighborhood set of the ith agent is described as $\mathcal{N}_i = \{j \in \mathcal{E}|(i, j) \in \mathcal{E}\}$ with $|\mathcal{N}_i|$ denoting the cardinality of the set $\mathcal{N}_i$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix with $a_{ij} = 1$, if $(\upsilon_j, \upsilon_i) \in \mathcal{E}$; and a_{ij} is zero, otherwise. We define the in-degree matrix $\mathcal{D} = \mathrm{diag}\,(d_1, \cdots, d_N)$, where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, and Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$. A diagonal matrix $G = \mathrm{diag}\,(g_1, \cdots, g_N)$ is the accessibility of the leader for the ith follower node with $g_i = 1$ if possible and $g_i = 0$ otherwise, $i = 1, \cdots, N$. We assume that at least one follower can access the information of the leader, that is, $G \neq 0$. It is also assumed that the graph $\mathcal{G}$ contains a spanning tree and $-(\mathcal{L} + G)$ is a nonsingular M-matrix. This implies that there exists a positive diagonal matrix $\Theta = \mathrm{diag}(\theta_1, \cdots, \theta_N)$ satisfying that $Y = \Theta^{-1}(\mathcal{L} + G) + (\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} > 0$. Define that $\beta = \frac{1}{2}\lambda_{\min}(Y)$, $\theta_{\max} = \max_{i=1,\ldots,N}\{\theta_i\}$, $\theta_{\min} = \min_{i=1,\ldots,N}\{\theta_i\}$, $\underline{\theta} = \min_{i=1,\ldots,N}\{\theta_i^{-1}\}$, $H = \Theta^{-1}(\mathcal{L} + G)(\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1}$, and $F = (\mathcal{L} + G)^{\mathrm{T}}(\mathcal{L} + G)$.

We aim to develop a distributed event-triggered control scheme to force each follower agent into tracking the leader's trajectory. For Agent i, a combinational state

$$s_i(t) = \sum_{j\in\mathcal{N}_i}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + g_i\left(x_l(t) - \hat{x}_i(t)\right) \tag{4}$$

is introduced here, where $i = 1, \cdots, N$, $\hat{x}_i$ is the estimation signal of x_i and is obtained from a state observer which will be given later.

Denote an event time sequence $\{t_0^i, t_1^i, \ldots\}$ for the ith follower agent, and it is assumed that its own triggered event only occurs at its individual triggered instant t_k^i, where $k = 0, 1, 2, \cdots$. Then, $s_i(t_k^i)$ represents the latest broadcast value of Agent i, and the measurement signal is given by $\breve{s}_i(t) = s_i(t_k^i)$, where $t \in [t_k^i, t_{k+1}^i)$. The proposed controller is based on sampling data, and we consider control scheme

$$u_i(t) = -K\breve{s}_i(t), t \in [t_k^i, t_{k+1}^i), \tag{5}$$

for the ith agent, where $K \in \mathbb{R}^{p\times n}$ is a gain matrix and it will be determined later.

In the above mentioned framework of data transmission, the leader-follower consensus is said to be achieved if the following properties hold, as $t \to \infty$,

$$\sum_{i=1}^{N}\|x_l(t) - x_i(t)\|^2 \leq \zeta, \tag{6}$$

and Zeno behavior is also excluded, where $\zeta > 0$. The control objective of this paper is to develop such a control strategy based on output information for the system (1) to satisfy (6).

3. Output-Based Distributed Event-Triggered Control Approach

In this part, an output-based leader-following consensus control scheme is designed for the system (1) in event-triggered manners, and then the stability of the closed-loop system as well as non-Zeno behaviour is presented.

3.1. Neural-Network-Based Observer Design

In this section, neural-network-based observers are employed for follower agents to estimate inner states based on the input and output information, where the neural networks (NNs) are utilized to parameterise the nonlinear dynamics and trained the weights on the basis of estimation errors.

In the light of the universal approximation property of the NNs, $f(x_i)$ can be represented as

$$f(x_i) = W_i^* \Xi_i(x_i) + \varepsilon_i(x_i), \tag{7}$$

where $W_i^* \in \mathbb{R}^{n \times n_h}$ is the ideal weight matrix and is bounded $\|W_i^*\| \le W_{i,M}$, $\Xi_i(\cdot) \in \mathbb{R}^{n_h}$ is the active function vector whose elements are chosen as a hyperbolic tangent function satisfying that $\|\Xi_i(\cdot)\| \le \Xi_{i,M}$, n_h is the number of hidden layer, $\varepsilon_i(\cdot)$ is the approximation error vector such that $\|\varepsilon_i(\cdot)\| \le \varepsilon_{i,M}$, and $W_{i,M}$, $\Xi_{i,M}$ and $\varepsilon_{i,M}$ are positive constants.

Then, a state observer is proposed for the ith agent

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i + \hat{W}_i \Xi_i(\hat{x}_i) + Bu_i + L_{ob}(y_i - \hat{y}_i), \\ \hat{y}_i = C\hat{x}_i, \end{cases} \tag{8}$$

with the adaptive tuning law

$$\dot{\hat{W}}_i = -\iota_i \left(\widetilde{y}_i^{\mathrm{T}} C A_c^{-1}\right)^{\mathrm{T}} \Xi_i^{\mathrm{T}}(\hat{x}_i) - \Pi_i \hat{W}_i, \tag{9}$$

where $\hat{y}_i$ is the reconstruction information for the ith agent, $L_{ob} \in \mathbb{R}^{n \times q}$ is the gain matrix of the observer, $\widetilde{y}_i = y_i - \hat{y}_i$, $A_c = A - L_{ob}C$, $\iota_i > 0$, and $\Pi_i > 0$.

The first main result of this paper can be summarized in the following theorem.

Theorem 1. *Given the state observer* (8) *for the system* (1)*, then the observer* (8) *guarantees that estimation errors are ultimately uniformly bounded, and the errors can be made arbitrarily small by appropriate design parameters.*

Proof 1. *Define error signals $\widetilde{x}_i = x_i - \hat{x}_i$ and $\widetilde{W}_i = W_i^* - \hat{W}_i$. Considering* (1) *and* (8)*, the observer error dynamics are*

$$\begin{cases} \dot{\widetilde{x}}_i = A_c \widetilde{x}_i + \widetilde{W}_i \Xi_i(\hat{x}_i) + \xi_i, \\ \widetilde{y}_i = C\widetilde{x}_i, \end{cases} \tag{10}$$

where $\xi_i = W_i^* (\Xi_i(x_i) - \Xi_i(\hat{x}_i)) + \varepsilon_i(x_i)$ is a bounded error term satisfying that $\|\xi_i\| \le \xi_{i,M}$, $\xi_{i,M}$ is a positive constant, and this due to the fact that the hyperbolic tangent functions, the approximation error of NNs $\varepsilon_i(\cdot)$ and ideal NNs' weights are bounded. We choose the Lyapunov function

$$V_{o,i} = \frac{1}{2}\widetilde{x}_i^{\mathrm{T}} \Gamma \widetilde{x}_i + \frac{1}{2} tr\left\{\widetilde{W}_i^{\mathrm{T}} \widetilde{W}_i\right\}, \tag{11}$$

where Γ is a positive matrix satisfying that, for certain positive definite matrix Λ,

$$A_c^{\mathrm{T}} \Gamma + \Gamma A_c = -\Lambda. \tag{12}$$

The derivative of $V_{o,i}$ is given by

$$\dot{V}_{o,i} = \frac{1}{2}\dot{\widetilde{x}}_i^{\mathrm{T}} \Gamma \widetilde{x}_i + \frac{1}{2}\widetilde{x}_i^{\mathrm{T}} \Gamma \dot{\widetilde{x}}_i + tr\left\{\widetilde{W}_i^{\mathrm{T}} \dot{\widetilde{W}}_i\right\}, \tag{13}$$

and one can get

$$\dot{V}_{o,i} = \frac{1}{2}\widetilde{x}_i^{\mathrm{T}}\left(A_c^{\mathrm{T}}\Gamma + \Gamma A_c\right)\widetilde{x}_i + \widetilde{x}_i^{\mathrm{T}}\Gamma\widetilde{W}_i\Xi_i(\hat{x}_i) + \widetilde{x}_i^{\mathrm{T}}\Gamma\xi_i + tr\left\{\widetilde{W}_i^{\mathrm{T}}\left[\iota_i\left(\widetilde{y}_i^{\mathrm{T}}CA_c^{-1}\right)^{\mathrm{T}}\Xi_i^{\mathrm{T}}(\hat{x}_i) + \Pi_i\hat{W}_i\right]\right\}. \tag{14}$$

Denoting that $a_i = \iota_i A_c^{-\mathrm{T}}CC^{\mathrm{T}}$, *and according the following inequalities*

$$tr\left\{\widetilde{W}_i^{\mathrm{T}}a_i\widetilde{x}_i\Xi(\hat{x}_i)\right\} \leq \Xi_{i,M}||\widetilde{W}_i||||a_i||||\widetilde{x}_i||, \tag{15}$$

$$tr\left\{\Pi_i\widetilde{W}_i^{\mathrm{T}}\hat{W}_i\right\} \leq \Pi_i\left(W_{i,M}||\widetilde{W}_i|| - ||\widetilde{W}_i||^2\right), \tag{16}$$

and completely square inequalities, it yields that

$$\dot{V}_{o,i} \leq -\frac{1}{2}\varphi_{o,i,1}||\widetilde{x}_i||^2 - \frac{1}{2}\varphi_{o,i,2}tr\left\{\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i\right\} + \psi_{o,i}, \tag{17}$$

where $\varphi_{o,i,1} = \lambda_{\min}(\Lambda) - \Xi_{i,M}(||\Gamma|| + ||a_i||) - 1$, $\varphi_{o,i,2} = \Pi_i - \Xi_{i,M}(||\Gamma|| + ||a_i||)$, $\psi_{o,i} = \frac{\xi_{i,M}^2}{2}||\Gamma||^2 + \frac{\Pi_i}{2}W_{i,M}^2$.

Choose $V_o = \sum_{i=1}^N V_{o,i}$, *and the derivative of it is*

$$\begin{aligned}\dot{V}_o &\leq -\frac{1}{2}\frac{\varphi_{o,1}}{\lambda_{\max}(\Gamma)}\widetilde{x}^{\mathrm{T}}\Gamma\widetilde{x} - \frac{1}{2}\varphi_{o,2}\sum_{i=1}^N tr\left\{\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i\right\} + \sum_{i=1}^N\psi_{o,i} \\ &\leq -\varphi_o V_o + \psi_o,\end{aligned} \tag{18}$$

where $\varphi_{o,1} = \min\{\varphi_{o,1,1},\cdots,\varphi_{o,N,1}\}$, $\varphi_{o,2} = \min\{\varphi_{o,1,2},\cdots,\varphi_{o,N,2}\}$, $\varphi_o = \min\{\varphi_{o,1}/\lambda_{\max}(\Gamma),\varphi_{o,2}\}$, *and* $\psi_o = \sum_{i=1}^N\psi_{o,i}$.

Therefore, the simple transposition of (11), (17) *and* (18) *leads to*

$$||\widetilde{x}_i|| \leq \sqrt{\frac{2\psi_{o,i}}{\lambda_{\min}(\Gamma)\varphi'_{o,i}}}, \tag{19}$$

where $\varphi'_{o,i} = \min\{\varphi_{o,i,1}/\lambda_{\max}(\Gamma),\varphi_{o,i,2}\}$. *This means that the size of compact* $\Omega_{\widetilde{x}_i} := \left\{\widetilde{x}_i \mid ||\widetilde{x}_i|| \leq \sqrt{2\psi_{o,i}/\left(\lambda_{\min}(\Gamma)\varphi'_{o,i}\right)}\right\}$ *can be made arbitrarily small by suitable choice of design parameters.*

3.2. Observer-Based Event-Triggered Control Law Design

In this part, on the basis of the information from observers, an output feedback event-triggered control scheme is presented as well as the stability for the closed-loop system. For the achievement of the control objective mentioned in Section 2, the measurement error

$$e_i(t) = \check{s}_i(t) - s_i(t) \tag{20}$$

and tracking error

$$\eta_i(t) = x_l(t) - \hat{x}_i(t) \tag{21}$$

are defined for the ith follower agent.

The triggered instant is defined as

$$t_{k+1}^i = \inf\left\{t > t_k^i : h_i \geq 0\right\}, \tag{22}$$

and the triggered function is chosen for the ith follower agent

$$h_i = \frac{||e_i||^2}{\beta b_1} - \mu_i\theta_i^{-1}s_i^{\mathrm{T}}Ps_i - \delta, \tag{23}$$

where $\mu_i > 0$, $\delta > 0$, $\beta_1 = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $\beta_2 = \frac{\lambda_{\max}(H)\lambda^2_{\max}(PBR^{-1}B^{\mathrm{T}}P)}{\theta\lambda_{\min}(P)}$, $\beta_3 = \frac{\lambda_{\max}(H)\lambda_{\max}(P^{\mathrm{T}}P)}{\theta\lambda_{\min}(P)}$, $\beta_4 = \frac{\lambda_{\max}(H)\lambda_{\max}(PL_{ob}C)}{\theta\lambda_{\min}(P)}$, $\mu_i = -\gamma_i + \beta_1 - \frac{b_1}{\beta}\beta_2 - 3\beta_3 - b_2\beta_4$, $\gamma_i > 0$, $b_1 > 0$, $b_2 > 0$, and the positive and symmetrical matrix P satisfying

$$PA + A^{\mathrm{T}}P - 2\theta_{\min}PBR^{-1}B^{\mathrm{T}}P + \rho^2\lambda_{\max}(D)\theta_{\max}I_n + Q \leq 0, \tag{24}$$

where D will be given in the following text, and I_n is an identify matrix with appropriate dimension. The diagram of the proposed control scheme is shown in Fig. 1. And then, another main result of this paper is presented in the following theorem.

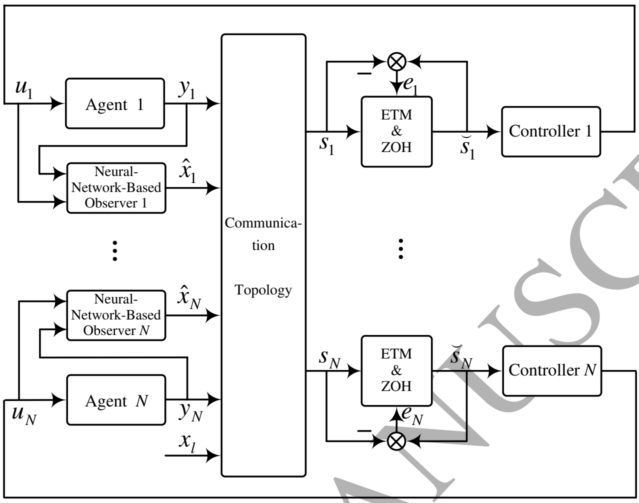

Figure 1: The diagram of the proposed control scheme.

Theorem 2. *Consider the multi-agent system including N followers in* (1) *and one leader in* (2) *connected through a directed graph and Assumption 1 holds. The control scheme* (5) *with $K = -\frac{1}{\beta}R^{-1}B^{\mathrm{T}}P$ and the observer* (8) *is able to steer the closed-loop system to achieve leader-following consensus target* (6) *and to exclude Zeno behavior if the triggered time sequence for each follower agent is defined by* (22) *with suitable choice of control parameters.*

Proof 2. *Recalling* (1), (20) *and* (21)*, the derivative of η_i becomes*

$$\dot{\eta}_i = A\eta_i + C\Big(f(x_l) - f(\hat{x}_i)\Big) + BK(e_i + s_i) - L_{ob}C\widetilde{x}_i + \widetilde{W}_i\Xi_i(\hat{x}_i) + \varepsilon_i. \tag{25}$$

Then, by denoting $\eta = \left[\eta_1^{\mathrm{T}}, \cdots, \eta_N^{\mathrm{T}}\right]^{\mathrm{T}}$, $e = \left[e_1^{\mathrm{T}}, \cdots, e_N^{\mathrm{T}}\right]^{\mathrm{T}}$, $s = \left[s_1^{\mathrm{T}}, \cdots, s_N^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\varepsilon = \left[\varepsilon_1^{\mathrm{T}}, \cdots, \varepsilon_N^{\mathrm{T}}\right]^{\mathrm{T}}$, we rewritten (25) *in vector as below*

$$\dot{\eta} = (I_N \otimes A)\eta + (I_N \otimes BK)e + (I_N \otimes BK)s + \widetilde{f} - (I_N \otimes L_{ob}C)\widetilde{x} + \widetilde{W}\Xi + \varepsilon, \tag{26}$$

where $\widetilde{f} = \left[\Big(f(x_l) - f(\hat{x}_1)\Big)^{\mathrm{T}}, \cdots, \Big(f(x_l) - f(\hat{x}_N)\Big)^{\mathrm{T}}\right]^{\mathrm{T}}$, $\widetilde{W} = \mathrm{diag}\Big(\widetilde{W}_1, \cdots, \widetilde{W}_N\Big)$, $\Xi = \left[\Xi_1^{\mathrm{T}}, \cdots, \Xi_N^{\mathrm{T}}\right]^{\mathrm{T}}$, and I_N is the Nth identify matrix.

We choose the Lyapunov function candidate

$$V_s = s^{\mathrm{T}}(\Theta^{-1} \otimes P)s, \tag{27}$$

where $s = ((\mathcal{L} + G) \otimes I_n)\,\eta$, and the time derivative of V_s yields

$$\begin{aligned}
\dot{V}_s =& \dot{s}^{\mathrm{T}}(\Theta^{-1} \otimes P)s + s^{\mathrm{T}}(\Theta^{-1} \otimes P)\dot{s} \\
=& \eta^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes A^{\mathrm{T}}P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes PA)\eta + s^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes (BK)^{\mathrm{T}}P)s \\
&+ s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes P(BK))s + e^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes (BK)^{\mathrm{T}}P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes P(BK))e \\
&+ \widetilde{f}^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes P)\widetilde{f} - \widetilde{x}^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes (L_{ob}C)^{\mathrm{T}}P)s \\
&+ (\widetilde{W}\Xi + \varepsilon)^{\mathrm{T}}((\mathcal{L} + G)^{\mathrm{T}}\Theta^{-1} \otimes P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes P)(\widetilde{W}\Xi + \varepsilon) - s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L} + G) \otimes PL_{ob}C)\widetilde{x}.
\end{aligned} \tag{28}$$

With the definition of η*, the relationship between* η *and* s*,* $K = -\frac{1}{\beta}R^{-1}B^{\mathrm{T}}P$*,* (3) *and completely square inequalities, one obtains that*

$$e^{\mathrm{T}}((\mathcal{L}+G)^{\mathrm{T}}\Theta^{-1}\otimes(BK)^{\mathrm{T}}P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L}+G)\otimes P(BK))e \le \frac{e^{\mathrm{T}}e}{\beta b_1} + \frac{b_1}{\beta}s^{\mathrm{T}}(H\otimes(PBR^{-1}B^{\mathrm{T}}P)^2)s, \tag{29}$$

$$\widetilde{f}^{\mathrm{T}}((\mathcal{L}+G)^{\mathrm{T}}\Theta^{-1}\otimes P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L}+G)\otimes P)\widetilde{f} \le s^{\mathrm{T}}\Theta^{-1}\otimes(\rho^2\lambda_{\max}(D)\theta_{\max})I_n s + s^{\mathrm{T}}(H\otimes(P^{\mathrm{T}}P))s, \tag{30}$$

$$s^{\mathrm{T}}((\mathcal{L}+G)^{\mathrm{T}}\Theta^{-1}\otimes(BK)^{\mathrm{T}}P)s + s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L}+G)\otimes P(BK))s \le -s^{\mathrm{T}}(\Theta^{-1}\otimes(2\theta_{\min}PBR^{-1}B^{\mathrm{T}}P))s, \tag{31}$$

$$s^{\mathrm{T}}(\Theta^{-1}(\mathcal{L}+G)\otimes P)(\widetilde{W}\Xi+\varepsilon)+(\widetilde{W}\Xi+\varepsilon)^{\mathrm{T}}((\mathcal{L}+G)^{\mathrm{T}}\Theta^{-1}\otimes P)s \le 2s^{\mathrm{T}}(H\otimes(P^{\mathrm{T}}P))s+\sum_{i=1}^{N}\left(\Xi_{i,M}\|\widetilde{W}_i\|^2+\varepsilon_{i,M}^2\right), \tag{32}$$

where $D = (\mathcal{L}+G)^{-\mathrm{T}}(\mathcal{L}+G)^{-1}$*. Then, it arrives*

$$\begin{aligned}\dot{V}_s \le & -s^{\mathrm{T}}(\Theta^{-1}\otimes Q)s + \frac{e^{\mathrm{T}}e}{\beta b_1} + s^{\mathrm{T}}(H\otimes(P^{\mathrm{T}}P))s - s^{\mathrm{T}}(\Theta^{-1}\otimes(2\theta_{\min}PBR^{-1}B^{\mathrm{T}}P))s + s^{\mathrm{T}}(\Theta^{-1}\otimes(\rho^2\lambda_{\max}(D)\theta_{\max})I_n)s \\ & + \frac{b_1}{\beta}\eta^{\mathrm{T}}(H\otimes(PBR^{-1}B^{\mathrm{T}}P)^2)\eta + \frac{1}{b_2}\widetilde{x}^{\mathrm{T}}\widetilde{x} + 3s^{\mathrm{T}}(H\otimes(P^{\mathrm{T}}P))s + b_2 s^{\mathrm{T}}(H\otimes(PL_{ob}C)^2)s + \sum_{i=1}^{N}\left(\Xi_{i,M}\|\widetilde{W}_i\|^2+\varepsilon_{i,M}^2\right).\end{aligned} \tag{33}$$

Since the triggered condition (22) *enforces the property, during the time interval* $[t_k^i, t_{k+1}^i)$*,*

$$\frac{\|e_i\|^2}{\beta b_1} \le \mu_i\theta_i^{-1}s_i^{\mathrm{T}}Ps_i + \delta, \tag{34}$$

(33) *can be rewritten as*

$$\dot{V}_s \le -\sum_{i=1}^{N}\gamma_i\theta_i^{-1}s_i^{\mathrm{T}}Ps_i + N\delta + \frac{1}{b_2}\widetilde{x}^{\mathrm{T}}\widetilde{x} + \sum_{i=1}^{N}\left(\Xi_{i,M}\|\widetilde{W}_i\|^2+\varepsilon_{i,M}^2\right). \tag{35}$$

Choose $V = V_s + V_o$*, and the time derivative of it yields*

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{N}\gamma_i\theta_i^{-1}s_i^{\mathrm{T}}Ps_i - \frac{1}{2}\left(\varphi_{o,1}-\frac{2}{b_2}\right)\widetilde{x}^{\mathrm{T}}\widetilde{x} - \frac{1}{2}\left(\varphi_{o,2}-2\Xi_M^2\right)\sum_{i=1}^{N}tr\left\{\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i\right\} + N\delta + \sum_{i=1}^{N}\left(\psi_{o,i}+\varepsilon_{i,M}^2\right) \\ \le & -\chi V + \varsigma,\end{aligned} \tag{36}$$

where $\chi = \min\left\{\left(\varphi_{o,1}-\frac{2}{b_2}\right), \varphi_{o,2}-2\Xi_M^2, \gamma_1, \cdots, \gamma_N\right\}$*,* $\varsigma = N\delta + \sum_{i=1}^{N}\left(\psi_{o,i}+\varepsilon_{i,M}^2\right)$*.*

Then, we can obtain that

$$V \le \mathrm{e}^{-\chi t}\left[V(0) - \frac{N\varsigma}{\chi}\right] + \frac{N\varsigma}{\chi} \le \omega, \tag{37}$$

where $\omega \triangleq V(0) + \dfrac{N\varsigma}{\chi}$ *is a constant. According to the Lyapunov function candidate* (27)*, it yields*

$$\underline{\theta}\lambda_{\min}(P)\sum_{i=1}^{N}\|s_i\|^2 \le s^{\mathrm{T}}(\Theta^{-1}\otimes P)s \le \omega, \tag{38}$$

and it means that $\|s_i\|$ *is bounded such that*

$$\|s_i\| \le \sqrt{\frac{\omega}{\underline{\theta}\lambda_{\min}(P)}}, \tag{39}$$

and we denote that $\bar{\omega} \triangleq \sqrt{\frac{\omega}{\underline{\theta}\lambda_{\min}(P)}} > 0$*.*

In the following text, we present the analysis about the Zeno-free behaviour of the system. For $t \in (t_k^i, t_{k+1}^i)$, we have

$$\frac{\mathrm{d}\|e_i\|}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(e_i^{\mathrm{T}} e_i)^{\frac{1}{2}} = \frac{e_i^{\mathrm{T}} \dot{e}_i}{\|e_i\|} \leq \|\dot{e}_i\|, \tag{40}$$

and $\|e_i\| > 0$. Taking the definition of e_i into account, it yields

$$\dot{e}_i = -\dot{s}_i, \tag{41}$$

and then utilizing (4), we obtain

$$\begin{aligned} \|\dot{e}_i\| =& \|\sum_{j\in\mathcal{N}_i}\left(\dot{\hat{x}}_j(t) - \dot{\hat{x}}_i(t)\right) + g_i\left(\dot{x}_l(t) - \dot{\hat{x}}_i(t)\right)\| \\ =& \Big\|As_i(t) + BK\Big[\sum_{j\in\mathcal{N}_i}\left(\breve{s}_i - \breve{s}_j\right) + g_i\breve{s}_i\Big] + \Big[\sum_{j\in\mathcal{N}_i}\left(\hat{f}(\hat{x}_j) - \hat{f}(\hat{x}_i)\right) + g_i\left(f(x_l) - \hat{f}(\hat{x}_i)\right)\Big] \\ &+ \sum_{j\in\mathcal{N}_i} L_{ob}C(\widetilde{x}_j - \widetilde{x}_i) - g_i L_{ob} C\widetilde{x}_i\Big\|. \end{aligned} \tag{42}$$

Substituting (19), (20) and Assumption 1 into (42) results in

$$\|\dot{e}_i\| \leq \|A\|\|e_i(t)\| + \epsilon_k^i + (2|\mathcal{N}_i| + s_i)\left(\rho\sqrt{N\bar{\kappa}} + \bar{W}_{i,M}\Xi_{i,M}\right) + \|L_{ob}C\|\left(\|\mathcal{L}\|\sqrt{N} + g_i\right)\frac{2\psi_{o,i}}{\varphi_{o,i}\lambda_{\min}(\Gamma)}, \tag{43}$$

where $\bar{\kappa} = \frac{\omega}{\underline{\theta}\lambda_{\min}(P)\lambda_{\min}(F)}$, $\epsilon_k^i = \max\limits_{t\in[t_k^i, t_{k+1}^i)}\left(\Big\|As_i(t_k^i) + BK\Big[\sum_{j\in\mathcal{N}_i}\left(s_i(t_k^i) - s_j(t_k^i)\right) + g_i s_i(t_k^i)\Big]\Big\|\right)$.

Using (22) and (43), one has

$$\sqrt{\beta b_1 \delta} \leq \|e(t_{k+1}^{i-})\| \leq \frac{\epsilon_k^i + \gamma_0}{\|A\|}\left(\mathrm{e}^{\|A\|(t_{k+1}^i - t_k^i)} - 1\right), \tag{44}$$

where $\gamma_0 = (2|\mathcal{N}_i| + s_i)\left(\rho\sqrt{N\bar{\kappa}} + \bar{W}_{i,M}\Xi_{i,M}\right) + \|L_{ob}C\|\left(\|\mathcal{L}\|\sqrt{N} + g_i\right)\frac{2\psi_{o,i}}{\varphi_{o,i}\lambda_{\min}(\Gamma)}$, and for any k, the property holds $\epsilon_k^i \leq (\|A\| + (2|\mathcal{N}|_i + s_i)\|BK\|)\,\bar{\omega} = \bar{\epsilon}$. Hence, it yields

$$t_{k+1}^i - t_k^i \geq \frac{1}{\|A\|}\ln\left(1 + \frac{\|A\|\sqrt{\beta b_1 \delta}}{\bar{\epsilon} + \gamma_0}\right) > 0. \tag{45}$$

Therefore, the Zeno behavior is excluded.

For the follower agents, it indicats that

$$\lim_{t\to\infty}\sum_{i=1}^{N}\|s_i\|^2 \leq \frac{N\varsigma}{\chi\underline{\theta}\lambda_{\min}(P)}, \tag{46}$$

and, according to the relationship between s and η, we further obtain that

$$\lim_{t\to\infty}\sum_{i=1}^{N}\|\eta_i\|^2 \leq \frac{N\varsigma}{\chi\underline{\theta}\lambda_{\min}(P)\lambda_{\min}(F)}. \tag{47}$$

By virtue of the fact

$$\|x_l - x_i\|^2 \leq 2\left(\|x_l - \hat{x}_i\|^2 + \|\hat{x}_i - x_i\|^2\right), \tag{48}$$

and (47), it follows that

$$\lim_{t\to\infty}\sum_{i=1}^{N}\|x_l - x_i\|^2 \leq \frac{2N\varsigma}{\chi\underline{\theta}\lambda_{\min}(P)\lambda_{\min}(F)} + \sum_{i=1}^{N}\frac{2\psi_{o,i}}{\varphi_{o,i}\lambda_{\min}(\Gamma)}, \tag{49}$$

which indicates that there exists T such that for all $t > T$, the consensus errors remain bounded.

This completes the proof of Theorem 2.

Remark 2. *Form the control objective* (6) *and the triggered function* (23)*, it is the fact that we only achieve the practical consensus but not the exact consensus in this paper. From* (49)*, it directly shows that the ultimate boundedness of the tracking errors might be adjustable by suitable choice of control parameters. Take the parameter δ for example. Decreasing this parameter might result in smaller tracking errors, that is to say, the consensus performance can be guaranteed if δ is sufficiently small. On the other hand, if δ is scaled down, the lower boundedness of the triggered intervals is shorter according to* (45) *and the events might occur frequently. This lead to more frequent data transmission and requirement for more bandwidth. Therefore, we have to make a compromise between control precision and communication events, and these parameters should be adjusted carefully for suitable performance in practical applications.*

4. Self-Triggered Scheme

From the event-triggered conditions (22) and (23) proposed in the previous section, it is apparent that the control scheme should monitor measurement information continuously to check whether triggered conditions are satisfied. Consequently, a kind of self-triggered schemes is proposed in the following section on the basis of control approaches presented in Section 3. To address this concern, we define that $q_k^i = \frac{c_i}{\sqrt{2+2c_i^2}}\|s_i(t_k^i)\|$, where $c_i^2 = 2\beta b_1\sqrt{\mu_i\delta\theta_i^{-1}\lambda_{\min}(P)}$. We set a conservative triggered condition as

$$h'\left(e_i(t), s_i(t_k^i)\right) = \|e_i\| - q_k^i = 0, \tag{50}$$

and the increasing rate of $\|e_i\|$ yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\|e_i\| \leq \alpha_i(t) \leq \|A\|q_k^i + \epsilon_k^i + \gamma_0, \tag{51}$$

where $\alpha_i(t) = \|A\|q_k^i + \|As_i(t_k^i) + BK\big[\sum_{j\in\mathcal{N}_i}\left(s_i(t_k^i) - s_j(t_k^i)\right) + g_i s_i(t_k^i)\big]\| + \gamma_0$. Denote $\alpha_k^i = \alpha_i(t_k^i)$, $\bar{\alpha}_k^i = \|A\|q_k^i + \epsilon_k^i + \gamma_0$, and it means that $\alpha_i(t)$ remains the same as α_k^i until other neighbouring agents' state $s_j(t_{k'(t)}^j)$ is updated. If no information from its neighbour is received before $\|e_i(t)\|$ reaches q_k^i, we obtain that $t_{k+1}^i - t_k^i = q_k^i/\alpha_k^i$; otherwise, the rate $\alpha_i(t)$ will be updated based on the newly received information, and one needs to calculate the remaining time for $\|e_i(t)\|$ to recover the rest of q_k^i. The primary result is summarized in the Theorem 3.

Theorem 3. *Given the MAS* (1)*, the system can reach consensus with the triggered time sequence $\{t_0^i, t_1^i, \cdots, t_k^i, \cdots\}$ stemmed from the Algorithm* 1.

Proof 3. *In light of the fact $\|e_i(t_k^i)\| = 0$, for $t \in (t_k^i, t_{k+1}^i]$, we have*

$$\begin{aligned}\|e_i(t)\| &= \int_{t_k^i}^{t} \frac{\mathrm{d}}{\mathrm{d}t}\|e_i\|\mathrm{d}t \\ &\leq \int_{t_k^i}^{t_{k+1}^i} \frac{\mathrm{d}}{\mathrm{d}t}\|e_i\|\mathrm{d}t \\ &\leq \alpha_i(t_k^i)\cdot\left(t_{m1} - t_k^i\right) + \alpha_i(t_{m1})\cdot\left(t_{m2} - t_{m1}\right) + \cdots + \alpha_i(t_{mp})\cdot\left(t_{k+1}^i - t_{mp}\right) = q_k^i,\end{aligned} \tag{52}$$

where t_{mp} is the updating time instant for $\alpha_i(t)$ over the interval $[t_k^i, t_{k+1}^i]$ on the basis of Algorithm 1.

From (20)*,* (50) *and* (52)*, it yields that*

$$\|e_i\|^2 \leq \frac{c_i^2}{2+2c_i^2}\|e_i(t) + s_i(t)\|^2 \leq \frac{c_i^2}{1+c_i^2}\left(\|e_i(t)\|^2 + \|s_i(t)\|^2\right), \tag{53}$$

Algorithm 1 Determination of Triggered Time t^j_{k+1}

Initialization

$t_0 = t^i_k$; $q_i = \frac{c_i}{\sqrt{2+2c_i^2}}\|s_i(t_0)\|$; $\alpha_i = \alpha_i(t_0)$;

$i = 1, \cdots, N$;

while $t < Time$

do $p = \arg\min_i \left(t^i_k + \frac{q_i}{\alpha_i}\right)$; $time = t^p_k + \frac{q_p}{\alpha_p}$;

for $j = 1, \cdots, N$

if $j = p$

update $t^j_{k+1} = time$; $k = k+1$; $q_j = \frac{c_j}{\sqrt{2+2c_j^2}}\|s_j(t^j_k)\|$;

$\alpha_j = \alpha_j(t^j_k)$; $\alpha_l = \alpha_l(t^j_k)$, where $l \in \mathcal{N}_j$;

else

$q_j = q_j - \alpha_j\left(time - t^j_k\right)$;

end if

end for

end while

return t^j_{k+1}.

and then, solving inequality (53) *and according to the definition of c_i, the following result comes*

$$\|e_i\|^2 \le c_i^2 s_i^2(t) \le \beta b_1\left(\mu_i \theta_i^{-1} s_i^{\mathrm{T}} P s_i + \delta\right), \tag{54}$$

which indicates that the inequality equation (54) *satisfies the triggered condition* (22) *and the analysis about consensus of MASs is the same as the previous one. This is omitted for the sake of space limitation. From the framework of the Algorithm 1, the inter-event time for Agent i holds the property that $t^i_k - t^i_{k+1} = q^i_k/\alpha^i_k \ge q^i_k/\bar{\alpha}^i_k$. It follows that $t^i_k - t^i_{k+1}$ is strictly positive as $s_i(t^i_k) \neq 0$. In the light of $s_i(t^i_k) = 0$, the limit of $q^i_k/\bar{\alpha}^i_k$ is also a positive number as $k \to \infty$. Thus, the Zeno-free behavior can be proved.*

5. Illustrative Example

In this section, we consider three single-link manipulators with flexible joints driven by direct-current (DC) motors, shown in Fig. 2, as follower agents, and the leader-following MASs are connected through a directed communication topology shown in Fig. 3. The dynamics of the ith follower manipulator are in the form of (1) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ f(x_i) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.33\sin(x_{i,3}) \end{bmatrix},\ x_i = \begin{bmatrix} x_{i,1} \\ x_{i,2} \\ x_{i,3} \\ x_{i,4} \end{bmatrix}$$

where $i = 1, 2, 3$. It is easy to check that the individual follower agent is observable and controllable. In view of Assumption 1, one can obtain that $\rho = 0.33$. The leader agent is identical and its model is given as (2), where $x_l = \left[x_{l,1}, x_{l,2}, x_{l,3}, x_{l,4}\right]^{\mathrm{T}}$, and $f(x_l) = \left[0, 0, 0, 0.33\sin(x_{l,3})\right]^{\mathrm{T}}$. The initial states are set as $x_l(0) = \left[0.2, 0.3, 0.2, 0.5\right]^{\mathrm{T}}$, $x_1(0) = \left[0, 3, 3, 0\right]^{\mathrm{T}}$, $x_2(0) = \left[1, 1, 0.1, 0.2\right]^{\mathrm{T}}$, $x_3(0) = \left[3, 0.1, 2, 0.4\right]^{\mathrm{T}}$. The control parameters are chosen as $Q = 5I_{2\times 2}$, $R = 50$, $\mu_i = 0.1$, $\nu_i = 2.5$, $\sigma_i = 0.39$, $\varsigma = 0.001$, $\delta = 7.97\times 10^{-3}$, $L_{ob} = \left[7, 1, 1, 1\right]^{\mathrm{T}}$, $K = \left[-1.46, -1.06, -0.02, -2.73\right]$,

$i = 1, 2, 3$. Figs. 4–10 show the trajectories of the leader and three follower agents, control inputs, and state estimations, respectively. During $[0\text{s}, 30\text{s}]$, the triggered events of individual agents are shown in Fig. 11, and the mean time intervals are 0.0127, 0.0119 and 0.0151, respectively, which are recorded in Tab. 1. From Fig. 11 and Tab. 1, it indicates that the control strategy with event-triggered manner proposed in this paper, compared with that without event-triggered case, substantially reduces the update of information transmission over connected networks.

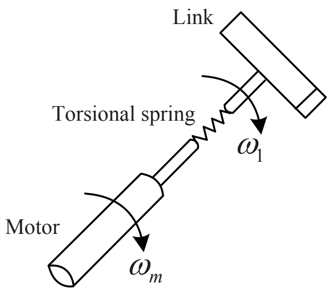

Figure 2: Single-link manipulator.

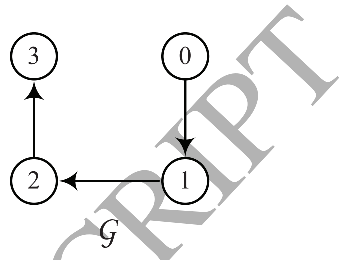

Figure 3: The communication topology.

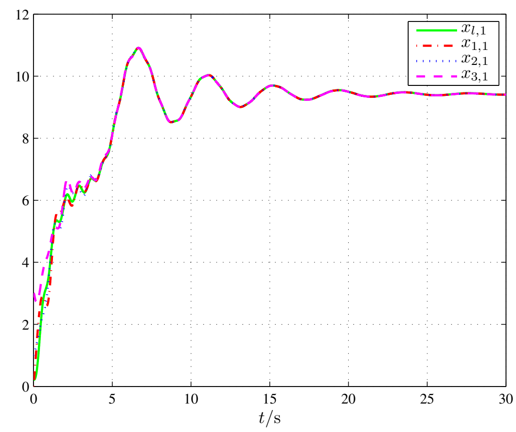

Figure 4: The trajectory of the leader $x_{l,1}$ and $x_{i,1}$, $i = 1, 2, 3$.

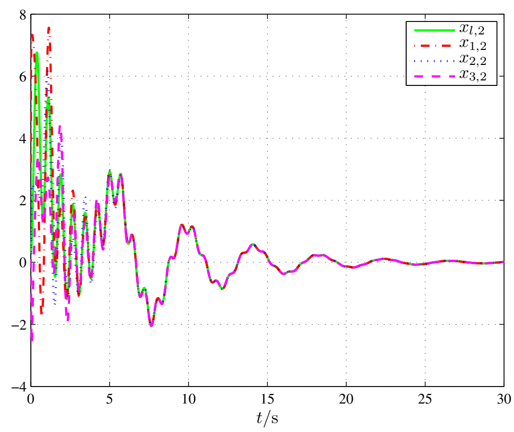

Figure 5: The trajectory of the leader $x_{l,2}$ and $x_{i,2}$, $i = 1, 2, 3$.

The relationship between the performance of consensus errors $\|x_l - x_i\|$ and δ are shown in Fig. 12, where $\delta_1 = 1.97 \times 10^{-2}$ (Case 1), $\delta_2 = 8.97 \times 10^{-3}$ (Case 2), $\delta_3 = 7.97 \times 10^{-5}$ (Case 3). Take the 1st and 2nd agents for illustration in this the example. From Fig. 12, it shows that one of feasible ways is to scale up the adjustable parameter δ if the event-triggered consensus tracking errors are desired to be lower. Further, numerous simulation results clarify that more triggered events occur along with the decreasement of δ, which means that the proposed algorithm have to make a compromise between the consensus performance and network communication resource. In addition, the triggered numbers in the first 15s regulated by event-triggered and self-triggered strategies are recorded in Tab. 2. From the comparison of update numbers of individual agent, we can observe that the number with the self-triggered scheme is more than that with the event-triggered policy. This is due to the fact that the triggered condition (50) is more conservative than (22), and occurrences of triggered events may happen with high probability, which leads to more execution instants. This illustrates the trade-off between the data updates and continuous monitors in the proposed control methods of this paper.

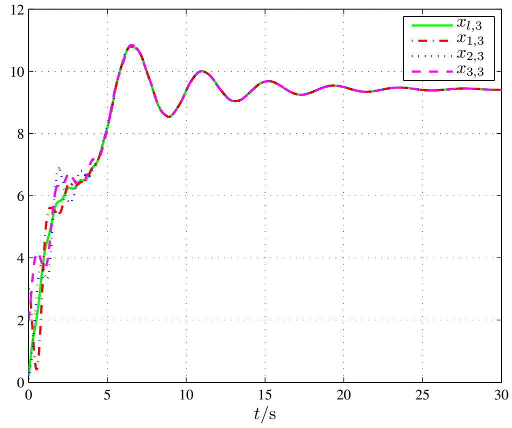

Figure 6: The trajectory of the leader $x_{l,3}$ and $x_{i,3}$, $i = 1, 2, 3$.

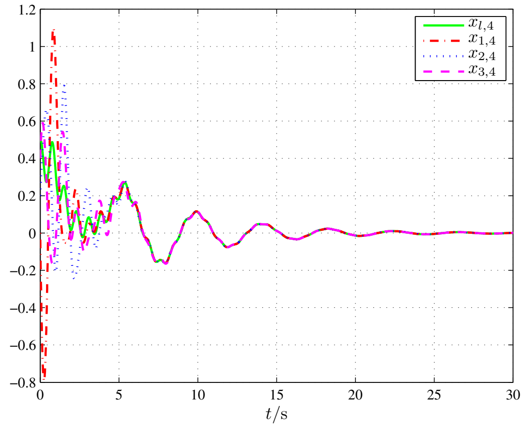

Figure 7: The trajectory of the leader $x_{l,4}$ and $x_{i,4}$, $i = 1, 2, 3$.

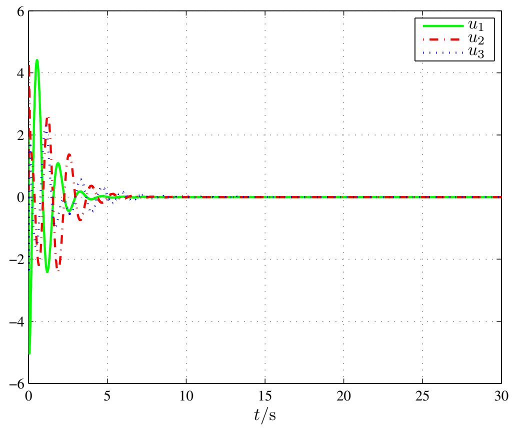

Figure 8: Control input signals.

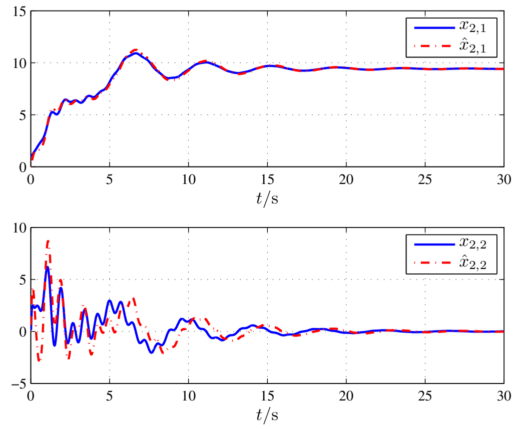

Figure 9: State estimations $\hat{x}_{2,1}$ and $\hat{x}_{2,2}$ for the 2nd agent.

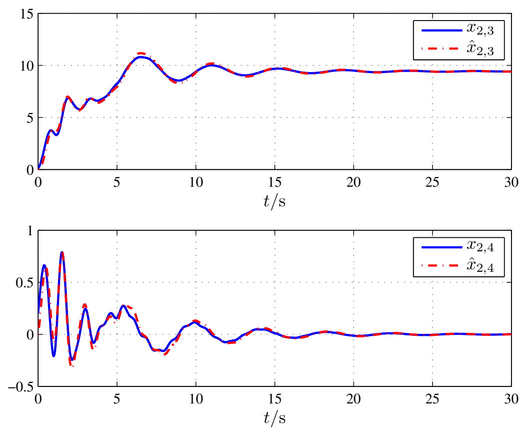

Figure 10: State estimations $\hat{x}_{2,3}$ and $\hat{x}_{2,4}$ for the 2nd agent.

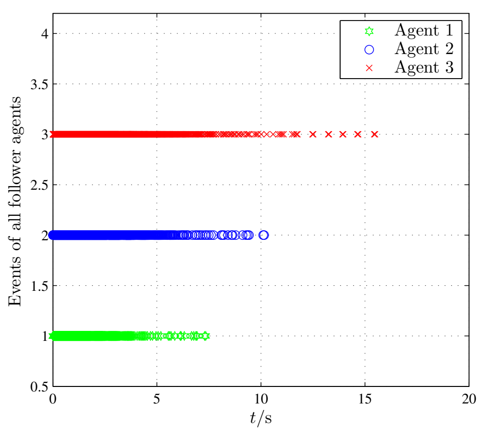

Figure 11: Triggered events of the three follower agents.

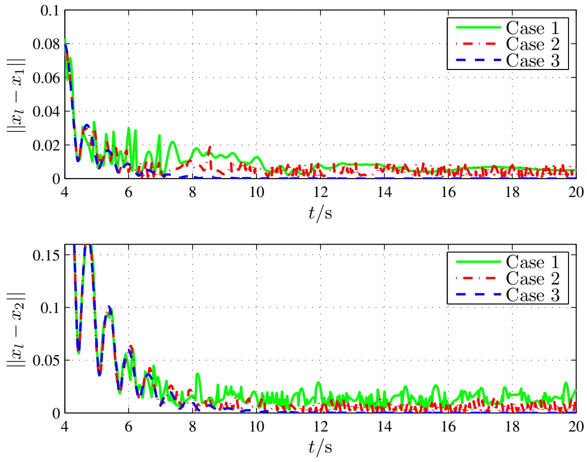

Figure 12: Consensus performance of with different parameters.

Table 1: Triggered Numbers and Intervals of Follower Agents

Case	Followers	Numbers of Triggered Events	Mean Time Interval
Event-Triggered	1	582	0.0127
	2	857	0.0119
	3	1029	0.0151
Without Event-Triggered	1&2&3	6001	0.005

Table 2: Performance Comparison of the Example

Control Strategy	Settling Time(s)	Numbers of Triggered Events of Agents		
		1	2	3
Event-Triggered	15	582	857	1029
Self-Triggered	15	790	1138	1376

6. Conclusion

An output-based event-driven scheme on leader-following consensus has been designed for a class of nonlinear MASs with Lipschitz-type dynamics connected through a directed communication topology. Neural-network-based observers have been constructed for individual follower agents and output control mechanisms have been presented within the event-triggered and self-triggered frameworks. Stability of the closed-loop system has been analyzed. It has been proved that the Zeno behavior is hardly inevitable even if a tight accuracy is required. The effectiveness of the proposed control schemes has been demonstrated via simulation examples. Further research directions would include formation control [13] of heterogeneous agents in presence of unknown communication delays and other complex networks.

Acknowledgment

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