

Second-order sliding mode fault-tolerant control of heat recovery steam generator boiler in combined cycle power plants



Saeid Aliakbari^a, Moosa Ayati^{b,*}, Johari H.S. Osman^a, Yahaya Md Sam^a

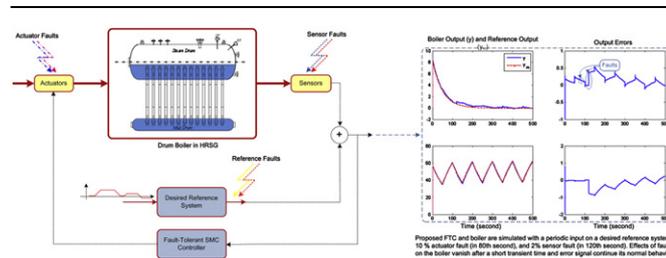
^a Faculty of Electrical Engineering, UTM, 81310 UTM Skudai, Johor, Malaysia

^b School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran, Iran

HIGHLIGHTS

- ▶ This paper proposes a PID-based adaptive second-order sliding mode controller (SMC).
- ▶ SMC is robust to actuator and sensor faults and tracks outputs of a reference system.
- ▶ SMC is used in fault tolerant control of a heat recovery steam generator boilers.
- ▶ Boiler and reference system have different number of states and inputs.
- ▶ Performance of SMC is investigated with different faults scenarios in simulations.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:

Received 16 April 2011

Accepted 24 April 2012

Available online 18 May 2012

Keywords:

Fault-tolerant control

Model reference control

Second-order sliding mode control

PID sliding surface

Heat recovery steam generator

ABSTRACT

Power generation plants are intrinsically complex systems due to their numerous internal components. Higher energy efficiency in power plants is now achieved through employing combined cycles. In this article, an **adaptive robust Sliding Mode** Controller (SMC) is designed to overcome the faults in Heat Recovery Steam Generator boilers (HRSG boilers) as one of the main parts of a combined cycle plant. On condition that a fault occurs in the HRSG boiler, the control system must be able to reconfigure its parameters to maintain the admissible thresholds in dynamic variables such as drum **pressure, steam temperature, and drum water level**. To achieve good performance for the boiler, the proposed adaptive robust SMC shall conquer the effects of **faults and uncertainties** by estimating their upper bounds **adaptively**, and **force the outputs of the multivariable boiler to track the outputs of a desired multivariable reference model**. Manipulating a suitable control input and using second-order sliding mode control strategy, **the output tracking error slides to zero on a PID sliding surface**. Besides tracking, the controlled boiler tolerates faults in **system matrix, faults in input matrix, and external disturbance signal**. Numerical simulations confirm the effectiveness of the proposed FTC (Fault-Tolerant Control) system for an **uncertain non-minimum phase HRSG boiler**.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

With growing advances in manufacturing and processing industries, and production developments in many practical control problems, such as petroleum, chemical industry, aviation and power plants, practical controlled systems (or plants)

* Corresponding author. Tel.: +98 21 82084313; fax: +24098 21 88633029.

E-mail addresses: Asaeid2@siswa.utm.my (S. Aliakbari), m.ayati@ut.ac.ir (M. Ayati), johari@fke.utm.my (J.H.S. Osman), yahaya@fke.utm.my (Y. Md Sam).

Nomenclature

v	volume	c_p	specific heat of the metal at constant pressure, (kJ/kg-k)
h	specific enthalpy	w	water
ρ	specific density	f	feed water
u	specific internal energy	p	drum pressure, (Pa or kg/cm ³)
q	mass flow rate	t	subscript for total system
s	subscript for steam	d	subscript for drum
m	subscript for metal	r	subscript for riser
LT	level transmitter	A_d	area of drum, (m ²)
PT	pressure transmitter	h_c	condensation enthalpy, (kJ/kg)
l	drum water level, (m)	h_f	enthalpy of feed water, (kJ/kg)
Q	heat flow rate to the risers, (kcal/s)	h_s	enthalpy of steam, (kJ/kg)
q_f	feed water flow rate, (kg/s)	h_w	enthalpy of water, (kJ/kg)
q_s	steam flow rate, (kg/s)	V_d	volume of boiler drum, (m ³)
q_{dc}	downwater mass flow rate, (kg/s)	V_{dc}	volume of downcomer, (m ³)
b	empirical parameter	V_r	volume of riser, (m ³)
ρ_s	density of steam, (kg/m ³)	V_{sd}	volume of steam in drum, (m ³)
ρ_w	density of water, (kg/m ³)	V_{wd}	volume of water in drum, (m ³)
α_r	steam-water mass fraction in riser	V_{wt}	total water volume, (m ³)
m_r	total riser mass, (kg)	T_d	residence time, (s)
m_t	total metal mass of the metal tubes and drum, (kg)	t_s	saturation temperature for steam, (°C)
k	friction coefficient in downcomer-riser loop (dimension less)	t_m	metal temperature, (°C)
		V_{sd}^*	volume of steam in drum at equilibrium point, (m ³)
		V_{st}	total steam volume, (m ³)
		V_t	total volume of drum, risers, and downcomers, (m ³)

become more elaborated and complex. Industrial control systems are prone to faults and failures with unknown time and magnitude in their sensors and actuators. These conditions may cause performance degradation or even malfunction of the whole process. Therefore, during the past three decades, fault-tolerant control systems have been noticed as a relief operation.

There are two classified approaches for designing FTC systems: **passive methods** [1,2], and **active methods** [3]. In the passive methods, it is desired to maintain stability and performance of the faulty process in an acceptable region by employing robust controllers **without online parameter regulation**. There are several number of approaches in this class such as algebraic Riccati equation [4], and LMI approach [5]. A fault-tolerant control system based on active approaches can **compensate faults, either by selecting one of the pre-computed control laws or by synthesizing a new control strategy adaptively**. Active methods rely on **updating controller parameters by adaptation mechanisms or using fault detection and isolation (FDI) methods**, to reconstruct or reconfigure the controller. Benefiting from the above approaches, **adaptive robust design methodology can be an efficient way to concur more interconnected and complicated faults especially, actuator faults**.

Moreover, considering **uncertainties** of industrial plants, which originate in errors and approximations in modeling physical characteristics and chemical behaviors of the plant, is a very important issue. Besides uncertainty and disturbances, suitable output tracking, fast fault compensation, **smooth and small control inputs**, and **stability** are other challenges in the control of any industrial plant especially power generation plants.

Power generation plants are delivering the heartbeat to the modern world with the increasing demand of energy. Boiler unit that produces steam is one of the critical components of a power

plant. This is because the steam flow rate, temperature, and pressure directly and highly affect the performance of the power plant. Although the steam production is varied during plant operation, output variables such as steam pressure, temperature, and water level of the drum must be maintained at their respected values. Because of the complexity and importance of the boiler control, several methods have been used to model [6–8], analysis [8–10], and control dynamical behaviors of boiler unit in a power plant.

The dynamic characteristics of a boiler–turbine in a power plant heavily depend on inner and outer disturbances, changes in set point, and operating point of turbine and consequently boiler. In Ref. [11] **a linear control strategy for control of boilers is proposed**. Also, in Refs. [12] and [13] **nonlinear control methods such as feedback linearization have been used** to control nonlinear boiler system. In Ref. [14] decentralized robust control and in Refs. [15] and [16] multivariable methods have been employed to control a boiler system. But these methods result in control input signals to the boiler with **large magnitudes and high powers which are not suitable in practice**.

In order to reduce the magnitude of control inputs and regulation errors, H_∞ optimal controllers [17–19] and evolutionary optimization methods [20] have been applied. However, these methods require **high computation expenses and in most cases they are offline methods** and unable to suppress the effects of time-varying disturbances and uncertainties of the boiler. To overcome uncertainties and disturbances, Refs. [15] and [21] suggested robust control methods.

Sliding mode control methods are very appropriate for boiler control since, they are applicable to **multivariable** systems and also, they have **fast response** which is very important in power plant boilers [22–26]. In addition, SMC is **intrinsically robust** to uncertainties and moreover it could be equipped by some **adaptive laws to increase the performance of the controller** [27]. contains

a comparative study on robust and sliding mode control of boiler system.

Objective of this work is to design a **second-order SMC with a PID sliding surface**. Also, issues related to practical implementation of the controller on a boiler are addressed to demonstrate applicability of the proposed sliding mode controller compared to traditional SMCs and PID controllers in industrial applications. Despite the fact that, under control boiler is multivariable (multi input- multi output) with strong interactions, the controller has other special features. For instance the controller adaptively **estimates the upper bound of the uncertainties**, and **external disturbances and reduces their harmful effects**. The closed-loop system with the proposed SMC is a **Model Reference Adaptive System (MRAS)** and forces the outputs of the boiler system to **follow outputs of a multivariable reference model on a desired PID sliding surface**.

Moreover, the controller of this paper is a second-order SMC for uncertain linear systems. Most of the second-order SMCs in literature are suitable for systems without uncertainties or systems with limited number of states, input, and outputs [28–30], while the proposed second-order SMC of this paper is applicable to **uncertain multivariable linear systems**. Using a second-order SMC not only **the state trajectories of the controlled system on the sliding surface converge to zero but also, the trajectories on the first-order time-derivative of the sliding surface slide to zero as time increases to infinity**. It should be mentioned, compared to the first-order SMC, the advantage of the second-order and higher-order SMCs is that **they attenuate chattering of control input and also, they have better convergence rate, robustness to uncertainties, and greater fault compensation capability**.

In the simulation section the proposed controller is used for FTC of a HRSG boiler system. In the first part, simulation results for the boiler without any fault or disturbance, demonstrates good tracking of the reference model output and suitable

performance of the controller. In the next parts, periodic input to the reference model, **input matrix faults, and output matrix faults are taken into account respectively**. Results illustrate effectiveness of the second-order adaptive robust SMC.

This paper is organized as follows. Section 2 gives an insight study on modeling and linearization of boiler system dynamical model. Section 3 contains the formulation of the second-order adaptive robust SMC and proves the stability of the closed-loop system when plant is affected by faults and uncertainties. Simulation results and analysis of the faults are presented in Section 4. Finally, conclusion notes are in Section 5.

Also, the following notations have been used through the paper.

2. Boiler dynamical model

A boiler–turbine unit is a configuration that is widely used in combined cycle power plants and many modern power plants. This configuration utilizes a single boiler to generate steam and directly feeds the steam to a steam turbine to generate electricity. Schematic diagram of the combined cycle power plant and its HRSG boiler is illustrated in Fig. 1. Fig. 2 depicts a HRSG boiler and boiler drum with details.

The boiler of Fig. 2 has two major parts, the steam drum and the mud drum. The heated water in the steam drum vapors and produces steam. The water in the steam drum is pumped down through downcomers to the mud drum. Then, it is pumped back to the steam drum via riser while impurities of water remain in the mud drum. The heat Q , supplied to the risers causes boiling. Gravity forces the saturated steam to rise causing a circulation in the riser-drum-downcomer loop. The pressure of the steam and level of the water in the steam drum are under control. Feed-water, q_f , is supplied to the drum and saturated steam, q_s , is taken from the drum to the superheaters and turbine. The

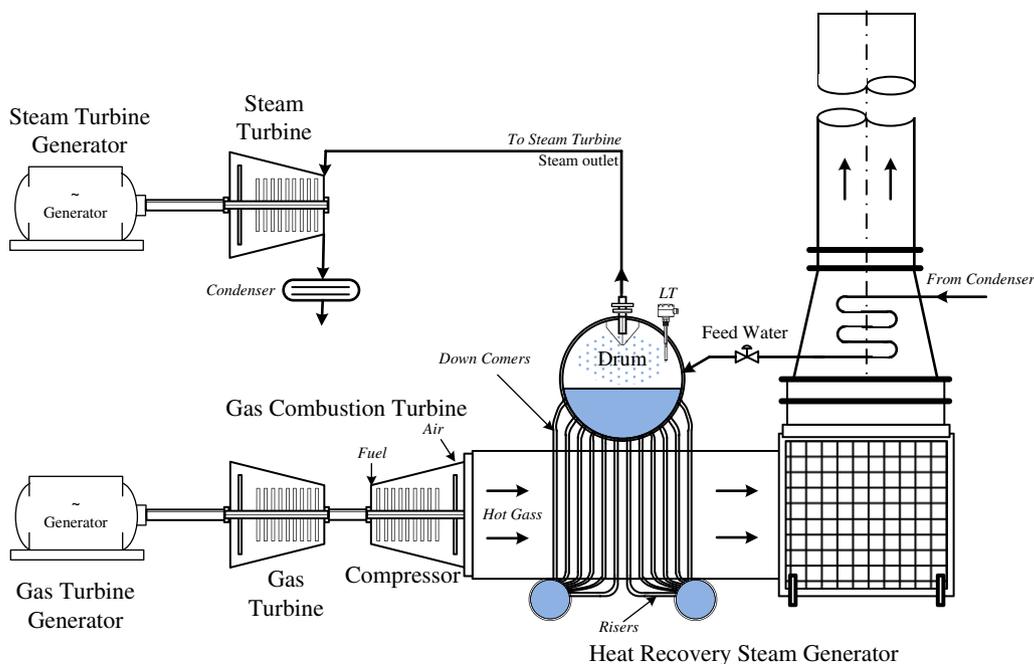


Fig. 1. Combined cycle power plant diagram.

$$\begin{aligned}
e_{11} &= \rho_w - \rho_s \\
e_{12} &= V_{wt} \frac{\partial \rho_w}{\partial p} + \frac{\partial \rho_s}{\partial p} \\
e_{21} &= \rho_w h_w - \rho_s h_s \\
e_{22} &= V_{wt} \left(h_w \frac{\partial \rho_w}{\partial p} + \rho_w \frac{\partial h_w}{\partial p} \right) + V_{st} \left(h_s \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p} \right) - V_t + m_t C_p \frac{\partial t_s}{\partial p} \\
e_{32} &= \left(\rho_w \frac{\partial h_w}{\partial p} - \alpha_r h_c \frac{\partial \rho_w}{\partial p} \right) (1 - \bar{\alpha}_v) V_r + \left((1 - \alpha_r) h_c \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p} \right) \bar{\alpha}_v V_r + (\rho_s + (\rho_w - \rho_s)) h_c V_r \frac{\partial \bar{\alpha}_v}{\partial p} - V_r + m_r \\
e_{32} &= ((1 - \alpha_r) \rho_s + \alpha_r \rho_w) h_c V_r \frac{\partial \bar{\alpha}_v}{\partial p} \\
e_{42} &= V_{sd} \frac{\partial \rho_s}{\partial p} + \frac{1}{h_c} \left(\rho_s V_{sd} \frac{\partial h_s}{\partial p} + \rho_w V_{wd} \frac{\partial h_w}{\partial p} - V_{sd} - V_{wd} + m_d C_p \frac{\partial t_s}{\partial p} \right) + \alpha_r (1 + b) V_r \left(\bar{\alpha}_v \frac{\partial \rho_s}{\partial p} + (1 - \bar{\alpha}_v) \frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w) \frac{\partial \bar{\alpha}_v}{\partial p} \right) \\
e_{42} &= \alpha_r (1 + b) (\rho_s - \rho_w) V_r \frac{\partial \bar{\alpha}_v}{\partial p} \\
e_{42} &= \alpha_r (1 + b) (\rho_s - \rho_w) V_r \frac{\partial \bar{\alpha}_v}{\partial p} \\
e_{44} &= \rho_s
\end{aligned} \tag{6}$$

The steam volume fraction, $\bar{\alpha}_v$, residence time, T_d , and down-comer mass flow rate, q_{dc} , are

$$\bar{\alpha}_v = \frac{\rho_w}{\rho_w - \rho_s} \left(1 - \frac{\rho_s}{(\rho_w - \rho_s) \alpha_r} \ln \left(1 + \frac{\rho_w - \rho_s}{\rho_s} \alpha_r \right) \right) \tag{7}$$

$$T_d = \frac{\rho_s V_{sd}^0}{q_{sd}} \tag{8}$$

$$0.5k q_{dc}^2 = \rho_w A_{dc} (\rho_w - \rho_s) g \bar{\alpha}_v V_r \tag{9}$$

The outputs of the system are chosen as the drum level l and the drum pressure p where

$$l = \frac{V_{sd} + V_{wd}}{A_d} = l_w + l_s \tag{10}$$

$l_w = V_{wd}/A_d$ represents level variations caused by changes of the amount of water in the drum and $l_s = V_{sd}/A_d$ represents level variations caused by the steam in the drum.

$$V_{wd} = V_{wt} - V_{dc} - (1 - \bar{\alpha}_v) V_r \tag{11}$$

In addition, steam tables are required to evaluate h_s , h_w , ρ_s , ρ_w , t_s , t_w , $\partial h_s / \partial p$, $\partial h_w / \partial p$, $\partial \rho_s / \partial p$, $\partial \rho_w / \partial p$, and $\partial t_s / \partial p$ at saturation pressure p . The following approximations have been used that represent the steam table over the desired operating points

$$\begin{aligned}
h_s &= a_{01} + (a_{11} + a_{21}(p - 10))(p - 10) \\
\rho_s &= a_{02} + (a_{12} + a_{22}(p - 10))(p - 10) \\
h_w &= a_{02} + (a_{12} + a_{23}(p - 10))(p - 10) \\
\rho_w &= a_{04} + (a_{14} + a_{24}(p - 10))(p - 10) \\
C_p &= a_{06} + a_{16}(p - 10)(t_{fw} - t_s)
\end{aligned} \tag{12}$$

where,

$$\begin{aligned}
a_{01} &= 2.728e6, a_{11} = 1.792e4, a_{21} = -924.0 \\
a_{02} &= 55.43, a_{12} = 7.136, a_{22} = 0.224 \\
a_{02} &= 1.408e6, a_{13} = 4.565e4, a_{31} = -1010.0 \\
a_{04} &= 691.35, a_{14} = -1.867, a_{24} = 0.081 \\
a_{05} &= 311.0, a_{15} = 7.822, a_{25} = -0.32 \\
a_{06} &= 5900, a_{16} = 250
\end{aligned} \tag{13}$$

The partial derivatives of the enthalpies, densities, and temperature with respect to pressure in equation (6), can be obtained from quadratic functions in equation (12).

A normal procedure for designing controllers starts by obtaining a linear model with constant parameters around an operating point by linearization methods. Since dynamic characteristics even for a reduced mathematical model is usually nonlinear, time-variant, and governed by strong cross coupling of inputs, the parameters of the **linearized model tend to be functions of time and operating point**.

In the rest of this paper and in the simulations the linearized HRSG boiler model is used. Consider a boiler with the following parameters, $V_d = 40 \text{ m}^3$, $V_r = 37 \text{ m}^3$, $V_{dc} = 11 \text{ m}^3$, $A_d = 20 \text{ m}^2$, $m_t = 300,000 \text{ kg}$, $m_r = 160,000 \text{ kg}$, $k = 25$, $b = 0.3$, and $T_d = 12 \text{ s}$. These parameters are of a Swedish 160 MW power plant named P16-G16.

Consider that the nonlinear HRSG boiler state space model is written as

$$\begin{aligned}
\dot{x} &= f(x, u) \\
y &= g(x, u)
\end{aligned} \tag{14}$$

where, the variables of the state vector x , input vector u , and output vector y are

$$\begin{aligned}
x^T &= [V_{wt} \quad p \quad V_{sd} \quad \alpha_r] \\
u^T &= [q_f \quad Q \quad q_s] \\
y^T &= [l \quad p]
\end{aligned} \tag{15}$$

The values of the states, inputs, and outputs in the nominal operating point are

$$\begin{aligned}
x^0 &= [57.5 \quad 8.5 \quad 0.051 \quad 4.8] \\
u^0 &= [32.00147798 \quad 32.00147798 \quad 80.40437506e6] \\
y^0 &= [1.2089 \quad 1]
\end{aligned} \tag{16}$$

x^0 is the nominal values if the states, u^0 is the nominal values if the inputs, and y^0 is the nominal values if the outputs. Linearization of state space model (14) around the nominal operating point is done by the Jacobean matrix approach. The approximated linearized state space model matrices are

$$\begin{aligned} \Delta \dot{x} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + D\Delta u \end{aligned}$$

$$A = \begin{bmatrix} 3.7451e-15 & 7.6548e-06 & 0 & 0 \\ -4.0887e-06 & -6.5527e-04 & 0 & 0 \\ 2.3773e-06 & 5.9026e-04 & 0.1426 & 0 \\ -8.1593e-14 & -5.5355e-02 & 18.216 & 0.083333 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0015 & -0.0015 & -6.9678e-12 \\ -5.9548e-05 & -9.0316e-05 & 5.9647e-11 \\ 3.4622e-05 & 5.2512e-05 & 3.2492e-11 \\ -0.0167 & 0.0239 & -1.0733e-09 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0500 & -0.0484 & 6.7129 & 0.0500 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

$\Delta x = x - x^0$, $u = u - u^0$, and $\Delta y = y - y^0$. The poles of the linearized model are at, -0.08333 0.08333 , -0.1426 0 , -0.000065527 , and 0.0 , and then boiler is marginally unstable. Also, it is worth mentioning that the linearized model is just valid in a neighborhood of the nominal operating point.

3. Formulation of the fault-tolerant controller

In the sequel, the formulations of the second-order adaptive robust SMC for uncertain linear systems are developed. Consider uncertain system

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(w_1(t))]x(t) + [B + \Delta B(w_2(t))]u(t) + D(w_a(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (18)$$

$x \in R^{n_x}$, $y \in R^{n_y}$, and $u \in R^{n_u}$, are state, output, and input vectors, respectively and constant matrices $A \in R^{n_x \times n_x}$, $B \in R^{n_x \times n_u}$, and $C \in R^{n_y \times n_x}$ have appropriate dimensions. $D(\cdot) \in R^{n_x}$ is a function of $w_2(t)$ and represents system disturbance and linearization errors. $w_1(t)$, $w_2(t)$, and $w_3(t)$ are Lebesgue measurable and take values in compact sets \mathcal{Q}_1 , \mathcal{Q}_2 , and \mathcal{Q}_3 . $\Delta A(w_1(t))$ and $\Delta B(w_2(t))$ are continuous functions of $w_1(t)$ and $w_2(t)$ respectively, and present system's time-varying uncertainties or linearization errors. The proposed SMC of this paper not only regulates the outputs of the system to a desired fixed value but also it tracks the outputs of a desired reference model,

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \quad (19)$$

where $x_m \in R^{n_{xm}}$, $y_m \in R^{n_{ym}}$ and $u_m \in R^{n_{um}}$ are state, output, and input vectors. Matrices $A_m \in R^{n_{xm} \times n_{xm}}$, $B_m \in R^{n_{xm} \times n_{um}}$, and $C_m \in R^{n_{ym} \times n_{xm}}$ are constant. Also, it is supposed that the time-derivative of the reference model states are always bounded, i.e. $\|\dot{x}_m\| \leq K_{x_m}$. This is not a strong condition because for a real plant all variables in the right hand side of (19) are bounded and then the left hand side is bounded.

Remark 1. The advantage of the proposed controller is that, system and reference model could have state and input vectors with different dimensions but, output vectors y and y_m have the same dimensions. Meanwhile, in many papers it is required that dimensions of state and input vectors shall be similar.

The proposed SMC is a model reference controller and therefore the overall closed-loop system is a Model Reference Adaptive System (MRAS). MRAS may be regarded as an adaptive servo system in which the desired performance is expressed in terms of a reference model, that gives the desired response to a command signal. In the MRAS the desired behavior of the system is specified by reference model, which is a convenient way to give specifications for a servo problem. The block diagram of the model reference SMC of this paper is given in Fig. 3.

The system has an ordinary feedback loop composed of the process and the SMC. Also, there is another feedback loop that tunes the controller parameters. The parameters are adjusted based on the output error, which is the difference between the output of the system and the output of the reference model. The ordinary feedback loop is called the inner loop, and the parameter adjustment loop is called the outer loop. The mechanism for adjusting the parameters in a model reference adaptive system can be obtained in different ways such as, gradient method or by applying stability theory [34].

Also, following assumptions hold for the system and reference model. These assumption help to design the desired model reference SMC.

Assumption 1. There exist constant matrices $G \in R^{n_x \times n_{xm}}$, $H \in R^{n_u \times n_{xm}}$, and $M \in R^{n_u \times n_{xm}}$ such that

$$\begin{bmatrix} A & B & 0 \\ C & 0 & 0 \\ 0 & 0 & B \end{bmatrix} \begin{bmatrix} G \\ H \\ M \end{bmatrix} = \begin{bmatrix} GA_m \\ C_m \\ GB_m \end{bmatrix}, \quad (20)$$

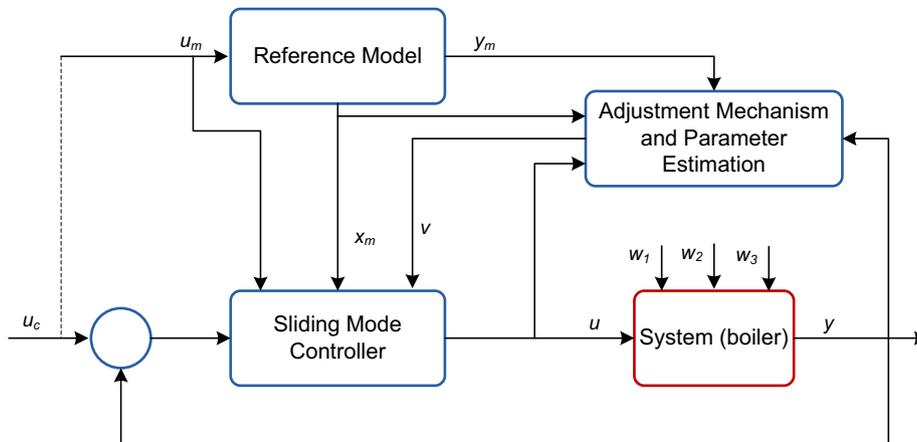


Fig. 3. Block diagram of closed-loop system with the proposed SMC.



Since choosing a suitable reference model is a design parameter and versatile, it is possible to select different A_m , B_m , and C_m then, mentioned constant matrices are achievable.

Assumption 2. The pair (A,B) is controllable.

Assumption 3. There exist continuous functions of suitable dimensions $\Xi_A(w_1(t))$, $\Xi_B(w_2(t))$, and $\Xi_D(w_3(t))$ such that $\Delta A(w_1(t)) = B\Xi_A(w_1(t))$, $\Delta B(w_2(t)) = B\Xi_B(w_2(t))$, and $D(w_3(t)) = B\Xi_D(w_3(t))$. If Assumption 3 holds then system (18) could be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + BF(w_1(t), w_2(t), w_3(t), x, t) \tag{21}$$

where,

$$F(w_1(t), w_2(t), w_3(t), x, t) = \Xi_A(w_1(t)) + \Xi_B(w_2(t)) + \Xi_D(w_3(t)) \tag{22}$$

is called lumped perturbation.

Assumption 4. For the lumped perturbation there exist unknown positive constants ρ_0 and ρ such that

$$\|\dot{F}(w_1(t), w_2(t), w_3(t), x, t)\| \leq \rho_0 + \rho \|\dot{x}\| \tag{23}$$

In order to have suitable output tracking of y_m , we propose the control input

$$u(t) = Hx_m(t) + Mu_m(t) + v(t) \tag{24}$$

$v(t)$ is an auxiliary control input and is introduced in the followings. Consider new state variable $z(t)$ which is a linear combination of $x(t)$ and $x_m(t)$.

$$z(t) = x(t) - Gx_m(t) \tag{25}$$

Taking time-derivative of (25) and using Assumption 1, equations (18) and (19), yield

$$\dot{z}(t) = \dot{x}(t) - G\dot{x}_m(t) = Az(t) + Bv(t) + B\tilde{F} \tag{26}$$

where $\tilde{F}(w_1(t), w_2(t), w_3(t), z, t) = F(w_1(t), w_2(t), w_3(t), x, t) - F(w_1(t), w_2(t), w_3(t), z + Gx_m(t), t)$. This system is called auxiliary error system or simply error system. Also, Assumption 4 yields,

$$\|\dot{F}(w_1(t), w_2(t), w_3(t), z, t)\| \leq \beta + \rho \|\dot{z}\|, \tag{27}$$

where $\beta > 0$ is constant. A good output tracking means $\lim_{t \rightarrow \infty} y(t) - y_m(t) = 0$. On the other hand $y(t) - y_m(t) = Cz(t)$ then. Since, C is bounded then, $\lim_{t \rightarrow \infty} z(t) = 0$ results in $\lim_{t \rightarrow \infty} Cz(t) = 0$, and then, $\lim_{t \rightarrow \infty} y(t) - y_m(t) = 0$. The designed SMC asymptotically stabilize states of error system (26) in the origin.

The auxiliary control signal has two parts

$$v(t) = v_{eq}(t) + v_{sw}(t) \tag{28}$$

$v_{eq}(t)$ is the equivalent control forcing the nominal system to the sliding surface and it is the main control action. Initially, the error is not on the sliding surface, and v_{eq} drives it to the sliding surface, this is called reaching phase. $v_{sw}(t)$ is the switching control keeping system states on the sliding surface and suppressing the effects of the uncertainties, disturbances, and un-modeled dynamics, this is called sliding phase. The proposed switching control law has an integral form

$$v_{sw}(t) = \int \{TS - (\gamma + \eta)\text{sign}(\dot{S})\} d\tau \tag{29}$$

T is a negative definite matrix and γ and η are positive constants where

$$\eta = \beta + \rho \|\dot{z}\| \tag{30}$$

and γ is arbitrary and is related to convergence rate. In equation (30) the constants β and ρ are unknown, but they are adaptively estimated by $\hat{\beta}$ and $\hat{\rho}$ respectively. The estimation errors $\tilde{\beta} = \hat{\beta} - \beta$ and $\tilde{\rho} = \hat{\rho} - \rho$ are calculated by adaptation laws

$$\dot{\hat{\beta}} = \alpha_0^{-1} \|\dot{S}\| \tag{31}$$

$$\dot{\hat{\rho}} = \alpha_1^{-1} \|\dot{S}\| \|\dot{z}\| \tag{32}$$

where α_0 and α_1 are positive constant adaptation gains. By using $\hat{\beta}$ and $\hat{\rho}$, the estimated η , i.e. $\hat{\eta}$, is

$$\hat{\eta} = \hat{\beta} + \hat{\rho} \|\dot{z}\| \tag{33}$$

Therefore, instead of (29) the following switching control law is used

$$v_{sw}(t) = \int \{TS - (\gamma + \hat{\eta})\text{sign}(\dot{S})\} d\tau \tag{34}$$

The sign function on the switching control causes high frequency chattering, especially when the time-derivative of the control input is close to zero. Chattering is harmful for system actuators and is not convenient in practical applications. To mitigate chattering, instead of (34), switching control law

$$v_{sw}(t) = \int \{TS - (\gamma + \hat{\eta})\tanh(\dot{S}/B)\} d\tau \tag{35}$$

is applied. $B \in R^+$ denotes the thickness of the boundary layer around the sliding surface, the larger bound (B) results in larger error magnitude and less chattering. In addition, for more chattering attenuation a second-order sliding control law is used. The proposed SMC employs PID sliding surface

$$\dot{S}(t) + K_S S(t) = K_P z(t) + K_I \int z(\tau) d\tau + K_D \dot{z}(t) \tag{36}$$

Constant matrices K_P , K_I , and K_D are the independent proportional, integral, and differential coefficients of the PID sliding surface. Also, constant matrix K_S helps damping of the sliding surface.

Assumption. 5 Matrix K_D should be chosen such that

$$\bar{\lambda}(K_D B) \geq 0 \text{ and } (1 + \bar{\lambda}(K_D B T)) \leq 0$$

Definition 1. Sliding surface S is from order r if $S = \dot{S} = \ddot{S} = \dots = S^{(r-1)} = 0$.

In this paper we are going to design a second-order SMC for the PID sliding surface, therefore, $S = 0$ and $\dot{S} = 0$. Using $v_{eq}(t)$, the states are forced toward the sliding surface and they are kept there by $\lim_{t \rightarrow \infty} z(t) = 0$, which yields $\dot{S} = 0$. $\dot{S} = 0$ is a necessary condition for second-order SMC which enforces the errors to stay on the sliding surface. In order to design $v_{eq}(t)$, first, time-derivative of (36) is taken,

$$\ddot{S}(t) + K_S \dot{S}(t) = K_P \dot{z}(t) + K_I z(t) + K_D \ddot{z}(t) \tag{37}$$

Substituting $\ddot{z}(t)$ from (26) yields

$$\ddot{S}(t) + K_S \dot{S}(t) = K_P \dot{z}(t) + K_I z(t) + K_D \{A\dot{z}(t) + B\dot{v}(t) + B\dot{\tilde{F}}\} \tag{38}$$

To have $\ddot{S} = 0$ for the nominal system (i.e. $\tilde{F} = 0$), $v_{eq}(t)$ should be

$$v_{eq}(t) = (K_D B)^{-1} \int \{K_S \dot{S}(\tau) - K_P \dot{z}(\tau) - K_I z(\tau) - K_D A \dot{z}(\tau)\} d\tau \quad (39)$$

$$K_P = \begin{bmatrix} 1.3731 & 0 & 0 & 1.3731 \\ 0 & 1.3731 & 0 & 0 \\ 0 & 0 & 1.3731 & 0 \end{bmatrix},$$

$$K_I = \begin{bmatrix} 2.6355 & 0 & 0 & 2.6355 \\ 0 & 2.6355 & 0 & 0 \\ 0 & 0 & 2.6355 & 0 \end{bmatrix},$$

$$K_D = \begin{bmatrix} 0.0691 & 0 & 0 & 0.0691 \\ 0 & 0.0691 & 0 & 0 \\ 0 & 0 & 0.0691 & 0 \end{bmatrix},$$

$$K_S = \begin{bmatrix} 1.4800 & 0 & 0 \\ 0 & 1.4800 & 0 \\ 0 & 0 & 1.4800 \end{bmatrix}.$$

Theorem 1. Let Assumption 1 to 5 hold for uncertain system (18) and reference model (19). Then, using the sliding mode control laws (24), (28), (34), (39), and adaptation laws (31) and (32), the error system (26) is asymptotically stable.

Proof. Asymptotic stability of (26) is proved via second Lyapunov stability theorem. Consider the Lyapunov function candidate

$$V(t) = \frac{1}{2} S^T S + \frac{1}{2} \dot{S}^T \dot{S} + \frac{1}{2} \alpha_0 \tilde{\beta}^2 + \frac{1}{2} \alpha_1 \tilde{\rho}^2 \quad (40)$$

Time-derivative of (40) is

$$\dot{V}(t) = \dot{S}^T S + \dot{S}^T \dot{S} + \alpha_0 \tilde{\beta} \dot{\tilde{\beta}} + \alpha_1 \tilde{\rho} \dot{\tilde{\rho}} \quad (41)$$

On the other hand, replacing (35) and (39) in (38) gives

$$\ddot{S}(t) = K_D B \Gamma S - K_D B (\gamma + \hat{\eta}) \text{sign}(\dot{S}) + K_D B \tilde{F} \quad (42)$$

Then, Substituting (42) in (41) and using equations (27) and (33) yield

$$\begin{aligned} \dot{V}(t) = & \dot{S}^T S + \dot{S}^T \{K_D B \Gamma S - K_D B (\gamma + \hat{\eta}) \text{sign}(\dot{S}) \\ & + K_D B \tilde{F}\} + \alpha_0 \tilde{\beta} \dot{\tilde{\beta}} + \alpha_1 \tilde{\rho} \dot{\tilde{\rho}} \leq \|\dot{S}\| \|S\| (1 + \bar{\lambda}(K_D B \Gamma)) - \|\dot{S}\| \bar{\lambda}(K_D B) \gamma + (\alpha_0 \dot{\tilde{\beta}} - \|\dot{S}\| \bar{\lambda}(K_D B)) \tilde{\beta} + (\alpha_1 \dot{\tilde{\rho}} - \|\dot{S}\| \bar{\lambda}(K_D B)) \tilde{\rho} \end{aligned} \quad (43)$$

Other parameters are

$$\gamma = 0.0013, \alpha_0 = 8.0314, \alpha_1 = 1.4713.$$

After designing the SMC controller and using it in the closed-loop MRAS, different fault signals have been applied and their effects on the tracking error are investigated. In the first part, the boiler without reference model input ($u_m = 0$), uncertainty, and disturbance is simulated. In the second part, the input of the reference model changes periodically. In the third and fourth parts, fault signals are applied to the boiler dynamics and their effects are investigated.

Assumption 5, (31), and (32) give $\dot{V}(t) \leq 0$ which shows asymptotic stability of the system (26) and the proof is complete.

Asymptotic stability of the error system ($\lim_{t \rightarrow \infty} z(t) = 0$) confirms that output of the system tracks the output of the reference model and output tracking error converges to zero.

4. Numerical simulations

In previous section an adaptive robust second-order SMC is proposed which is applicable to systems with faults, uncertainties, and time-varying disturbances. In this section, second-order SMC is designed for fault-tolerant control of the HRSG boiler given in Section 2. For control the following reference model is considered,

$$\begin{aligned} A_m = & \begin{bmatrix} -0.01 & 0 & 0 \\ 0 & -0.02 & 0 \\ 0 & 0 & -0.03 \end{bmatrix}, B_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ C_m = & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, D_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (44)$$

This model has the desired properties such as acceptable response speed, zero steady state error to step inputs, and it is without overshoot. Then, using the system matrices (17) and reference matrices (44), equality (20) is solved. The PID sliding surface parameters are calculated based on the assumptions and conditions of Section 3 by using evolutionary algorithms.

Part I: In this part, the boiler and controller are simulated as regulator in the nominal state which means reference model input is zero and there is no uncertainty or fault. Fig. 4 and Fig. 5 depict the states of the boiler and reference model.

In Fig. 4 from top to down the states are drum pressure, p , total water volume, V_{wt} , steam-mass fraction in the risers, α_r , and steam volume in the drum, V_{sd} . The states of the linearized boiler go to zero ($\Delta x = 0$) and therefore the states of the boiler are converged to the operating point values of (16). Fig. 6 illustrates the outputs (drum level and pressure) and output errors of the boiler and reference model. Output errors decrease gradually and boiler outputs follow reference model outputs without overshoot or bias.

States of the auxiliary system are depicted in Fig. 7. The SMC pushes the states of this system to sliding surface and zero and therefore the output error will converges to zero.

Part II: In this part a periodic square signal is used as the second input of the reference model. Fig. 8 depicts the outputs and outputs error of the boiler and reference model. Boiler outputs track the reference model outputs and the tracking errors are relatively small.

Part III: In this part, beside reference model input, it is supposed that because of a fault in the boiler, the boiler dynamics and therefore, input matrix of the linearized boiler model suddenly changes. This type of faults is called actuator or input fault. The variation of input matrix is

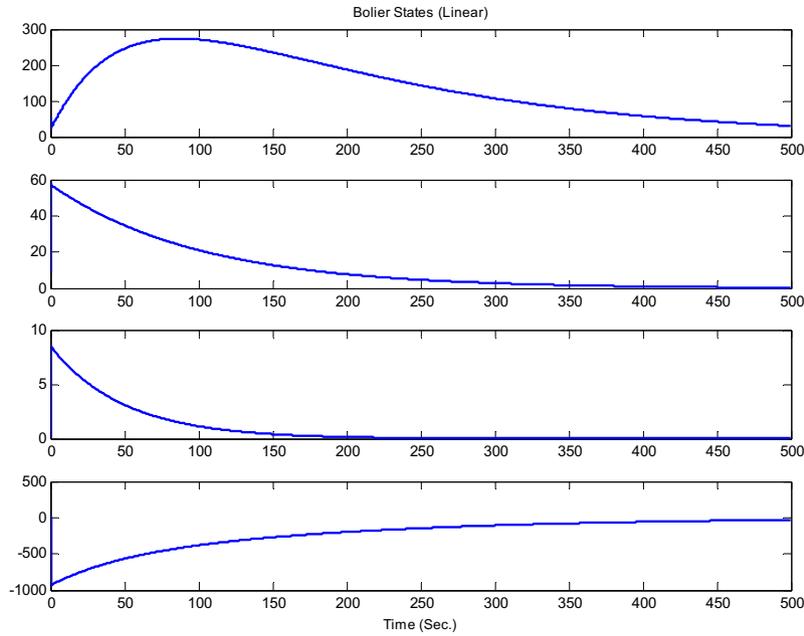


Fig. 4. States of the linearized boiler.

$$\Delta B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 20 & -20 & 0 \end{bmatrix}$$

Fault causes a variation about 10% of the input matrix B , and has effects on two inputs of the system q_f (feedwater flow rate) and Q (heat flow rate). Also, fault happens at 80th second and stays until the end of the simulation time. Outputs and output errors of the boiler and reference model are illustrated in Fig. 9. Right after the fault, there is a very small transient in the error signals but as time goes to infinity the output errors converge to

their ordinary values. This confirms that the SMC is robust against input faults.

Part IV: In this part, beside reference model periodic input and input matrix fault, it is supposed that because of an unintentional change in the boiler sensors or parameters, the boiler output dynamics and therefore, output matrix of the boiler changes. This type of fault is called sensor or output fault. The variation of output matrix is

$$\Delta C = \begin{bmatrix} 0.002 & -0.001 & 0 & 0 \\ -0.003 & 0.002 & 0 & 0.001 \end{bmatrix}$$

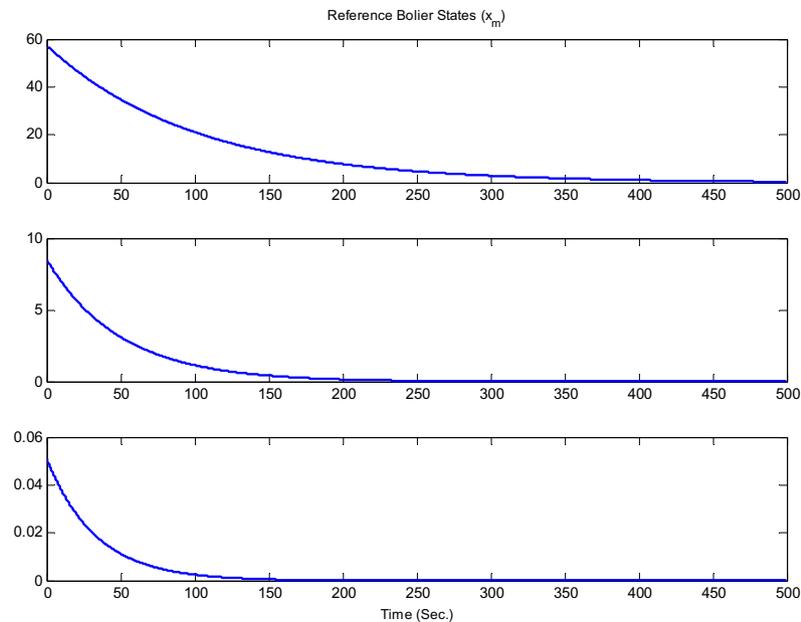


Fig. 5. States of the reference model.

- [21] A. Swarnakar, H.J. Marquez, T. Chen, A new scheme on robust observer-based control design for interconnected systems with application to an industrial utility boiler, *IEEE Transaction on Control Systems Technology* 16 (3) (2008) 539–548.
- [22] M.C. Pai, Design of adaptive sliding mode controller for robust tracking and model following, *Journal of the Franklin Institute* 347 (2010) 1837–1849.
- [23] M. Abedi, F. Bakhtiyari-Nejad, M. Saffar-Avval, M. Abedi, A. Alasty, A comparative study between linear and sliding mode adaptive controllers for a hot gas generator, *Applied Thermal Engineering* 30 (2010) 413–426.
- [24] S. Yang, C. Qian, J. Lu, Adaptive Control Design for a Nonlinear Drum-boiler Turbine System (September 3–5, 2008) 17th IEEE International Conference on Control Applications.
- [25] S.Y. Chen, F.J. Lin, Robust nonsingular terminal sliding-mode control for nonlinear magnetic bearing system, *IEEE Transaction on Control System Technology* (2010) doi: 10.1109/TCST.2010.2050484.
- [26] Y.M. Sam, J.H.S. Osmana, M. Ruddin, A. Ghanib, A class of proportional-integral sliding mode control with application to active suspension system, *Systems & Control Letters* 51 (4) (2004) 217–223.
- [27] R.H. Moradi, F. Bakhtiyari-Nejad, M. Saffar-Avval, Robust control of an industrial boiler system, a comparison between two approaches: sliding mode control & H_{∞} technique, *Energy Conversion and Management* 50 (2009) 1401–1410.
- [28] I. Eker, Second-order sliding mode control with experimental application, *IAS Transaction* 49 (2010) 394–405.
- [29] Y. Feng, X. Yu, X. Zheng, Second-order terminal sliding mode control of input-delay systems, *Asian Journal of Control* 8 (1) (2006) 12–20.
- [30] I. Fei, C. Batur, A class of adaptive sliding mode controller with proportional-integral sliding surface, *Proceeding, IMechE* 223 (2008) 989–993.
- [31] I. Mrunalini, P. Kundu, K.K. Dutta, State space model for drum boiler system, *IE(1) Journal-EL* 86 (2006) 260–267.
- [32] I.J. Åström, R.D. Bell, Drum-boiler dynamics, *Automatica* 36 (3) (2000) 363–378.
- [33] H.G. Kwatny, J. Berg, Drum level regulation at all loads, in: *Preprints IFAC 12th World Congress*, vol. 3, 1993, pp. 405–408 Sydney, Australia.
- [34] K.J. Åström, B. Wittenmark, *Adaptive Control*, second ed. Prentice Hall, 1994.