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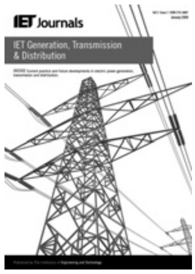

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Analytical approach for placement and sizing of distributed generation on distribution systems

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Abstract: An analytical method for placement and sizing of distributed generation on power distribution systems for loss reduction is introduced. The proposed analytical method is developed based on a new formulation for the power flow problem, which is non-iterative, direct, and involves no convergence issues even for systems with high *R*/*X* branch ratios. Further, this power flow solution is extremely useful whenever fast and repetitive power flow estimations are required. A priority list based on loss sensitivity factors is developed to determine the optimal locations of the candidate distributed generation units. Sensitivity analysis is performed to estimate the optimal size and power factor of the candidate distributed generation units. Various types of distributed generators (DGs) have been dealt with and viable solutions are proposed to reduce total system loss. The proposed method has been tested on 33-bus and 69-bus distribution systems, which are extensively used as examples in solving the placement and sizing problem of DGs. Exhaustive power flow routines are also performed to verify the sizes obtained by the analytical method. The test results show that the proposed analytical method could lead to optimal or near-optimal solution, while requiring lower computational effort.

1 Introduction

Distribution systems have been operated in a vertical and centralised manner for many years for best control and coordination of their protective devices. Distribution systems are characterised by high *R*/*X* branch ratios with radial or weakly-meshed topological structure [1–4]. In fact, the radial topological structure makes distribution systems the most extensive part in the entire power system. The poor voltage regulation and the high line resistance both play a significant role in increasing total power losses of distribution systems. Minimisation of power losses of distribution systems is constantly achieved by feeder reconfiguration techniques [5–11]. However, distributed generators (DGs) [12] have been recently proposed to minimise distribution system power losses. The potential benefits of DG installation on distribution networks include total system losses reduction, voltage profile improvement, peak load shaving and reliability enhancement [12–14]. Given such advantages, DG can play vital role in reducing losses and improving voltage profile of distribution systems, if they are properly located, sized and their penetration level is also identified.

The problem of allocating and sizing of DG is essentially a non-linear complex mathematical optimisation problem. Multiple solutions, with various scenarios, are constantly sought while handling the allocation and sizing problem. A great variety of solution techniques are proposed in the literature to handle the problem of placement and sizing of DG on distribution systems. These solution techniques can be broadly classified as population-based optimisation methods or heuristic and analytical-based techniques. Population-based optimisation methods may include genetic algorithms [15, 16], artificial bee colony algorithm [17], tabu search [18] and particle swarm optimisation [19]. Population-based optimisation methods are widely adapted in both operational and planning studies and have given satisfactory results over the years. DG placement and sizing based on mathematical programming such as mixed-integer linear programming are also presented in the literature [20].

Over the past few years, there has been great interest in using analytical approaches to handle the allocation and sizing problem of DGs [21–24]. A common objective function used in such approaches is distribution system loss reduction and voltage profile improvement. The vast majority of the analytical methods available in the literature were developed based on the exact loss formula developed by Elgerd [25]. The exact loss formula is an equation relating voltage magnitude and voltage angle at a bus with the active power and reactive power injections to that bus in a highly non-linear fashion. In fact, analytical approaches are less complicated than the heuristic techniques mentioned above. However, exhaustive power flow tasks are still being performed in the solution procedures. The number of power flows performed in DG placement and sizing based on analytical approaches, for instance, could possibly attain $(n_s - 1)$ for a radial system with n_s buses. Performing full AC power flow (FACPF), on one hand, gives high calculation precision but requires a quite extensive computational burden and storage. On the other hand, with the distinctive properties of distribution systems such as high *R*/*X* branch ratios, there is a good chance that the power flow solution might fail to converge. Therefore, it would be appropriate if the non-linear power flow equations are simplified so that certain applications that require fast and repetitive solutions can be handled more expeditiously with small, but acceptable, sacrifices in accuracy.

This paper proposes an analytical approach for placement and sizing of DG on distribution systems for loss reduction. The motivation in this paper was developed based on the premise that analytical approaches are reliable, require less computational effort and are also suitable for planning studies such as distributed generation planning. Furthermore, analytical approaches could lead to an optimal or near-optimal global solution. The main contributions of this paper include:

1. The development of a novel linearised power flow model, in which the coupling between active power and voltage magnitude as well as the coupling between reactive power and voltage angle is maintained. The proposed power flow solution is very fast, robust and does not encounter convergence issues even with ill-conditioned systems that have high R/X branch ratios.
2. The use of sensitivity analysis to select the appropriate locations for DG units, and thereby reducing the search space and computational burden.
3. This work is different from that presented in the literature, in the sense that it proposes direct and understandable expressions to estimate the optimal, or near-optimal size of the DG unit. The work presented in this paper does not utilise the complicated exact loss formula.
4. This work considers various types of DGs and incorporates the optimal power factor in the sizing problem.

It is appropriate to point out here that the proposed power flow solution is not intended to replace the powerful non-linear power flow methods, such as Newton–Raphson method [26], Gauss–Seidel method [26], fast-decoupled method [27–29] and its modified versions [30, 31] or the backward/forward sweeping method [32]. It rather allows us to get fast and reliable power flow estimations, which are highly sought in certain applications that require repetitive and reliable solutions. We also understand that most distribution systems are unbalanced. However, balanced operation is a very common assumption that is being widely adapted while handling the allocation and sizing problem of distributed generation [7–11, 17, 19, 22–24]. It is worth noting here that the proposed power flow solution is applicable for various systems including unbalanced systems. The modifications in case of unbalanced distribution networks are straightforward and largely lie in certain elements in the bus admittance matrix; thus the advantages obtained with balanced operation are preserved. We applied the proposed analytical method on 33-bus and 69-bus distribution systems, which are broadly used as examples in solving the DG placement and sizing problem. We verified the results obtained by the proposed analytical method with those obtained by exhaustive power flow routines and also compared them with some other analytical methods available in the literature. The test results show that the proposed analytical solution could lead to optimal or near-optimal solution, while requiring lower computational effort.

2 Development of models and methods

This section first presents the objective function and constraints. It also provides the network model formulation utilised in this paper.

2.1 Objective function and constraints

The objective function of the problem aims at minimising the real power loss and improving the voltage profile at all system buses. Mathematically, the problem can be posed as

$$\text{Total loss} = \min\left(\sum_{k=1}^{N_b} |I_k^2| \times R_k\right) \tag{1}$$

subject to

1. Real and reactive power injections

$$\begin{aligned} B'\delta - GV + P_G &= P_D \\ G'\delta + BV + Q_G &= Q_D \end{aligned} \tag{2}$$

2. Voltage limits constraints

$$|V_k^{\min}| \le |V_k| \le |V_k^{\max}| \tag{3}$$

3. Feeder capacity constraints

$$\begin{aligned} |I_F| &\le |I_F^{\max}| \\ |I_R| &\le |I_R^{\max}| \end{aligned} \tag{4}$$

4. DG real and reactive power constraints

$$\begin{aligned} P_{\mathrm{DG}}^{\min} &\le P_{\mathrm{DG}} \le P_{\mathrm{DG}}^{\max} \\ Q_{\mathrm{DG}}^{\min} &\le Q_{\mathrm{DG}} \le Q_{\mathrm{DG}}^{\max} \end{aligned} \tag{5}$$

where N_b is number of buses, I_k is the current flowing out of branch k, R_k is the resistance of branch k. Further, B', B, G' and G are as developed in Section 2.2 with dimensions $(N_\mathrm{b} \times N_\mathrm{b})$, δ represents the bus voltage angles $(N_\mathrm{b} \times 1)$, $V^{\max}$ and $V^{\min}$ represent the maximum and minimum allowable voltages $(N_\mathrm{b} \times 1)$, P_G and Q_G represent the real and reactive power generation of the substation $(N_\mathrm{b} \times 1)$, P_D and Q_D represent the real and reactive power loads $(N_\mathrm{b} \times 1)$, $P_{\mathrm{DG}}^{\min}$ and $Q_{\mathrm{DG}}^{\min}$ represent the available real and reactive power capacities of the DGs $(N_\mathrm{b} \times 1)$, $I_\mathrm{F}^{\max}$ and $I_\mathrm{R}^{\max}$ represent the forward and reverse flow capacities of distribution lines $(N_\mathrm{f} \times 1)$, with N_f being the number of distribution feeders.

In the above formulation, (2) represents the sum of the power at any arbitrary bus, which is simply the power balance equation for real and reactive power. The formulation of this constraint is given below in Section 2.2.

2.2 Network model formulation

This section presents a linearised AC power flow formulation (LACPF). Let us start by writing down the well-known real and reactive power injections at bus k [26]. That is

$$P_k = V_k \sum_{m\in\Psi_k} V_m\left(G_{km}\cos\delta_{km} + B_{km}\sin\delta_{km}\right) \tag{6}$$

$$Q_k = V_k \sum_{m\in\Psi_k} V_m\left(G_{km}\sin\delta_{km} - B_{km}\cos\delta_{km}\right) \tag{7}$$

where Ψ_k is the set of buses adjacent to bus k, including bus k.

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P_k and Q_k are the real and reactive power injections at bus k. V_k and V_m are the voltage magnitudes at buses k and m, respectively. Also, G_{km} is the real element (k, m) of the bus admittance matrix, B_{km} is the imaginary element (k, m) of the bus admittance matrix and δ_{km} is the voltage angle difference between buses k and m, respectively.

In practice, we keep bus voltages around 1 p.u. with a pre-specified value (usually ±5%). The voltage magnitudes at buses k and m can alternatively be represented as

$$\begin{aligned} V_k &= 1.0 \pm \Delta V_k \\ V_m &= 1.0 \pm \Delta V_m \end{aligned} \tag{8}$$

where ΔV_k and ΔV_m are both expected to be small quantities.

If we ignore the small portion ΔV_k, the magnitude of the voltage at bus k can be approximated by 1 p.u. This approximation will not prevent V_k from being calculated, rather it would approximate the power injection at bus k. Therefore, (6) and (7) can be written as

$$P_k = V_k \sum_{m\in\Psi_k} V_m\left(G_{km}\cos\delta_{km} + B_{km}\sin\delta_{km}\right) \tag{9}$$

$$Q_k = V_k \sum_{m\in\Psi_k} V_m\left(G_{km}\sin\delta_{km} - B_{km}\cos\delta_{km}\right) \tag{10}$$

It is important to note that this is only an approximation that enables the linearisation; it is not an assumption that the voltage magnitude equals 1 p.u. Equations (9) and (10) can now be expressed as follows

$$P_k \simeq \sum_{m\in\Psi_k} V_m\left(G_{km}\cos\delta_{km} + B_{km}\sin\delta_{km}\right) \tag{11}$$

$$Q_k \simeq \sum_{m\in\Psi_k} V_m\left(G_{km}\sin\delta_{km} - B_{km}\cos\delta_{km}\right) \tag{12}$$

Further, let us assume that the phase angle difference between bus (lateral or sub-lateral) k and bus m is small. Equations (11) and (12) can be now expressed as

$$P_k \simeq \sum_{m\in\Psi_k} V_m\left(G_{km} + B_{km}\delta_{km}\right) \tag{13}$$

$$Q_k \simeq \sum_{m\in\Psi_k} V_m\left(G_{km}\delta_{km} - B_{km}\right) \tag{14}$$

Equations (13) and (14) can also be written as

$$P_k \simeq \sum_{m\in\Psi_k} \left(V_m G_{km} + V_m B_{km}\delta_{km}\right) \tag{15}$$

$$Q_k \simeq \sum_{m\in\Psi_k} \left(V_m G_{km}\delta_{km} - V_m B_{km}\right) \tag{16}$$

As before, let us make a further approximation, $V_m \simeq 1.0$ p.m., in the second term of (15). This implies that

$$P_k \simeq \sum_{m\in\Psi_k} \left(V_m G_{km} + B_{km}\delta_{km}\right) \tag{17}$$

Let us expand (17) as the following

$$P_k \simeq \sum_{m\in\Psi_k} V_m G_{km} + \sum_{m\in\Psi_k} B_{km}\delta_{km} \tag{18}$$

which can be rewritten as

$$P_k \simeq \sum_{m\in\Psi_k} V_m G_{km} + \sum_{m\in\Psi_k} B_{km}\left(\delta_k - \delta_m\right) \tag{19}$$

Now, (19) can be further broken to two parts as follows

$$P_k \simeq P_{ku} + P_{kv} \tag{20}$$

where

$$P_{ku} = \sum_{m\in\Psi_k} V_m G_{km} \tag{21}$$

and

$$P_{kv} = \sum_{m\in\Psi_k} B_{km}\delta_k - \sum_{m\in\Psi_k} B_{km}\delta_m \tag{22}$$

where

$$B_{km} = \begin{cases} \sum b_{km} + b_{kk}, & \text{for} \quad m = k \\ -b_{km}, & \text{for} \quad m \neq k \end{cases} \tag{23}$$

Here, b_{kk} is the total susceptance of the shunt elements connected at bus k. It is evident from (23) that summing the B_{km} terms for all $m \in \Psi_k$ yields

$$\sum_{m\in\Psi_k} B_{km} = -b_{k1} - b_{k2} - \cdots + \left(\sum_{m\neq k} b_{km} + b_{kk}\right) - \cdots - b_{kN} = b_{kk} \tag{24}$$

Hence, (22) can be rewritten as

$$P_{kv} = -\sum_{m\neq k} B_{km}\delta_m - (B_{kk} - b_{kk})\delta_k \tag{25}$$

Therefore, (20) will have the following form

$$P_k \simeq \sum_{m\in\Psi_k} V_m G_{km} - \sum_{m\neq k} B_{km}\delta_m - \left(B_{kk} - b_{kk}\right)\delta_k \tag{26}$$

For the reactive power equation, we will have

$$Q_k \simeq -\sum_{m\in\Psi_k} V_m B_{km} + \sum_{m\in\Psi_k} G_{km}\left(\delta_k - \delta_m\right) \tag{27}$$

Now, in a similar fashion to what it was done with real power, (27) can be further broken to

$$Q_k \simeq Q_{ku} + Q_{kv} \tag{28}$$

where

$$Q_{ku} = -\sum_{m\in\Psi_k} V_m B_{km} \tag{29}$$

and

$$Q_{kv} = \sum_{m\in\Psi_k} G_{km}(\delta_k - \delta_m) \tag{30}$$

Now, Q_{kv} is obtained in a similar manner as

$$Q_{kv} = -\sum_{m\neq k} G_{km}\delta_m - (G_{kk} - g_{kk})\delta_k \tag{31}$$

where

$$G_{km} = \begin{cases} \sum g_{km} + g_{kk}, & \text{for} \quad m = k \\ -g_{km}, & \text{for} \quad m \neq k \end{cases} \tag{32}$$

Here, g_{kk} is the total conductance of the shunt elements connected at bus k. It is evident from (32) that summing the G_{km} terms for all $m \in \Psi_k$ yields

$$\sum_{m\in\Psi_k} G_{km} = -g_{k1} - g_{k2} - \cdots + \left(\sum_{m\neq k} g_{km} + g_{kk}\right) - \cdots - g_{kN} = g_{kk} \tag{33}$$

Hence, (28) will have the following form

$$Q_k \simeq -\sum_{m\neq k} G_{km}\delta_m - (G_{kk} - g_{kk})\delta_k - \sum_{m\in\Psi_k} V_m B_{km} \tag{34}$$

In matrix notation, the proposed linearised AC power flow equations in general form can be expressed as

$$\begin{bmatrix} P_K \\ Q_K \end{bmatrix} = \begin{bmatrix} -B' & G \\ -G' & -B \end{bmatrix} \begin{bmatrix} \delta_K \\ V_K \end{bmatrix} \tag{35}$$

where $\boldsymbol{B}'$ is a modified susceptance matrix, $\boldsymbol{G}'$ is a modified conductance matrix, $\boldsymbol{G}$ is the conventional conductance matrix and $\boldsymbol{B}$ is the conventional susceptance matrix. These matrices are, respectively, defined as

$$\boldsymbol{B}' = \begin{bmatrix} (B_{11} - b_{11}) & B_{12} & \cdots & B_{1N} \\ B_{21} & (B_{22} - b_{22}) & \cdots & B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N1} & B_{N2} & \cdots & (B_{NN} - b_{NN}) \end{bmatrix} \tag{36}$$

$$\boldsymbol{G}' = \begin{bmatrix} (G_{11} - g_{11}) & G_{12} & \cdots & G_{1N} \\ G_{21} & (G_{22} - g_{22}) & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & (G_{NN} - g_{NN}) \end{bmatrix} \tag{37}$$

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \tag{38}$$

$$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{21} & B_{22} & \cdots & B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N1} & B_{N2} & \cdots & B_{NN} \end{bmatrix} \tag{39}$$

3 Detailed solution procedures

A priority list based on loss sensitivity factors is used in this section to determine the candidate buses for DG allocation. Further, DG injection-based sensitivity analysis is performed to determine the optimal sizes of the DG units. A methodology to select the corresponding optimal power factor is also provided in this section.

3.1 Identification of penetration level

In this paper, we define the penetration level of the distributed generation as

$$\text{Penetration level} = \frac{S_{\text{DG}}}{S_{\text{TD}}} \times 100\% \tag{40}$$

Here, S_{DG} and S_{TD} are the output power of the distributed generation unit and the total system demand, respectively.

3.2 Selection of optimal location: a priority list

In this paper, the active power loss sensitivity factor λ_p has been identified and used to determine the optimal locations for DG units for total power loss reduction. To estimate this sensitivity factor, let us consider the simple radial distribution feeder shown below in Fig. 1. From Fig. 1, the line power losses can be calculated as

$$P_{\text{loss}} = \frac{\left(P_{Lk,\text{eff}}^2 + Q_{Lk,\text{eff}}^2\right)}{|V_k^2|} \times R_k \tag{41}$$

Here, $P_{Lk,\text{eff}}$ and $Q_{Lk,\text{eff}}$ are the effective real and reactive loads beyond bus k.

We define the active power loss sensitivity factor λ_p as [33]

$$\lambda_p = \frac{\partial P_{\text{loss}}}{\partial P_{Lk,\text{eff}}} = \frac{2 \times P_{Lk,\text{eff}} \times R_k}{|V_k^2|} \tag{42}$$

The active power loss sensitivity factor is calculated using

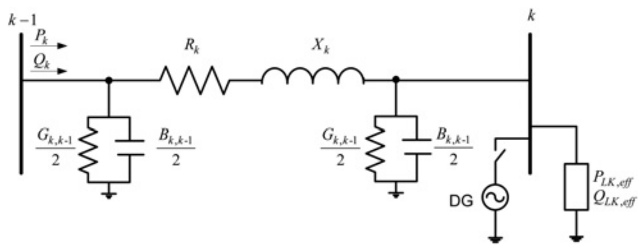

Fig. 1 *Simple radial distribution feeder*

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(42) for all buses, and then the calculated values are arranged in a priority list with a descending order so that buses with high λ_p are considered first for allocations for the available DG units.

3.3 Selection of optimal size

To determine the optimal DG sizes, let us start by considering the distribution feeder depicted below in Fig. 1. As depicted, a DG of size P_{DG} and Q_{DG} is arbitrarily placed at bus k of the system. Before the installation of this DG unit, the power losses in the line section k, $k-1$ is estimated as in (41). That is

$$P_L^- = \frac{\left(P_{Lk,\text{eff}}^2 + Q_{Lk,\text{eff}}^2\right)}{\left|V_k^2\right|} \times R_k \tag{43}$$

However, after the DG is installed at bus k, power losses in the line section k, $k-1$ can be estimated as

$$P_L^+ = \left[\frac{\left(P_{DG} - P_{Lk,\text{eff}}\right)^2}{\left|V_k^2\right|} + \frac{\left(Q_{DG} - Q_{Lk,\text{eff}}\right)^2}{\left|V_k^2\right|}\right] \times R_k \tag{44}$$

Alternatively, (44) can be represented as

$$P_L^+ = \left[\frac{P_{DG}^2 - 2\cdot P_{DG}\cdot P_{Lk,\text{eff}} + P_{Lk,\text{eff}}^2}{\left|V_k^2\right|} + \frac{Q_{DG}^2 - 2\cdot Q_{DG}\cdot Q_{Lk,\text{eff}} + Q_{Lk,\text{eff}}^2}{\left|V_k^2\right|}\right] \times R_k \tag{45}$$

Consequently, the difference in power losses before and after the installation of the DG at bus k can be estimated as

$$\Delta P_L = P_L^+ - P_L^- \tag{46}$$

Alternatively, (46) can be written as

$$\Delta P_L = \left[\frac{P_{DG}^2 + Q_{DG}^2 - 2\cdot P_{DG}\cdot P_{Lk,\text{eff}} - 2\cdot Q_{DG}\cdot Q_{Lk,\text{eff}}}{\left|V_k^2\right|}\right] \times R_k \tag{47}$$

Therefore, for the total real power losses to be minimum in the feeder section $(k-1,\ k)$, the first derivative of (47) with respect to the active power injected by the DG unit at this particular bus should be driven to zero. That is

$$\frac{\partial \Delta P_L}{\partial P_{DG}} = 0 \tag{48}$$

Hence, after performing the partial derivatives of (47) and rearranging, the optimal size of the DG unit can be written as

$$S_{DG} = \sqrt{P_{DG}^2 + Q_{DG}^2} \tag{49}$$

The optimal DG size in watt is given below as

$$P_{DG} = \frac{P_{Lk,\text{eff}} + \alpha\cdot Q_{Lk,\text{eff}}}{1+\alpha^2} \tag{50}$$

The optimal DG size in var is given as

$$Q_{DG} = \frac{P_{Lk,\text{eff}} + \alpha\cdot Q_{Lk,\text{eff}}}{\alpha+\beta} \tag{51}$$

where α and β are, respectively, defined as

$$\alpha = \tan\theta = \frac{Q_{DG}}{P_{DG}} \tag{52}$$

$$\beta = \cot\theta = \frac{P_{DG}}{Q_{DG}} \tag{53}$$

where θ is the power factor angle of the candidate DG unit.

From (49)–(51), the DG sizes are obtained. Conventionally, DG size is represented in VA and this has been given by (49). As a rule of thumb, it was determined that the total losses of the system are minimum when the DG size matches the effective load connected to the DG bus. In fact, this concurs with our derivations in (50) and (51). For instance, if the selected DG unit was a photovoltaic that supplies real power only, that is, the power factor of the DG is unity, according to (50), the optimal DG size in watt (P_{DG}) will be equal to that of the total effective load connected to that bus. The optimal DG size in Var (Q_{DG}) will be equal to zero in this case. On the other hand, if the selected DG was a synchronous condenser that regulates the bus voltage by injecting reactive power only, accordingly, the optimal DG size in Var (Q_{DG}) will be equal to that of the total effective load connected to that bus. However, the optimal DG size in watt (P_{DG}) will be equal to zero in this case. The same principle applies for various DG units as can be seen from (49) to (51).

3.4 Selection of optimal power factor

As can be clearly seen for the formulation given by (49)–(51), the power factor of the DG unit plays a significant role in determining its optimal size. However, finding the optimal power factor that contributes to minimising the total system losses is not an easy task. For instance, DG units should operate at practical power factors to maintain the upper and lower voltage constraints, and thereby contribute to a reduction of the total real power losses. However, the majority of the DG units used at distribution level are not dispatched and not well-controlled since their dispatching could cause certain operational problems in the protection system of the distribution network. Furthermore, DG units are not favoured in regulating bus voltages at the installation node according to IEEE standard 1547-2033 [34]. The strategy that is widely used to keep bus voltages within the permissible range is through capacitor banks. However, even when capacitor banks are used, violations of voltage limits happen commonly in distribution systems.

Most of the methods used to estimate the optimal power factor of DG units are developed based on expert systems. For instance, the optimal power factor of the DG unit has been assumed to be equal to that of the load connected to the same bus, with reverse operating strategy in [17]. Furthermore, the optimal power factor of the DG unit was assumed to be equal to that of the total downstream load at which the DG unit is connected in [23]. It is shown in [23] that the losses are minimum when the power factor of the DG is selected to be equal to that of the total downstream load. In fact, this assumption seems to be more realistic, and hence, it is adapted in this work. Depending on their capabilities of injecting active and reactive power, DG units can be classified as [23]:

1. *DG type I:* This DG is capable of injecting active and reactive power such as synchronous generator.
2. *DG type II:* This DG is capable of injecting active power but absorbs reactive power from the system such as induction generator.
3. *DG type III:* This DG is capable of injecting active power only such as photovoltaic.
4. *DG type IV:* This DG is capable of injecting reactive power only such as synchronous condenser.

The sign of the power factor has also been adapted from the same reference. That is, α will be considered positive if the DG injects reactive power, as in Type I and Type II, for instance. Thus, if a DG unit is going to be installed at bus k of the distribution feeder shown in Fig. 1, the power factor of this unit is calculated as

$$\mathrm{PF}_{\mathrm{DG}} = \frac{P_{Lk,\mathrm{eff}}}{\sqrt{P_{Lk,\mathrm{eff}}^2 + P_{Lk,\mathrm{eff}}^2}} \qquad (54)$$

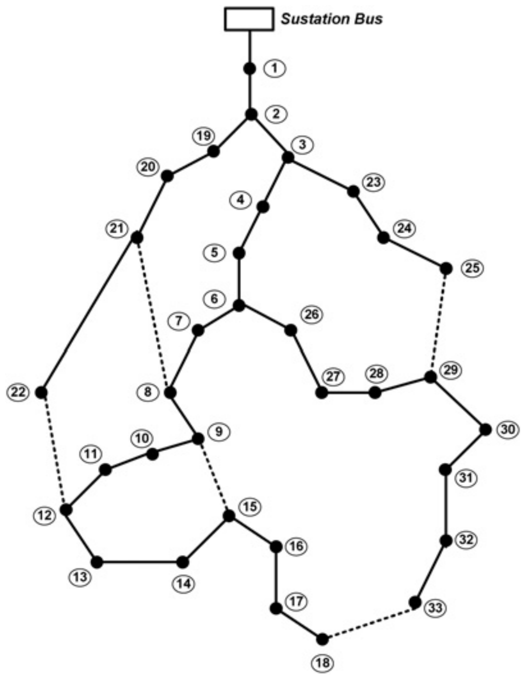

Fig. 2 *Single-line diagram of the 33-bus system*

Dotted lines denote normally open tie switches

4 Demonstration and discussion

To demonstrate the effectiveness of the proposed analytical method, it has been tested on different distribution networks. We first validate the proposed power flow solution, and then we utilised the analytical expressions to determine the optimal locations and sizes of the candidate distributed generation units.

4.1 Validation of the proposed power flow solution

We tested the proposed power flow solution on several test systems. These systems include 33-bus system [7], 69-bus system [35], 119-bus system [36], 76-bus [4] unbalanced system and an 8-bus unbalanced system [37], and we obtained very promising results. For the sake of validation, we provided the results of the 33-bus system and the 69-bus system, which are used in this paper, in the following section. We did not provide further discussion about power flows of the other systems as we felt that power flow is one contribution among other contributions of this research.

4.1.1 33-Bus distribution system test case: The first test system is a 100 kVA, 12.66 kV, radial distribution system [7]. This system has 33 buses, 32 branches and 5 tie lines as can be seen from the single-line diagram depicted below in Fig. 2. The total real and reactive power loads on this system are 3715 kW and 2300 kVar, respectively. The voltage profile obtained by the proposed LACPF method is depicted below in Fig. 3. The FACPF results obtained by the forward/backward sweeping method [32] are also shown for comparison purposes. As can be seen, the simulation results correspond closely to those obtained by performing FACPF. The error in voltage obtained by the proposed LACPF approach was less than 1% at all buses. The maximum voltage error obtained by performing the proposed method was $(-0.008 - 0.001i)$ or 0.843% and

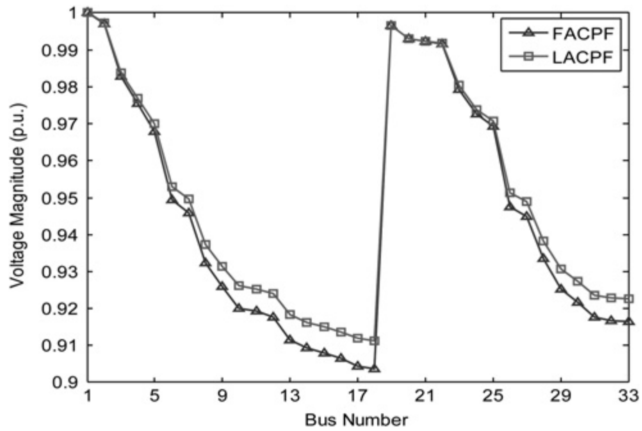

Fig. 3 *Voltage profile of the 33-bus system obtained by the proposed LACPF method and the FACPF method*

occurred at bus 18, the farthest bus from the substation. The detailed results are also given below in Table 1. It is worth pointing out here that various case studies with different loading conditions are carried out on the 33-bus system while verifying the proposed power flow solution. The maximum error for different loading conditions is still found to be quite small, provided that this system has an R/X ratio equal to 3. This small value of error indicates that the power flow solution proposed in this paper can be applied for the applications in which repetitive solutions and exhaustive power flow solutions are required as in the DG placement and sizing problem.

4.2 69-Bus distribution system test case

The second test network is a 69-bus, 12.66 kV medium voltage radial distribution network [35]. This network has been considered by many researchers as a relatively

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Table 1 33-Bus distribution system power flow results

Bus	FACPF		Proposed		Error	
	$\|V\|$ (p.u.)	Angle (radian)	$\|V\|$ (p.u.)	Angle (radian)	Vector (p.u.)	Absolute (%)
1	1	0	1	0	0	0
2	0.9970	0.0002	0.9972	0.0002	−0.000 + 0.000*i*	0.020
3	0.9828	0.0017	0.9839	0.0015	−0.001 + 0.000*i*	0.102
4	0.9753	0.0028	0.9769	0.0025	−0.002 + 0.000*i*	0.155
5	0.9679	0.0040	0.9700	0.0035	−0.002 + 0.000*i*	0.219
6	0.9494	0.0024	0.9530	0.0020	−0.004 + 0.000*i*	0.369
7	0.9459	−0.0017	0.9498	−0.0014	−0.004 − 0.000*i*	0.412
8	0.9322	−0.0044	0.9372	−0.0036	−0.005 − 0.000*i*	0.526
9	0.9259	−0.0057	0.9314	−0.0046	−0.006 − 0.000*i*	0.605
10	0.920	−0.0068	0.9261	−0.0055	−0.007 − 0.000*i*	0.653
11	0.9192	−0.0067	0.9253	−0.0054	−0.007 − 0.000*i*	0.664
12	0.9177	−0.0065	0.9239	−0.0052	−0.007 − 0.000*i*	0.676
13	0.9115	−0.0081	0.9183	−0.0065	−0.008 − 0.001*i*	0.747
14	0.9092	−0.0095	0.9162	−0.0075	−0.008 − 0.001*i*	0.782
15	0.9078	−0.0102	0.9149	−0.0080	−0.008 − 0.001*i*	0.795
16	0.9064	−0.0106	0.9137	−0.0083	−0.008 − 0.001*i*	0.807
17	0.9043	−0.0119	0.9118	−0.0093	−0.008 − 0.001*i*	0.843
18	0.9037	−0.0121	0.9113	−0.0094	−0.008 − 0.001*i*	0.843
19	0.9964	0.0001	0.9966	0.0000	−0.000 + 0.000*i*	0.010
20	0.9929	−0.0011	0.9931	−0.0011	−0.000 − 0.000*i*	0.020
21	0.9922	−0.0015	0.9924	−0.0015	−0.000 + 0.000*i*	0.023
22	0.9915	−0.0018	0.9918	−0.0018	−0.000 − 0.000*i*	0.020
23	0.9793	0.0011	0.9804	0.0010	−0.001 + 0.000*i*	0.113
24	0.9726	−0.0004	0.9739	−0.0005	−0.001 + 0.000*i*	0.134
25	0.9693	−0.0012	0.9707	−0.0012	−0.001 + 0.000*i*	0.145
26	0.9475	0.0031	0.9512	0.0026	−0.004 + 0.000*i*	0.391
27	0.9449	0.0040	0.9488	0.0034	−0.004 + 0.000*i*	0.403
28	0.9335	0.0055	0.9383	0.0045	−0.005 + 0.000*i*	0.515
29	0.9253	0.0068	0.9307	0.0056	−0.006 + 0.000*i*	0.585
30	0.9217	0.0087	0.9274	0.0070	−0.006 + 0.001*i*	0.620
31	0.9175	0.0072	0.9236	0.0059	−0.007 + 0.000*i*	0.655
32	0.9166	0.0068	0.9228	0.0055	−0.008 + 0.000*i*	0.666
33	0.9163	0.0067	0.9225	0.0054	−0.008 + 0.000*i*	0.666

large-scale network. The total real and reactive power loads on this network, respectively, are 3802.19 kW and 2694.06 kVar, but the R/X ratios for some network's branches are greater than 3. The voltage profile obtained by the proposed power flow method is depicted below in Fig. 4. It is interesting to highlight here that the error in voltage was less than 1% at all buses, and only few buses (exactly 10 buses) had an error between 0.27 and 0.936%. The maximum vector error is $(-0.009 + 0.004i)$ or 0.936% and had occurred at bus 65, which is a quite far from the substation bus. Owing to space constraints, we only provide selected power flow results for the buses that had errors between 0.813 and 0.936%, which are given in Table 2.

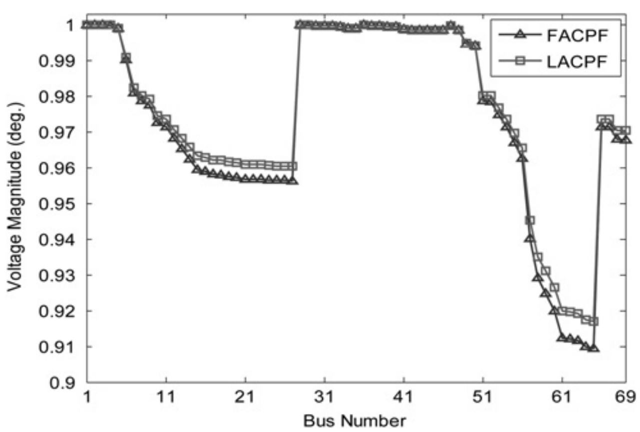

Fig. 4 *Voltage profile of the 69-bus system obtained by the proposed LACPF method and the FACPF method*

5 33-Bus system optimal locations and sizes

The optimal results of allocating and sizing of the DG units on the 33-bus distribution system using the proposed analytical solution are presented in this section. These results are also verified by performing exhaustive power flow routines.

5.1 *Analytical method*

The proposed LACPF is utilised to determine the optimal location, size and power factor of DG units for the 33-bus distribution system. We first performed sensitivity analysis using (42) to determine the optimal locations of the candidate DG units. The results are depicted below in Fig. 5. As can be seen from Fig. 5, the optimal location for the first DG unit is found to be at bus 6. The power factor of the downstream load connected at bus 6 is also obtained from the power flow solution and it was equal to 0.8907 lagging. Even though this paper is concerned with active power losses minimisation, the four DG types mentioned earlier in Section 3.4 have been sized according to (49)–(51) and installed at bus 6, one at a time, to select the DG type which has much impact on total loss reduction. As expected, DG type I and DG type II contributed significantly to loss reduction not only because both of

Table 2 69-Bus distribution system selected power flow results

Bus	FACPF		Proposed		Error	
	|V| (p.u.)	Angle (radian)	|V| (p.u.)	Angle (radian)	Vector (p.u.)	Absolute (%)
61	0.9130	0.0179	0.9198	0.0179	−0.007 + 0.004*i*	0.813
62	0.9120	0.0179	0.9196	0.0180	−0.008 + 0.004*i*	0.886
63	0.9120	0.0189	0.9192	0.0180	−0.008 + 0.004*i*	0.8542
64	0.9100	0.0201	0.9175	0.0183	−0.008 + 0.004*i*	0.881
65	0.9090	0.0202	0.9170	0.0184	−0.009 + 0.004*i*	0.936

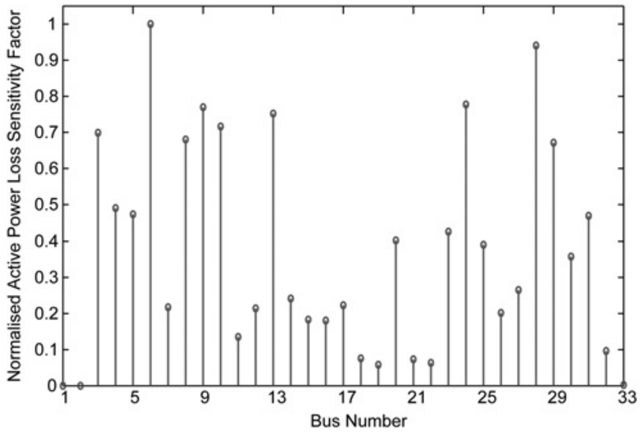

Fig. 5 *Active power loss sensitivity factor of the 33-bus system*

them inject active power to the system, but also the DG locations were chosen in accordance to active power losses reduction. Therefore, the number of DG units used for loss reduction will be limited to two distributed generation units of type I and type II and the results of these types will be presented and discussed in the following sections.

The results obtained by the proposed analytical approach are presented below in Tables 3 and 4, respectively. We performed exhaustive power flow solutions to verify the results obtained by the analytical approach. All simulations are carried out using MATLAB 7.9 on an Intel core, 4 GB, 800 MHz computer.

Table 3 shows the results obtained by both methods for the case of DG type I. This DG type injects both real and reactive power to the bus at which it is connected. When a single DG is selected to be installed at bus 6, the optimal size obtained by the proposed analytical approach is 2487 kVA. The reduction in power losses due to this injection was 62.08%. As indicated previously, we are interested in allocating and sizing of two DG units in this research, thus another DG of the same type is connected at bus 28, which is identified from sensitivity analysis as an optimal location for the second DG unit, as can be seen from Fig. 5. The DG size for this case is given below in Table 3. The total reduction in total power losses achieved by installing the second DG is 69.32%. It is worth mentioning here that once we determined the size of the DG unit from the analytical solution, we changed the obtained DG size in stepwise fashion, that is, in very small steps to refine the optimal size. We then selected the size that yields to total minimum power losses. It is also important to point out here that while refining the size of the DG size we almost got the nearest integer size obtained by the analytical method as we have chosen small step size to change the size of the DG units. This refining stage has given satisfactory results for the test systems used in this paper as a result of the selected small step size. During the entire process, if any constraint is violated, we considered the next available DG location for further evaluation.

Table 4 shows the results obtained by the analytical approach and the exhaustive power flow routines for the case of type II DG, which injects real power only. As expected, the reduction in power losses when this DG type is used is less than that obtained by deploying type I DG. Using the proposed analytical method, the total loss reduction when a single DG is utilised is 43.79%, while that obtained by using two DG units is 45.46%.

The general solution procedures of the analytical method are summarised as follows:

1. Enter the number and the type of candidate DG units.
2. Obtain the initial system loss using the LACPF.
3. Use the priority list and sensitivity analysis to determine the optimal locations of the candidate DG units.
4. Use (49)–(53) and the LACPF to determine the optimal DG size and the corresponding power factor.
5. Change the optimal DG size obtained in (4) in small stepwise fashion to estimate the suitable size. Choose the size that gives minimum losses.
6. Check constraints. If any of the constraints given in Section 2.1 is violated, consider the next available DG location.

Table 3 Results of the 33-bus distribution system – DG type I

Method	Analytical approach		Exhaustive power flow	
Item	Single DG	Two DG	Single DG	Two DG
optimal location	6	28	6	28
S_{DG}, kVA	2487	1204	2485	1200
power loss, kW	69.01	55.85	69.00	55.73
% loss reduction	62.08	69.32	62.03	69.17

Table 4 Results of the 33-bus distribution system – DG type II

Item	Analytical approach		Exhaustive power flow	
	Single DG	Two DG	Single DG	Two DG
optimal location	6	28	6	28
S_{DG}, kVA	2215	785	2214	785
power loss, kW	102	99	102	97
% loss reduction	43.79	45.46	43.8	45.4

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5.2 Exhaustive power flow

To verify the sizes of the DG units obtained by the proposed analytical expressions, we have compared them with those obtained by performing exhaustive power flow solutions. In this case, we first identify the penetration level of the DG units using (40). We assumed a penetration level of 15–60%. Accordingly, the allowable minimum and maximum DG output power for the 33-bus system is varying between 556 and 2620 kVA. Since DG sizes are given in discrete values, we have changed the DG sizes in small steps and installed the candidate DG units at bus 6, which is identified as an optimal location for DG installation from sensitivity analysis. We performed power flow solution several times and determined the DG size that gives minimum losses. This size is found to be 2485 kVA, which gives a reduction in total losses of 62.03%, respectively. The same procedures are repeated for the second DG (type II), which is connected at bus 28 according to the sensitivity analysis. The DG size obtained for this case is also given below in Table 3. Further, the total loss reduction achieved in this case is 69.17%.

As can also be seen from Tables 3 and 4, the results obtained by the proposed analytical method correspond closely to those obtained by performing exhaustive power flow solutions. It is worth noting here that while the results obtained from both methods were almost similar, the proposed analytical method is shown to be considerably faster than the exhaustive power flow as it requires few power flow solutions to determine the optimal size.

6 69-Bus system optimal locations and sizes

The optimal results of allocating and sizing of the DG units on the 69-bus distribution system using the proposed analytical solution are presented in this section. These results are also verified by performing exhaustive power flow routines.

6.1 Analytical method

The proposed power flow is used to determine the optimal locations and sizes of DG units for the 69-bus distribution system. Then, sensitivity analysis is carried out to determine the optimal locations for the candidate DG units, and the results are shown below in Fig. 6. The optimal location for the first DG is found to be at bus 61. The power factor of the downstream load connected at bus 61 is also obtained

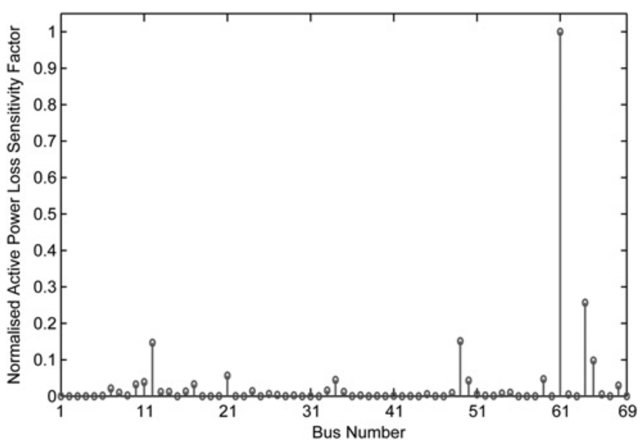

Fig. 6 *Active power loss sensitivity factor of the 69-bus system*

Table 5 Results of the 69-bus distribution system – DG type I

Item	Analytical approach		Exhaustive power flow	
	Single DG	Two DG	Single DG	Two DG
optimal location	61	49	61	49
S_{DG}, kVA	1918	945	1916	945
power loss, kW	23.92	21.63	23.90	21.63
% loss reduction	86.86	88.10	86.87	88.09

from the power flow solution and it was found equal to 0.82 lagging. Table 5 shows the results obtained by the proposed analytical solution and those obtained from performing exhausted power flows for the case of DG type I. This DG type injects both real and reactive power to the bus at which it is connected. A single DG is selected to be installed at bus 61. As can be seen from Table 5, the sizes obtained by both methods are almost the same. This very close agreement can be attributed, in part, to the fact that the step size in both analytical and exhausted power flow is chosen to be very small, that is, numerous step sizes are used in both methods.

As can be seen from Table 5, when a single DG is concerned, the percentage loss reduction achieved by the analytical method is 86.86%. The percentage reduction in losses by performing exhaustive power flows is almost the same. The DG sizes for this case study are also given in Table 5. Now, a second DG, of same type, and of size 945 kVA is connected at bus 49, which is identified as an optimal location for DG installation from sensitivity analysis. The total loss reduction obtained by the analytical approach is 88.10%.

Table 6 shows the results obtained by both methods for the case of type II DG, which injects real power only. As expected, the total loss reduction when such DG type is used is less than that obtained by deploying type I DG. The total loss reduction when a single DG is utilised is 57.06%. On the other hand, when two DG units are installed at the specified buses, the total loss reduction obtained by the proposed analytical method is 87.91%.

6.2 Exhaustive power flow

To verify the sizes of the DG units obtained by the proposed analytical method, we have compared them with those obtained by performing exhaustive power flow solutions. In this case, we first identify the penetration level of the DG units using (40). We assumed a penetration level of 15–60%. Accordingly, the allowable minimum and maximum DG output power for the 69-bus system is varying between 700 and 2795 kVA. Since DG sizes are given in discrete values, we have changed the DG sizes in small steps and

Table 6 Results of the 69-bus distribution system – DG type II

Item	Analytical approach		Exhaustive power flow	
	Single DG	Two DG	Single DG	Two DG
optimal location	61	49	61	49
S_{DG}, kVA	1580	760	1580	760
power loss, kW	78	22	78	22
% loss reduction	57.06	87.91	57.06	87.91

installed the candidate DG units at bus 61, which is identified as an optimal location for DG installation from sensitivity analysis. We performed power flow solution several times and determined the size that gives minimum loss. This size is found to be 1916 kVA, which gives a reduction in total losses of 86.86%. The same procedures are repeated for the second DG (type II), which is connected at bus 49 according to the sensitivity analysis. The DG size obtained for this case is given below in Table 6.

It is worth mentioning here that the similar procedures used for the exhaustive power flow of the 33-bus system are also applied to the 69-bus system. Moreover, as we have chosen small step size to change the size of the DG units, we used the nearest size to the one obtained by the analytical method. Such approximation has given satisfactory results for the test systems used in this paper as a result of the selected small step size. Further, if any constraint is violated, the next DG unit is considered for further evaluation.

7 Voltage profile improvement

Table 7 shows the minimum bus voltages obtained before and after the DG installation for the 33-bus and 69-bus systems, respectively. The minimum bus voltage of the 33-bus system before the introduction of the DG was 0.9113 p.u. and had occurred at bus 18. The maximum bus voltage was equal to 1.0 p.u., which is the substation bus voltage. The minimum bus voltage of the 33-bus distribution system has been boosted to 0.9509 p.u. when a single DG of type I is installed, and boosted to 0.9693 p.u. when two DG units of type I are installed, and had occurred at node 18 for both cases. This means that the improvement in the voltage profile for both cases is about 4.35% and 6.36%, respectively.

For the 69-bus distribution system, the minimum bus voltage was 0.9170 p.u. and had occurred at bus 65, whereas the maximum bus voltage was equal to 1.0 p.u., which is the substation bus voltage. After the installation of single DG and two DG units of type I, the system's minimum bus voltage became 0.9728 p.u., which means an improvement of 6.09%. On the other hand, the maximum bus voltage after the installation of these DG units became 1.011 p.u. and occurred at bus 27.

8 Comparative study

To test the effectiveness of the proposed analytical approach, it has been compared with some other methods presented in the literature. The 33-bus system is considered in this comparative study. The comparison results are shown below in Table 8. As can be clearly seen from Table 8, the proposed analytical method could lead to optimal or near-optimal solution, while requiring less computational burden and effort.

Table 7 Minimum bus voltages before and after DG installation

Test system	DG type I		DG type II	
	Single DG	Two DG	Single DG	Two DG
33-bus distribution system	0.9509	0.9693	0.9411	0.9516
voltage improvement, %	4.35	6.36	3.27	4.42
69-bus distribution system	0.9728	0.9729	0.9695	0.9697
voltage improvement, %	6.09	6.10	5.73	5.75

Table 8 Performance comparison of the proposed method

Comparison	Method	Method	Proposed analytical method	
Item	Ref. [22]	Ref. [23]	Based FACPF	Based LACPF
optimal location	6	6	6	6
optimal size, kW	2490	2601	2236	2215
% loss reduction	47.33	47.39	46.30	43.79
iterations/power flow	5	5	5	—
NET, ms/power flow	290.52	287.50	295.87	51.23

9 Conclusion

This paper proposes an analytical approach for placement and sizing of DGs on power distribution systems with an objective of loss reduction. The motivation in this paper was developed based on the principle that analytical approaches require less computational effort than other heuristic approaches. The solution procedures presented in this paper began by introducing a novel power flow solution, which is very fast, robust and does not encounter convergence issues even with ill-conditioned systems that have high *R*/*X* branch ratios. This power flow solution method has then been used to perform sensitivity analysis to determine the optimal location, size and power factor of the candidate DG unit. One advantage of the work presented here is that it incorporates the power factor of the distributed generation units into the sizing problem while the vast majority of the work presented in the literature has only considered distributed generation units operating at unity power factor. Various DG types are considered in this paper and techniques to determine the optimal size and the corresponding power factor are also proposed. The proposed analytical method has been tested on two distribution systems broadly used as examples in solving the DG placement and sizing problem. The obtained results were also verified by exhaustive power flow routines. The results have shown that the proposed method could lead to optimal or near-optimal global solution, while requiring lower computational effort.

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