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Adaptive Fault-Tolerant Control of Uncertain Nonlinear Large-Scale Systems With Unknown Dead Zone

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***Abstract*—In this paper, an adaptive neural fault-tolerant control scheme is proposed and analyzed for a class of uncertain nonlinear large-scale systems with unknown dead zone and external disturbances. To tackle the unknown nonlinear interaction functions in the large-scale system, the radial basis function neural network (RBFNN) is employed to approximate them. To further handle the unknown approximation errors and the effects of the unknown dead zone and external disturbances, integrated as the compounded disturbances, the corresponding disturbance observers are developed for their estimations. Based on the outputs of the RBFNN and the disturbance observer, the adaptive neural fault-tolerant control scheme is designed for uncertain nonlinear large-scale systems by using a decentralized backstepping technique. The closed-loop stability of the adaptive control system is rigorously proved via Lyapunov analysis and the satisfactory tracking performance is achieved under the integrated effects of unknown dead zone, actuator fault, and unknown external disturbances. Simulation results of a mass–spring–damper system are given to illustrate the effectiveness of the proposed adaptive neural fault-tolerant control scheme for uncertain nonlinear large-scale systems.**

***Index Terms*—Adaptive fault control, backstepping control, disturbance observer, large-scale systems, neural network (NN).**

I. Introduction

MANY physical systems such as power system, aerospace system, chemical engineering system, and telecommunication network are composed of interconnections of lower-dimensional subsystems [1]–[4]. Unique technical issues naturally arise from control of the large-scale systems to achieve the corresponding control goals under subsystem interactions. At the same time, the large-scale system usually possesses nonlinear and uncertain characteristic which will further enhance the control design difficulty [5]. It is well known that the decentralized control structure naturally alleviates the computational burden associated with a centralized control scheme. Hence, the decentralized robust control as an effective control approach has been extensively studied for the uncertain nonlinear large-scale system [6]. Up to now, there has been an increasing interest in the development of decentralized control theories for large-scale systems [7], [8]. In [9], a fuzzy adaptive decentralized output-feedback control design was proposed for large-scale nonlinear systems with dynamical uncertainties. Adaptive fuzzy decentralized control was studied for large-scale nonlinear systems with time-varying delays and unknown high-frequency gain sign in [10]. In [11], a global decentralized robust stabilization was studied for interconnected uncertain nonlinear systems with multiple inputs. In the practice, the large-scale system may suffer the integrated effects coming from actuator fault, external disturbance, and unknown dead zone. Thus, the adaptive decentralized fault-tolerant control should be further investigated for uncertain nonlinear large-scale systems.

Control systems are often subjected to faults which can be caused by actuators, sensors, or system faults [12]. It is important to design the efficient fault-tolerant control scheme to keep the stable and acceptable control performance when faults occur [13]. An adaptive fault diagnosis and fault-tolerant control was developed for multi-input and multi-output (MIMO) nonlinear uncertain systems in [14]. Fault detection and fault-tolerant control were studied for a civil aircraft using the sliding mode method in [15]. Actuator faults as a common fault widely exist in control systems, which should be efficiently tackled to guarantee closed-loop system stability and tracking performance when actuator faults occur [16]. An adaptive actuator failure compensation controller was proposed using output feedback in [17]. In [18], a virtual grouping based adaptive actuator failure compensation technique was studied for MIMO nonlinear systems. In particular, some fault-tolerant control results have been achieved for the nonlinear large-scale system. In [19], an observer-based adaptive decentralized fuzzy fault-tolerant controller was proposed for nonlinear large-scale systems with actuator failures. Decentralized fault-tolerant control was studied for a class of interconnected nonlinear systems in [20]. However, the fault-tolerant control considering the unknown dead zone and the unknown external disturbance needs to be further investigated.

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The dead zone characteristic as one of the most important nonsmooth input nonlinearity widely exists in lots of practical systems. The related robust control problem has attracted extensive attentions [21]–[24]. An adaptive dead zone compensation method was proposed for output-feedback canonical systems in [25]. In [26], a neural-hybrid control was developed for systems with sandwiched dead zones. Adaptive output control schemes were developed for nonlinear systems with uncertain dead zone nonlinearity in [27] and for uncertain nonlinear systems with nonsymmetric dead zone input in [28]. For a large-scale system with actuator faults, the input dead zone will further degrade the tracking control performance. Thus, the decentralized control considering the unknown dead zone should be further developed for the uncertain nonlinear large-scale system. In [29], a decentralized variable structure controller was proposed for uncertain large-scale systems containing a dead zone. Adaptive fuzzy decentralized output feedback control was studied for nonlinear large-scale systems with unknown dead zone inputs in [30]. However, the unknown external disturbance needs to be efficiently tackled in the adaptive fault-tolerant control design for uncertain nonlinear large-scale systems.

One of the key challenges in decentralized control for uncertain nonlinear large-scale systems is the development of techniques for dealing with the interaction uncertainties and the unknown disturbances. In this paper, the radial basis function neural network (RBFNN) is employed to approximate the unknown nonlinear continuous interconnection functions [31]–[33]. In many existing works of the control area, universal function approximators [such as fuzzy logic systems and neural networks (NNs)] are employed to tackle the system uncertainty [34]–[39]. To efficiently handle the unknown approximation error and the unknown disturbance, the disturbance observer is developed to estimate the compounded disturbance which consists of the unknown approximation error, the effect of the unknown dead zone, and the unknown disturbance. In the past decades, various disturbance observer design and disturbance-observer-based control schemes have been extensively studied for system with external disturbances [40]. In [41], a sliding mode controller using disturbance observer was proposed for systems with mismatched uncertainties. An adaptive fuzzy tracking controller was developed for a class of uncertain MIMO nonlinear systems using disturbance observer in [42]. The disturbance attenuation and rejection issue were investigated for a class of MIMO nonlinear systems based on the disturbance observer [43]. However, the disturbance observer needs to be further developed for fault-tolerant control of uncertain nonlinear large-scale systems with unknown dead zone.

To develop fault-tolerant control scheme of uncertain nonlinear large-scale systems, a decentralized backstepping control technique is employed in this paper. During the last decades, various backstepping control schemes have been developed [44]–[49]. Decentralized dynamic surface control was studied for large-scale interconnected systems in strict-feedback form using NNs in [50]. In [51], an adaptive NN decentralized backstepping output-feedback control was proposed for nonlinear large-scale systems with time delays. However, there are few backstepping control results for uncertain nonlinear large-scale systems with unknown dead zone, actuator fault, and unknown disturbances.

This paper aims at developing an adaptive neural fault-tolerant control scheme to track the desired trajectories of uncertain nonlinear large-scale systems under the integrated effects of the unknown dead zone, actuator fault, and unknown external disturbances. The organization of this paper is as follows. The problem statement is given in Section II. The adaptive neural fault-tolerant control scheme based on the disturbance observer and the RBFNN is developed using the backstepping method in Section III. Simulation studies are provided in Section IV to demonstrate the effectiveness of the proposed adaptive neural fault-tolerant control approach, followed by some concluding remarks in Section V.

II. Problem Formulation

Consider a nonlinear large-scale system that is composed of N subsystems and the ith subsystem $\Sigma_i (i = 1, \dots, N)$ can be described by

$$\Sigma_i \begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(\bar{y}, \bar{x}_{i,1}) + d_{i,1}(t) \\ \dot{x}_{i,2} = x_{i,3} + f_{i,2}(\bar{y}, \bar{x}_{i,2}) + d_{i,2}(t) \\ \vdots \\ \dot{x}_{i,n_i-1} = x_{i,n_i} + f_{i,n_i-1}(\bar{y}, \bar{x}_{i,n_{i-1}}) + d_{i,n_i-1}(t) \\ \dot{x}_{i,n_i} = f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) + \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) u_i + d_{i,n_i}(t) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $x_{i,j}, i = 1, \dots, N, j = 1, \dots, n_i$ are the subsystem states; $\bar{x}_{i,j} = [x_{i,1}, x_{i,2}, \dots, x_{i,j}]^T \in R^j$ is the group state vector; $u_i \in R$ is the control input; $y_i \in R$ is the subsystem output and $\bar{y} = [y_1, \dots, y_N]$ is the whole system output vector; $f_{i,j}(\bar{y}, \bar{x}_{i,j}) \in R$ are the unknown smooth functions; $\bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) \in R$ is an unknown control gain function; and $d_{i,j}(t) \in R$ are the system unknown external disturbances. The system states $x_{i,j}$ are assumed to be measurable.

However, in practical engineering, actuators may become faulty. Bias faults and gain faults are two kinds of actuator faults that are commonly occurring in the practice. An actuator bias fault can be described as

$$u_i^f(t) = u_i(t) + \zeta_i(t) \tag{2}$$

where $\zeta_i(t)$ denotes a bounded signal, and an actuator gain fault can be described as

$$u_i^f(t) = \rho_i(\bar{x}_{i,n_i}) u_i(t) \tag{3}$$

where $0 < \rho_i(\bar{x}_{i,n_i}) \leq 1$, which denotes the remaining control rate. Here, $\rho_i(\bar{x}_{i,n_i})$ at the failure time instant t_f is assumed to be unknown.

The two actuator faults can be uniformly described as

$$u_i^f(t) = \rho_i u_i(t) + \zeta_i(t). \tag{4}$$

On the other hand, there may exist the actuator dead zone nonlinearity in an uncertain nonlinear large-scale system. In such case, the control input u_i can be expressed as

$$u_i(t) = \mathrm{DZ}(v_i(t)) \tag{5}$$

where v_i is the input of the dead zone and DZ$(\cdot)$ denotes a dead zone operator.

In this paper, the considered dead zone is described as [23]

$$u_i(t) = \mathrm{DZ}(v_i(t)) = \begin{cases} m_i(v_i(t) - b_{ir}), & \text{for } v_i(t) \geq b_{ir} \\ 0, & \text{for } b_{il} < v_i(t) < b_{ir} \\ m_i(v_i(t) - b_{il}), & \text{for } v_i(t) \leq b_{il} \end{cases} \quad (6)$$

where $m_i > 0$, $b_{ir} > 0$, and $b_{il} < 0$ are the unknown dead zone parameters.

To develop a robust fault-tolerant control scheme, the dead zone model (6) is rewritten as [23]

$$\mathrm{DZ}(v_i(t)) = m_i v_i(t) + \eta_i(v_i(t)) \quad (7)$$

where $\eta_i(v_i(t))$ is given by

$$\eta_i(v_i(t)) = \begin{cases} -m_i b_{ir}, & \text{for } v_i(t) \geq b_{ir} \\ -m_i v_i(t), & \text{for } b_{il} < v_i(t) < b_{ir} \\ -m_i b_{il}, & \text{for } v_i(t) \leq b_{il}. \end{cases} \quad (8)$$

For a practical system, we know that the parameter m_i of the dead zone is bounded. Thus, from (8), we have

$$|\eta_i(v_i(t)| \leq \eta_{im} \quad (9)$$

where $\eta_{im} = \max\{m_{i\max} b_{ir\max}, -m_{i\max} b_{il\min}\}$, $m_{i\max}$ is the upper-bound of the parameter m_i, $b_{ir\max}$ is the upper-bound of the parameter b_{ir}, and $b_{il\min}$ is the lower-bound of the parameter b_{il}.

Considering (4), the uncertain nonlinear large-scale system (1) with the unknown dead zone and the actuator fault can be rewritten as

$$\Sigma_i \begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(\bar{y}, \bar{x}_{i,1}) + d_{i,1}(t) \\ \dot{x}_{i,2} = x_{i,3} + f_{i,2}(\bar{y}, \bar{x}_{i,2}) + d_{i,2}(t) \\ \vdots \\ \dot{x}_{i,n_i-1} = x_{i,n_i} + f_{i,n_i-1}(\bar{y}, \bar{x}_{i,n_{i-1}}) + d_{i,n_i-1}(t) \\ \dot{x}_{i,n_i} = f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) v_i \\ \qquad + D_{i,n_i}(t) \\ y_i = x_{i,1} \end{cases} \quad (10)$$

where $g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) = \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\rho_i m_i$ and $D_{i,n_i}(t) = d_{i,n_i}(t) + \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\rho_i\eta_i + \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\zeta_i$.

For the given desired trajectory $y_{i,d}$, the control objective is that the adaptive fault-tolerant control input v_i is designed to ensure the tracking error of the uncertain nonlinear large-scale system convergent in the presence of the system uncertainties, unknown dead zone, actuator faults, and time-varying unknown external disturbances.

In this paper, RBFNNs are employed to approximate the uncertain interconnection functions $f_{i,j}(\bar{y}, \bar{x}_{i,j})$ of the nonlinear large-scale system (1) and the nonlinear disturbance observer is developed to estimate the compounded disturbance by combing the NN approximation error with the unknown external disturbance. To facilitate proceed the design of adaptive neural fault-tolerant control scheme, the following lemmas and assumptions are required.

Lemma 1 [52]: As a class of linearly parameterized NN, RBFNNs are employed to approximate the continuous function $f(Z) : R^q \to R$ which can be written as

$$\hat{f}(Z) = \hat{\theta}^T \phi(Z) + \varepsilon \quad (11)$$

where $Z = [z_1, z_2, \ldots, z_q]^T \in R^q$ is the input vector of the NN, $\hat{\theta} \in R^p$ is a weight vector of the NN, $\phi(Z) = [\phi_1(Z), \phi_2(Z), \ldots, \phi_p(Z)]^T \in R^p$ is the basis function vector, and ε is the approximation error of the NN. The optimal weight value θ of RBFNN is expressed as

$$\theta = \arg \min_{\hat{\theta} \in \Omega_f} \left[\sup_{z \in S_Z} \left| \hat{f}\left(Z|\hat{\theta}\right) - f(Z) \right| \right] \quad (12)$$

where $\Omega_f = \{\hat{\theta} : \|\hat{\theta}\| \leq \bar{M}\}$ is a valid field of the estimate parameter $\hat{\theta}$, $\bar{M}$ is a design parameter, and $S_Z \subset R^n$ is an allowable set of the state vector. Using the optimal weight value, we have

$$f(Z) = \theta^T \phi(Z) + \varepsilon^* \qquad |\varepsilon^*| \leq \bar{\varepsilon} \quad (13)$$

where ε^* is the optimal approximation error and $\bar{\varepsilon} > 0$ is the upper bound of the approximation error.

Lemma 2 [52]: For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\pi_1(\|x\|) \leq V(x) \leq \pi_2(\|x\|)$, such that $\dot{V}(x) \leq -c_1 V(x) + c_2$, where $\pi_1, \pi_2 : R^n \to R$ are class K functions and c_1 and c_2 are positive constants, then the solution $x(t)$ is uniformly bounded.

Assumption 1 [52]: There exist positive constants $\underline{g}_i$, $i = 1, \ldots, N$ and $\bar{g}_i$, such that $\underline{g}_i \leq |\bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})| \leq \bar{g}_i$. Without losing generality, we shall assume that $\bar{g}_{i,n_i}$ are positive in the adaptive fault-tolerant control design.

Assumption 2: There exist the unknown positive constants $\beta_{0i,j}, i = 1, \ldots, N, j = 1, \ldots, n_i$ and $\beta_{1i,j}$ such that the external disturbance satisfy $|d_{i,j}| \leq \beta_{0i,j}$ and $|\dot{d}_{i,j}| \leq \beta_{1i,j}$.

Assumption 3 [52]: There exist constants $g_i^d > 0, i = 1, 2, \ldots, n$ such that $|\dot{\bar{g}}_{i,n_i}(.)| \leq g_i^d$ in the compact set Ω_j.

Assumption 4: For the desired system trajectory $y_{i,d}$, there exist unknown positive constants $\bar{\tau}_i$ such that $|y_{i,d}^{(n_i)}| \leq \bar{\tau}_i$, $i = 1, \ldots, N$.

Remark 1: Considering $0 < \rho_i < 1$, we have $|g_{i,n_i}(.)| = |\bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\rho_i m_i| \leq |\bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) m_i|$. Invoking Assumption 1 yields $\underline{g}_{i0} \leq |g_{i,n_i}| \leq \bar{g}_{i0}$, where $\underline{g}_{i0}$ and $\bar{g}_{i0}$ are positive constants. For a practical system, the remaining control rate ρ_i satisfies $\dot{\rho}_i \leq \rho_{i0}$ with $\rho_{i0} > 0$. Considering the unknown constant m_i of dead zone and Assumption 3, we have $|\dot{g}_{i,n_i}| \leq \vartheta_i$ with $\vartheta_i > 0$. In Assumption 2, we assume that the external disturbance and the time derivative of the external disturbance are bounded. Usually, the time-dependent disturbance $d_{i,j}(t)$ is bounded. At the same time, if the external disturbance is fast changing, the system can become uncontrollable in the practice. Hence, it is reasonable to assume that the time derivation of the external disturbance $d_{i,j}(t)$ is bounded. Otherwise, the nonlinear larger scale system may be uncontrollable.

Remark 2: Comparing with the existing actuator fault model, the state-dependent nonlinear actuator gain fault model is considered using $\rho_i(\bar{x}_{i,n_i})$ in this paper. Generally speaking, the actuator fault will degrade the control performance and further affect the system states. On the contrary, the undesirable system states may lead to more serious actuator fault. Namely, there exists interaction between the fault and the system states.

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Thus, the nonlinear remaining control rate ρ_i given in (3) is more suitable for a practical control problem.

III. Adaptive Neural Fault-Tolerant Control Design Using Disturbance Observer

In this section, the adaptive neural fault-tolerant control scheme will be proposed based on the output of the designed disturbance observer and RBFNN by using the standard back-stepping technique. The detailed design process of adaptive neural fault-tolerant control consists of: 1) the disturbance observer design in each step for the ith subsystem; 2) the virtual control law design; and 3) the stability analysis of error systems. The detailed design of the ith subsystem $\Sigma_i (i = 1, \ldots, N)$ is presented as follows.

Step 1: To design the adaptive neural fault-tolerant control, we define

$$e_{i,1} = x_{i,1} - y_{i,d} \tag{14}$$

$$e_{i,2} = x_{i,2} - \alpha_{i,1} - \dot{y}_{i,d} \tag{15}$$

where $\alpha_{i,1}$ is a designed virtual control law.

Considering (10) and differentiating $e_{i,1}$ with respect to time yields

$$\dot{e}_{i,1} = \dot{x}_{i,1} - \dot{y}_{i,d} = x_{i,2} + f_{i,1}(\bar{y}, \bar{x}_{i,1}) + d_{i,1}(t) - \dot{y}_{i,d}. \tag{16}$$

The RBFNN is employed to approximate the unknown term $\lambda_{i,1} f_{i,1}(\bar{y}, \bar{x}_{i,1})$. According to (13) yields

$$\dot{e}_{i,1} = x_{i,2} + \lambda_{i,1}^{-1}\theta_{i,1}^T\phi(Z_{i,1}) + \lambda_{i,1}^{-1}\varepsilon_{i,1}^* + d_{i,1}(t) - \dot{y}_{i,d} \tag{17}$$

where $\lambda_{i,1} > 0$ is a design parameter and $Z_{i,1} = [\bar{y}, \bar{x}_{i,1}]^T$.

Defining $D_{i,1} = \lambda_{i,1}^{-1}\varepsilon_{i,1}^* + d_{i,1}(t)$, we have

$$\dot{e}_{i,1} = x_{i,2} + \lambda_{i,1}^{-1}\theta_{i,1}^T\phi(Z_{i,1}) + D_{i,1}(t) - \dot{y}_{i,d}. \tag{18}$$

A. Disturbance Observer Design

For enhancing the disturbance rejection ability, the nonlinear disturbance observer is adopted to estimate the unknown compounded disturbance in this paper. From the approximation ability of the RBFNN, we know that the time derivative of $\varepsilon_{i,1}^*$ is bounded. At the same time, the time derivative of $d_{i,1}(t)$ is also bounded according to Assumption 2. Thus, the time derivative of the compounded disturbance $D_{i,1}(t)$ is bounded. Namely, we have

$$|D_{i,1}| \le \beta_{i,10}, \quad |\dot{D}_{i,1}| \le \beta_{i,11} \tag{19}$$

where $\beta_{i,10}$ and $\beta_{i,11}$ are unknown positive constants.

The nonlinear disturbance observer is proposed as

$$\begin{aligned}\hat{D}_{i,1} &= \lambda_{i,1}(e_{i,1} - \xi_{i,1})\\ \dot{\xi}_{i,1} &= x_{i,2} + \lambda_{i,1}^{-1}\hat{\theta}_{i,1}^T\phi(Z_{i,1}) + \hat{D}_{i,1} - \dot{y}_{i,d}.\end{aligned} \tag{20}$$

Considering (18) and (20), we obtain

$$\begin{aligned}\dot{\hat{D}}_{i,1} &= \lambda_{i,1}(\dot{e}_{i,1} - \dot{\xi}_{i,1})\\ &= \lambda_{i,1}\Big(D_{i,1} - \hat{D}_{i,1} - \lambda_{i,1}^{-1}\hat{\theta}_{i,1}^T\phi(Z_{i,1}) + \lambda_{i,1}^{-1}\theta_{i,1}^T\phi(Z_{i,1})\Big)\\ &= \lambda_{i,1}\tilde{D}_{i,1} - \tilde{\theta}_{i,1}^T\phi(Z_{i,1})\end{aligned} \tag{21}$$

where $\tilde{\theta}_{i,1} = \hat{\theta}_{i,1} - \theta_{i,1}$, and $\tilde{D}_{i,1} = D_{i,1} - \hat{D}_{i,1}$.

Considering (21) yields

$$\dot{\tilde{D}}_{i,1} = \dot{D}_{i,1} - \dot{\hat{D}}_{i,1} = \dot{D}_{i,1} - \lambda_{i,1}\tilde{D}_{i,1} + \tilde{\theta}_{i,1}^T\phi(Z_{i,1}). \tag{22}$$

B. Adaptive Virtual Control Law

The parameter updated law of $\hat{\theta}_{i,1}$ is designed as

$$\dot{\hat{\theta}}_{i,1} = \gamma_{i,1}\Big(\lambda_{i,1}^{-1}\phi(Z_{i,1})e_{i,1} - \delta_{i,1}\hat{\theta}_{i,1}\Big) \tag{23}$$

where $\gamma_{i,1} > 0$ and $\delta_{i,1} > 0$ are design parameters.

Invoking (15) and (18), we have

$$\dot{e}_{i,1} = e_{i,2} + \alpha_{i,1} + \lambda_{i,1}^{-1}\theta_{i,1}^T\phi(Z_{i,1}) + D_{i,1}(t). \tag{24}$$

The virtual control law $\alpha_{i,1}$ is proposed as

$$\alpha_{i,1} = -k_{i,1}e_{i,1} - \lambda_{i,1}^{-1}\hat{\theta}_{i,1}^T\phi(Z_{i,1}) - \hat{D}_{i,1}(t) \tag{25}$$

where $k_{i,1} > 0$ is a designed parameter.

Substituting (25) into (24), we obtain

$$\begin{aligned}\dot{e}_{i,1} &= -k_{i,1}e_{i,1} + e_{i,2} + \lambda_{i,1}^{-1}\theta_{i,1}^T\phi(Z_{i,1})\\ &\quad - \lambda_{i,1}^{-1}\hat{\theta}_{i,1}^T\phi(Z_{i,1}) + D_{i,1}(t) - \hat{D}_{i,1}(t)\\ &= -k_{i,1}e_{i,1} + e_{i,2} - \lambda_{i,1}^{-1}\tilde{\theta}_{i,1}^T\phi(Z_{i,1}) + \tilde{D}_{i,1}.\end{aligned} \tag{26}$$

Step j ($2 \le j \le n_i - 1$): Define

$$e_{i,j} = x_{i,j} - \alpha_{i,j-1} - y_{i,d}^{(j-1)} \tag{27}$$

$$e_{i,j+1} = x_{i,j+1} - \alpha_{i,j} - y_{i,d}^{(j)} \tag{28}$$

where $y_{i,d}^{(i-1)}$ is the $(i-1)$th order time-derivative of $y_{i,d}$ and $y_{i,d}^{(i)}$ is the ith order time-derivative of $y_{i,d}$, respectively. $\alpha_{i,j-1}$ and $\alpha_{i,j}$ are the virtual control laws of the $(j-1)$th step and the jth step of the ith subsystem, respectively.

Considering (10) and differentiating $e_{i,j}$ with respect to time yields

$$\begin{aligned}\dot{e}_{i,j} &= \dot{x}_{i,j} - \dot{\alpha}_{i,j-1} - y_{i,d}^{(j)} = x_{i,j+1} + f_{i,j}\big(\bar{y}, \bar{x}_{i,j}\big)\\ &\quad + d_{i,j}(t) - \dot{\alpha}_{i,j-1} - y_{i,d}^{(j)}.\end{aligned} \tag{29}$$

The RBFNN is employed to approximate the unknown term $\lambda_{i,j} f_{i,j}(\bar{y}, \bar{x}_{i,j})$. According to (13) yields

$$\begin{aligned}\dot{e}_{i,j} &= x_{i,j+1} + \lambda_{i,j}^{-1}\theta_{i,j}^T\phi\big(Z_{i,j}\big) + \lambda_{i,j}^{-1}\varepsilon_{i,j}^*\\ &\quad + d_{i,j}(t) - \dot{\alpha}_{i,j-1} - y_{i,d}^{(j)}\end{aligned} \tag{30}$$

where $\lambda_{i,j} > 0$ is a design parameter and $Z_{i,j} = [\bar{y}, \bar{x}_{i,j}]^T$.

C. Disturbance Observer Design

Defining $D_{i,j} = \lambda_{i,j}^{-1}\varepsilon_{i,j}^* + d_{i,j}(t)$, we have

$$\dot{e}_{i,j} = x_{i,j+1} + \lambda_{i,j}^{-1}\theta_{i,j}^T\phi\big(Z_{i,j}\big) + D_{i,j}(t) - \dot{\alpha}_{i,j-1} - y_{i,d}^{(j)}. \tag{31}$$

From the definition of the compounded disturbance $D_{i,j}(t)$, we know that its time derivative is bounded. Namely, we have

$$\big|D_{i,j}\big| \le \beta_{i,j0}, \quad \big|\dot{D}_{i,j}\big| \le \beta_{i,j1} \tag{32}$$

where $\beta_{i,j0}$ and $\beta_{i,j1}$ are unknown positive constants.

The nonlinear disturbance observer is proposed as

$$\begin{aligned}\hat{D}_{i,j} &= \lambda_{i,j}(e_{i,j} - \xi_{i,j})\\ \dot{\xi}_{i,j} &= x_{i,j+1} + \lambda_{i,j}^{-1}\hat{\theta}_{i,j}^T\phi(Z_{i,j}) + \hat{D}_{i,j}\\ &\quad - \dot{\alpha}_{i,j-1} - y_{i,d}^{(j)}.\end{aligned} \tag{33}$$

Considering (31) and (33), we obtain

$$\begin{aligned}\dot{\hat{D}}_{i,j} &= \lambda_{i,j}(\dot{e}_{i,j} - \dot{\xi}_{i,j})\\ &= \lambda_{i,j}\Big(D_{i,j} - \hat{D}_{i,j} - \lambda_{i,j}^{-1}\hat{\theta}_{i,j}^T\phi(Z_{i,j}) + \lambda_{i,j}^{-1}\theta_{i,j}^T\phi(Z_{i,j})\Big)\\ &= \lambda_{i,j}\tilde{D}_{i,j} - \tilde{\theta}_{i,j}^T\phi(Z_{i,j})\end{aligned} \tag{34}$$

where $\tilde{\theta}_{i,j} = \hat{\theta}_{i,j} - \theta_{i,j}$ and $\tilde{D}_{i,j} = D_{i,j} - \hat{D}_{i,j}$.

Considering (34) yields

$$\dot{\tilde{D}}_{i,j} = \dot{D}_{i,j} - \dot{\hat{D}}_{i,j} = \dot{D}_{i,j} - \lambda_{i,j}\tilde{D}_{i,j} + \tilde{\theta}_{i,j}^T\phi(Z_{i,j}). \tag{35}$$

D. Adaptive Virtual Control Law

The parameter updated law of $\hat{\theta}_{i,j}$ is designed as

$$\dot{\hat{\theta}}_{i,j} = \gamma_{i,j}\Big(\lambda_{i,j}^{-1}\phi(Z_{i,j})e_{i,j} - \delta_{i,j}\hat{\theta}_{i,j}\Big) \tag{36}$$

where $\gamma_{i,j} > 0$ and $\delta_{i,j} > 0$ are design parameters.

Invoking (28) and (31), we have

$$\dot{e}_{i,j} = e_{i,j+1} + \alpha_{i,j} + \lambda_{i,j}^{-1}\theta_{i,j}^T\phi(Z_{i,j}) + D_{i,j}(t) - \dot{\alpha}_{i,j-1}. \tag{37}$$

The virtual control law $\alpha_{i,j}$ is proposed as

$$\alpha_{i,j} = -k_{i,j}e_{i,j} - \lambda_{i,j}^{-1}\hat{\theta}_{i,j}^T\phi(Z_{i,j}) - \hat{D}_{i,j}(t) - e_{i,j-1} + \dot{\alpha}_{i,j-1} \tag{38}$$

where $k_{i,j} > 0$ is a design constant.

Substituting (38) into (37), we obtain

$$\begin{aligned}\dot{e}_{i,j} &= -k_{i,j}e_{i,j} + e_{i,j+1} - e_{i,j-1} + \lambda_{i,j}^{-1}\theta_{i,j}^T\phi(Z_{i,j})\\ &\quad - \lambda_{i,j}^{-1}\hat{\theta}_{i,j}^T\phi(Z_{i,j}) + D_{i,j}(t) - \hat{D}_{i,j}(t)\\ &= -k_{i,j}e_{i,j} + e_{i,j+1} - e_{i,j-1}\\ &\quad - \lambda_{i,j}^{-1}\tilde{\theta}_{i,j}^T\phi(Z_{i,j}) + \tilde{D}_{i,j}.\end{aligned} \tag{39}$$

Step n_i: Define

$$e_{i,n_i} = x_{i,n_i} - \alpha_{i,n_i-1} - y_{i,d}^{(n_i-1)} \tag{40}$$

where $y_{i,d}^{(n_i-1)}$ is the (n_i-1)th order time-derivative of $y_{i,d}$. α_{i,n_i-1} is the virtual control law of the (n_i-1)th step of the ith subsystem.

Considering (10) and (40) and differentiating e_{i,n_i} with respect to time yields

$$\begin{aligned}\dot{e}_{i,n_i} &= \dot{x}_{i,n_i} - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n)} = f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\\ &\quad + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})v_i + D_{i,n_i} - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)}.\end{aligned} \tag{41}$$

Considering $D_{i,n_i}(t) = d_{i,n_i}(t) + \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\rho_i\eta_i + \bar{g}_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\zeta_i$ and Remark 1, we have

$$|D_{i,n_i}| \le \beta_{i,n_i0}, \quad |\dot{D}_{i,n_i}| \le \beta_{i,n_i1} \tag{42}$$

where β_{i,n_i0} and β_{i,n_i1} are unknown positive constants.

E. Desired Control Law

The desired fault-tolerant control v_i^* can be designed as

$$\begin{aligned}v_i^* &= -k_{i,n_i}e_{i,n_i} - D_{i,n_i} - e_{i,n_i-1}\\ &\quad - \frac{1}{g_{i,n_i}}\Big(f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)}\Big)\end{aligned} \tag{43}$$

where $k_{i,n_i} > 0$ is a design constant.

Define $h_i(x) = 1/(g_{i,n_i})(f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)})$. By employing RBFNNs to approximate $h_i(x)$ and considering (13), v_i^* can be expressed as

$$v_i^* = -k_{i,n_i}e_{i,n_i} - \theta_{i,n_i}^T\phi(Z_{i,n_i}) - \varepsilon_{i,n_i}^* - D_{i,n_i} - e_{i,n_i-1} \tag{44}$$

where $Z_{i,n_i} = [\bar{x}_{i,n_i}, \bar{y}, \dot{\alpha}_{i,n_i-1}, y_{id}^{(n_i)}]^T$.

F. Adaptive Control Law

Considering the unknown θ_{i,n_i}, ε_{i,n_i}^*, and D_{i,n_i}, the adaptive fault-tolerant control law v is proposed as

$$v_i = -k_{i,n_i}e_{i,n_i} - \hat{\theta}_{i,n_i}^T\phi(Z_{i,n_i}) - \hat{D}_{i,n_i} - e_{i,n_i-1} \tag{45}$$

where $\hat{D}_{i,n_i}$ is the estimate of D_{i,n_i} and $\hat{\theta}_{i,n_i}$ is the estimate of θ_{i,n_i} which is updated by

$$\dot{\hat{\theta}}_{i,n_i} = \gamma_{i,n_i}\Big(\phi(Z_{i,n_i})e_{i,n_i} - \delta_{i,n_i}\hat{\theta}_{i,n_i}\Big) \tag{46}$$

where $\gamma_{i,n_i} > 0$ and $\delta_{i,n_i} > 0$ are design parameters.

Considering (41) and (45), we obtain

$$\begin{aligned}\dot{e}_{i,n_i} &= g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})(\theta_{i,n_i}^T\phi(Z_{i,n_i}) + \varepsilon_{i,n_i}^*)\\ &\quad + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\Big(-k_{i,n_i}e_{i,n_i} - \hat{\theta}_{i,n_i}^T\phi(Z_{i,n_i})\Big)\\ &\quad + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\Big(-\hat{D}_{i,n_i} - e_{i,n_i-1}\Big) + D_{i,n_i}.\end{aligned} \tag{47}$$

Define $\tilde{\theta}_{i,n_i} = \hat{\theta}_{i,n_i} - \theta_{i,n_i}$, we have

$$\begin{aligned}\dot{e}_{i,n_i} &= g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\Big(-\tilde{\theta}_{i,n_i}^T\phi(Z_{i,n_i}) + \varepsilon_{i,n_i}^* - k_{i,n_i}e_{i,n_i}\Big)\\ &\quad + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})\Big(-\hat{D}_{i,n_i} - e_{i,n_i-1}\Big) + D_{i,n_i}.\end{aligned} \tag{48}$$

G. Disturbance Observer Design

To facilitate the design of nonlinear disturbance observer, (41) can be also written as

$$\begin{aligned}\dot{e}_{i,n_i} &= l_{i,n_i}^{-1}\varphi_i(\bar{x}_{i,n_i}, v_i) + D_{i,n_i} - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)}\\ &= l_{i,n_i}^{-1}\theta_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + l_{i,n_i}^{-1}\varepsilon_{i\varphi}^*\\ &\quad + D_{i,n_i} - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)}\end{aligned} \tag{49}$$

where $\varphi_i(\bar{x}_{i,n_i}, v_i) = l_{i,n_i}(f_{i,n_i}(\bar{y}, \bar{x}_{i,n_i}) + g_{i,n_i}(\bar{y}, \bar{x}_{i,n_i})v_i)$, $\theta_{i\varphi}$ is optimal weight value of the NN, $\varepsilon_{i\varphi}^*$ is an optimal approximation error of the RBFNN, and $l_{i,n_i} > 0$ is a design parameter of the developed nonlinear disturbance observer.

For the error system (49), an auxiliary variable is defined as [53]

$$s_i = e_{i,n_i} - \xi_{i,n_i} \tag{50}$$

and the intermedial variable ξ_{i,n_i} is proposed as

$$\dot{\xi}_{i,n_i} = q_i s_i + l_{i,n_i}^{-1}\hat{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) - \dot{\alpha}_{i,n_i-1} - y_{i,d}^{(n_i)} \tag{51}$$

where $q_i > 0$ is a designed parameter and $\hat{\theta}_{i\varphi}$ is the estimate of the optimal weight value $\theta_{i\varphi}$.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Differentiating (50) and considering (51), we have

$$\begin{aligned}\dot{s}_i &= \dot{e}_{i,n_i} - \dot{\xi}_{i,n_i}\\ &= l_{i,n_i}^{-1}\theta_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + l_{i,n_i}^{-1}\varepsilon_{i\varphi}^* + D_{i,n_i}\\ &\quad - \left(q_i s_i + l_{i,n_i}^{-1}\hat{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\right) = -q_i s_i\\ &\quad - l_{i,n_i}^{-1}\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + l_{i,n_i}^{-1}\varepsilon_{i\varphi}^* + D_{i,n_i}\end{aligned} \tag{52}$$

where $\tilde{\theta}_{i\varphi} = \hat{\theta}_{i\varphi} - \theta_{i\varphi}$.

Considering (52) yields

$$\begin{aligned}s_i\dot{s}_i &= -q_i s_i^2 - l_{i,n_i}^{-1}s_i\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\\ &\quad + l_{i,n_i}^{-1}s_i\varepsilon_{i\varphi}^* + s_i D_{i,n_i}\\ &\le -(q_i - 1.0)s_i^2 - l_{i,n_i}^{-1}s_i\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\\ &\quad + 0.5 l_{i,n_i}^{-2}\varepsilon_{i\varphi}^{*2} + 0.5\beta_{i,n_i 0}^2.\end{aligned} \tag{53}$$

Using the auxiliary variable s_i, the nonlinear disturbance is designed as

$$\hat{D}_{i,n_i} = l_{i,n_i}(s_i - \psi_i) \tag{54}$$

and the intermedial variable ψ_i is given by

$$\dot{\psi}_i = -q_i s_i + \hat{D}_{i,n_i}. \tag{55}$$

Define, $\tilde{D}_{i,n_i} = D_{i,n_i} - \hat{D}_{i,n_i}$. Differentiating (54), and considering (52) and (55) yield

$$\begin{aligned}\dot{\hat{D}}_{i,n_i} &= l_{i,n_i}(\dot{s}_i - \dot{\psi}_i)\\ &= l_{i,n_i}\left(-q_i s_i - l_{i,n_i}^{-1}\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\right)\\ &\quad + l_{i,n_i}\left(l_{i,n_i}^{-1}\varepsilon_{i\varphi}^* + D_{i,n_i} - \left(-q_i s_i + \hat{D}_{i,n_i}\right)\right)\\ &= -\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + \varepsilon_{i\varphi}^* + l_{i,n_i}\left(D_{i,n_i} - \hat{D}_{i,n_i}\right)\\ &= -\tilde{\theta}_{i\varphi}^T\phi_{i\psi}(\bar{x}_{i,n_i}, v_i) + \varepsilon_{i\varphi}^* + l_{i,n_i}\tilde{D}_{i,n_i}.\end{aligned} \tag{56}$$

Considering (56) yields

$$\begin{aligned}\dot{\tilde{D}}_{i,n_i} &= \dot{D}_{i,n_i} - \dot{\hat{D}}_{i,n_i}\\ &= \dot{D}_{i,n_i} - l_{i,n_i}\tilde{D}_{i,n_i} + \tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) - \varepsilon_{i\varphi}^*.\end{aligned} \tag{57}$$

Invoking (57), we have

$$\begin{aligned}\tilde{D}_{i,n_i}\dot{\tilde{D}}_{i,n_i} &= \tilde{D}_{i,n_i}\dot{D}_{i,n_i} - l_{i,n_i}\tilde{D}_{i,n_i}^2\\ &\quad + \tilde{D}_{i,n_i}\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) - \tilde{D}_{i,n_i}\varepsilon_{i\varphi}^*.\end{aligned} \tag{58}$$

Considering the following fact:

$$\begin{aligned}2\tilde{D}_{i,n_i}\tilde{\theta}_{i\varphi}^T\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) &\le 2\left|\tilde{D}_{i,n_i}\right|\left\|\tilde{\theta}_{i\varphi}\right\|\left\|\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\right\|\\ &\le \tau_{i,n_i}\mu_{i,n_i}^2\tilde{D}_{i,n_i}^2 + \frac{1}{\tau_{i,n_i}}\left\|\tilde{\theta}_{i\varphi}\right\|^2\end{aligned} \tag{59}$$

yields

$$\begin{aligned}\tilde{D}_{i,n_i}\dot{\tilde{D}}_{i,n_i} &\le \tilde{D}_{i,n_i}^2 + 0.5\dot{D}_{i,n_i}^2 - l_{i,n_i}\tilde{D}_{i,n_i}^2\\ &\quad + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\tilde{D}_{i,n_i}^2 + \frac{1}{2\tau_{i,n_i}}\left\|\tilde{\theta}_{i\varphi}\right\|^2 + 0.5\varepsilon_{i\varphi}^{*2}\\ &\le -\left(l_{i,n_i} - \left(1.0 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2\\ &\quad + \frac{1}{2\tau_{i,n_i}}\left\|\tilde{\theta}_{i\varphi}\right\|^2 + 0.5\beta_{i,n_i 1}^2 + 0.5\varepsilon_{i\varphi}^{*2}\end{aligned} \tag{60}$$

where $\|\phi_{i\varphi}(\bar{x}_{i,n_i}, v_i)\| \le \mu_{i,n_i}$ and $\tau_{i,n_i} > 0$ is a design parameter.

The parameter updated law $\hat{\theta}_{i\varphi}$ is designed as

$$\dot{\hat{\theta}}_{i\varphi} = \gamma_{i\varphi}\left(l_{i,n_i}^{-1}\varphi_{i\varphi}(\bar{x}_{i,n_i}, v_i)s_i - \delta_{i\varphi}\hat{\theta}_{i\varphi}\right) \tag{61}$$

where $\gamma_{i\varphi} > 0$ and $\delta_{i\varphi} > 0$ are the design parameters.

H. Main Results

The above adaptive fault-tolerant control design procedure and stability analysis can be summarized in the following theorem, which contains the results for the uncertain nonlinear large-scale systems (1) with the external disturbance, unknown dead zone, and actuator fault using the nonlinear disturbance observer and backstepping technique.

Theorem 1: Consider a class of uncertain large-scale systems (1) with the external disturbance, unknown dead zone, and actuator fault and suppose that full-state information is available. The nonlinear disturbance observers of each subsystems are designed as (20), (33), (50), (51), (54), and (55). The updated laws of the NN weight are chosen as (23), (36), (46), and (61). The nonlinear disturbance observer based fault-tolerant control scheme is proposed as (45). Given for all initial conditions, the appropriate design parameters $k_{i,j}$, q_i, $l_{i,j}$, $\delta_{i,j}$, $\delta_{i\varphi}$, and $\tau_{i,j}$ can be chosen according to (A.23) such that all closed-loop signals uniformly ultimately bounded under the proposed adaptive fault-tolerant control based on the nonlinear disturbance observer.

IV. Simulation Results

In this section, simulation results of the mass–spring–damper system are presented to illustrate the effectiveness of the proposed fault-tolerant control scheme using the disturbance observer. The considered mass–spring–damper system is illustrated in Fig. 1 and the motion equations of mechanical system can be expressed as [54]

$$\begin{cases} M_1\ddot{y}_1 = u_1 - f_{K1}(x) - f_{B1}(x) + f_{K2}(x) + f_{B2}(x)\\ \qquad\quad - f_{C1}(x) + f_{C2}(x) + \Delta_1\\ M_2\ddot{y}_2 = u_2 - f_{K2}(x) - f_{B1}(x) - f_{C2}(x) + \Delta_2\end{cases} \tag{62}$$

where $x = [y_1, \dot{y}_1, y_2, \dot{y}_2]^T$ are the state variables: $f_{K1}(x) = K_{10}y_1 + \Delta K_1 y_1{}^3$ and $f_{K2}(x) = K_{20}(y_2 - y_1) + \Delta K_2(y_2 - y_1)^3$ denote the spring forces; $f_{B1}(x) = 2\dot{y}_1 + 0.2\dot{y}_1^2$ and $f_{B2}(x) = 2.2(\dot{y}_2 - \dot{y}_1) + 0.15(\dot{y}_2 - \dot{y}_1)^2$ denote the friction forces. $f_{C1}(x) = 0.02\text{sgn}(\dot{y}_1)$ and $f_{C2}(x) = 0.02\text{sgn}(\dot{y}_2 - \dot{y}_1)$ are the coulomb friction forces. The parameters Δ_1 and Δ_2 are given as $\Delta_1 = 0.2\sin(3t)\exp(-0.2t)$ and $\Delta_2 = 0.2\cos(3t)\exp(-0.1t)$, respectively. The parameters are given as $M_1 = 0.25$ kg, $M_2 = 0.2$ kg, $K_{10} = 1(N/m)$, and $K_{20} = 2(N/m)$. The perturbations are given as $\Delta K_1 = 0.1$ and $\Delta K_2 = 0.12$ [54].

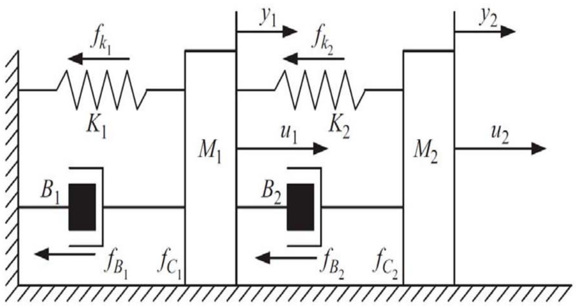

Fig. 1. Mass–spring–damper system.

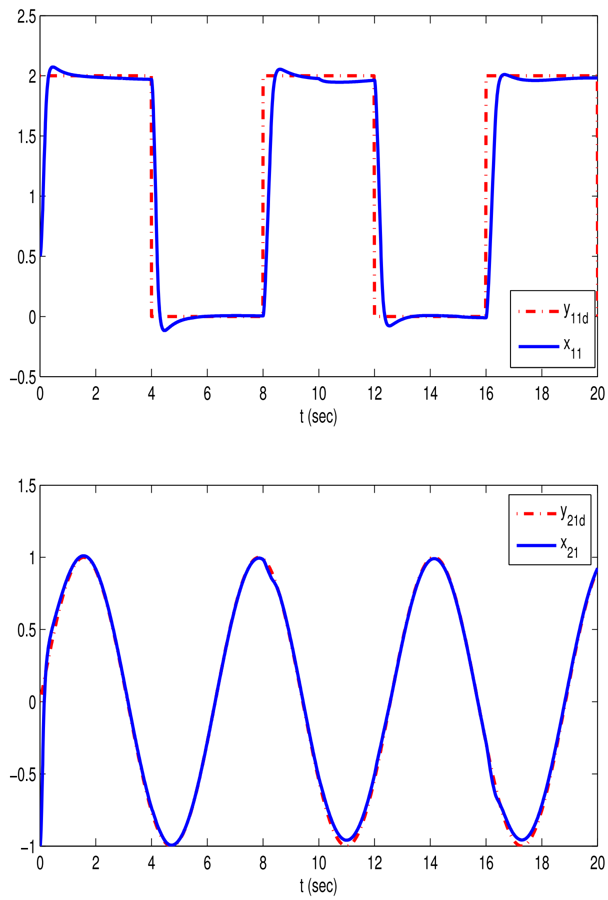

Fig. 2. Tracking results of x_1 and x_{1d} of case 1.

Let $x_{11} = y_1, x_{12} = \dot{y}_1, x_{21} = y_2$, and $x_{22} = \dot{y}_2$. Then, (62) can be transformed into the following form:

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \frac{1}{M_1}u_1 + \frac{1}{M_1}\big[-f_{K1}(x) - f_{B1}(x) + f_{K2}(x) \\ \qquad\qquad + f_{B2}(x) - f_{C1}(x) + f_{C2}(x) + \Delta_1\big] \\ y_1 = x_{11} \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = \frac{1}{M_2}u_2 + \frac{1}{M_2}[-f_{K2}(x) - f_{B1}(x) - f_{C2}(x) + \Delta_2] \\ y_2 = x_{21}. \end{cases} \tag{63}$$

The control objective is that the system outputs can track the reference signals

$$y_{1,d} = \begin{cases} 2 & 8k \le t \le 4(2k+1) \\ 0 & 4(2k+1) \le t \le 8(k+1) \end{cases}$$

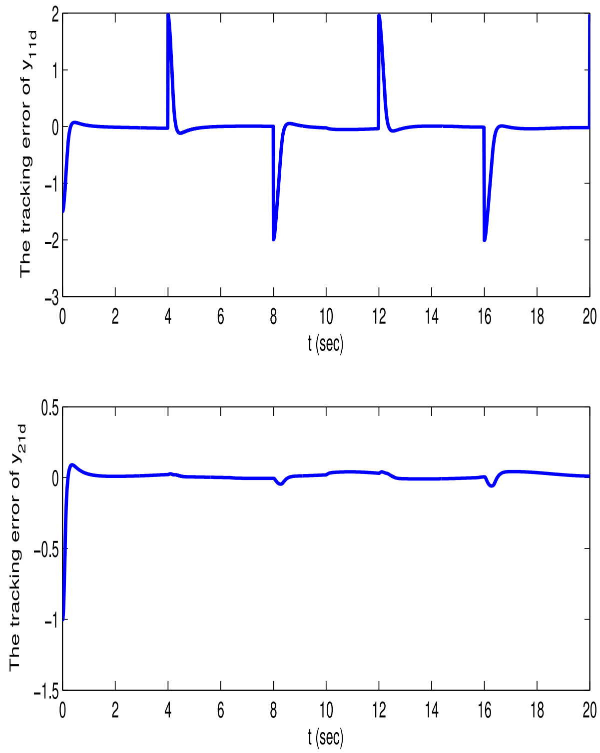

Fig. 3. Tracking errors of case 1.

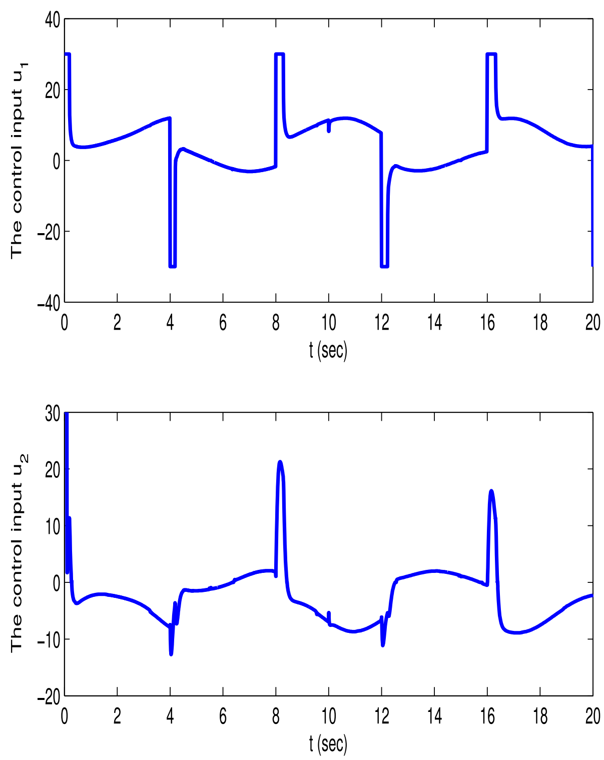

Fig. 4. Control inputs of case 1.

and $y_{2,d} = \sin(t)$ to a bounded compact set and all the signals in the closed-loop system are guaranteed to be bounded. The dead zone parameters are chosen as $b_{i,r} = 0.5$, $b_{i,l} = -0.5$, and $m_1 = 1$. At the same time, the control input boundaries are assumed as $u_{i\max} = 30$ and $u_{i\max} = -30$ to satisfy the practical system requirement.

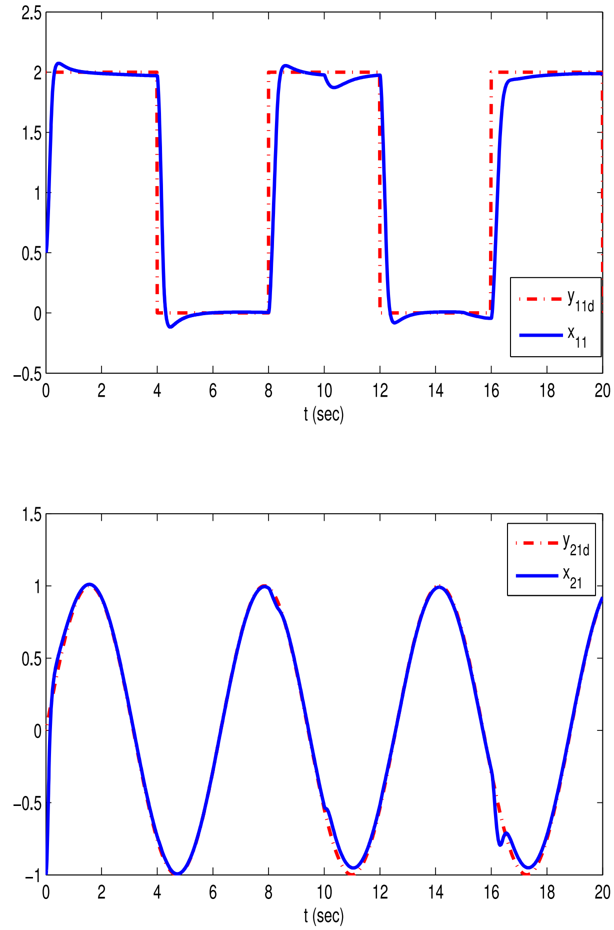

Fig. 5. Tracking results of x_1 and x_{1d} of case 2.

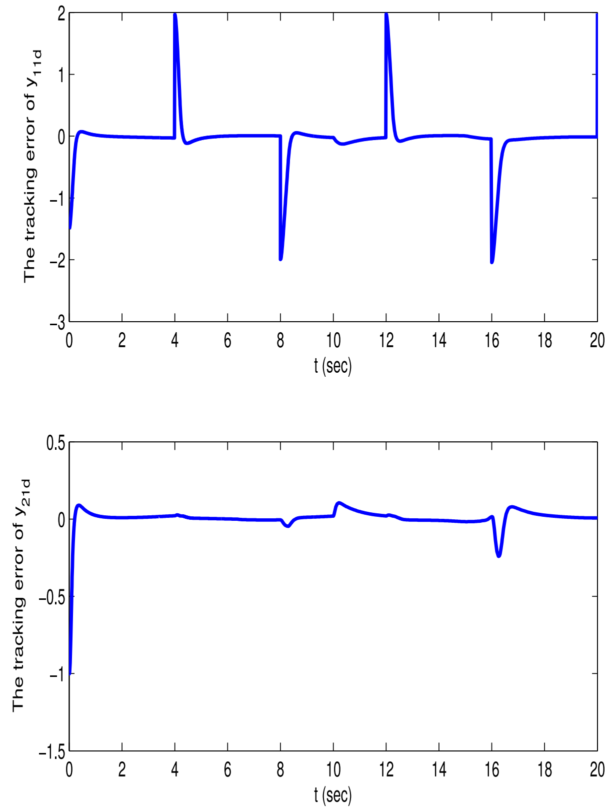

Fig. 6. Tracking errors of case 2.

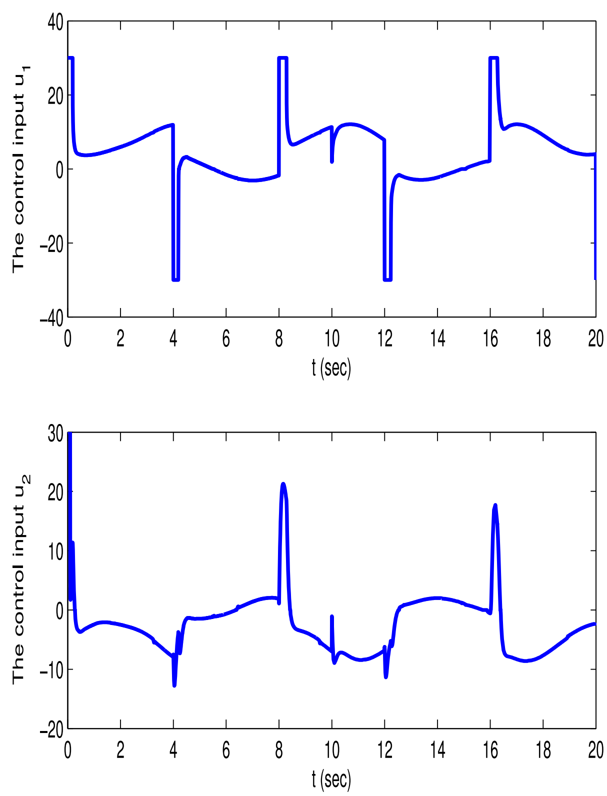

Fig. 7. Control inputs of case 2.

In the simulation study, the nonlinear disturbance observers of each subsystems are designed as (20), (33), (50), (51), (54), and (55). The updated laws of the NN weight are chosen as (23), (36), (46), and (61). The nonlinear disturbance observer based fault-tolerant control scheme is proposed as (45). To design the adaptive fault-tolerant control scheme, all design parameters are chosen as $k_1 = 10, k_2 = 40$, $\lambda_1 = \lambda_2 = 10$, $\delta_1 = \delta_2 = 0.2$, $l_2 = 2$, $q_2 = 3$, and $\delta_\varphi = 0.5$. The initial state conditions are chosen as $x_{11} = 0$, $x_{22} = 0$, and $\hat{\theta}_1 = \hat{\theta}_2 = 0$.

To illustrate the effectiveness of the proposed adaptive fault-tolerant control scheme, the tracking control simulation results of two cases are given. In two cases, the actuator bias fault is chosen as $\zeta_i = 0.2\sin(0.5t)$ and the actuator fault appear at $t_f = 10$ s. At the same time, the unknown nonlinear remaining control rate coefficient ρ_i are chosen as $\rho_i = 1/(1+e^{-0.2})$ and $\rho_i = 1/(1+e^{-2[\sin(x_{11})^2+\cos(x_{21})^2]})$, respectively.

Case 1: The nonlinear remaining control rate coefficient ρ_i is chosen as a constant $\rho_i = 1/(1+e^{-0.2})$.

In this case, under the proposed adaptive fault-tolerant controller based on the disturbance observer and the NN, the tracking control result of the uncertain nonlinear plant (63) is shown in Fig. 2 and the tracking error is given in Fig. 3. If all actuators do not have any faults for $t < 10$ s, we can see that the satisfactory control performance has been obtained. Meanwhile, there is not the overshooting for the tracking performance when actuators lose constant efficiency at 10 s. We can note that the satisfactory tracking control performance is always guaranteed under the time-varying external disturbance, the dead zone, and the actuator faults from Figs. 2 and 3. In Fig. 4, the control input is presented which is bounded.

Case 2: The nonlinear remaining control rate coefficient ρ_i is chosen as a time-varying parameter $\rho_i = 1/(1+e^{-2[\sin(x_{11})^2+\cos(x_{21})^2]})$.

In this case, under the proposed adaptive fault-tolerant controller based on the disturbance observer and the NN, the tracking control result of the uncertain nonlinear plant (63) is shown in Fig. 5 and the tracking error is given in Fig. 6. If all actuators do not have any faults for $t < 10$ s, we note that the satisfactory control performance has been obtained, while the small overshooting will appear when actuators have time-varying loss efficiency at 10 s. From Figs. 5 and 6, the satisfactory tracking control performance is achieved under the time-varying external disturbance, the dead zone, and the actuator fault. The bounded control input is presented in Fig. 7.

Based on the simulation results of the simulation example, we can conclude that the developed adaptive fault-tolerant control based on the disturbance observer and the RBFNN is effective for the uncertain nonlinear large-scale system with the time-varying external disturbance and unknown dead zone.

V. Conclusion

An adaptive fault-tolerant control scheme has been developed for uncertain nonlinear large-scale systems with unknown dead zones, unknown external disturbances, and actuator faults in this paper. The RBFNN has been employed to approximate the unknown nonlinear continuous interconnection function terms and the nonlinear disturbance observer has been used to estimate the compounded disturbance consisting of the unknown approximation error, the unknown dead zone, and the unknown disturbance. Using outputs of the RBFNN and the disturbance observer, the adaptive fault-tolerant controller has the desired capacity to control uncertain nonlinear large-scale systems with the unknown dead zone, unknown external disturbances, and actuator faults including the time-varying bias fault and the nonlinear gain fault. The uniformly ultimately bounded convergence of all closed-loop signals has been established. Finally, simulation results have illustrated the effectiveness of the proposed adaptive fault-tolerant control scheme. In the future, the application of the developed adaptive fault-tolerant control scheme should be further studied.

Appendix

A. Error System Analysis in Step 1

Proof: Choose the Lyapunov function candidate as

$$V_{i,1} = \frac{1}{2}e_{i,1}^2 + \frac{1}{2\gamma_{i,1}}\tilde{\theta}_{i,1}^T\tilde{\theta}_{i,1} + \frac{1}{2}\tilde{D}_{i,1}^2. \tag{A.1}$$

Invoking (22) and (26), the time derivative of $V_{i,1}$ is given by

$$\begin{aligned}\dot{V}_{i,1} &= e_{i,1}\dot{e}_{i,1} + \gamma_{i,1}^{-1}\tilde{\theta}_{i,1}^T\dot{\tilde{\theta}}_{i,1} + \tilde{D}_{i,1}\dot{\tilde{D}}_{i,1}\\ &= -k_{i,1}e_{i,1}^2 + e_{i,1}e_{i,2} - \lambda_{i,1}^{-1}e_{i,1}\tilde{\theta}_{i,1}^T\phi(Z_{i,1}) + e_{i,1}\tilde{D}_{i,1}\\ &\quad + \tilde{D}_{i,1}\dot{D}_{i,1} - \lambda_{i,1}\tilde{D}_{i,1}^2 + \tilde{D}_{i,1}\tilde{\theta}_{i,1}^T\phi(Z_{i,1})\\ &\quad + \gamma_{i,1}^{-1}\tilde{\theta}_{i,1}^T\dot{\hat{\theta}}_{i,1}.\end{aligned} \tag{A.2}$$

Substituting (23) into (A.2) yields

$$\begin{aligned}\dot{V}_{i,1} &= -k_{i,1}e_{i,1}^2 + e_{i,1}e_{i,2} - \delta_{i,1}\tilde{\theta}_{i,1}^T\hat{\theta}_{i,1} + e_{i,1}\tilde{D}_{i,1}\\ &\quad + \tilde{D}_{i,1}\dot{D}_{i,1} - \lambda_{i,1}\tilde{D}_{i,1}^2 + \tilde{D}_{i,1}\tilde{\theta}_{i,1}^T\phi(Z_{i,1}).\end{aligned} \tag{A.3}$$

Considering (19) and the following facts:

$$\begin{aligned}2\tilde{\theta}_{i,1}^T\hat{\theta}_{i,1} &= \left\|\tilde{\theta}_{i,1}\right\|^2 + \left\|\hat{\theta}_{i,1}\right\|^2 - \left\|\theta_{i,1}\right\|^2\\ &\ge \left\|\tilde{\theta}_{i,1}\right\|^2 - \left\|\theta_{i,1}\right\|^2\end{aligned} \tag{A.4}$$

$$2\tilde{D}_{i,1}\tilde{\theta}_{i,1}^T\phi(Z_{i,1}) \le \tau_{i,1}\mu_{i,1}^2\tilde{D}_{i,1}^2 + \frac{\left\|\tilde{\theta}_{i,1}\right\|^2}{\tau_{i,1}} \tag{A.5}$$

we have

$$\begin{aligned}\dot{V}_{i,1} &\le -(k_{i,1}-0.5)e_{i,1}^2 - \left(\frac{\delta_{i,1}}{2} - \frac{1}{2\tau_{i,1}}\right)\left\|\tilde{\theta}_{i,1}\right\|^2\\ &\quad - \left(\lambda_{i,1} - 1 - 0.5\tau_{i,1}\mu_{i,1}^2\right)\tilde{D}_{i,1}^2 + 0.5\beta_{i,11}^2\\ &\quad + \frac{\delta_{i,1}}{2}\left\|\theta_{i,1}\right\|^2 + e_{i,1}e_{i,2}\end{aligned} \tag{A.6}$$

where $\tau_{i,1} > 0$ is a design parameter and $\|\phi(Z_{i,1})\| \le \mu_{i,1}$. This concludes the proof. ■

B. Error System Analysis in Step j

Proof: Choose the Lyapunov function candidate as

$$V_{i,j} = \frac{1}{2}e_{i,j}^2 + \frac{1}{2\gamma_{i,j}}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} + \frac{1}{2}\tilde{D}_{i,j}^2. \tag{A.7}$$

Invoking (35) and (39), the time derivative of $V_{i,j}$ is given by

$$\begin{aligned}\dot{V}_{i,j} &= e_{i,j}\dot{e}_{i,j} + \gamma_{i,j}^{-1}\tilde{\theta}_{i,j}^T\dot{\tilde{\theta}}_{i,j} + \tilde{D}_{i,j}\dot{\tilde{D}}_{i,j}\\ &= -k_{i,j}e_{i,j}^2 + e_{i,j}e_{i,j+1} - \lambda_{i,j}^{-1}e_{i,j}\tilde{\theta}_{i,j}^T\phi(Z_{i,j})\\ &\quad + e_{i,j}\tilde{D}_{i,j} + \tilde{D}_{i,j}\dot{D}_{i,j} - \lambda_{i,j}\tilde{D}_{i,j}^2 + \tilde{D}_{i,j}\tilde{\theta}_{i,j}^T\phi(Z_{i,j})\\ &\quad + \gamma_{i,j}^{-1}\tilde{\theta}_{i,j}^T\dot{\hat{\theta}}_{i,j} - e_{i,j}e_{i,j-1}.\end{aligned} \tag{A.8}$$

Substituting (36) into (A.8) yields

$$\begin{aligned}\dot{V}_{i,j} &= -k_{i,j}e_{i,j}^2 + e_{i,j}e_{i,j+1} - \delta_{i,j}\tilde{\theta}_{i,j}^T\hat{\theta}_{i,j}\\ &\quad + e_{i,j}\tilde{D}_{i,j} + \tilde{D}_{i,j}\dot{D}_{i,j} - \lambda_{i,j}\tilde{D}_{i,j}^2\\ &\quad + \tilde{D}_{i,j}\tilde{\theta}_{i,j}^T\phi(Z_{i,j}) - e_{i,j}e_{i,j-1}.\end{aligned} \tag{A.9}$$

Considering (32) and the following facts:

$$\begin{aligned}2\tilde{\theta}_{i,j}^T\hat{\theta}_{i,j} &= \left\|\tilde{\theta}_{i,j}\right\|^2 + \left\|\hat{\theta}_{i,j}\right\|^2 - \left\|\theta_{i,j}\right\|^2\\ &\ge \left\|\tilde{\theta}_{i,j}\right\|^2 - \left\|\theta_{i,j}\right\|^2\end{aligned} \tag{A.10}$$

$$2\tilde{D}_{i,j}\tilde{\theta}_{i,j}^T\phi(Z_{i,j}) \le \tau_{i,j}\mu_{i,j}^2\tilde{D}_{i,j}^2 + \frac{\left\|\tilde{\theta}_{i,j}\right\|^2}{\tau_{i,j}} \tag{A.11}$$

we have

$$\begin{aligned}\dot{V}_{i,j} &\le -(k_{i,j}-0.5)e_{i,j}^2 - \left(\frac{\delta_{i,j}}{2} - \frac{1}{2\tau_{i,j}}\right)\left\|\tilde{\theta}_{i,j}\right\|^2\\ &\quad - \left(\lambda_{i,j} - 1 - 0.5\tau_{i,j}\mu_{i,j}^2\right)\tilde{D}_{i,j}^2 + 0.5\beta_{i,j1}^2\\ &\quad + e_{i,j}e_{i,j+1} - e_{i,j}e_{i,j-1} + \frac{\delta_{i,j}}{2}\left\|\theta_{i,j}\right\|^2\end{aligned} \tag{A.12}$$

where $\tau_{i,j} > 0$ is a design parameter and $\|\phi(Z_{i,j})\| \le \mu_{i,j}$. This concludes the proof. ■

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

C. Error System Analysis in Step n_i

Proof: Consider the Lyapunov function candidate as

$$V_{i,n_i} = \frac{1}{2g_{i,n_i}} e_{i,n_i}^2 + \frac{1}{2\gamma_{i,n_i}} \tilde{\theta}_{i,n_i}^T \tilde{\theta}_{i,n_i} + \frac{1}{2\gamma_{i\varphi}} \tilde{\theta}_{i\varphi}^T \tilde{\theta}_{i\varphi} + \frac{1}{2} s_i^2 + \frac{1}{2} \tilde{D}_{i,n_i}^2. \quad \text{(A.13)}$$

Invoking (48), (53), (60) and Remark 1, the time derivative of V_{i,n_i} is

$$\begin{aligned}\dot{V}_{i,n_i} &= \frac{1}{g_{i,n_i}} e_{i,n_i}\dot{e}_{i,n_i} - \frac{\dot{g}_{i,n_i}}{2g_{i,n_i}^2} e_{i,n_i}^2 + \frac{1}{\gamma_{i,n_i}} \tilde{\theta}_{i,n_i}^T \dot{\hat{\theta}}_{i,n_i} \\ &\quad + \frac{1}{\gamma_{i\varphi}} \tilde{\theta}_{i\varphi}^T \dot{\hat{\theta}}_{i\varphi} + s_i\dot{s}_i + \tilde{D}_{i,n_i}\dot{\hat{D}}_{i,n_i} \\ &\le e_{i,n_i}\left(-k_{i,n_i}e_{i,n_i} - \tilde{\theta}_{i,n_i}^T \phi_{i,n_i}(Z_{i,n_i})\right) \\ &\quad - e_{i,n_i}\left(\hat{D}_{i,n_i} - \varepsilon_{i,n_i}^* + e_{i,n_i-1}\right) + \frac{e_{i,n_i}D_{i,n_i}}{g_{i,n_i}} \\ &\quad + \frac{\vartheta_i}{2\underline{g}_{i0}^2} e_{i,n_i}^2 + \frac{1}{\gamma_{i,n_i}} \tilde{\theta}_{i,n_i}^T \dot{\hat{\theta}}_{i,n_i} + \frac{1}{\gamma_{i\varphi}} \tilde{\theta}_{i\varphi}^T \dot{\hat{\theta}}_{i\varphi} \\ &\quad - (q_i - 1.0)s_i^2 - l_{i,n_i}^{-1} s_i \tilde{\theta}_{i\varphi}^T \phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + 0.5 l_{i,n_i}^{-2} \varepsilon_{i\varphi}^{*2} \\ &\quad + 0.5\beta_{i,n_i0}^2 - \left(l_{i,n_i} - \left(1.0 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\ &\quad + \frac{1}{2\tau_{i,n_i}} \left\|\tilde{\theta}_{i\varphi}\right\|^2 + 0.5\beta_{i,n_i1}^2 + 0.5\varepsilon_{i\varphi}^{*2}. \end{aligned} \quad \text{(A.14)}$$

Considering $\tilde{D}_{i,n_i} = D_{i,n_i} - \hat{D}_{i,n_i}$ and Remark 1, we have

$$\begin{aligned}\dot{V}_{i,n_i} &\le e_{i,n_i}\left(-k_{i,n_i}e_{i,n_i} - \tilde{\theta}_{i,n_i}^T \phi_{i,n_i}(Z_{i,n_i}) + \varepsilon_{i,n_i}^*\right) \\ &\quad + \frac{\vartheta_i}{2\underline{g}_{i0}^2} e_{i,n_i}^2 + e_{i,n_i}\tilde{D}_{i,n_i} + e_{i,n_i}D_{i,n_i}\left(\frac{1}{g_{i,n_i}} - 1\right) \\ &\quad - e_{i,n_i}e_{i,n_i-1} + \frac{1}{\gamma_{i,n_i}} \tilde{\theta}_{i,n_i}^T \dot{\hat{\theta}}_{i,n_i} + \frac{1}{\gamma_{i\varphi}} \tilde{\theta}_{i\varphi}^T \dot{\hat{\theta}}_{i\varphi} \\ &\quad - (q_i - 1.0)s_i^2 - l_{i,n_i}^{-1} s_i \tilde{\theta}_{i\varphi}^T \phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) + 0.5 l_{i,n_i}^{-2} \varepsilon_{i\varphi}^{*2} \\ &\quad + 0.5\beta_{i,n_i0}^2 - \left(l_{i,n_i} - \left(1.0 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\ &\quad + \frac{1}{2\tau_{i,n_i}} \left\|\tilde{\theta}_{i\varphi}\right\|^2 + 0.5\beta_{i,n_i1}^2 + 0.5\varepsilon_{i\varphi}^{*2} \\ &\le -\left(k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2}\right) e_{i,n_i}^2 - e_{i,n_i}\tilde{\theta}_{i,n_i}^T \phi_{i,n_i}(Z_{i,n_i}) \\ &\quad - e_{i,n_i}e_{i,n_i-1} + \frac{1}{\gamma_{i,n_i}} \tilde{\theta}_{i,n_i}^T \dot{\hat{\theta}}_{i,n_i} + \frac{1}{\gamma_{i\varphi}} \tilde{\theta}_{i\varphi}^T \dot{\hat{\theta}}_{i\varphi} \\ &\quad - (q_i - 1.0)s_i^2 - l_{i,n_i}^{-1} s_i \tilde{\theta}_{i\varphi}^T \phi_{i\varphi}(\bar{x}_{i,n_i}, v_i) \\ &\quad - \left(l_{i,n_i} - \left(1.5 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\ &\quad + \frac{1}{2\tau_{i,n_i}} \left\|\tilde{\theta}_{i\varphi}\right\|^2 + \left(0.5 + 0.5\left|\frac{1}{\underline{g}_{i0}} - 1\right|^2\right)\beta_{i,n_i0}^2 \\ &\quad + \left(0.5 + 0.5 l_{i,n_i}^{-2}\right)\varepsilon_{i\varphi}^{*2} + 0.5\beta_{i,n_i1}^2 + 0.5\varepsilon_{i,n_i}^{*2}. \end{aligned} \quad \text{(A.15)}$$

Substituting (46) and (61) into (A.15), we obtain

$$\begin{aligned}\dot{V}_{i,n_i} &\le -\left(k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2}\right) e_{i,n_i}^2 \\ &\quad - e_{i,n_i}e_{i,n_i-1} - \delta_{i,n_i}\tilde{\theta}_{i,n_i}^T \hat{\theta}_{i,n_i} - \delta_{i\varphi}\tilde{\theta}_{i\varphi}^T \hat{\theta}_{i\varphi} \\ &\quad - (q_i - 1.0)s_i^2 - \left(l_{i,n_i} - \left(1.5 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\ &\quad + \frac{1}{2\tau_{i,n_i}} \left\|\tilde{\theta}_{i\varphi}\right\|^2 + \left(0.5 + 0.5\left|\frac{1}{\underline{g}_{i0}} - 1\right|^2\right)\beta_{i,n_i0}^2 \\ &\quad + \left(0.5 + 0.5 l_{i,n_i}^{-2}\right)\varepsilon_{i\varphi}^{*2} + 0.5\beta_{i,n_i1}^2 + 0.5\varepsilon_{i,n_i}^{*2}. \end{aligned} \quad \text{(A.16)}$$

Considering the following facts:

$$\begin{aligned} 2\tilde{\theta}_{i,n_i}^T \hat{\theta}_{i,n_i} &= \left\|\tilde{\theta}_{i,n_i}\right\|^2 + \left\|\hat{\theta}_{i,n_i}\right\|^2 - \left\|\theta_{i,n_i}\right\|^2 \\ &\ge \left\|\tilde{\theta}_{i,n_i}\right\|^2 - \left\|\theta_{i,n_i}\right\|^2 \end{aligned} \quad \text{(A.17)}$$

and

$$\begin{aligned} 2\tilde{\theta}_{i\varphi}^T \hat{\theta}_{i\varphi} &= \left\|\tilde{\theta}_{i\varphi}\right\|^2 + \left\|\hat{\theta}_{i\varphi}\right\|^2 - \left\|\theta_{i\varphi}\right\|^2 \\ &\ge \left\|\tilde{\theta}_{i\varphi}\right\|^2 - \left\|\theta_{i\varphi}\right\|^2 \end{aligned} \quad \text{(A.18)}$$

we have

$$\begin{aligned}\dot{V}_{i,n_i} &\le -\left(k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2}\right) e_{i,n_i}^2 - e_{i,n_i}e_{i,n_i-1} \\ &\quad - \frac{\delta_{i,n_i}}{2}\left\|\tilde{\theta}_{i,n_i-1}\right\|^2 - \left(\frac{\delta_{i\varphi}}{2} - \frac{1}{2\tau_{i,n_i}}\right)\left\|\tilde{\theta}_{i\varphi}\right\|^2 \\ &\quad - (q_i - 1.0)s_i^2 - \left(l_{i,n_i} - \left(1.5 + 0.5\tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\ &\quad + \left(0.5 + 0.5\left|\frac{1}{\underline{g}_{i0}} - 1\right|^2\right)\beta_{i,n_i0}^2 \\ &\quad + 0.5\beta_{i,n_i1}^2 + \left(0.5 + 0.5 l_{i,n_i}^{-2}\right)\varepsilon_{i\varphi}^{*2} \\ &\quad + \frac{\delta_{i,n_i}}{2}\left\|\theta_{i,n_i}\right\|^2 + \frac{\delta_{i\varphi}}{2}\left\|\theta_{i\varphi}\right\|^2 + 0.5\varepsilon_{i,n_i}^{*2}. \end{aligned} \quad \text{(A.19)}$$

Thus, this concludes the proof. ■

D. Proof of Theorem 1

Proof: For the large-scale systems (1) consisting of N subsystems, the Lyapunov function candidate is chosen as

$$V = \sum_{i=1}^{N} \sum_{j=1}^{n_i} V_{i,j}. \quad \text{(A.20)}$$

Invoking (A.6), (A.12), and (A.21), we have

$$\begin{aligned}\dot{V} &\le -\sum_{i=1}^{N}\sum_{j=1}^{n_i-1}(k_{i,j} - 0.5)e_{i,j}^2 \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(\frac{\delta_{i,j}}{2} - \frac{1}{2\tau_{i,j}}\right)\left\|\tilde{\theta}_{i,j}\right\|^2 \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{n_i-1}\left(\lambda_{i,j} - 1 - 0.5\tau_{i,j}\mu_{i,j}^2\right)\tilde{D}_{i,j}^2 \end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=1}^{N}\left(k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2}\right)e_{i,n_i}^2 \\
&- \sum_{i=1}^{N}\frac{\delta_{i,n_i}}{2}\left\|\tilde{\theta}_{i,n_i-1}\right\|^2 - \sum_{i=1}^{N}\left(\frac{\delta_{i\varphi}}{2} - \frac{1}{2\tau_{i,n_i}}\right)\left\|\tilde{\theta}_{i\varphi}\right\|^2 \\
&- \sum_{i=1}^{N}(q_i - 1.0)s_i^2 - \sum_{i=1}^{N}\left(l_{i,n_i} - \left(1.5 + \tau_{i,n_i}\mu_{i,n_i}^2\right)\right)\tilde{D}_{i,n_i}^2 \\
&+ \sum_{i=1}^{N}\left[\left(0.5 + 0.5\left|\frac{1}{\underline{g}_{i0}} - 1\right|^2\right)\beta_{i,n_i0}^2 \right.\\
&\qquad \left. + \left(0.5 + 0.5 l_{i,n_i}^{-2}\right)\varepsilon_{i\varphi}^{*2} + 0.5\varepsilon_{i,n_i}^{*2}\right] \\
&+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(0.5\beta_{i,j1}^2 + \frac{\delta_{i,j}}{2}\left\|\theta_{i,j}\right\|^2\right) + \sum_{i=1}^{N}\frac{\delta_{i\varphi}}{2}\left\|\theta_{i\varphi}\right\|^2 \\
&\le -\kappa V + C
\end{aligned}
\tag{A.21}
$$

where κ and C are given by

$$
\kappa := \min\begin{pmatrix} k_{i,j} - 0.5,\ \lambda_{i,j} - 1 - 0.5\tau_{i,j}\mu_{i,j}^2, \\ k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2} \\ q_i - 1.0,\ l_{i,n_i} - \left(1.5 + \tau_{i,n_i}\mu_{i,n_i}^2\right) \\ \frac{\delta_{i,j}}{\gamma_{i,j}} - \frac{1}{\gamma_{i,j}\tau_{i,j}},\ \frac{\delta_{i,n_i}}{\gamma_{i,n_i}},\ \frac{\delta_{i\varphi}}{\gamma_{i\varphi}} - \frac{1}{\gamma_{i\varphi}\tau_{i,n_i}} \end{pmatrix}
$$

$$
\begin{aligned}
C :&= \sum_{i=1}^{N}\left[\left(0.5 + 0.5\left|\frac{1}{\underline{g}_{i0}} - 1\right|^2\right)\beta_{i,n_i0}^2\right] \\
&= \sum_{i=1}^{N}\left[\left(0.5 + 0.5 l_{i,n_i}^{-2}\right)\varepsilon_{i\varphi}^{*2} + 0.5\varepsilon_{i,n_i}^{*2}\right] + \sum_{i=1}^{N}\frac{\delta_{i\varphi}}{2}\left\|\theta_{i\varphi}\right\|^2 \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(0.5\beta_{i,j1}^2 + \frac{\delta_{i,j}}{2}\left\|\theta_{i,j}\right\|^2\right).
\end{aligned}
\tag{A.22}
$$

To ensure the closed-loop system stability, the corresponding design parameters $k_{i,j}$, $\lambda_{i,j}$, $\tau_{i,j}$, k_{i,n_i}, q_i, l_{i,n_i}, $\delta_{i,j}$, and $\delta_{i\varphi}$ should be chosen to make the following inequalities hold:

$$
\begin{aligned}
k_{i,j} - 0.5 &> 0 \\
\lambda_{i,j} - 1 - 0.5\tau_{i,j}\mu_{i,j}^2 &> 0 \\
k_{i,n_i} - 1.5 - \frac{\vartheta_i}{2\underline{g}_{i0}^2} &> 0 \\
q_i - 1.0 &> 0 \\
l_{i,n_i} - (1.5 + \tau_{i,n_i} &> 0 \\
\frac{\delta_{i,j}}{\gamma_{i,j}} - \frac{1}{\gamma_{i,j}\tau_{i,j}} &> 0 \\
\frac{\delta_{i\varphi}}{\gamma_{i\varphi}} - \frac{1}{\gamma_{i\varphi}\tau_{i,n_i}} &> 0.
\end{aligned}
\tag{A.23}
$$

According to (A.21), we have

$$
0 \le V \le \frac{C}{\kappa} + \left[V(0) - \tfrac{C}{\varrho}\right]e^{-\kappa t}. \tag{A.24}
$$

From (A.21), we can know that V is exponentially convergent, i.e., $\lim_{t\longrightarrow\infty} V = (C/\kappa)$. According to (A.23), it may directly show that the signals $e_{i,j}$ and $\tilde{D}_{i,j}$ are semiglobally uniformly bounded when $t \to 0$. Hence, the tracking errors $e_{i,1}$ and the approximation error $\tilde{D}_{i,j}$ of the closed-loop system are bounded. This concludes the proof. ■

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