System Identification Using Chaos With Application to Equalization of a Chaotic Modulation System

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***Abstract*— In this brief, we considered the problem of blind identification of an autoregressive (AR) system driven by a chaotic signal. Because of the inherently deterministic nature of a chaotic signal, a new dynamic-based estimation approach called minimum phase space volume (MPSV) technique was applied to identify an AR system. It was shown that not only could this chaotic approach provide an accurate identification, but it was also more effective than the conventional statistic method in the sense that the chaotic approach had a smaller mean squares error (MSE), and it was so robust that it did not require an order determination procedure. In a chaotic modulation communication system, since the signal of transmission is modulated by a chaotic dynamical system, equalization of the transmitted signal through a communication channel is therefore a problem of system identification with a chaotic probing signal. It was observed that the equalization performance of the chaotic approach was superior to the conventional statistical method. This is another benefit for using chaos in a spread spectrum communication system.**

***Index Terms*—Chaos, communication, equalization, estimation, spread spectrum, system identification.**

I. Introduction

Chaos has recently drawn a great deal of attentions in the signal processing and communication communities [1]–[3]. Not only a wide range of signal processes such as radar [4], speech [5], and indoor propagation [6] have been demonstrated to be chaotic rather than purely random, but a chaotic system can also be applied to various problems such as secure communications [2], [3], adaptive array [7], and control [8]. Here we consider the problem of system identification using chaos.

System identification is an important issue in many areas including process control [9] and communications [10], [11]. In some cases, we want to identify the system without any access of the input probing sequence, e.g., the dereverberation problem of a hands-free telephone. In other scenarios such as process control and communications, we may have some control on the input sequences. For example, in equalizing a spread spectrum (SS) or code-division multiple access (CDMA) system, the spreading sequence (i.e., the input sequence) can be selected to achieve the best equalization performance though we may not have the access of the exact input sequence for equalization. Nevertheless, system identification with a chaotic input sequence is found intriguing in both cases. In the former one, although the input sequences are uncontrollable, the fact that many real-life signal processes are chaotic makes the problem of system identification with chaotic input sequence a practical necessity. For the latter case where a chaotic signal is employed in a system, the problem of system identification with a chaotic driven signal is encountered in many situations such as channel identification [11], narrowband interference cancellation of a chaotic spread spectrum system [12], and equalization of a chaotic modulation system.

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Conventional identification techniques such as maximum likelihood (ML), minimum variance (MV), and higher order statistics (HOS) are usually studied in the context of statistics. When the signal is chaotic, probability density is no longer the most fundamental representation from an informational viewpoint [13], [14]. Although these statistical methods can be used in a chaotic identification problem, they do not explicitly take into account the inherent deterministic properties peculiar to a chaotic input sequence. Therefore, their identification results may not be "optimum." A dynamic-based, rather than the classical statistic-based, processing technique is needed to fully exploit the information carried in a chaotic signal.

In this paper, a dynamic-based estimation technique called the minimum phase space volume (MPSV) method [15] is applied to identify an unknown system. The MPSV method is based on the idea that a chaotic signal occupies a finite "volume" in a finitely dimensional phase space. On the other hand, a random signal will not display any regular pattern in such a phase space, and hence have a relatively large "volume." This PSV complexity measure is used here as the objective function for system identification with a chaotic driven sequence.

In Section II, the phase space volume (PSV) complexity measure is applied to parameter estimation. Section III presents the work on system identification using chaos. The system considered here is the widely used autoregressive (AR) model because any linear system can be represented as an AR model [16]. It is shown that not only can the chaotic approach generate an accurate estimate of the system parameters, but it also does not require an order determination procedure for system identification. We compare the identification results using the MPSV estimator with a chaotic input sequence and the standard least squares (LS) method with a white Gaussian probing signal. Monte Carlo simulations conclude that the chaotic identification method is superior to the statistical approach. In Section IV, we apply this chaotic identification method to equalize a chaotic modulation communication system. Assuming the channel can be described by an AR model [17], the MPSV method is found effective in equalizing a chaotic modulated signal which is distorted by an AR channel.

II. System Identification with a Chaotic Input Sequence

Given an autoregressive (AR) model

$$x_t = \sum_{i=1}^{p} a_i x_{t-i} + n_t \tag{1}$$

where p is the order of the AR model, a_i's are the parameters, n_t and x_t are the input and output of the system, respectively, identification of the above system usually composes of two procedures: 1) order determination, i.e., estimation of p, and 2) parameter estimation, i.e., estimation of a_i's. The conventional approach uses a white Gaussian signal n_t and determine p and a_i by an information criterion and the least squares (LS) method [18], respectively. Here we consider an alternative that n_t is a chaotic sequence generated by a nonlinear map:

$$n_t = f(n_{t-1}, n_{t-2}, \cdots, n_{t-d}). \tag{2}$$

Since n_t is inherently deterministic, a dynamic-based complexity measure is used here to estimate a_i and p.

Definition: The phase space volume (PSV) of a signal x_t in the embedded Euclidean space $\mathbb{R}^d$ at resolution ε is defined as

$$V_\varepsilon(x_t) = V_\varepsilon(A) = \inf\left\{\sum_{i=1}^{\infty} |U_i|^d\right\} \tag{3}$$

where $A = \{\mathbf{x}_t = (x_t, x_{t+1}, \cdots, x_{t+d-1}) | t = 1, 2, \cdots\}$ and $\{U_i\}$ is an ε-cover of A, i.e., $A \subset \cup_{i=1}^{\infty} U_i$ with $0 < |U_i| \le \varepsilon$ and the diameter $|U_i| = \sup\{|\mathbf{a} - \mathbf{b}|: \mathbf{a}, \mathbf{b} \in U_i\}$.

Note that the concept of phase space volume is not entirely new, when $\varepsilon \to 0$, $V_\varepsilon(A) \to V(A)$, which is equivalent to the Hausdorff measure or the d-dimensional Lebesque measure if A is a Borel subset of $\mathbb{R}^d$. In fact, it is not hard to see that V_ε is also a measure for nonzero ε. In practice, V_ε must be used in the computation. The accuracy, therefore, depends on ε, and the tradeoff for using a small ε is a high computational cost. Here we use an approximation of V which does not require choosing ε explicitly [15]

$$V' = \sum_{i=1}^{n} \min_{j \ne i} |n_{i1} - n_{j1}| \times \cdots \times |n_{id} - n_{jd}|. \tag{4}$$

Assuming that n_t is unknown, the inverse filtering approach is employed here to estimate a_i from x_t. That is,

$$u_t = \sum_{i=0}^{q} b_i x_{t-i}. \tag{5}$$

The coefficients a_i's are then estimated by minimizing the PSV of u_t with respect to $\mathbf{b} = (b_0, b_1, \cdots, b_q)$. It is shown that $\hat{\mathbf{b}} = \mathbf{a} = (a_1, a_2, \cdots, a_p)$ if and only if the PSV of the output of the inverse filter u_t goes to the minimum [15]. Note that minimizing V' in (4) is a nonlinear optimization problem, in general. Here we used the random search technique to look for the global optimization solution. In particular, we used 3000 different points to initiate the optimization and took the best one as the solution. We summarize the identification procedure as follows:

THE MPSV System Identification Algorithm:

1) Given the received signal $\{x_t, t = 1, 2, \cdots, N\}$ where x_t is given by (1), construct an inverse system u_t as given in (5).
2) Embed u_t into a d-dimensional phase space using the delay coordinate, i.e., $\boldsymbol{u}_t = (u_t, u_{t+\tau'}, \cdots, u_{t+(d-1)\tau})$ where d is an embedding dimension of n_t. (Here $\tau = 1$.)
3) Minimize the PSV of the filter output vector $\boldsymbol{u}_t$ with respect to $\boldsymbol{b} = (b_1, \cdots, b_q)$, i.e.,

$$\min_b V'(u_t) = \min_b \sum_{i=1}^{N-d+1} \min_{j \ne i} |u_i - u_j| \times \cdots \times |u_{i+d-1} - u_{j+d-1}|. \tag{6}$$

III. Analysis of the Chaotic Identification Technique

To understand the effectiveness of the chaotic identification approach, we first considered a widely used second order AR model for illustration [18]:

$$x_t = 0.195 x_{t-1} - 0.95 x_{t-2} + n_t \tag{7}$$

where the chaotic driven signal n_t is chosen as the logistic map:

$$n_t = \lambda n_{t-1}(1 - n_{t-1}). \tag{8}$$

When $\lambda = 4$, n_t is white [19]. Therefore, it is informative enough to drive an AR model for identification [9]. The coefficients were chosen to have a stable AR model. We used the mean squares error (MSE) defined as

$$\text{MSE} = \frac{1}{2T} \sum_{j=1}^{T} \left\{[\hat{b}_1(j) - a_1]^2 + [\hat{b}_2(j) - a_2]^2\right\} \tag{9}$$

TABLE I
Identification of an AR System with the Logistic Driven Signal Using the MPSV Estimation Technique

order (q)	b_1	b_2	b_3	b_4	MSE	PSV
1	0.1957				9.02e-01	14.48
2	0.1956	-0.9518			1.86e-06	2.68e-02
3	0.1955	-0.9493	1.21e-03		7.41e-07	2.21e-02
4	0.1970	-0.9501	1.78e-04	1.25e-04	1.04e-06	2.85e-02

TABLE II
Identification of an AR System with a White Gaussian Signal Using the LS Estimation Technique

order (q)	b_1	b_2	b_3	b_4	MSE
1	0.0899				9.13e-01
2	0.2124	-0.9658			2.77e-04
3	0.0813	-0.9587	-1.21e-01		9.25e-03
4	0.2345	-0.9604	1.67e-02	2.40e-02	6.33e-04

where T is the number of trials, as a measure of the estimation accuracy. We report the results of the AR coefficient estimates using different inverse filter order q in Table I. 256 points were used in the experiment, and the embedding dimension d was chosen as 2. We set $T = 1$ to understand the efficiency of this new system identification method in a practical situation such as communication channel identification where identification is based on a single trial.

Apparently, the chaotic approach provided an accurate estimate for the AR coefficients. To compare the performance, we used a white Gaussian signal to drive the same system and used the LS technique to estimate a_is from x_t. Since the Gaussian plus LS method is an optimum approach in the statistical sense, it is a good comparison to evaluate the efficiency of the chaotic identification method. Table II reports the identification results based on the Gaussian plus LS method. Again, 256 points were used in this case. For all values of q, except for $q = 1$, the chaotic technique produced much more accurate estimates than the LS plus Gaussian approach. One reason is that the chaotic signal is inherently deterministic, an effective estimator such as MPSV can therefore exploit more useful information for identification. On the other hand, the Gaussian plus LS method is only statistically unbiased and hence requires an average of many trials to obtain a good estimate. To see that, we averaged the estimated coefficients from 1 to 200 trials in Fig. 1. Although the mean values of the estimated coefficients based on these two methods got closer when T became larger, the statistical method required much more trials to get an averaged estimate close to the true value. In fact, the chaotic method generated good coefficient estimates even for $T = 1$. In addition to the fast convergent rate, the MSE of the chaotic method was also much smaller than that of the statistical approach. In Tables I and II, except for $q = 1$, where both methods had the same level of MSE, the MSE of the chaotic approach was much smaller than that of the statistical method for all other values of q. Fig. 1 also illustrated that the estimated coefficients of the chaotic approach were much closer to the true coefficients than those of the statistical method.

Another interesting observation was that the chaotic identification was so robust that all extra b_is, that is, b_is for $i = 3, \cdots, q$ were very close to zero. In fact, this observation could be justified by the following lemma.

Lemma 1: Given that $u_t = \sum_{i=0}^{q} b_i x_{t-i}$ and $n_t = x_t - \sum_{i=1}^{p} a_i x_{t-i}$ where $p \le q$ and n_t is generated from the logistic map, the necessary and sufficient condition for the difference between u_t

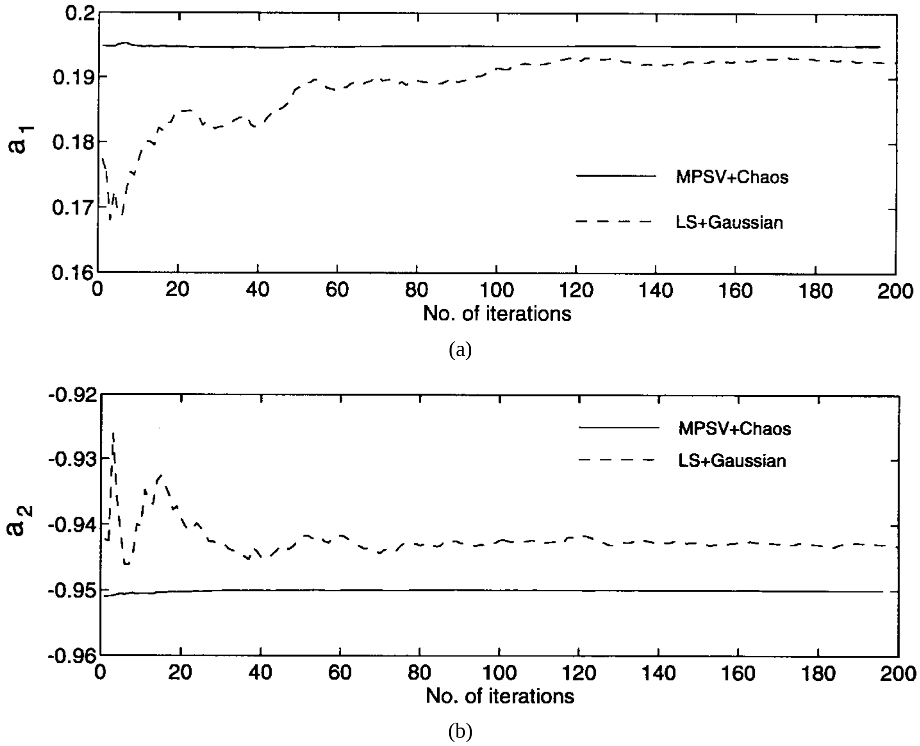

Fig. 1. Comparison of the convergence of the system coefficients using the MPSV plus chaos and the LS plus Gaussian approaches.

and n_t, i.e.,

$$\begin{aligned} u_t - n_t &= \sum_{i=0}^{q} b_i x_{t-i} - \left(x_t - \sum_{i=1}^{p} a_i x_{t-i}\right) \\ &= (b_0 - 1)x_t + \sum_{i=1}^{p}(b_i + a_i)x_{t-i} + \sum_{i=p+1}^{q} b_i x_{t-i} \quad (10) \end{aligned}$$

equal to zero is $b_i = 1$ for $i = 0$, $b_i = -a_i$ for $i = 1, \cdots, p$, and $b_i = 0$ for $i = p+1, \cdots, q$.

Proof: The sufficient condition is straightforward. Substituting b_is into the right side of (10), we have $u_t - n_t = 0$. To show the necessary condition, consider (10) when $u_t - n_t = 0$. Equation (10) becomes

$$(b_0 - 1)x_t + \sum_{i=1}^{p}(b_i + a_i)x_{t-i} + \sum_{i=p+1}^{q} b_i x_{t-i} = 0. \quad (11)$$

Substituting x_t of (1) into (8), we have

$$\begin{aligned} x_t =& \lambda\left(x_{t-1} - \sum_{i=1}^{p} a_t x_{t-i-1}\right)\left(1 - x_{t-1} + \sum_{i=1}^{p} a_i x_{t-i-1}\right) \\ &+ \sum_{i=1}^{p} a_i x_{t-i}. \quad (12) \end{aligned}$$

Equation (12) indicates that x_t is a nonlinear function of its history $x_{t-1}, x_{t-2}, \cdots, x_{t-p}$. In other words, $\{x_t\}$ is a linearly independent process. Therefore, all coefficients of (11) must equal zero, and the result follows. ■

Lemma 1 was important since it indicated that all extra coefficients in the inverse filter went to zero in the chaotic identification procedure. In other words, the chaotic identification did not require a separate order determination process as the conventional statistical identification methods. This property is not only valid for the logistic map, but it is also true for general chaotic signals.

Lemma 2: If n_t is chaotic, the identification of an AR model does not require an order determination procedure. That is, $b_i = 1$ for $i = 0$, $b_i = -a_i$ for $i = 1, \cdots, p$ and $b_i = 0$ for $i = p+1, \cdots, q$.

Proof: The proof is the same as that of Lemma 1 except that $n_t = f(n_{t-1}, n_{t-2}, \cdots, n_{t-d})$ instead of the logistic map. Substituting x_t of (1) into f, we have

$$\begin{aligned} x_t =& f\left(x_{t-1} - \sum_{i=1}^{p} a_i x_{t-i-1}, \cdots, x_{t-d} - \sum_{i=1}^{p} a_i x_{t-i-d}\right) \\ &+ \sum_{i=1}^{p} a_i x_{t-1}. \quad (13) \end{aligned}$$

Since f must be nonlinear to generate chaos, x_t is a nonlinear function of $x_{t-1}, \cdots, x_{t-p-d}$. Therefore, all coefficients of (11) must be equal to zero, and the results follow. ■

Monte Carlo simulation was performed to understand the effectiveness of the chaotic identification method. We considered the MSE of both identification approaches using different number of points for estimation. The results are plotted in Fig. 2. In this experiment, $d = 2$ and $T = 30$. As expected, the MSE decreased with increasing number of points, N. When 20 points were used in the identification, the MSE of the chaotic method was about -20 dB, whereas for 200 points, the MSE was about -42 dB. It should be noted that the MSE of the chaotic approach reached the level of -40 dB when $N = 60$ and could not be improved further. It was because the computation used single precision only, and hence the accuracy of the estimates was limited by the precision. The performance of the stochastic method also improved as N increased. For $N = 20$, the MSE of the statistical approach was about -13 dB, and for $N = 200$, it decreased to -26 dB. Comparing the performance of the two approaches, chaotic identification consistently outperformed the LS plus Gaussian method in all cases. The difference ranged from 7–20 dB. For the chaotic method to achieve the same accuracy of the stochastic method, say -26 dB for $N = 200$, the chaotic method required only 30 data

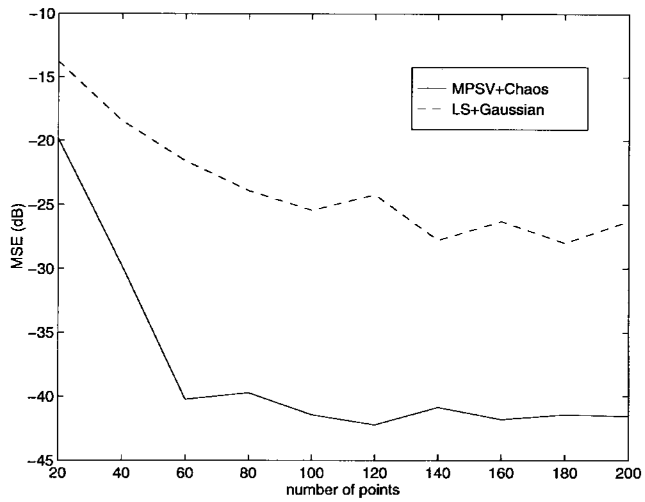

Fig. 2. MSE of the estimated coefficients of the MPSV plus chaos and the LS plus Gaussian method using different numbers of points.

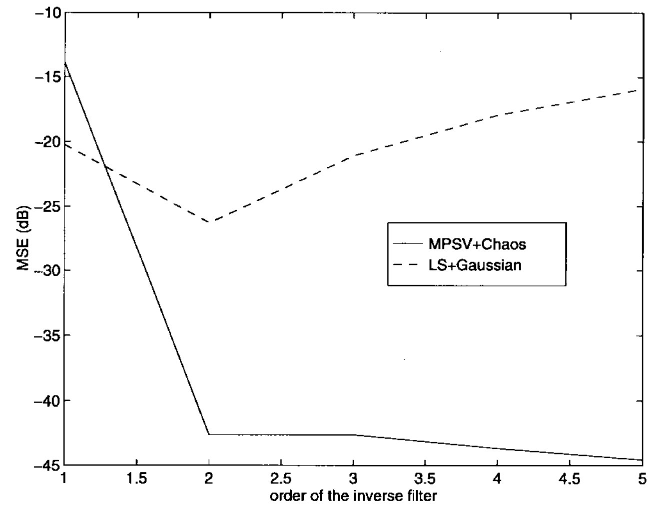

Fig. 4. MSE of the two identification methods using different orders for the inverse filter.

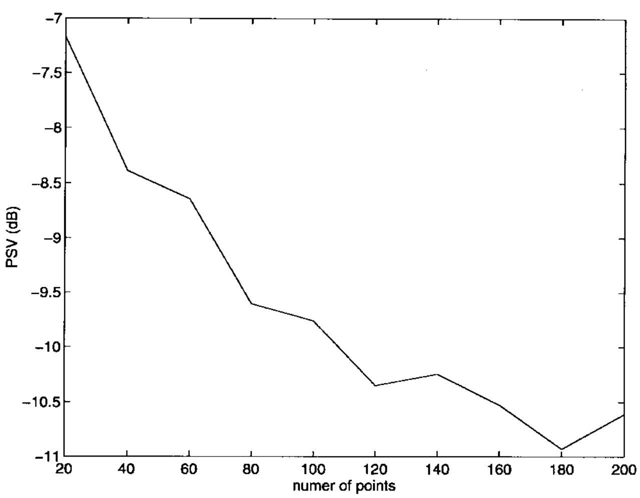

Fig. 3. Phase space volume of the MPSV plus chaos method using different numbers of points.

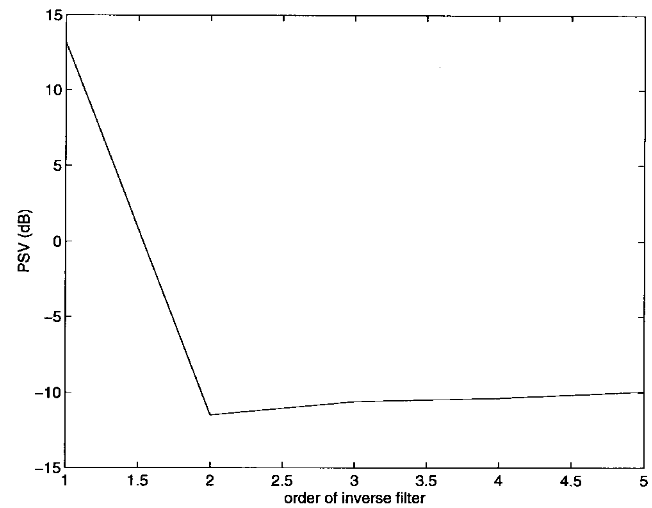

Fig. 5. Phase space volume of the MPSV plus chaos method using different orders for the inverse filter.

points. The effectiveness of the chaotic identification in dealing with short data sequences made it particularly attractive in a nonstationary environment such as wireless communications. In Fig. 3, we plot the corresponding PSV of the chaotic identification approach. Apparently, the PSV had a similar behavior as that of the MSE. That is, the PSV decreased as the number of points increased.

Fig. 4 presents the results of using different AR model orders for inverse filtering. The performance of the chaotic approach followed closely with Lemma 1. More precisely, the MSE remained about the same level for order $q \geq 2$. The slight difference was simply a result of numerical errors. On the other hand, the LS plus white Gaussian approach was more sensitive to the order of the inverse filter. The difference between the MSE for $q = 2$ and 4 was about 10 dB. It indicated that the stochastic approach could not perform satisfactorily when a wrong inverse filter order was used. Therefore, an order determination process such as an information criterion must be used with a statistical estimation method. However, when the chaotic identification was used, we might simply choose a sufficiently large inverse filter order, and it would not affect the accuracy of the identification. In Fig. 5, we plot the PSV versus q. We found that the PSV also remained the same for any q greater than or equal to two.. In other words, the PSV could also be used to determine the correct order of the original system when this was required in some applications.

IV. Application to Equalization of a Chaotic Modulation System

Spread spectrum (SS) and code division multiple access (CDMA) is a means of multiple access transmission which will be used in the next generation wireless communication system [20]. Recently there is considerable interest in the use of chaos in a SS/CDMA system. One approach uses a chaotic system to generate spreading codes to replace the conventional pseudonoise (PN) sequence [2], [7], [21]. In this case, chaos is mainly used for spreading code design. The basic communication structure remains the same as that of a conventional SS/CDMA system. Another approach employs a chaotic dynamical system to modulate the signal of transmission to achieve the purpose of wide-band transmission [22]–[24]. The signal of transmission is stored in the bifurcating parameter of a chaotic dynamical system. By keeping this bifurcating parameter in the chaotic regime, the output of the dynamical system which is the transmitted signal, is therefore chaotic and hence occupies a wide bandwidth required for SS/CDMA communication.

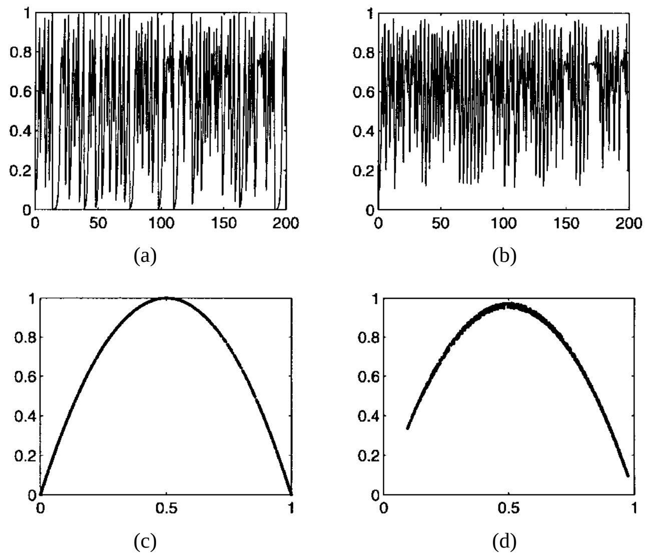

Fig. 6. (a) Trajectory of the logistic map in time domain. (b) Trajectory of the logistic modulated signal in time domain. (c) Two-dimensional phase space portrait of the trajectory in (a). (d) Two-dimensional phase space portrait of the trajectory in (b).

This chaotic modulation communication scheme is not only theoretically interesting, but it also offers several advantages to the conventional SS/CDMA system. First, it is expected to have a higher capacity than the conventional SS/CDMA for multi-user communication because the bifurcating parameter (or initial condition) has a wide range of real values for signal transmission but the capacity of a conventional SS/CDMA system is usually limited by the orthogonality requirement. Second, the chaotic modulation technique does not require any code synchronization procedure which is a major problem of a conventional SS/CDMA system. Since the chaotic modulation method creates a wide-band transmitted signal by embedding the signal of transmission into the bifurcating parameter of a chaotic system, code synchronization is not required in the receiver to decode the signal.

Assume that s_t is the signal of transmission, a chaotic map

$$n_t = f(n_{t-1}, \cdots, n_{t-d}, \lambda) \tag{14}$$

where λ is the bifurcating parameter, is used to modulate s_t by setting $\lambda = s_t$. To have a wide-band signal n_t for transmission, s_t is kept in the chaotic regime. In this paper, the logistic map was used to modulate the signal, and the chaotic transmitter became

$$n_t = s_{t-1} n_{t-1}(1 - n_{t-1}). \tag{15}$$

The signal s_t was controlled in the range $|s_t| \in [3.7, 4]$ which is the chaotic regime for the logistic map. For illustration, a sine wave was used as s_t here:

$$s_t = 3.85 + 0.05 \sin\left(\frac{t}{5}\right). \tag{16}$$

Since λ is now a function of time, the signal waveform of (15) and its corresponding phase space behavior may be quite different from that of the original logistic map. To see this, we plot the trajectory and the corresponding two-dimensional phase space portrait of the logistic map and the modulated logistic map in Fig. 6. Although the trajectories look different, their phase space behaviors are quite similar. The only difference is that the parabolic curve of the chaotic modulated signal is wider in the vertical direction than that of the logistic signal, and the trajectory does not go to the center region. This would be the result of the fluctuation of the bifurcating parameter. The regular phase space pattern of the logistic modulated signal indicates that the MPSV estimation can be applied to this system.

Various issues on this chaotic modulation scheme such as the receiver design [23] and applications to telecommunication [24] have been investigated. Here we considered the problem of equalization of this chaotic modulation communication system (Fig. 7). More precisely, after passing through a communication channel, the transmitted signal n_t is usually distorted. The objective is to recover n_t from the received signal x_t. Assuming that the channel can be represented as an AR model, the equalization problem becomes the blind deconvolution of the following AR model:

$$x_t = \sum_{i=1}^{p} a_i x_{t-i} + n_t. \tag{17}$$

The problem is basically the same as the identification problem described in the previous sections. The only difference is that n_t comes from a time-varying dynamical system (15) instead of a standard time-invariant nonlinear dynamical system given in (2).

We applied the MPSV identification algorithm to (17). The more accurate the AR channel coefficient estimates, the better n_t can be obtained from x_t. MSE was used again as the performance measure to evaluate the equalization result. First, we computed the MSE of the equalization using different numbers of points. d was equal to 2, and T was equal to 30. Again, the LS plus Gaussian driven signal method was used for comparison. The results are plotted in Fig. 8. Apparently, the chaotic equalization approach consistently outperformed the optimum statistical method for all number of points. In fact, the performance of the MPSV plus chaotic modulated signal was very close to that of the MPSV plus chaos method. For $N = 20$, the MSE of the MPSV plus chaotic modulated signal approach was about −20 dB, and it decreased to −42 dB when $N = 200$. The only difference was that the MPSV plus chaotic modulated signal needed more points (about $N = 100$) to reach the level of −40 dB while the MPSV plus chaos method required just 60 points. Again, when the MSE decreased to the level of −40 dB, the performance saturated due to numerical precision. Comparing to the statistically optimal performance, the chaotic equalization approach had an improvement of 7–20 dB. In other words, using chaotic modulation for SS/CDMA communication had a much better equalization performance than the conventional SS/CDMA system in which random signals are used as spreading codes. Our analysis showed that even a statistically optimal method was still inferior to the chaotic equalization approach. In Fig. 9, we plot the corresponding PSV of the chaotic approach. The PSV in this case was similar to that of the MPSV plus chaos method. When more points were used, the PSV of the estimation went down monotonically.

Next we investigated the effect of the inverse filter order on the equalization performance. The MSE was depicted in Fig. 10. Again, the performance of the MPSV plus chaotic modulated signal approach was very close to that of the MPSV plus chaos method shown in Fig. 4. The MSE decreased drastically from $q = 1$ to 2 and remained about the same for $q \geq 2$. The slight difference was the result of numerical error. This showed that Lemma 1 is also applied to the chaotic modulation system. Comparing to the LS plus white Gaussian approach, the difference in MSE was about 20 dB for $q = 2$, and it became 27 dB for $q = 5$. The robustness of the chaotic approach was another favorable characteristic of a chaotic SS/CDMA system. In Fig. 11, we plot the PSV versus q. We found that the PSV remained

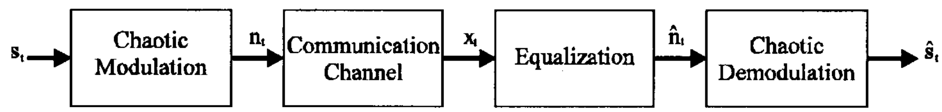

Fig. 7. Block diagram of the chaotic modulation communication system.

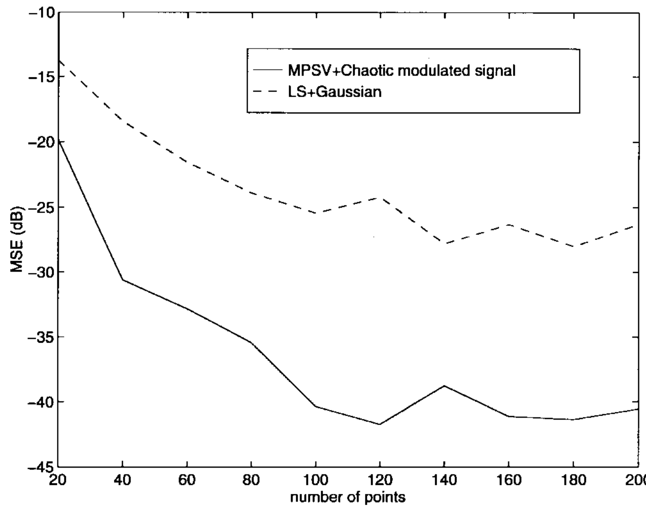

Fig. 8. MSE of the estimated coefficients of the MPSV plus chaotic modulated signal and the LS plus Gaussian methods using different numbers of points.

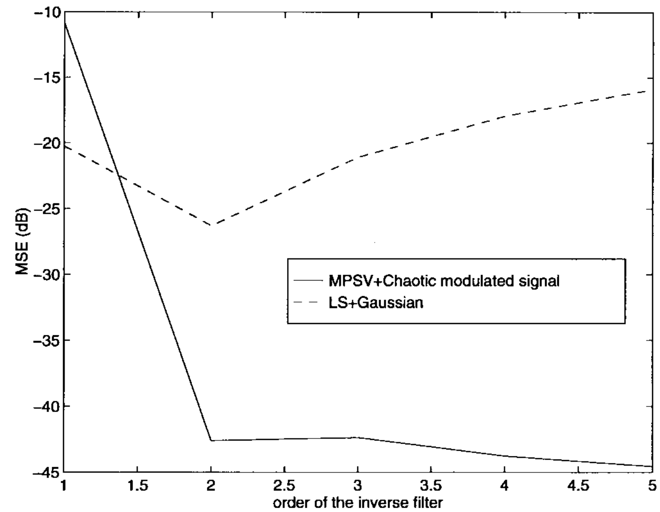

Fig. 10. MSE of the equalization of the chaotic modulated signal using different orders for the inverse filter.

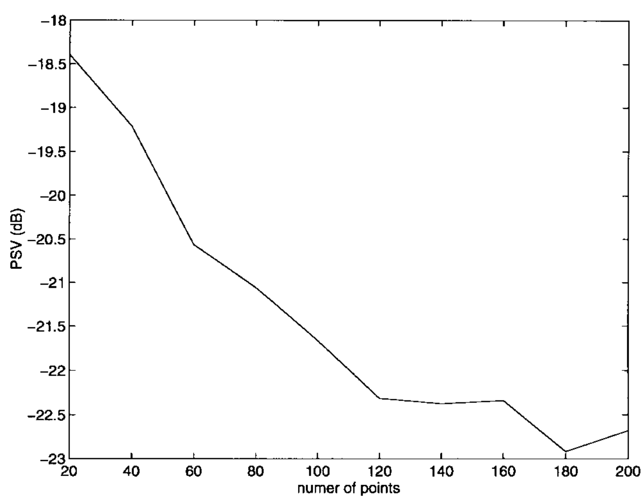

Fig. 9. Phase space volume of the MPSV plus chaotic modulated signal method using different numbers of points.

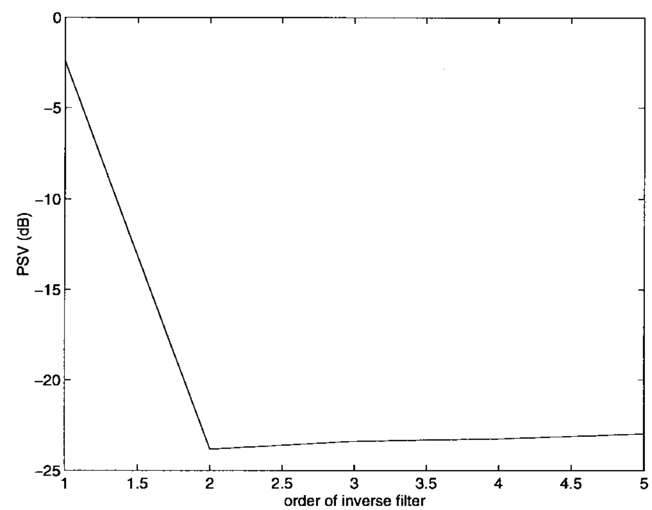

Fig. 11. Phase space volume of the MPSV plus chaotic modulated signal method using different orders for the inverse filter.

the same for any q greater than or equal to two. The result was almost exactly the same as that of the MPSV plus chaos method.

V. Discussion

This paper considered the problem of blind identification of an autoregressive (AR) system driven by a chaotic signal. Not only will this problem be encountered in the situation where the real signal such as speech and radar is chaotic, but it can also be used in the applications such as chaotic communication where chaotic signals are involved. A novel dynamic-based estimation approach called the minimum phase space volume technique was used here to estimate the AR coefficients using the inverse filter approach. It was shown here that not only could the chaotic approach identify an AR system accurately, but it was also more effective than the statistically optimal identification method, which used a least squares estimator with a white Gaussian driven signal. By more effective, the MSE of the estimates was smaller, and it was so robust that it did not require an order determination procedure. We applied this to equalize a chaotic modulation communication system. It was shown that the equalization performance based on the chaotic approach was also superior to that conventional statistical method. This showed another advantage of using chaos in spread spectrum and code division multiple assess communication.

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