Mixed H_2/H_∞ Approach of Full Order State Observer Design for Satellite Attitude Control System

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Abstract: This paper investigates the design method of full order state observer for satellite attitude control system using mixed H_2/H_∞ optimal approach. A linear system described by state space form is introduced first to state the problem of full order state observer design. The designed observer should meet the requirement of H_2 performance and H_∞ performance so that the disturbance would influence the observer as little as possible. Thus, we convert the satellite attitude control system to the same state space form, and deduce the mixed H_2/H_∞ approach based on linear matrix inequalities (LMIs). By solving LMIs, we can obtain the solution of corresponding convex optimization problem. Then, the observer gain can be decided. The simulation results based on satellite attitude control system show that the magnitude of observer error is 10^{-6}rad/s within 10000s. We can conclude that the mixed H_2/H_∞ approach of full order state observer design is effective and the external disturbance has little influence on the observation error.

Key Words: Satellite, Mixed H_2/H_∞ Approach, State Observer, LMIs

1. INTRODUCTION

The problems of state observer design have received a large amount of attention during the last decades. Although the past two decades have witnessed several important developments in state observer design [1, 2], there still remain certain problems that are of great practical and theoretical significance. A reduced-order nonlinear observer design approach is presented in [3], it allows adjustment of the decay rate of observer error which has the characteristic of global asymptotic stability. Besides, many reduced-order state observers have been studied recently. A nonlinear observer design that allows the estimation of state variables and unknown process or sensor disturbances is proposed in [4], by solving the system of singular PDEs, a state-dependent gain is computed, and both state and disturbance observation errors converge to zero. The state observer has been applied in many aspects, such as orbit uncertainty estimation [5], unknown disturbance inputs estimation [6], flight control design [7] and so on.

Different performance criteria are utilized in different conditions [8]. H_2 and H_∞ are two common optimization criteria, which have been used in many references, but the use of H_2 and H_∞ in state observer design is very rare. In [9], the problem of H_∞ observer design for uncertain linear discrete-time systems with time delay is considered, but it is solved in the form of the matrix Riccati-like equations/inequalities, which is much more complicated than LMIs. However, few studies focused on the design of full order state observer for satellite attitude control system have been finished. Previous research failed to consider the mixed H_2/H_∞ condition, thus, these previous results are unsatisfactory.

In this paper, a full order state observer design method for satellite attitude control system using mixed H_2/H_∞ approach is proposed, and the mixed H_2/H_∞ performance constraint is converted to a convex optimization problem based on LMIs. With the MATLAB LMI toolbox, we can easily obtain the solution of the problem, then, the observer gain can be decided to be applied in the simulation of observation error.

2. PROBLEM STATEMENT

Consider the linear system described by the following state space form:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1u(t) + B_2d(t) \\ y(t) = C_1x(t) + D_1u(t) + D_2d(t) \\ z(t) = C_2x(t) \end{cases} \tag{1}$$

Where, x is the state vector, u is the control input vector, d is the disturbance input vector, y and z are the measured output vector and the interested output vector, respectively, others are the system coefficient matrices of appropriate dimensions.

Design a full order state observer

$$\dot{\hat{x}} = (A + LC_l)\hat{x} - Ly + (B_1 + LD_l)d \tag{2}$$

such that the effect of disturbance d(t) to estimate error can be controlled at a desired level. Where, $\hat{x}$ is the state observation vector, L is the observer gain. Then, the estimate of the interested output is

$$\hat{z}(t) = \hat{C}_2x(t) \tag{3}$$

Which is desired to be affected by the disturbance d(t) as little as possible.
With (1), we have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u(t) + B_2d(t) \\ &= Ax(t) + Ly - Ly + B_1u(t) + B_2d(t) \\ &= Ax(t) + L(C_1x(t) + D_1u(t) + D_2d(t)) - Ly + B_1u(t) + B_2d(t) \\ &= (A + LC_1)x(\text{t}) - Ly + (B_1 + LD_l)u(t) + (B_2 + LD_2)d(t) \end{aligned} \tag{4}$$

Let

$$\begin{cases} e(t) = x(t) - \hat{x}(t) \\ \overline{z}(t) = z(t) - \hat{z}(t) \end{cases} \tag{5}$$

Equation(2), (3), (4), (5) yield the following observation error equation:

$$\begin{cases} \dot{e}(t) = (A + LC_1)e(t) + (B_2 + LD_2)d(t) \\ \hat{z}(t) = C_2 e(t) \end{cases} \tag{6}$$

with some possibly small γ_∞ and γ_2, the following H_∞ and H_2 performance conditions

$$\begin{aligned} \|G_{\bar{z}d}\|_\infty &= \|C_2(sI - A - LC_1)^{-1}(B_2 + LD_2)\|_\infty \le \gamma_\infty \\ \|G_{\bar{z}d}\|_2 &= \|C_2(sI - A - LC_1)^{-1}(B_2 + LD_2)\|_2 \le \gamma_2 \end{aligned} \tag{7}$$

are satisfied.

As a consequence, the system (6)is asymptotically stable, namely

$$e(t) \to 0, \text{ when } t \to \infty$$

Which means that $\hat{x}(t)$ is an asymptotic estimate of $x(t)$.

3. SATELLITE ATTITUDE DYNAMICS

In the inertial coordinate system, the attitude dynamics of a satellite can be described as

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z = T_{cx} + T_{gx} + T_{dx} \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x = T_{cy} + T_{gy} + T_{dy} \\ I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y = T_{cz} + T_{gz} + T_{dz} \end{cases} \tag{8}$$

Where, T_c, T_g and T_d are the control torque, the gravitational torque and the disturbance torque, respectively.

Three Euler angles φ, θ and ψ are roll, pitch and yaw attitude angle of satellite, respectively. Under small angle approximation, the angular velocity is given as follows

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\varphi} - \omega_0\psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0\varphi \end{bmatrix} \tag{9}$$

Where, ω_0 is the orbital angular velocity.

As is known,

$$\begin{bmatrix} T_{gx} \\ T_{gy} \\ T_{gz} \end{bmatrix} = \begin{bmatrix} -3\omega_0^2(I_y - I_z)\varphi \\ -3\omega_0^2(I_x - I_z)\theta \\ 0 \end{bmatrix} \tag{10}$$

Combining(8), (9) and(10), the final attitude dynamic equation can be obtained

$$\begin{cases} I_x\ddot{\varphi} + 4(I_y - I_z)\omega_0^2\varphi + (I_y - I_z - I_x)\omega_0\dot{\psi} = T_{cx} + T_{dx} \\ I_y\ddot{\theta} + 3\omega_0^2(I_x - I_z)\theta = T_{cy} + T_{dy} \\ I_z\ddot{\psi} + (I_y - I_x)\omega_0^2\psi + (I_x + I_z - I_y)\omega_0\dot{\varphi} = T_{cz} + T_{dz} \end{cases} \tag{11}$$

3.1 Second-Order System Form

Equation (11) can be written in the following second-order matrix form:

$$M\ddot{q} + H\dot{q} + Gq = L_1 u + L_2 d \tag{12}$$

where,

$$q = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}, u = \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix}, d = \begin{bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{bmatrix}$$

$$M = diag(I_x, I_y, I_z),\ L_1 = L_2 = I_{3\times3},$$

$$H = \omega_0(I_y - I_x - I_z)\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix},$$

$$G = diag(4\omega_0^2(I_y - I_z), 3\omega_0^2(I_x - I_z), \omega_0^2(I_y - I_x))$$

3.2 State Space Form

Define

$$x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T, y = 10^{-3}M\ddot{q}, z = q$$

Then system(12) can be converted into the following state-space form:

$$\begin{cases} \dot{x} = Ax + B_1u + B_2d \\ z_\infty = C_1x + D_1u + D_2d \\ z_2 = C_2x \end{cases} \tag{13}$$

Where,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_0^2 I_x^{-1}(I_y - I_z) & 0 & 0 & 0 & 0 & -\omega_0 I_x^{-1}(I_y - I_x - I_z) \\ 0 & -3\omega_0^2 I_y^{-1}(I_x - I_z) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_0^2 I_z^{-1}(I_y - I_x) & \omega_0 I_z^{-1}(I_y - I_x - I_z) & 0 & 0 \end{bmatrix}$$

$$C_1 = 10^{-3}\begin{bmatrix} -4\omega_0^2(I_y - I_z) & 0 & 0 & 0 & 0 & -\omega_0(I_y - I_x - I_z) \\ 0 & -3\omega_0^2(I_x - I_z) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_0^2(I_y - I_x) & \omega_0(I_y - I_x - I_z) & 0 & 0 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0_{3\times3} & diag(I_x^{-1}, I_y^{-1}, I_z^{-1}) \end{bmatrix}^T, C_2 = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \end{bmatrix}, D_1 = 10^{-3} \times L_1, D_2 = 10^{-3} \times L_2$$

4. MIXED H_2/H_∞ STATE OBSERVER DESIGN BASED ON LMI

Before we solve the problem of state observer design of satellite attitude system, some lemmas are introduced. Based on these lemmas, we conclude the mixed H_2/H_∞ method for state observer design.

Theorem 1 H_∞ performance constraint

$$\|G(s)\|_\infty = \|C(sI - A)^{-1}B + D\|_\infty < \gamma_\infty$$

holds if and only if there exists a matrix X>0, such that the following inequality holds:

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{14}$$

Theorem 2 H_2 performance constraint

$$\|G(s)\|_2 = \|C(sI - A)^{-1}B\|_2 < \gamma_2$$

holds if and only if there exists a symmetric positive definite matrix X such that

$$\begin{cases} AX + XA^T + BB^T < 0 \\ trace(CXC^T) < \gamma_2^2 \end{cases} \tag{15}$$

Lemma 1 Let $A(x) \in S^m$ be a matrix function in R^n, and γ be a positive scalar. Then, the following statements are equivalent:

(1) $\exists x \in R^n$, such that $trace(A(x)) < \gamma$

(2) $\exists x \in R^n, z \in S^m$, such that $A(x) < Z$, while $trace(Z) < \gamma$

Lemma 2[10] (Schur complement lemma) Let the partitioned matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$$

be symmetric. Then

$$\begin{aligned} A < 0 &\Leftrightarrow A_{11} < 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} < 0 \\ &\Leftrightarrow A_{22} < 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T < 0 \end{aligned} \tag{16}$$

or

$$\begin{aligned} A > 0 &\Leftrightarrow A_{11} > 0, A_{22} - A_{12}^T A_{11}^{-1} A_{12} > 0 \\ &\Leftrightarrow A_{22} > 0, A_{11} - A_{12} A_{22}^{-1} A_{12}^T > 0 \end{aligned} \tag{17}$$

Theorem 3 Equation(7) has a solution if and only if there exist a matrix W, a symmetric positive definite matrix X, a symmetric matrix Q such that

$$\begin{cases} X > 0 \\ \begin{bmatrix} A^T X + C_1^T W^T + XA + WC_1 & XB_2 + WD_2 & C_2^T \\ (XB_2 + WD_2)^T & -\gamma_\infty I & 0 \\ C_2 & 0 & -\gamma_\infty I \end{bmatrix} < 0 \\ \begin{bmatrix} XA + WC_1 + (XA + WC_1)^T & XB_2 + WD_2 \\ (XB_2 + WD_2)^T & -I \end{bmatrix} < 0 \\ \begin{bmatrix} -Q & C_2 \\ C_2^T & -X \end{bmatrix} < 0 \\ trace(Q) < \rho \end{cases} \tag{18}$$

By minimizing $c_\infty \gamma_\infty + c_2 \rho$, where $\rho = \gamma_2^2$, the observer gain can be constructed as $L = X^{-1}W$.

Proof As to H_∞ condition, it follows from Theorem 1 that the problem has a solution if and only if there exists a matrix X>0, such that the following inequality holds:

$$\begin{bmatrix} (A + LC_1)^T X + X(A + LC_1) & X(B_2 + LD_2) & C_2^T \\ (B_2 + LD_2)^T X & -\gamma_\infty I & 0 \\ C_2 & 0 & -\gamma_\infty I \end{bmatrix} < 0 \tag{19}$$

By defining

$$W = XL$$

The inequality (19) can be turned into the LMI form as follow:

$$\begin{bmatrix} A^T X + C_1^T W^T + XA + WC_1 & XB_2 + WD_2 & C_2^T \\ (XB_2 + WD_2)^T & -\gamma_\infty I & 0 \\ C_2 & 0 & -\gamma_\infty I \end{bmatrix} < 0$$

As to H_2 condition, it follows from Theorem 2 that the problem has a solution if and only if there exists a symmetric positive definite matrix P, such that

$$\begin{aligned} &(A + LC_1)P + P(A + LC_1)^T + (B_2 + LD_2)(B_2 + LD_2)^T < 0 \\ &trace(C_2 P C_2^T) < \gamma_2^2 \end{aligned} \tag{20}$$

Multiplied by P^{-1} at both sides of the first inequality in (20) simultaneously, we can obtain

$$P^{-1}(A + LC_1) + (A + LC_1)^T P^{-1} + P^{-1}(B_2 + LD_2)(B_2 + LD_2)^T P^{-1} < 0 \tag{21}$$

According to Lemma 2, (21) can be converted into the following form

$$\begin{bmatrix} P^{-1}(A + LC_1) + (A + LC_1)^T P^{-1} & P^{-1}(B_2 + LD_2) \\ (B_2 + LD_2)^T P^{-1} & -I \end{bmatrix} < 0 \tag{22}$$

Let

$$X = P^{-1}, W = XL$$

Then, we can have

$$\begin{bmatrix} XA + WC_1 + (XA + WC_1)^T & XB_2 + WD_2 \\ (XB_2 + WD_2)^T & -I \end{bmatrix} < 0 \tag{23}$$

With Lemma 1, the second inequality in (20) can be converted into

$$C_2PC_2^T < Q, trace(Q) < \gamma^2$$

With Lemma 2, let $X = P^{-1}$, we can have

$$\begin{bmatrix} -Q & C_2 \\ C_2^T & -X \end{bmatrix} < 0, trace(Q) < \gamma^2 \quad (24)$$

Then the proof of Theorem 3 is completed.

5. SIMULATION AND ANALYSIS

To validate the performance of full order state observer in satellite attitude control system, take in-orbit service microsatellite flying in the orbit height of 300km for example to make simulations. Choose simulation parameters as follows:

$$I_x = 20kg\cdot m^2, I_y = 12kg\cdot m^2, I_z = 15kg\cdot m^2,$$

Choose the initial state as

$$e(0) = [0\ 0\ 0\ 0\ 0\ 0]^T$$

Where, the unit of angle is rad and the unit of angular velocity is rad/s. Meanwhile d is chosen as

$$d = 1.5\times10^{-5}\times\begin{bmatrix} 3\sin(\omega t)+1 \\ 3\sin(\omega t)+2\cos(\omega t) \\ 4\sin(\omega t)+1 \end{bmatrix} N\cdot m$$

With MATLAB LMI Toolbox, the mixed H_2/H_∞ optimal state observer condition (18) gives the following optimal parameters:

$$X = \begin{bmatrix} 7.0735\times10^{-5} & 0 & -3.5131\times10^{-8} & -12.5289 & 0 & -0.0280 \\ 0 & 8.3629\times10^{-5} & 0 & 0 & -10.5275 & 0 \\ -3.5131\times10^{-8} & 0 & 1.5776\times10^{-5} & -0.0055 & 0 & -5.3282 \\ -12.5289 & 0 & -0.0055 & 4.4412\times10^{6} & 0 & 5.8157\times10^{3} \\ 0 & -10.5275 & 0 & 0 & 2.6549\times10^{6} & 0 \\ -0.0280 & 0 & -5.3282 & 5.8157\times10^{3} & 0 & 3.3696\times10^{6} \end{bmatrix}$$

$$W = \begin{bmatrix} 626.4139 & 0 & 1.8638 \\ 0 & 877.3237 & 0 \\ 0.2762 & 0 & 355.1803 \\ -2.2206\times10^{8} & 0 & -3.8771\times10^{5} \\ 0 & -2.2124\times10^{8} & 0 \\ -2.9080\times10^{5} & 0 & -2.2464\times10^{8} \end{bmatrix}$$

Then, the corresponding observer gain is

$$L = X^{-1}W = \begin{bmatrix} -944.6756 & 0 & -4.7873 \\ 0 & 838.0993 & 0 \\ -27.7384 & 0 & -4.4915\times10^{3} \\ -50.0027 & 0 & -8.7362\times10^{-6} \\ 0 & -83.3300 & 0 \\ -5.2120\times10^{-5} & 0 & -66.6738 \end{bmatrix}$$

Once we obtain the observer gain, we can establish the simulink structure of observation error, which can be seen in Figure 1, and matrices A, C_1, B_2, D_2 are all known previously. As a result, we can obtain the observation error. Here, we take error of angular velocity for example, which can be seen in Figure 2. Figure 2 shows that the observation error of angular velocity is as small as the magnitude of 10^{-6}rad/s within 10000s. Namely, the observer error of angular velocity can converge to about zero which can be neglected within about 3 hours. It is also seen from Figure 2 that the external disturbance indeed has little influence on the observation error.

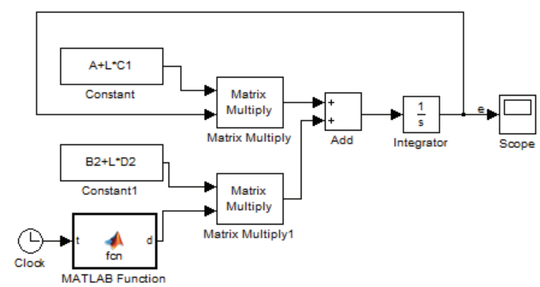

Fig 1. Simulink structure of observation error

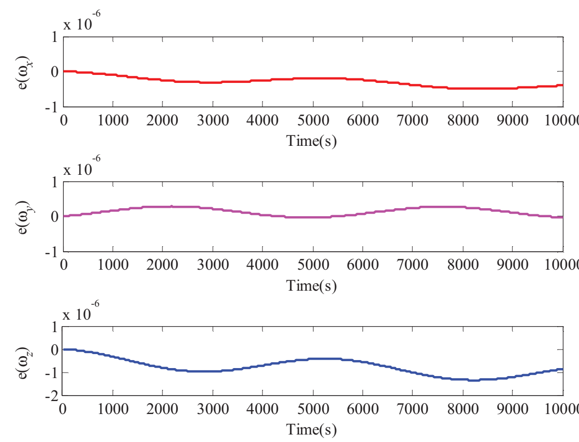

Fig 2. Observation error in terms of angular velocity(Unit: rad/s)

6. CONCLUSION

The design method of full order state observer for satellite attitude control system is proposed using mixed H_2/H_∞ optimal approach. We convert the problem of full order state observer design to a convex optimal problem based on LMIs. Once we obtain the solution of LMIs, we can get the observer gain. The mixed H_2/H_∞ approach can ensure the excellent performance of the designed observer and little influence of external disturbances on the observer. The simulation results demonstrate the effectiveness of the proposed full order state observer design method using the mixed H_2/H_∞ approach, and the external disturbance indeed has little effect on the observation error. Further investigations may focus on the problem of observer design with time delay.

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