

# The process of isothermal compression of gasses at sub-atmospheric pressures through regulated water injection in Braysson cycles

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## ABSTRACT

Although the Braysson cycle constitutes the ideal limit for the Combined Cycle Power Plants, its actual implementation has not been achieved due to the difficulty in building the required isothermal compressor. The present study proposes the incorporation of regulated water injection during the final compression, which could maintain the temperature constant due to the evaporation. The analysis for the thermodynamic implications of the injection on the ideal version of the Braysson cycle indicates that the (ideal cycle) efficiency reduction will be minimal. The study provides an analysis for the water injection rate that will permit such a process and shows that the additional work needed to drive the process will not be affected significantly by the injection. In addition, it shows that the minimum temperature of the Braysson cycle will be lower than the corresponding level of the conventional (Gas–Steam turbine Combined cycle plants), something that could improve the efficiency as well. Finally it shows that the process may be expressed by a polytropic relationship of the type  $pv^\beta = \text{constant}$ , where  $\beta \approx 1.06$ .

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## 1. Introduction

Bottoming cycles constitute additions to the standard (topping) ones by exploiting the high temperature enthalpy carried along by the exhausted flue gases. Such Topping–Bottoming cycle combinations that permit expansion to a temperature equal to the atmospheric one may be called “full expansion” ones. Kerrebrok [1] has shown that the incorporation of the sequence isentropic expansion–isothermal compression as a Bottoming cycle constitutes the ideal limit for all such combined cycles. Over the last 10 or so years the incorporation of the isothermal compression–cooling process into a thermodynamic cycle was evaluated by a number of relevant studies. Goto et al. [2] have examined the ideal versions of the Diesel and Brayton cycles incorporating such bottoming additions to create the corresponding “full” expansion ones. They showed that the combined ideal cycle thermal efficiencies could exceed the 80% level for the case of the perfect air, when their simple (conventional) versions are limited around the 60% level (for practical pressure ratios). Recent reports call the single stage version of the full expansion cycle “the Braysson cycle”. Second law and finite rate analysis of this cycle have been provided by Zheng et al. [3] and Zhou et al. [4], respectively.

Unfortunately, it is hard to achieve isothermal compression through conventional heat exchangers, since it requires a very close co-ordination between the heat and work transfer operations. Actual implementation so far has been extremely limited. The cryogenic engine built by Ordonez [5] is probably the best-known case where a tubular heat exchanger was inserted inside a reciprocating engine cylinder for the purposes of implementing an isothermal expansion process. No wonder that the actual results were rather disappointing. Frost et al. [6] have proposed a combined cycle that is equivalent to a full expansion Brayton one. The topping one was a typical Brayton cycle, while the bottoming (termed the “Ericsson Cycle” by the authors) receives heat from the topping cycle (Gas Turbine or Diesel Engine) through a counter current, constant pressure heat exchanger. The rest of the cycle is completed by an isentropic expansion (up to the atmospheric temperature)—isothermal compression–cooling sequence. The last process involves a rotating heat pipe. They predicted a real plant total efficiency above the 50% level.

Water evaporation has been considered for quite some time as a heat sink mechanism for isothermal (or nearly so) compression processes, for Reciprocating Engine and Compressors as well as Gas Turbine based plants. The earliest extensive tests employing Water Injection in reciprocating compressors appears to have been conducted in the Soviet Union, starting around 1955. Gal'perin et al. [7] provide the relevant biographical data about these efforts for the period before 1967. In the same paper they developed the

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thermodynamic analysis for the modifications imposed by the water injection on the polytropic exponent of a general (not isothermal) compression process. Kabakov and Sicherba [8] and Dobrokhov et al. [9] published similar analytical and experimental studies. More recently Zhao et al. [10] examined the “isothermal” compression for a scroll compressor (forming part of an automotive fuel cell system) incorporating injection of water and compared the generated experimental data against an (rather complex) analytical model for the entire process. Similarly, a more detailed study of the water injection rate inside a reciprocating air compressor has been carried out by the iso-engine concept group (e.g. Coney et al. [11]), by incorporating modern CFD codes and advanced optical experimental techniques. Of course, all these studies refer to compressions above the atmospheric conditions.

Far greater attention has been paid on the injection of water in the compression process of Gas Turbine plants. The recent review by Claus [12] covers most variations on the ideas that have been tested so far. The application closest to the present study is the so called “wet or foggy compression” [13]. Here the water not only cools the incoming air just before the engine inlet, but it also carries along “large” diameter droplets to be evaporated inside the compressor blade channels. Unfortunately, inside a Gas Turbine the air pressure is always well above the atmospheric one. As the pressure of the (humid) air rises, this tends to saturate rather quickly. Under such conditions perfectly isothermal compression is not possible for temperatures close to the atmospheric. Zheng et al. [14] studied the thermodynamic processes for the fogging cooling application in Gas Turbines and concluded that for a compression ratio above 10:1, the exit (humid) air temperature will exceed the 400 K level (starting from the reference 288 K one) even in the limit where the (humid) air is maintained at saturated conditions throughout the entire compressor length. In addition, the momentum losses introduced by the “large” diameter droplets introduce additional entropy, so that the actual compression process may be represented by a  $pv^n = \text{constant}$  relationship where  $n = 1.12$ , if the thermodynamic properties of the atmospheric air are maintained constant to those at  $T = 288$  K. For the higher temperatures corresponding to the constantly saturated mixture scenario this will be increased somewhat. Roumeliotis and Mathioudakis [15,16] have provided experimental data for the single and multistage axial compressors under foggy conditions. They concluded that the water evaporation affects not only the total pressure losses but it modifies the performance map of the compressor as well.

The present study positions the water injection on the opposite side of the cycle, where the (flue) gas pressures are well below that of the atmosphere (as a result of the need to expand the gases sufficiently, so as to bring their temperature close to the atmospheric). At those pressures the (water) steam specific internal energy and enthalpy are essentially only functions of the temperature. This leads to an analytical expression of the (perfectly) isothermal compression process in the well-known polynomial formulation ( $pv^n = \text{constant}$ ), where  $n$  is of order 1.06. This, however, requires a regulated water mass flux. By employing this formulation, the analysis proceeds to estimate the implications of the water injection on the ideal Braysson cycle thermal efficiency. Although this level of analysis does not provide accurate energy transfer estimates, the calculated relative variation for the depended cycle parameters (efficiency, specific work, etc) against the free ones (pressure ratio, maximum to minimum temperature ratio, constant  $\gamma$ , etc) usually is fairly good. The results show that the work consumed for the injection assisted compression is increased by a relatively small magnitude over the corresponding process of the ideal air. Next, the analysis establishes the parameter levels for which such a compression may be implemented, i.e. the air temperature, pressure and specific humidity values at the start of

the process. These levels are shown to be of the order of 330–340 K for (isothermal) compression ratios of the order of 10:1, i.e. comparable with the levels of the bottoming Rankine cycle steam condensers. In the proposed Braysson plant the compressor exhaust temperature constitutes the exhaust temperature of the topping cycle flue gases, while in the conventional Brayton–Rankine Combine cycle plants the corresponding temperature is of order 370 K. This implies that the proposed plant will deliver a slightly better efficiency than the conventional plants. Of course, the great advantage of the proposed cycle comes from the absence of any Waste Heat Recovery Boiler. This not only will reduce the plant construction costs but it will also permit a load acceleration capability similar to that of the single Gas Turbine (or Diesel Engine) plants. In other words, the proposed plant will not need separate “transient load” supporting plants.

## 2. Some introductory aspects

### 2.1. The isothermal compression process

The isothermal process of compressing ideal gases implies that  $T = \text{constant}$ , which in turn leads to  $pv = \text{constant}$ . The corresponding equation for the work transfer between states 1 (start) and 2 (finish) is equal to

$$\Delta W_{12} = MRT \ln \left( \frac{p_2}{p_1} \right) = MRT \ln \left( \frac{v_1}{v_2} \right) \quad (1)$$

The parameters  $p$ ,  $v$ ,  $T$  refer to the pressure, specific volume and temperature of the perfect gas, respectively, while  $M$  and  $R$  refer to the mass and the gas constant, respectively.

### 2.2. The humid mixture and the saturation condition

If the humid mixture is assumed to be composed of perfect gases, the saturation specific humidity ( $\omega$ ) may be evaluated through the application of the perfect gas law, which leads to the following relationship

$$\omega_S = \left( \frac{m_W}{m_{DG}} \right)_S = \frac{R_W}{R_{DG}} \frac{\phi p_{SAT}(T)}{p - \phi p_{SAT}(T)} \quad (2)$$

Here  $p_{SAT}(T)$  is the saturation pressure of the water at the temperature of the gaseous mixture ( $T$ ), while  $\phi$  is the relative humidity. Water injection increases the specific humidity ( $\omega$ ) as the regulated compression process evolves. Limit condition is reached when the mixture becomes saturated. Beyond this point no more water is evaporated.

### 2.3. The ideal full expansion cycles

The two widely employed cycles of Otto and Brayton are considered as references for their “full expansion” counterparts, as illustrated in Fig. 1. Fig. 1a illustrates the Otto cycle in both the standard version (defined by the sequence 01–02–03–04–01) and the new, “full” expansion one, (the sequence 01–02–03–05–01). The corresponding cycle structures for the Brayton are illustrated in Fig. 1b. It can easily be shown that the thermal efficiency of the ideal full expansion version for both cycles is given by

$$\eta_{TH} = 1 - \frac{1}{\theta} \frac{\ln(X)}{X-1} = 1 - \frac{f(X)}{\theta} \quad (3)$$

where  $\theta = r^{\gamma-1}$  is the “compression” temperature ratio,  $\theta = T_3/T_1$  is the ratio between the maximum and the minimum temperatures,  $X = \theta/\theta$ , and  $f(X) = \ln(X)/(X-1)$ .

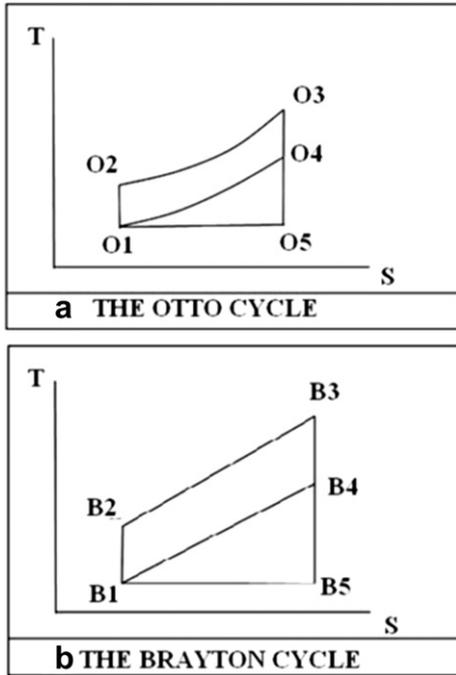


Fig. 1. The full expansion versions of the (a) Otto cycle and (b) the Brayton cycle.

As illustrated in Fig. 2, for atmospheric air (i.e. when  $\gamma = 1.4$ ) the function  $f(X) < 1$  as long as  $X > 1$ . In other words, the thermal efficiencies of the full expansion cycles are always larger than the corresponding values of the standard ones, which are equal and given by

$$\eta_{OTTO} = \eta_{BRAYTON} = 1 - \frac{1}{\theta} \quad (4)$$

The ideal thermal efficiencies of the full expansion cycles may exceed the 80% levels, when the corresponding standard cycles reach efficiencies in the 60% level.

2.4. The water injection effects in the Braysson and Gas Turbine cycles

The cycle modifications imposed by the water injection in the relevant processes of the Braysson cycle and the typical Gas Turbine (Brayton) one are illustrated in Fig. 3. The basic Brayton cycle (i.e. Power Generating Gas Turbine) is defined by the thermodynamic

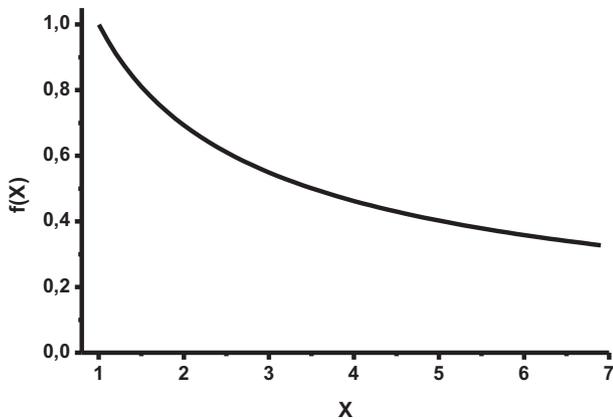


Fig. 2. The function  $f(X)$ .

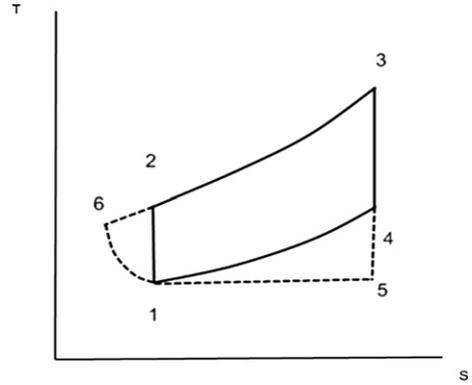


Fig. 3. The modification imposed by water injection in the Braysson and the Gas Turbine cycles.

state sequence 1–2–3–4–1. The Fog Compression process inter-cools the Compressor, so that the Gas Turbine cycle is now defined by the sequence 1–6–3–4–1. The Braysson cycle is defined by the processes 1–2–3–4–5–1. The sequence 3–4–5 represents the expansion inside the turbine of the Gas Turbine (sequence 3–4) as well as the additional expansion inside the “bottoming cycle” turbine (sequence 4–5). It is apparent that the area of the Braysson cycle is much larger than that of the “intercooled” Gas Turbine.

The typical structure of a Power Plant operating on the Braysson cycle as modified by the water injection is illustrated in Fig. 4. Most of the plant components are widely employed in the conventional plants (the second turbine will not be much different from the corresponding units of a Steam Turbine plant). The new “component” is associated with the exhaust of the flue gases. The best solution is given by a conventional axial compressor assisted by water injection between its stages. This configuration allows for the entire expansion (turbine)–(isothermal) compression apparatus to be built around a common shaft, in a structure reminiscent of the low-pressure steam turbine stages. Given the very low pressures near the end of the expansion–start of the compression, axial turbomachinery provides the most appropriate (non-dimensional) “specific speed”. However, for large plants the high volumetric rates and the restricted shaft angular frequency (50–60 Hz) lead to a rather large axial velocity (a well-known problem in the low-pressure section of the Steam Turbines). This, in turn, leads into a very small diameter for the injected water droplets, if the droplet atomization–evaporation rates are to be implemented within the spacing separating two successive compressor stages. Such compression magnitudes and the rather large water mass flux imply a rather serious thermal efficiency reduction for the plant, of the order of 2–3% as discussed earlier. Of course, on top of this one should include the additional losses coming from the mixing interaction between the high-speed flue gases and the water. Until such an axial compressor is developed that would eliminate the above problem, it is proposed to repressurize the exhaust gases by the method illustrated in Fig. 4. This involves a chamber (EC) that will collect the turbine exhaust and distribute it among the numerous isothermal compressors (IsC). The structure of this section will be quite sizable but the low temperatures (of order 340 K) and pressures (of order 0.1–1 bar) involved could be managed by a construction based on cheap materials (cement for the chamber and plastics for the compressors). In addition, it avoids the solid matter concentration limits imposed for the water entering the hot section of the Gas Turbine (or Diesel Engine). The proposed plant could operate even on seawater. In the last scenario, the subsequent cooling of the exhaust (outside the compressors) by the atmospheric air (in a free convection Cooling Tower unit) could lead to a very cheap desalination plant.

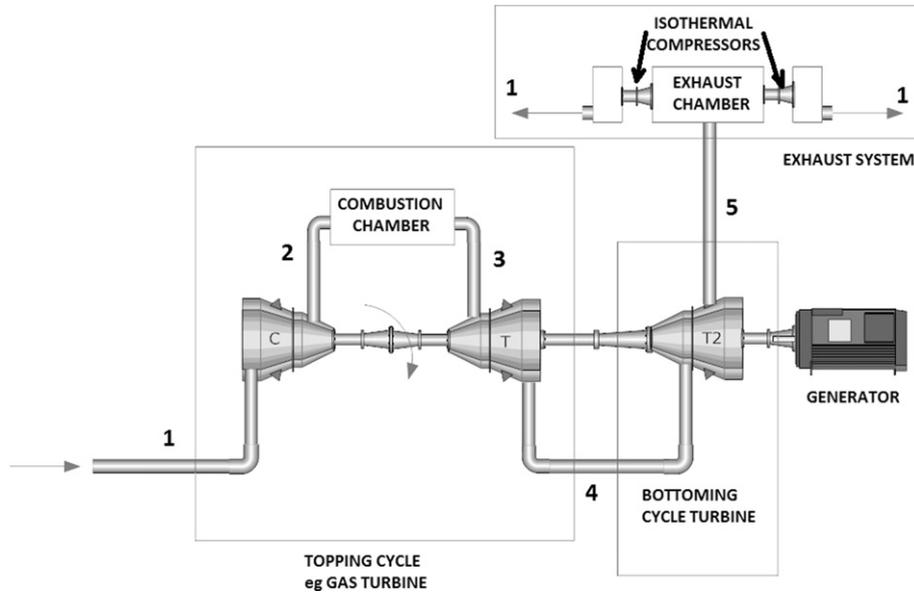


Fig. 4. The typical structure of a Braysson cycle incorporating water injection for the isothermal compression process.

**3. Isothermal compression in a closed system**

As discussed above the isothermal compression process may be implemented in either an open system (i.e. axial compressor) or a closed system (positive displacement) unit. Since the second is easier to construct, the following analysis will be based on a closed thermodynamic system approach. Given that the specific internal energy of the slightly superheated–saturated water steam differs only 1–2% from the corresponding enthalpy at the sub-atmospheric pressures of interest here, it is quite straightforward to show that both thermodynamic system approaches give (almost) the same results. The closed system approach will incorporate inlet–exhaust processes in addition to the compression one. Now the regulated water injection rate (actually water evaporation one) cannot be implemented easily by employing a single nozzle injecting into a single cylinder, due to the time delay between the injection and the evaporation. In our laboratory such a compressor is under development employing a Vane Rotary concept, as illustrated in Fig. 5a. This combines the roles of both the turbine and the compressor. Water is injected at a given cavity just after the separating rib passes in front of the injection nozzle. The analysis follows one such cavity as it traverses the entire compressor arc. The humid unsaturated gaseous mixture is composed of dry (perfect) gases and water steam. The volume change inside the cavity is represented by the motion of a (fictitious) piston (Fig. 5b). The upward motion of the piston compresses the gasses while the regulated water injection (followed by an instantaneous evaporation) maintains a constant temperature. The mass of the dry gases remains constant throughout the process, while the water mass (and the corresponding humidity) increases due to the injection.

The law of mass conservation gives the following relationship

$$dm = m_{DG}d\omega = dm_W \tag{5}$$

Here  $m$ ,  $m_{DG}$  and  $m_W$  are, respectively, the masses of the entire mixture, the dry gasses and the water steam. The first law applied to this configuration gives

$$dU = \delta Q - pdV + dH_{in} \tag{6}$$

where  $dU$  = the total internal energy change of the mixture, as the piston moves to reduce the volume by  $dV$ ;  $\delta Q$  = the heat entering

the system through the walls or due to the friction caused by the motion of the piston;  $p$  = the pressure of the mixture; and  $dH_{in}$  = the enthalpy of the injected water (liquid) mass.

The internal energy for the mixture ( $U$ ) is equal to the sum of the corresponding internal energies of the dry gasses and the evaporated steam. The internal energies for the dry gasses and the liquid water are assumed to be equal to zero at  $T = 273.15 \text{ K} = 0 \text{ }^\circ\text{C}$ . Hence

$$U = m_{DG}C_{VDG}(T - 273.15) + m_W u_{SHS} \tag{9}$$

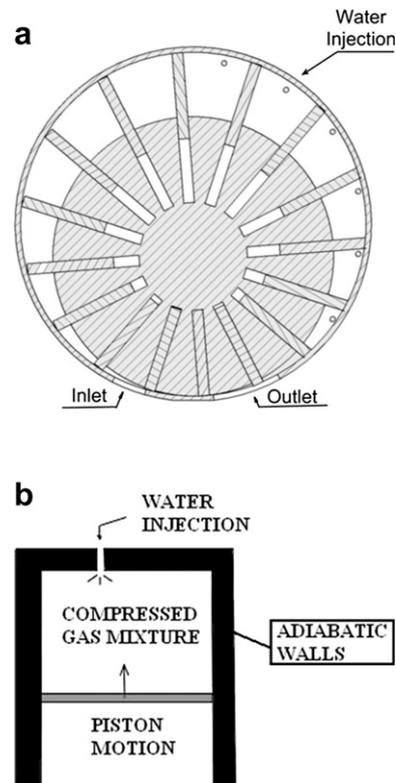


Fig. 5. A typical isothermal compressor configuration and the simplified control volume. (a) The expander–compressor concept studied in our laboratory. (b) The simplified control volume geometry.

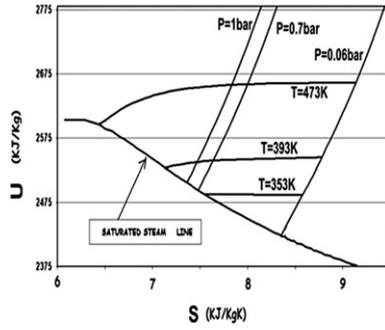


Fig. 6. The variation of the specific internal energy of water steam with pressure and temperature.

where  $c_{VDG}$  = (average) constant volume specific heat of the dry gasses (kJ/kgK);  $T$  = mixture temperature (K);  $u_{SHS}$  = specific internal energy of the superheated (or saturated) steam contained within the humid mixture (kJ/kg);  $dh_{in} = h_W dm_W = m_{DG} h_W d\omega$ ;  $h_W$  = specific enthalpy of the injected water (kJ/kg).

As illustrated in Fig. 6 (which represents the corresponding region from the Mollier diagram), the specific internal energy of the superheated steam ( $u_{SHS}$ ) is a function only of the temperature, since it is practically independent of the pressure at levels around or below the atmospheric one. This amounts for the isothermal lines to be (practically) parallel to the entropy axis right off the saturation line. On the other side, at high pressures and low temperatures (i.e. the conditions occurring during the Gas Turbine intercooling process) this is not so. For an adiabatic frictionless piston system  $\delta Q = 0$ . Hence, for a perfectly isothermal process, Eq. (6) is reduced into

$$d\{m_W(u_{SHS} - h_W)\} = -pdV \quad (10)$$

With the help of the perfect gas law this may be transformed into

$$m_{DG}(u_{SHS} - h_W) d\omega = -pdV = -\frac{m_{DG}R_{DG}\left(1 + \frac{R_W}{R_{DG}}\omega\right)T}{V}dV \quad (11)$$

or

$$\frac{\left(\frac{u_{SHS} - h_W}{T}\right)d\omega}{R_{DG}\left(1 + \frac{R_W}{R_{DG}}\omega\right)} = -\frac{dV}{V} \quad (11a)$$

This equation may be simplified further by introducing the modified specific humidity ( $\omega^*$ ) parameter, defined by

$$\omega^* = 1 + \frac{R_W}{R_{DG}}\omega \quad (12a)$$

By employing this new parameter, Eq. (2) is transformed into

$$\omega^* = \frac{1}{1 - \frac{\phi p_{SAT}(T)}{p}} \quad (12b)$$

In other words, the modified humidity parameter depends only on the mixture state (pressure, temperature and relative humidity) and the saturation curve of the water steam. The parameter is not influenced by the “Dry Gas” mixture composition. Eq. (11a) becomes:

$$a_V \frac{d\omega^*}{\omega^*} = -\frac{dV}{V} \quad (13)$$

where

$$a_V = \frac{u_{SHS} - h_W}{R_W T} \quad (14)$$

Parameter  $a_V$  does not depend on the “Dry Gas” composition either. For a given mixture temperature ( $T$ ) and injected water conditions ( $h_W$ ), the parameter  $a_V$  remains constant throughout an isothermal compression process. Hence Eq. (13) may be integrated between stations 5 (the initial state) and 1 (the final state) to give

$$\left(\frac{\omega_1^*}{\omega_5^*}\right) = \left(\frac{V_5}{V_1}\right)^{\frac{1}{a_V}} \quad (15)$$

The above equation holds for any (perfect gas mixture of) flue gasses. It provides the rate of the injection that sustains the isothermal compression. By assuming that the mixture is always composed of perfect gasses, it is quite straightforward to relate the humidity ratio to the corresponding pressure one, since

$$\frac{V_{1,*} p_1}{V_5 p_5} = \frac{m_{DG} R_{DG} \left(1 + \frac{R_W}{R_{DG}} \omega_1\right) T}{m_{DG} R_{DG} \left(1 + \frac{R_W}{R_{DG}} \omega_5\right) T} = \frac{\omega_1^*}{\omega_5^*} \quad (16)$$

The combination of Eqs. (15) and (16) leads to the following polytropic relationship for the isothermal compression through regulated water injection

$$p_1 V_1^{b_V} = p_5 V_5^{b_V} = p V^{b_V} \quad (17)$$

where

$$b_V = 1 + \frac{1}{a_V} \quad (18)$$

Eq. (17) shows that the isothermal compression through the action of a regulated water injection maintains the form of a polytropic relationship, instead of the logarithmic one observed in the corresponding dry (perfect) gas process. The  $a_V(T)$  and  $b_V(T)$  functions are illustrated in Fig. 7 (for the temperature range expected in Braysson cycle applications). Typical values are  $a_V = 16.5$  and  $b_V = 1.06$ . It is clear that the order of magnitude of the two parameters will not be affected significantly ( $a_V$  has a small tendency to increase as  $T$  drops) within the expected temperature range. These values imply that the pressure–volume relationship is much closer to the dry gas isothermal rather than the corresponding iso-saturated or the isentropic ones. Relationships (15) and (17) are valid only as long as the mixture does not exceed the saturation condition during the entire compression process. From a practical point of view this means that the mixture may reach the saturation state just at the end of the compression. At this limiting state the specific humidity will be given by:

$$\omega_{1,SAT} = \frac{R_{DG}}{R_W} \frac{p_{SAT}(T)}{p_1 - p_{SAT}(T)} \quad (19)$$

The corresponding modified (humidity) parameter becomes

$$\omega_{1,SAT}^* = \omega_{SAT}^*(T) = 1 + \frac{p_{SAT}(T)}{p - p_{SAT}(T)} = \frac{1}{1 - \frac{p_{SAT}(T)}{p}} \quad (20)$$

In combination with Eqs. (15) and (18), this leads to the following relationship

$$\frac{p}{(\omega^*)^{1+a_V}} = \frac{p}{(\omega^*)^{b_V/(b_V-1)}} = \text{const.} \quad (21)$$

By referring to Fig. 3, Eqs. (20) and (21) define the limiting starting conditions (i.e. pressure  $p_5$  and modified humidity  $\omega_5^*$ ) of the compression for which the isothermal compression (through regulated water injection) will terminate (at point 1) with

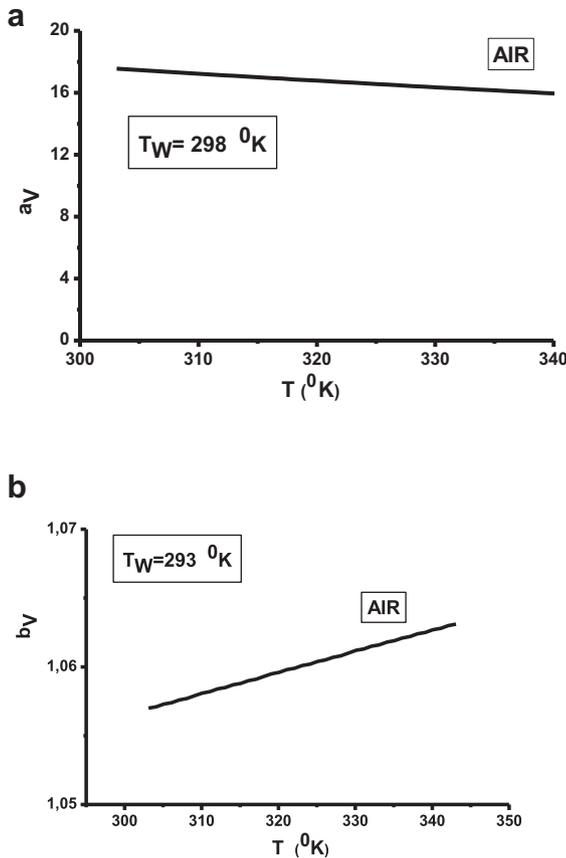


Fig. 7. The influence of the compression temperature on the parameters  $a_v$  and  $b_v$ . (a) The parameter  $a_v$ . (b) The parameter  $b_v$ .

a saturated gaseous mixture. In other words, if  $p_5$  is fixed as well, the modified humidity  $\omega_5^*$  should not exceed a certain value. The relationship  $(\omega_5^*)_{\text{LIMIT}} = f(p_5, T)$  is illustrated in Fig. 8 for the case  $p_1 = 1$  bar. Mathematically, this is given by the equation

$$\begin{aligned}
 (\omega_5^*)_{\text{LIMIT}} &= \omega_{1,\text{SAT}}^* \left(\frac{p_5}{p_1}\right)^{a_v+1} = \frac{1}{1 - \frac{p_{\text{SAT}}(T_5)}{p_1}} \left(\frac{p_5}{p_1}\right)^{a_v+1} \\
 &= \frac{1}{1 - \frac{p_{\text{SAT}}(T_5)}{p_5}} \left(\frac{p_5}{p_1}\right)^{a_v} \quad (22)
 \end{aligned}$$

In practice, due to  $\text{H}_2\text{SO}_4$  condensation considerations the temperature during the compression process will be increased by

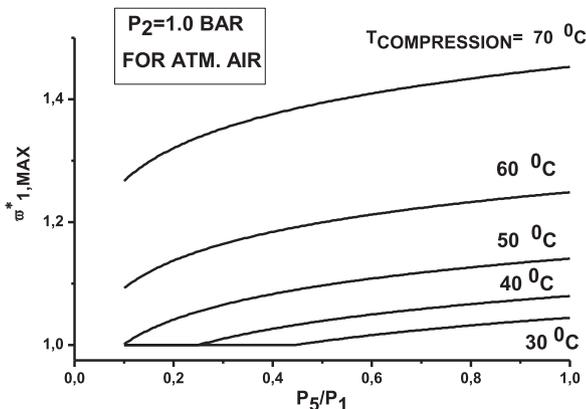


Fig. 8. The maximum (modified) humidity at the start of the compression.

5–10 K (for the case of a conventional Power Plant based on a Gas Turbine Topping cycle Plant). For the typical flue gas composition of the Natural Gas burning Gas Turbines (mole fractions of the order of 0.06 for the water and  $10^{-5}$  for the  $\text{SO}_3$ ), the Okkes [17] equation predicts dew point temperatures of the order of  $70 \text{ }^{\circ}\text{C}$  (i.e. 343 K), when the expansion reaches the 0.1 bar level. This implies that the low-pressure (bottoming) turbine is not expected to face any serious sulfuric acid corrosion problem. The isothermal compressor, however, must be designed appropriately. Of course, for Topping cycle machines operating on a smaller  $T_3$  these limitations will not apply.

The efficiency evaluation of the proposed Braysson plant starts with the calculation of the “coefficient of performance” ( $\text{cop}_v$ ) for the isothermal compressors in the exhaust system. This parameter compares the work consumed for the entire exhaust process (volume charging–isothermal compression–discharging to the atmosphere) by employing water injection ( $W_{V51}$ ) over the corresponding work when the isothermal compression does not employ water cooling ( $W_{\text{DRY}}$ ). The first work is given by

$$\text{cop}_v = \frac{\Delta W_{V51}}{\Delta W_{\text{DRY}}} = \frac{Y - 1}{\ln(Y)} \quad (23)$$

The  $\text{cop}_v$  represents the work rise due to the injection (actually is defined as the ratio of the compression work in the presence of the injection over that of the isothermal compression of the dry gasses alone). Now, the work done for the wet mixture compression is given by

$$\begin{aligned}
 W_{V51} &= p_5 v_5 - \left( - \int_{V_5}^{V_1} p dv \right) - p_1 v_1 \\
 &= (\omega_5^* - \omega_1^*) MRT_5 - \frac{p_1 V_1}{b_v - 1} \left\{ \left(\frac{V_1}{V_5}\right)^{b_v - 1} - 1 \right\} \\
 &= (\omega_5^* - \omega_1^*) MRT_5 - \frac{m_1 R_1 T_5}{b_v - 1} \{ r^{b_v - 1} - 1 \} \quad (24)
 \end{aligned}$$

In the above equation  $r = v_5/v_1$  and  $m_i R_i T = \omega_i MRT$ , where  $M$  represents the mass of the dry flue gases while  $R$  their gas constant. The work transfer ( $W_{\text{DRY}}$ ) is calculated the same way, except for the fact that the isothermal compression of the constant composition mixture obeys the  $p v = \text{constant}$  rule. Of course the second process reaches the same exit pressure ( $p_1$ ) but (as a result) the final volume will be smaller than  $v_1$ . It is a straightforward process to prove that the ratio between these two work transfers is equal to

$$\text{cop}_v = \frac{W_{V51}}{W_{\text{DRY}}} = \frac{Y - 1}{\ln(Y)} \quad (25)$$

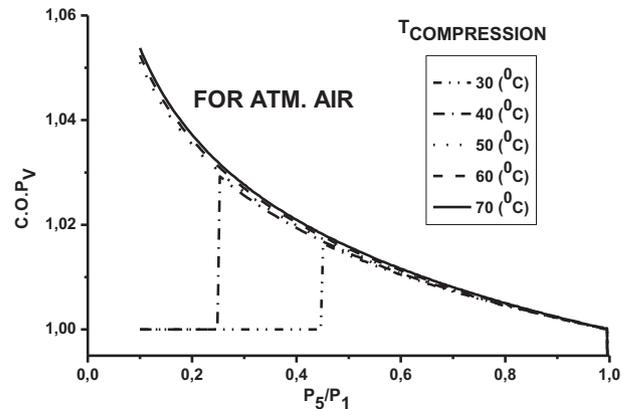


Fig. 9. The coefficient of performance.

where  $Y = r^{b_V-1}$ . The relationship  $\text{cop}_V(Y)$  is illustrated in Fig. 9. It is quite apparent that the  $\text{cop}_V$  is dominated by the required compression ratio. The influence of the temperature during the compression plays a much smaller role. At a given temperature there exists a (low) limit for the starting pressure ( $p_5$ ). Below it the humid gas mixture saturates, hence no water evaporation may take place. The water injection increases the work consumed for the exhaust process by a very small amount (less than 4%) over that of the “dry” isothermal process.

The efficiency of the entire Braysson plant may be evaluated by the equation

$$\eta_{\text{BRS}} = \frac{\text{NET WORK}}{\text{HEAT INPUT}} = \frac{W_{35} - W_{12} - W_{V51}}{Q_{32}} = \frac{\omega_5^* \text{MRT}_1 \left( \frac{\gamma}{\gamma-1} (\Theta - \theta) - \text{cop}_V \ln(p_1/p_5) \right)}{\omega_5^* \text{MRT}_1 (\Theta - \theta) \left( \frac{\gamma}{\gamma-1} \right)} \quad (26)$$

The parameters  $\Theta$  and  $\theta$  were defined in Section 2.3. By employing the parameter  $X = \Theta/\theta$  (as in Eq. (3)), it is quite simple to transform Eq. (26) into

$$\eta_{\text{BRS}} = 1 - \frac{1}{\theta} \frac{\ln(X^{\text{cop}_V})}{X-1} = 1 - \frac{f_1(X)}{\theta} \quad (27)$$

Since

$$\frac{p_1}{p_5} = \frac{p_1}{p_2} \frac{p_2}{p_3} \frac{p_3}{p_4} = \left( \frac{\Theta}{\theta} \right)^{\frac{\gamma}{\gamma-1}} \quad (28)$$

the pressure ratio  $p_1/p_5$  becomes a function of the one of the (free)  $\theta$  parameters, if the other is fixed. Usually it is  $\Theta$  that becomes fixed, as a result of the technological limits imposed by the cooling of the blades. For typical Gas Turbine compression ratios of the order of 30:1 (i.e.  $\theta = 2.65$ ) the corresponding  $p_5/p_1 = 0.06$  as long as  $\Theta = 6$ . Most conventional power generating Gas Turbines operate with a high temperature around 1550 K (i.e.  $\Theta = 5.35$ ), so that a minimum pressure ( $p_5$ ) around the 0.1 bar is the expected operational limit.

The variation of the plant (ideal) efficiency is illustrated in Fig. 10, where  $\Theta = 6$ . In addition to the efficiency of the proposed Braysson cycle version, this figure includes the corresponding efficiencies of the conventional Braysson cycle and the Brayton cycle with the same  $\theta$  value (i.e. Gas Turbine compression ratio). Clearly, the modified cycle efficiency is very close to that of the conventional Braysson one and much higher than the simple Brayton cycle.

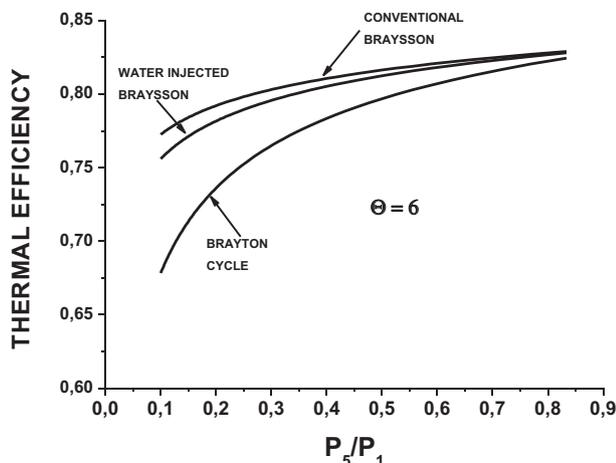


Fig. 10. The injection effect on the cycle thermal efficiency.

## 4. Conclusions

The ideal Braysson cycle was estimated for the case of employing regulated water injection rate for the isothermal compression process of the cycle. For the expected parameter range in typical applications, the analysis lead to a polytropic relationship for the process of the form  $pV^\beta = \text{constant}$ , where  $\beta \approx 1.06$ . The analysis predicts a very small reduction in the thermal efficiency of the cycle against that of the conventional Braysson cycle and much larger than the corresponding Brayton (gas turbine) one.

## Nomenclature

$a_V$	constant (Eq. (14))
$b_V$	constant (Eq. (18))
$c_p$	specific heat (at constant pressure)
$c_V$	specific heat (at constant volume)
$\text{cop}_V$	coefficient of performance (Eq. (23))
$H$	enthalpy
$h$	specific enthalpy
$P$	pressure
$Q$	heat transfer
$r$	compression ratio
$R$	gas constant
$s$	entropy
$T$	temperature
$U$	internal energy
$u$	specific internal energy
$V$	volume
$W$	work transfer
$X$	parameter (Eq. (5))
$Y$	parameter (Eq. (25))

## Greek

$\gamma$	the ratio $c_p/c_V$
$\eta$	efficiency
$\theta$	temperature ratio in the compression (Eq. (5))
$\Theta$	temperature ratio (Eq. (5))
$\varphi$	relative humidity (Eq. (12b))
$\omega$	specific humidity
$\omega^*$	modified specific humidity (Eq. (12a))

## Subscripts

1, 2	endpoints of the compression process
DG	dry gas mixture
fg	evaporation
SAT	saturation
SHS	superheated
TH	thermal
W	water

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