Article

Journal of Vibration and Control
17(7) 1009–1020

DOI: 10.1177/1077546310367890
jvc.sagepub.com

Discrete-time adaptive sliding mode control for a class of uncertain time delay systems

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Abstract

This paper is concerned with a discrete-time adaptive sliding mode control for a class of uncertain time delay systems. It is assumed that the dynamic systems are described by a discrete-time time delay state equation with mismatched uncertainties, and that the states are measured in the contamination with independent random noises. The augmented state equations are derived based on the state of the time delay, and the weighted extended Kalman filter and the weighted least squares estimator to take the estimates for the augmented states and the uncertainties are proposed. The discrete-time adaptive sliding mode control is designed using the integral-type sliding surface and the output information obtained from the estimators. It is verified that the estimation errors converge to zero as time increases, and the states for the dynamic systems are ultimately bounded under the action of the proposed adaptive sliding mode control. The effectiveness of the proposed method is indicated by the simulation experiment in a simple numerical example.

Keywords

Adaptive sliding mode control, uncertain time delay system, weighted extended Kalman filter, weighted least squares estimator

Received: 17 February 2009; accepted: 2 March 2010

1. Introduction

The sliding mode control (SMC), which is a particular mode of the variable structure control (VSC), is very effective for designing robust controllers for dynamic systems in the presence of uncertainties. Although most parts of the methodologies of the SMC have been presented on the fact that the dynamic systems are insensitive to the uncertainties satisfying the matching condition, it is frequently encountered in realistic situations that the uncertainties do not satisfy the matching condition, and the complete information of the states for the dynamic systems is untenable by the measurement restriction of available sensors and/or by the contamination with measurement noises. Numerous methodologies and their applications of the SMC have been devoted to dynamic systems with uncertainties (Edwards and Spurgeon, 1998; Utkin et al., 1999; Cheng et al., 2000; Furuta and Pan, 2000; Feng et al., 2002; Bartolini et al., 2003; Edwards et al., 2003; Wang et al., 2003; Efe et al., 2004; Huang and Chen, 2004; Hwang, 2004; Chen, 2006; Janardhanan and Bandyopadhyay, 2006; Lai et al., 2007; Yoshimura, 2008).

The researches for the dynamic systems with time delays are considerably important because time delays frequently exist in various engineering systems such as automotive systems, electrical networks, aircraft systems, nuclear reactors, pneumatic and hydraulic systems, biological systems, chemical processes, etc. The existence of the time delays causes system instability and the degraded control performance as a default so that considerable attention is directed to the control of such dynamic systems. Various kinds of methodologies of the SMC for dynamic systems with time delays have been recently presented as follows. The methodology of the SMC has been developed in dynamic systems with

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unknown constant or time-varying time delays via the linear matrix inequalities (LMIs) (Gouaisbaut et al., 2002; Xia and Jia, 2003; Niu et al., 2005; Xia et al., 2008). Some approaches of the adaptive SMC have been presented for uncertain time delay systems (Chou and Cheng, 2001; Li and Decarlo, 2003; Qucheriah, 2005). Moreover, the observer-based SMC (Niu et al., 2004) and the memory-less state feedback SMC (Hua et al., 2008) have been proposed for such systems. However, in spite of engineering importance, there are few investigations related to the adaptive SMC for discrete-time uncertain time delay systems.

This paper proposes a discrete-time adaptive SMC for a class of uncertain time delay systems. It is assumed that the dynamic systems are described by a time delay state equation with mismatched uncertainties, and the complete information of the states is untenable by the measurement restriction of available sensors and by the contamination with independent random noises. The augmented state equation is derived on the basis of the time delay of the states, and the weighted extended Kalman filter (WEKF) and the weighted least squares estimator (WLSE) to take the estimates for the augmented states and the uncertainties are designed. The discrete-time adaptive SMC is developed by using the integral-type sliding surface and the output information obtained from the estimators. It is verified that the estimation errors obtained from the WEKF converge to zero as time increases, and the states for the time delay systems are ultimately bounded under the action of the proposed adaptive SMC. It is assumed that the time delay is known so that the robustness concerned with the amounts of the time delay is not discussed. However, the adaptive or robust SMC for a class of discrete-time uncertain systems with unknown constant or time-varying time delays is left to future investigations.

2. Description of discrete-time uncertain time delay systems

Consider the dynamic systems described by a discrete-time uncertain time delay state equation with mismatched uncertainties where the time delay l represents a known positive integer as follows.

$$\begin{gathered}\mathbf{x}(k+1)=\mathbf{F}\mathbf{x}(k)+\mathbf{F}_l\mathbf{x}(k-l)+\mathbf{G}\mathbf{u}(k)+\mathbf{f}(\mathrm{x},k)+\mathbf{w}(k)\\ \mathbf{x}(k)=\mathbf{x}_0(k)\qquad k\in[-l,\ 0],\quad \mathbf{u}(0)=\mathbf{u}_0(0)\end{gathered}\tag{1}$$

The following notations are used in equation (1): $\mathbf{x}(k)\in\mathbf{R}^n$ and $\mathbf{x}(k-l)\in\mathbf{R}^n$ are, respectively, the state vectors; $\mathbf{u}(k)\in\mathbf{R}^m$ is the control vector $(m\le n)$; $\mathbf{x}_0(k)\in\mathbf{R}^n$ and $\mathbf{u}_0(0)\in\mathbf{R}^m$ represent, respectively, the initial conditions for $\mathbf{x}(k)$ and $\mathbf{u}(k)$; $\mathbf{w}(k)\in\mathbf{R}^n$ represents unknown bounded and persistent disturbance vector; $\mathbf{F}\in\mathbf{R}^{n\times n}$, $\mathbf{F}_l\in\mathbf{R}^{n\times n}$ and $\mathbf{G}\in\mathbf{R}^{n\times m}$ are, respectively, known constant matrices; $\mathbf{f}(\mathbf{x},k)\in\mathbf{R}^n$ represents the unknown modeling error vector. It is assumed that the pair $(\mathbf{F},\mathbf{G})$ is controllable, and that mismatched uncertainties are, respectively, expressed in a parameterized form as

$$\mathbf{f}(\mathbf{x},k)=\boldsymbol{\Phi}(\mathbf{x},k)\boldsymbol{\alpha}\tag{2}$$

$$\mathbf{w}(k)=\boldsymbol{\Psi}(k)\boldsymbol{\beta}\tag{3}$$

where

$$\boldsymbol{\Phi}(\mathbf{x},k)=\begin{bmatrix}\boldsymbol{\varphi}_1^T(\mathbf{x},k) & \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T\\ \mathbf{0}^T & \boldsymbol{\varphi}_2^T(\mathbf{x},k) & \mathbf{0}^T & \cdots & \cdots\\ \cdots & \mathbf{0}^T & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \mathbf{0}^T\\ \mathbf{0}^T & \cdots & \cdots\ \mathbf{0}^T & & \boldsymbol{\varphi}_n^T(\mathbf{x},k)\end{bmatrix}$$

$$\boldsymbol{\alpha}=\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \vdots\\ \alpha_n\end{bmatrix}$$

$$\boldsymbol{\Psi}(k)=\begin{bmatrix}\boldsymbol{\psi}_1^T(k) & \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T\\ \mathbf{0}^T & \boldsymbol{\psi}_2^T(k) & \mathbf{0}^T & \cdots & \cdots\\ \cdots & \mathbf{0}^T & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \mathbf{0}^T\\ \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T & \boldsymbol{\psi}_n^T(k)\end{bmatrix}$$

$$\boldsymbol{\beta}=\begin{bmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\beta}_2\\ \vdots\\ \vdots\\ \boldsymbol{\beta}_n\end{bmatrix}$$

The $\boldsymbol{\Phi}(\mathbf{x},k)\in\mathbf{R}^{n\times q_1}$ and $\boldsymbol{\Psi}(k)\in\mathbf{R}^{n\times q_2}$ are, respectively, known matrices whose elements are, respectively, expressed as sets of state function and time-varying function vectors, and $\boldsymbol{\alpha}\in\mathbf{R}^{q_1}$ and $\boldsymbol{\beta}\in\mathbf{R}^{q_2}$ are, respectively, unknown vectors whose components are expressed as sets of constant parameter vectors (Yoshimura, 2008, 2010). Moreover, it is assumed that the vectors included in the above matrices satisfy the following inequalities:

$$\left\|\boldsymbol{\varphi}_i(\mathbf{x},k)\right\|\le L_{1i}\left\|\mathbf{x}(k)\right\|\qquad i=1,\ 2,\ \ldots n\tag{4}$$

$$\left\|\boldsymbol{\psi}_i(k)\right\|\le L_{2i}\qquad i=1,\ 2,\ \ldots n\tag{5}$$

where L_{1i} and L_{2i} are, respectively, positive constants, and $\|\,\|$ denotes the Euclidean norm of vectors. Rewriting equation (1) by using equations (2) and (3) gives

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{F}\mathbf{x}(k)+\mathbf{F}_l\mathbf{x}(k-l)+\mathbf{G}\mathbf{u}(k)+\boldsymbol{\Phi}(\mathbf{x},k)\boldsymbol{\alpha}+\boldsymbol{\Psi}(k)\boldsymbol{\beta}\\ &= \mathbf{F}\mathbf{x}(k)+\mathbf{F}_l\mathbf{x}(k-l)+\mathbf{G}\mathbf{u}(k)+\boldsymbol{\Gamma}(\mathbf{x},k)\boldsymbol{\theta}\end{aligned} \tag{6}$$

where $\boldsymbol{\Gamma}(\mathbf{x},k)\in\mathbf{R}^{n\times q}$ $(q=q_1+q_2)$ and $\boldsymbol{\theta}\in\mathbf{R}^q$ are, respectively, defined as

$$\boldsymbol{\Gamma}(\mathbf{x},k)=\begin{bmatrix}\boldsymbol{\Phi}(\mathbf{x},k) & \boldsymbol{\Psi}(k)\end{bmatrix} \qquad \boldsymbol{\theta}=\begin{bmatrix}\boldsymbol{\alpha}^T & \boldsymbol{\beta}^T\end{bmatrix}^T$$

The measurement equation is in a discrete form taken as

$$\mathbf{y}(k)=\mathbf{H}\mathbf{x}(k)+\mathbf{v}(k) \tag{7}$$

where $\mathbf{y}(k)\in\mathbf{R}^p$ is the measurement vector $(p\le n)$, $\mathbf{v}(k)\in\mathbf{R}^p$ is the zero-mean independent measurement noise vector, and $\mathbf{H}\in\mathbf{R}^{p\times n}$ is the known constant matrix satisfying the pair $(\mathbf{F},\ \mathbf{H})$ to be observable.

3. Discrete-time adaptive SMC

To design the discrete-time adaptive SMC, the augmented state vector $\mathbf{x}(k)\in\mathbf{R}^{(l+1)n}$ is defined as

$$\mathbf{x}_a(k)=[\mathbf{x}^T(k-l)\ \mathbf{x}^T(k-l+1)\cdots\cdots\mathbf{x}^T(k-1)\ \mathbf{x}^T(k)]^T$$

so that the augmented state and measurement equations are, respectively, obtained as

$$\mathbf{x}_a(k+1)=\mathbf{F}_a\mathbf{x}_a(k)+\mathbf{G}_a\mathbf{u}(k)+\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)\boldsymbol{\theta} \tag{8}$$

$$\mathbf{y}(k)=\mathbf{H}_a\mathbf{x}_a(k)+\mathbf{v}(k) \tag{9}$$

where $\mathbf{F}_a\in\mathbf{R}^{(l+1)n\times(l+1)n}$, $\mathbf{G}_a\in\mathbf{R}^{(l+1)n\times m}$, $\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)\in\mathbf{R}^{(l+1)n\times q}$ and $\mathbf{H}_a\in\mathbf{R}^{p\times(l+1)n}$ are, respectively, defined as

$$\mathbf{F}_a=\begin{bmatrix}\mathbf{O}&\mathbf{I}&\mathbf{O}&\cdot&\cdot&\cdot&\mathbf{O}\\ \cdot&\mathbf{O}&\mathbf{I}&\mathbf{O}&\cdot&\cdot&\cdot\\ \cdot&\cdot&\mathbf{O}&\cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\mathbf{O}\\ \mathbf{O}&\cdot&\cdot&\cdot&\cdot&\mathbf{O}&\mathbf{I}\\ \mathbf{F}_l&\mathbf{O}&\cdot&\cdot&\cdot&\mathbf{O}&\mathbf{F}\end{bmatrix} \qquad \mathbf{G}_a=\begin{bmatrix}\mathbf{O}\\ \mathbf{O}\\ \cdot\\ \cdot\\ \cdot\\ \mathbf{O}\\ \mathbf{G}\end{bmatrix}$$

$$\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)=\begin{bmatrix}\mathbf{O}\\ \mathbf{O}\\ \cdot\\ \cdot\\ \cdot\\ \mathbf{O}\\ \boldsymbol{\Gamma}(\mathbf{x},k)\end{bmatrix} \qquad \mathbf{H}_a=\begin{bmatrix}\mathbf{O}&\mathbf{O}&\cdot&\cdot&\cdot&\mathbf{O}&\mathbf{H}\end{bmatrix}$$

The sliding surface is proposed as

$$\boldsymbol{\sigma}(k)=\mathbf{S}\mathbf{x}_a(k)-\mathbf{S}\sum_{i=0}^{k-1}(\mathbf{F}_a+\mathbf{G}_a\mathbf{T}-\mathbf{I})\mathbf{x}_a(i) \tag{10}$$

where $\boldsymbol{\sigma}(k)\in\mathbf{R}^m$ is the sliding surface vector, $\mathbf{S}\in\mathbf{R}^{m\times(l+1)n}$ and $\mathbf{T}\in\mathbf{R}^{m\times(l+1)n}$ are, respectively, constant sliding coefficient matrices, and $\mathbf{I}\in\mathbf{R}^{(l+1)n\times(l+1)n}$ is the unit matrix. The proposed sliding surface $\boldsymbol{\sigma}(k)$ is derived based on the integral-type sliding surface in continuous systems (Sam et al., 2004; Niu et al., 2005).

It is assumed that the discrete-time adaptive SMC $\mathbf{u}_s(k)$ is composed of two kinds of controls given as

$$\mathbf{u}_s(k)=\mathbf{u}_e(k)+\mathbf{u}_n(k) \tag{11}$$

where the first and second terms on the right hand of equation (11), respectively, correspond to the equivalent and switching controls. The equivalent control is obtained by substituting equation (8) into the time difference of $\boldsymbol{\sigma}(k)$ defined as $\Delta\sigma(k+1)=\boldsymbol{\sigma}(k+1)-\boldsymbol{\sigma}(k)$ and by setting the resultant equation to zero. Then, $\mathbf{u}_e(k)$ is obtained as

$$\mathbf{u}_e(k)=\mathbf{T}\mathbf{x}_a(k)-(\mathbf{S}\mathbf{G}_a)^{-1}\mathbf{S}\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)\boldsymbol{\theta} \tag{12}$$

According to the SMC in continuous uncertain systems, the switching control $\mathbf{u}_n(k)$ is assumed to be

$$\mathbf{u}_n(k)=-\bar{u}_n(\mathbf{x}_a,k)(\mathbf{S}\mathbf{G}_a)^{-1}\frac{\boldsymbol{\sigma}(k)}{\|\boldsymbol{\sigma}(k)\|} \tag{13}$$

where $\bar{u}_n(\mathbf{x}_a,k)$ is a positive function of $\mathbf{x}_a(k)$. Hence, substituting equations (12) and (13) into equation (11) provides $\mathbf{u}_s(k)$ as

$$\begin{aligned}\mathbf{u}_s(k)&=\mathbf{T}\mathbf{x}_a(k)-(\mathbf{S}\mathbf{G}_a)^{-1}\mathbf{S}\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)\boldsymbol{\theta}\\ &\quad-\bar{u}_n(\mathbf{x}_a,k)(\mathbf{S}\mathbf{G}_a)^{-1}\frac{\boldsymbol{\sigma}(k)}{\|\boldsymbol{\sigma}(k)\|}\end{aligned} \tag{14}$$

Unlike to the SMC in continuous uncertain systems, it is generally not easy for the states of the discrete-time systems to be maintained completely on the sliding surface under the action of the SMC even if the switching control is included in equation (14), because the control signal is generated only at a certain sampling instant and is kept over the entire sampling period. Therefore, assuming that $\bar{u}(\mathbf{x}_a,k)$ is $\gamma\|\boldsymbol{\sigma}(k)\|$ where γ denotes constant, equation (14) is rewritten as

$$\mathbf{u}_s(k)=\mathbf{T}\mathbf{x}_a(k)-(\mathbf{S}\mathbf{G}_a)^{-1}\left[\mathbf{S}\boldsymbol{\Gamma}_a(\mathbf{x}_a,k)\boldsymbol{\theta}+\gamma\boldsymbol{\sigma}(k)\right] \tag{15}$$

Since $\mathbf{x}(k)$, $\boldsymbol{\theta}$ and $\boldsymbol{\sigma}(k)$ included in equation (15) cannot be directly measured, the variables are, respectively, replaced by their estimates, $\hat{\mathbf{x}}(k)$, $\hat{\boldsymbol{\theta}}(k)$ and $\hat{\boldsymbol{\sigma}}(k)$ where these are obtained as a set of the measurements $\{\mathbf{y}(1), \mathbf{y}(2)\cdots\cdots\mathbf{y}(k)\}$ is given. Then, the discrete-time adaptive SMC $\mathbf{u}_s(k)$ is designed as

$$\mathbf{u}_s(k) = \mathbf{T}\hat{\mathbf{x}}_a(k) - (\mathbf{S}\mathbf{G}_a)^{-1}[\mathbf{S}\boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\hat{\boldsymbol{\theta}}(k) + \gamma\hat{\boldsymbol{\sigma}}(k)] \tag{16}$$

where

$$\hat{\boldsymbol{\sigma}}(k) = \mathbf{S}\hat{\mathbf{x}}_a(k) - \mathbf{S}\sum_{i=0}^{k-1}(\mathbf{F}_a + \mathbf{G}_a\mathbf{T} - \mathbf{I})\hat{\mathbf{x}}_a(i|k) \tag{17}$$

and $\hat{\mathbf{x}}_a(i|k)$ included in equation (17) denotes the smoothed estimate for $\mathbf{x}_a(i)$ as a set of the measurements $\{\mathbf{y}(1), \mathbf{y}(2)\cdots\cdots\mathbf{y}(k)\}$ is given. It is a well-known fact that the trajectory of $\mathbf{x}_a(k)$ is kept on the vicinity of $\boldsymbol{\sigma}(k) = \mathbf{0}$ and slides with the direction to the origin if the following inequality is satisfied (Yoshimura, 2008):

$$\boldsymbol{\sigma}^T(k+1)\boldsymbol{\sigma}(k+1) - \boldsymbol{\sigma}^T(k)\boldsymbol{\sigma}(k) < 0 \tag{18-1}$$

or equivalently,

$$\boldsymbol{\sigma}^T(k)\Delta\boldsymbol{\sigma}(k+1) + \frac{1}{2}\left\|\Delta\boldsymbol{\sigma}(k+1)\right\|^2 < 0 \tag{18-2}$$

Substituting equations (8), (10) and (16) into inequality (18-2), the resultant equation becomes

$$\begin{aligned}\boldsymbol{\sigma}^T(k)\Delta\boldsymbol{\sigma}(k+1) &+ \frac{1}{2}\left\|\boldsymbol{\sigma}(k+1)\right\|^2 \\ &= \boldsymbol{\sigma}^T(k)[\boldsymbol{\delta}(k) - \gamma\hat{\boldsymbol{\sigma}}(k)] + \frac{1}{2}\left\|\boldsymbol{\delta}(k) - \gamma\hat{\boldsymbol{\sigma}}(k)\right\|^2 < 0\end{aligned} \tag{19}$$

where

$$\begin{aligned}\boldsymbol{\delta}(k) &= -\mathbf{S}[\mathbf{G}_a\mathbf{T}\tilde{\mathbf{x}}_a(k) + \boldsymbol{\Gamma}_a(\mathbf{x}_a, k)\tilde{\boldsymbol{\theta}}(k) + \tilde{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\hat{\boldsymbol{\theta}}(k)] \\ \tilde{\mathbf{x}}_a(k) &= \mathbf{x}_a(k) - \hat{\mathbf{x}}_a(k), \quad \tilde{\boldsymbol{\theta}}(k) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(k), \\ \tilde{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k) &= \boldsymbol{\Gamma}_a(\mathbf{x}_a, k) - \boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\end{aligned}$$

Assuming that $\hat{\boldsymbol{\sigma}}(k)$ and $\boldsymbol{\delta}(k)$, respectively, approach to $\boldsymbol{\sigma}(k)$ and zero as k increases, inequality (19) becomes

$$\begin{aligned}\boldsymbol{\sigma}^T(k)\Delta\boldsymbol{\sigma}(k+1) &+ \frac{1}{2}\left\|\boldsymbol{\sigma}(k+1)\right\|^2 \\ &= -\frac{1}{2}\gamma(2-\gamma)\boldsymbol{\sigma}^T(k)\boldsymbol{\sigma}(k) < 0\end{aligned} \tag{20}$$

Then, inequality (20) is satisfied if $0 < \gamma < 2$.

4. Design of estimators

4.1. Proposed weighted extended kalman filter (WEKF)

The extended Kalman filter (EKF) is very popular as an estimator to take the estimates for $\mathbf{x}_a(k)$ and $\boldsymbol{\theta}$ in discrete-time uncertain systems as a set of the measurements $\{\mathbf{y}(1), \mathbf{y}(2)\cdots\cdots\mathbf{y}(k)\}$ is given (Jazwinski, 1970). Defining the augmented state vector composed of $\mathbf{x}_a(k)$ and $\boldsymbol{\theta}$ in equation (8), the augmented state and measurement equations are respectively derived as

$$\mathbf{x}_e(k+1) = \mathbf{f}_e(\mathbf{x}_e, k) + \mathbf{G}_e\mathbf{u}(k) \tag{21}$$

$$\mathbf{y}(k) = \mathbf{H}_e\mathbf{x}_e(k) + \mathbf{v}(k) \tag{22}$$

where $\mathbf{x}_e(k) \in \mathbf{R}^{(l+1)n+q}$, $\mathbf{f}_e(\mathbf{x}_e, k) \in \mathbf{R}^{(l+1)n+q}$, $\mathbf{G}_e \in \mathbf{R}^{[(l+1)n+q]\times m}$ and $\mathbf{H}_e \in \mathbf{R}^{p\times[(l+1)n+q]}$ are, respectively, defined as

$$\mathbf{x}_e(k) = \begin{bmatrix}\mathbf{x}_a(k) \\ \boldsymbol{\theta}\end{bmatrix} \quad \mathbf{f}_e(\mathbf{x}_e, k) = \begin{bmatrix}\mathbf{F}_a\mathbf{x}_a(k) + \boldsymbol{\Gamma}_a(\mathbf{x}_a, k)\boldsymbol{\theta} \\ \boldsymbol{\theta}\end{bmatrix}$$

$$\mathbf{G}_e = \begin{bmatrix}\mathbf{G}_a \\ \mathbf{O}\end{bmatrix} \quad \mathbf{H}_e = \begin{bmatrix}\mathbf{H}_a & \mathbf{O}\end{bmatrix}$$

Assuming that the measure to design the EKF is given as

$$\begin{aligned}J_e = \sum_{i=1}^{k}&[\mathbf{y}(i) - \mathbf{H}_e\mathbf{x}_e(i)]^T\mathbf{R}_v^{-1}[\mathbf{y}(i) - \mathbf{H}_e\mathbf{x}_e(i)] \\ &+ [\mathbf{x}_e(0) - \hat{\mathbf{x}}_e(0)]^T\mathbf{P}_e^{-1}(0)[\mathbf{x}_e(0) - \hat{\mathbf{x}}_e(0)]\end{aligned} \tag{23}$$

where $\mathbf{R}_v \in \mathbf{R}^{p\times p}$ is a positive definite constant matrix corresponding to the covariance for $\mathbf{v}(k)$ in equation (22), and $\hat{\mathbf{x}}_e(0) \in \mathbf{R}^{(l+1)n+q}$ and $\mathbf{P}_e(0) \in \mathbf{R}^{[(l+1)n+q]\times[(l+1)n+q]}$ are, respectively, the initial estimate for $\mathbf{x}_e(0)$ and a positive definite matrix corresponding to the covariance for $\mathbf{x}_e(0)$. Then, the EKF is derived by minimizing the measure subject to the constraint of equation (21) and taking a linear approximation to the nonlinearity $\mathbf{f}_e(\mathbf{x}_e, k)$ (Jazwinski, 1970). It is well-known that the EKF has improved performance in the estimation of the states and the uncertainties under the assumption that the nonlinearities included in the augmented state equation (21) are negligibly small, a number of unknown parameters are relatively limited and enough information about the system output is obtained by the measurement equation (22). However, it is frequently experienced that the estimation errors obtained from the EKF are much degraded or diverged if the above situation is not satisfied. The main reason is that the dynamics between the models to design the EKF and the exact systems are very different because the higher order terms of the nonlinearities are ignored in the models. As an approach to improve the degradation or the divergence of the estimation errors due to the EKF, the higher weight for more recent output information has been placed to modify the EKF (Jazwinski, 1970; Yoshimura, 2008, 2010). The approach simply states that $\mathbf{R}_v^{-1}$ is replaced as $\mathbf{exp}[-c_e(k-i)]\mathbf{R}_v^{-1}$ at $(k-i)th$ sampling instant in (23) where c_e is a positive constant denoting the weighting factor of the output measurement information so that the modified EKF is called the weighted extended Kalman filter (WEKF) (Jazwinski, 1970; Yoshimura, 2008, 2010). The proposed WEKF takes the estimate $\hat{\mathbf{x}}_e(k)$ for $\mathbf{x}_e(k)$ as

$$\hat{\mathbf{x}}_e(k+1) = \hat{\mathbf{x}}_e(k+1|k) + \mathbf{K}_e(k+1)[\mathbf{y}(k+1) - \mathbf{H}_e\hat{\mathbf{x}}_e(k+1|k)] \tag{24}$$

$$\hat{\mathbf{x}}_e(k+1|k) = \mathbf{f}_e(\hat{\mathbf{x}}_e, k) + \mathbf{G}_e\mathbf{u}_s(k) \tag{25}$$

$$\mathbf{K}_e(k+1) = \mathbf{exp}(c_e)\mathbf{P}_e(k+1|k)\mathbf{H}_e^T \times [\mathbf{exp}(c_e)\mathbf{H}_e\mathbf{P}_e(k+1|k)\mathbf{H}_e^T + \mathbf{R}_v]^{-1}$$

$$\mathbf{P}_e(k+1) = \mathbf{exp}(c_e)\mathbf{P}_e(k+1|k) - \mathbf{exp}(c_e)\mathbf{K}_e(k+1)\mathbf{H}_e\mathbf{P}_e(k+1|k) \tag{27}$$

$$\mathbf{P}_e(k+1|k) = \mathbf{F}_e(\hat{\mathbf{x}}_e, k)\mathbf{P}_e(k)\mathbf{F}_e^T(\hat{\mathbf{x}}_e, k) \tag{28}$$

where $\mathbf{F}_e(\hat{\mathbf{x}}_e, k) \in \mathbf{R}^{[(l+1)n+q]\times[(l+1)n+q]}$ is defined as the partial derivative of $\mathbf{f}_e(\mathbf{x}_e, k)$ with respect to $\mathbf{x}_e(k)$ evaluated at $\mathbf{x}_e(k) = \hat{\mathbf{x}}_e(k)$, and it is assumed nonsingular and bounded matrix based on the inequalities (4) and (5). It is easily seen that the WEKF is in a structure identical to the EKF if c_e is set to zero. It is verified in Appendix that the estimation errors obtained from the WEKF converge to zero as time k increases. Therefore, $\mathbf{u}_s(k)$ is designed by equations (16) and (17) where the estimates, $\hat{\mathbf{x}}_a(k)$ and $\hat{\boldsymbol{\theta}}(k)$, are obtained by the WEKF, and the smoothed estimate $\hat{\mathbf{x}}_a(i|k)$ is derived by the smoothing equation derived from the WEKF (Meditch, 1969).

4.2. Proposed weighted least squares estimator (WLSE)

It is seen that the dimension of the augmented state vector $\mathbf{x}_e(k)$ is more increased if the time delay l is longer. As a result, the WEKF given by equations (24) to (28) becomes quite complicated in a structure and necessitates much computational load, and hence it is not suitable in a realistic situation because the time delay l is relatively long. Therefore, a weighted least squares estimator (WLSE) that decreases the computational load is proposed for the time delay systems. The proposed WLSE takes the estimates, $\hat{\mathbf{x}}_a(k)$ and $\hat{\boldsymbol{\theta}}(k)$, for $\mathbf{x}_a(k)$ and $\boldsymbol{\theta}$ as

$$\hat{\mathbf{x}}_a(k+1) = \mathbf{F}_a\hat{\mathbf{x}}_a(k) + \mathbf{G}_a\mathbf{u}_s(k) + \boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\hat{\boldsymbol{\theta}}(k+1) \tag{29}$$

$$\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \mathbf{K}_\theta(k+1)[\mathbf{y}(k+1) - \mathbf{H}_a\hat{\mathbf{x}}_a(k+1|k)] \tag{30}$$

$$\hat{\mathbf{x}}_a(k+1|k) = \mathbf{F}_a\hat{\mathbf{x}}_a(k) + \mathbf{G}_a\mathbf{u}_s(k) + \boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\hat{\boldsymbol{\theta}}(k) \tag{31}$$

$$\mathbf{K}_\theta(k+1) = \mathbf{exp}(c_e)\mathbf{P}_\theta(k)\boldsymbol{\Gamma}_a^T(\hat{\mathbf{x}}_a, k)\mathbf{H}_a^T \times [\mathbf{exp}(c_e)\mathbf{H}_a\boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\mathbf{P}_\theta(k)\boldsymbol{\Gamma}_a^T(\hat{\mathbf{x}}_a, k)\mathbf{H}_a^T + \mathbf{R}_v]^{-1} \tag{32}$$

$$\mathbf{P}_\theta(k+1) = \mathbf{exp}(c_e)\mathbf{P}_\theta(k) - \mathbf{exp}(c_e)\mathbf{P}_\theta(k)\boldsymbol{\Gamma}_a^T(\hat{\mathbf{x}}_a, k) \times \mathbf{H}_a^T[\mathbf{exp}(c_e)\mathbf{H}_a\boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\mathbf{P}_\theta(k) \times \boldsymbol{\Gamma}_a^T(\hat{\mathbf{x}}_a, k)\mathbf{H}_a^T + \mathbf{R}_v]^{-1}\mathbf{exp}(c_e)\mathbf{H}_a\boldsymbol{\Gamma}_a(\hat{\mathbf{x}}_a, k)\mathbf{P}_\theta(k) \tag{33}$$

It is easily seen that the proposed WLSE is derived by tending the covariance $\mathbf{P}_x(0)$ for the initial state $\mathbf{x}(0)$ to infinite in the WEKF given by equations (24) to (28), and that the WLSE decreases the computational load more when compared with the WEKF because the corrective gain matrix $\mathbf{K}_e(k)$ in the WLSE can only be computed by using $\mathbf{P}_\theta(k)$. However, it is noted that $\hat{\mathbf{x}}_a(i|k)$ in equation (17) is not obtained because the smoothing equation cannot be derived in the WLSE. Hence, $\hat{\mathbf{x}}_a(i|k)$ is replaced by $\hat{\mathbf{x}}_a(i)$ in equation (17). Then, $\mathbf{u}_s(k)$ is designed by using equation (16) and the equation given as

$$\hat{\boldsymbol{\sigma}}(k) = \mathbf{S}\hat{\mathbf{x}}_a(k) - \mathbf{S}\sum_{i=0}^{k-1}(\mathbf{F}_a + \mathbf{G}_a\mathbf{T} - \mathbf{I})\hat{\mathbf{x}}_a(i) \tag{34}$$

5. System stability and selection of the sliding surface

The trajectory of $\mathbf{x}(k)$ is kept in the vicinity of $\boldsymbol{\sigma}(k) = \mathbf{0}$ and slides with the direction to the origin in finite sampling instant under the action of the proposed adaptive SMC if the sliding surface $\boldsymbol{\sigma}(k)$ is suitably selected. As the dynamic systems described by equation (8) are subject to $\mathbf{u}_s(k)$ given by (16), those are expressed as follows:

$$\mathbf{x}_a(k+1) = (\mathbf{F}_a + \mathbf{G}_a\mathbf{T})\mathbf{x}_a(k) + [\mathbf{I} - \mathbf{G}_a(\mathbf{S}\mathbf{G}_a)^{-1}\mathbf{S}] \times \boldsymbol{\Gamma}_a(\mathbf{x}_a, k)\boldsymbol{\theta} + \boldsymbol{\xi}(k) = \bar{\mathbf{F}}_a\mathbf{x}_a(k) + \bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\boldsymbol{\theta} + \boldsymbol{\xi}(k) \tag{35}$$

where

$$\bar{\mathbf{F}}_a = \mathbf{F}_a + \mathbf{G}_a\mathbf{T} \qquad \bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k) = [\mathbf{I} - \mathbf{G}_a(\mathbf{S}\mathbf{G}_a)^{-1}\mathbf{S}]\boldsymbol{\Gamma}_a(\mathbf{x}_a, k)$$

and

$$\begin{aligned}\boldsymbol{\xi}(k) = &- \mathbf{G}_a\{\gamma(\mathbf{S}\mathbf{G}_a)^{-1}\hat{\boldsymbol{\sigma}}(k) + \mathbf{T}\tilde{\mathbf{x}}_a(k) - (\mathbf{S}\mathbf{G}_a)^{-1} \\ &\times \mathbf{S}[\boldsymbol{\Gamma}_a(\mathbf{x}_a, k)\tilde{\boldsymbol{\theta}}(k) + \tilde{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\hat{\boldsymbol{\theta}}(k)]\}\end{aligned} \tag{36}$$

It is seen that $\boldsymbol{\xi}(k)$ approaches to $-\gamma\mathbf{G}_a(\mathbf{S}\mathbf{G}_a)^{-1}\boldsymbol{\sigma}(k)$ as k increases, and it is bounded because $\boldsymbol{\sigma}(k)$ is kept on the vicinity of $\boldsymbol{\sigma}(k) = \mathbf{0}$.

To verify the stability of the dynamic systems described by equation (35), the Lyapunov function candidate is defined as

$$V_s(k) = \mathbf{x}_a^T(k)\mathbf{P}_s\mathbf{x}_a(k) \tag{37}$$

where $\mathbf{P}_s \in \mathbf{R}^{(l+1)n\times(l+1)n}$ is some symmetric positive definite matrix. Defining the time difference $\Delta V_s(k+1)$ of $V_s(k)$ as

$$\begin{aligned}\Delta V_s(k+1) &= V_s(k+1) - V_s(k) \\ &= \mathbf{x}_a^T(k+1)\mathbf{P}_s\mathbf{x}_a(k+1) - \mathbf{x}_a^T(k)\mathbf{P}_s\mathbf{x}_a(k)\end{aligned} \tag{38}$$

the dynamic systems described by equation (35) are stable if $\Delta V_s(k+1)$ is always negative except at $\mathbf{x}_a(k) = \mathbf{0}$. Substituting equation (35) into equation (38) yields

$$\begin{aligned}\Delta V_s(k+1) = &-\mathbf{x}_a^T(k)(\mathbf{P}_s - \bar{\mathbf{F}}_a^T\mathbf{P}_s\bar{\mathbf{F}}_a)\mathbf{x}_a(k) \\ &+ \boldsymbol{\theta}^T\bar{\boldsymbol{\Gamma}}_a^T(\mathbf{x}_a, k)\mathbf{P}_s\bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\boldsymbol{\theta} \\ &+ \boldsymbol{\xi}^T(k)\mathbf{P}_s\boldsymbol{\xi}(k) + 2\mathbf{x}_a^T(k)\bar{\mathbf{F}}_a^T\mathbf{P}_s\bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\boldsymbol{\theta} \\ &+ 2\mathbf{x}_a^T(k)\bar{\mathbf{F}}_a^T\mathbf{P}_s\boldsymbol{\xi}(k) + 2\boldsymbol{\theta}^T\bar{\boldsymbol{\Gamma}}_a^T(\mathbf{x}_a, k)\mathbf{P}_s\boldsymbol{\xi}(k)\end{aligned} \tag{39}$$

Therefore, the first term on the right hand of (39) means that there exist some symmetric positive definite matrix $\mathbf{Q}_s$ satisfying

$$\mathbf{P}_s - \bar{\mathbf{F}}_a^T\mathbf{P}_s\bar{\mathbf{F}}_a = \mathbf{Q}_s \tag{40-1}$$

if the term is always negative except at $\mathbf{x}_a(k) = \mathbf{0}$. Using the inequalities (4) and (5), the 2nd–6th terms on the right hand of (39) are, respectively, denoted as the following inequalities:

$$\boldsymbol{\theta}^T\bar{\boldsymbol{\Gamma}}_a^T(\mathbf{x}_a, k)\mathbf{P}_s\bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\boldsymbol{\theta} \le C_{11}\|\mathbf{x}_\mathrm{a}(k)\|^2 + C_{21}\|\mathbf{x}_\mathrm{a}k)\| + C_{31} \tag{40-2}$$

$$\boldsymbol{\xi}^T(k)\mathbf{P}_s\boldsymbol{\xi}(k) \le C_{32} \tag{40-3}$$

$$\left|2\mathbf{x}_a^T(k)\bar{\mathbf{F}}_a^T\mathbf{P}_s\bar{\boldsymbol{\Gamma}}_a(\mathbf{x}_a, k)\boldsymbol{\theta}\right| \le C_{13}\|\mathbf{x}_\mathrm{a}(k)\|^2 + C_{23}\|\mathbf{x}_\mathrm{a}(k)\| \tag{40-4}$$

$$\left|2\mathbf{x}_a^T(k)\bar{\mathbf{F}}_a^T\mathbf{P}_s\boldsymbol{\xi}(k)\right| \le C_{24}\|\mathbf{x}_a(k)\| \tag{40-5}$$

$$\left|2\boldsymbol{\theta}^T\bar{\boldsymbol{\Gamma}}_a^T(\mathbf{x}_a, k)\mathbf{P}_s\boldsymbol{\xi}(k)\right| \le C_{25}\|\mathbf{x}_a(k)\| + C_{35} \tag{40-6}$$

where the coefficients, $C_{11} \sim C_{35}$, in (40-2) to (40-6) are, respectively, positive constants. Using (40-1) to (40-6), $\Delta V_s(k+1)$ becomes

$$\Delta V_s(k+1) \le -[\lambda_{\min}(\mathbf{Q}_s) - C_1]\|\mathbf{x}_a(k)\|^2 + C_2\|\mathbf{x}_a(k)\| + C_3 \tag{41}$$

where $\lambda_{\min}(\mathbf{Q}_s)$ is the minimum eigenvalue of $\mathbf{Q}_s$, and

$$C_1 = C_{11} + C_{13} \quad C_2 = C_{21} + C_{23} + C_{24} + C_{25}$$
$$C_3 = C_{31} + C_{32} + C_{35}$$

It is seen on the right hand of equation (41) that $\Delta V_s(k+1)$ is always negative if the following inequalities are satisfied:

$$\lambda_{\min}(\mathbf{Q}_s) > C_1 \tag{42-1}$$

$$\begin{aligned}\|\mathbf{x}_a(k)\| &> \frac{C_2 + \sqrt{C_2^2 + 4[\lambda_{\min}(\mathbf{Q}_s) - C_1]C_3}}{2[\lambda_{\min}(\mathbf{Q}_s) - C_1]} \\ &= C_V\end{aligned} \tag{42-2}$$

Then, the trajectory of $\mathbf{x}_a(k)$ approaches with the direction to the origin on the vicinity of $\boldsymbol{\sigma}(k) = \mathbf{0}$ as k increases if $\Delta V_s(k+1)$ is negative, but it lies inside of the domain centered at the origin given by $\|\mathbf{x}_a(k)\| \le C_V$ if $\Delta V_s(k+1)$ is positive or zero. Therefore, the inequality (42-1) corresponds to the stability condition denoting that $\mathbf{x}_a(k)$ is ultimately bounded. Without loss of generality, the inequalities (40-2) to (40-6) can be evaluated by partitioning $\mathbf{G} = [\mathbf{O}^T\ \mathbf{G}_2^T]^T$ in equation (6) where $\mathbf{G}_2 \in \mathbf{R}^{m\times m}$ and $\mathbf{O} \in \mathbf{R}^{n\times m}$ are respectively nonsingular and null matrices (Yoshimura, 2008).

It is not easy to select suitably $\mathbf{T}$ so as to satisfy (42-1) if the dimension of $\mathbf{x}_a(k)$ is relatively large. Therefore, $\mathbf{T}$ is selected as the steady-state control gain matrix in the LQ (Linear Quadratic) control (Meditch, 1969) where the pair ($\mathbf{F}_a$, $\mathbf{G}_a$) is assumed controllable. It is derived by minimizing the given performance index with respect to the control subject to the constraint of the system dynamics. The performance index is assumed as

$$J_c = \frac{1}{N}\sum_{k=1}^{N}\left[\mathbf{x}_a^T(k)\mathbf{Q}_c\mathbf{x}_a(k) + \mathbf{u}^T(k-1)\mathbf{R}_c\mathbf{u}(k-1)\right] \quad (43)$$

where $\mathbf{Q}_c \in \mathbf{R}^{(l+1)n\times(l+1)n}$ and $\mathbf{R}_c \in \mathbf{R}^{m\times m}$ are, respectively, some positive semi-definite and positive definite matrices denoting the trade-off between the state and the control, and N is the total sampling number. Minimizing J_c with respect to $\mathbf{u}(k)$ subject to the constraint of equation (35) where the second and third terms on the right hand of the equation are ignored, the LQ control $\mathbf{u}_L(k)$ is obtained as $\mathbf{u}_L(k) = \mathbf{T}(k)\mathbf{x}_a(k)$ where $\mathbf{T}(k)$ corresponds to the time-varying control gain matrix. The $\mathbf{T}(k)$ is obtained by performing the recursive relation given as

$$\mathbf{T}(k) = -[\mathbf{G}_a^{\mathbf{T}}\mathbf{U}(k+1)\mathbf{G}_a + \mathbf{R}_c]^{-1}\mathbf{G}_a^{\mathbf{T}}\mathbf{U}(k+1)\mathbf{F}_a \quad (44\text{-}1)$$

$$\mathbf{U}(k) = \mathbf{F}_a^{\mathbf{T}}\mathbf{U}(k+1)\mathbf{F}_a + \mathbf{F}_a^{\mathbf{T}}\mathbf{U}(k+1)\mathbf{G}_a\mathbf{T}(k) + \mathbf{Q}_c \quad (44\text{-}2)$$

for $k = N-1,\ N-2,\ldots,0$ where $\mathbf{U}(N) = \mathbf{Q}_c$. Performing the recursive relation where N is assumed relatively large, $\mathbf{T}(k)$ approaches to $\mathbf{T}$ in the steady state. It is noted that $\mathbf{T}$ is modified so as to satisfy the stability condition given by (42-1) by changing the trade-off, $\mathbf{Q}_c$ and $\mathbf{R}_c$.

While $\mathbf{S}$ is selected so as to reduce the amplitude of $\mathbf{x}_a(k)$ as small as possible and to satisfy the stability condition given by (42-1). However, since $\mathbf{S}$ is generally not possible to select as the dimension of $\mathbf{x}_a(k)$ is relatively large, it is performed by the simulation experiment.

6. Numerical example

Consider the simple dynamic system described by a discrete-time uncertain time delay state equation and the measurement equation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 0.3 & 0.4 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}\begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k) + \begin{bmatrix} \mathbf{sin}\ 0.5k & 0 \\ 0 & \mathbf{tanh}\ x_2(k) \end{bmatrix} \times \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (45)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \quad (46)$$

Then, the augmented state and the measurement equations are, respectively, obtained as

$$\mathbf{x}_a(k+1) = \mathbf{F}_a\mathbf{x}_a(k) + \mathbf{G}_a u(k) + \boldsymbol{\Gamma}_a(\mathbf{x}_a, k)\boldsymbol{\theta} \quad (47)$$

$$\mathbf{y}(k) = \mathbf{H}_a\mathbf{x}_a(k) + \mathbf{v}(k) \quad (48)$$

where

$$\mathbf{x}_a(k) = \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_1(k) \\ x_2(k) \end{bmatrix} \quad \mathbf{F}_a = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.4 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.4 \end{bmatrix}$$

$$\mathbf{G}_a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_a(\mathbf{x}_a, k) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathbf{sin}\ 0.5k & 0 \\ 0 & \mathbf{tanh}\ x_2(k) \end{bmatrix} \quad \boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\mathbf{H}_a = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The uncertainties are, respectively, given as $\theta_1 = 0.2$ and $\theta_2 = 0.5$, and the measurement noises, $v_1(k)$ and $v_2(k)$, are, respectively, zero-mean independent random sequences with the standard deviations given as 10^{-3}. Assuming that $\mathbf{Q}_c = diag[10^2\ 10^2\ 10^3\ 10^3]$ and $R_c = 1$ on J_c given by (43), $\mathbf{T} \in \mathbf{R}^{1\times4}$ and $\mathbf{U} \in \mathbf{R}^{4\times4}$ are, respectively, calculated by performing the recursive relation given by (44-1) and (44-2) as

$$\mathbf{T} = [-0.377 \quad -0.388 \quad -0.740 \quad -0.642]$$

$$\mathbf{U} = \begin{bmatrix} 0.415 & 0.158 & 0.526 & 0.321 \\ 0.158 & 0.179 & 0.281 & 0.161 \\ 0.562 & 0.281 & 2.259 & 0.650 \\ 0.321 & 0.161 & 0.650 & 1.465 \end{bmatrix} \times 10^3$$

where $\mathbf{U}$ is denoted as $\mathbf{P}_s \in \mathbf{R}^{4\times4}$, and $\mathbf{S}$ is selected as $[0.1 \quad 0.1 \quad 0.1 \quad 0.1]$ by performing the simulation experiment. Then, $\lambda_{\min}(\mathbf{Q}_s) = 100$ and $C_1 = 0$ are, respectively, evaluated from (40-1), (40-2) and (40-4) so that the stability condition given by (42-1) is satisfied. Defining the augmented state vector as

$$x_e(k) = [x_1(k-1) \quad x_2(k-1) \quad x_1(k) \quad x_2(k) \quad \theta_1 \quad \theta_2]^T$$

the augmented state and measurement equations for $\mathbf{x}_e(k)$ corresponding to equations (21) and (22) are, respectively, derived, and the proposed WEKF and WLSE are respectively designed.

The simulation experiment is carried out with the initial conditions given by

$$\mathbf{x}_a(0) = [\,0 \quad 0 \quad 1 \quad 1\,]^T$$
$$u_o(0) = 0 \quad \hat{\mathbf{x}}_e(0) = [\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]^T$$
$$\mathbf{P}_e(0) = diag[\,1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\,] \quad \hat{\boldsymbol{\theta}}(0) = [\,0 \quad 0\,]^T$$
$$\mathbf{P}_\theta(0) = diag[\,1 \quad 1\,]$$

The time evolutions of the state $x_1(k)$ and its estimate $\hat{x}_1(k)$, the state $x_2(k)$ and its estimate $\hat{x}_2(k)$, the estimates for the uncertainties, $\hat{\theta}_1(k)$ and $\hat{\theta}_2(k)$, and the adaptive SMC $u_s(k)$ and the sliding surface $\sigma(k)$ are respectively shown in Figures 1(a) to 1(d) for Method 1 (Proposed adaptive SMC using the WEKF: $\gamma = 0.1, \quad c_e = 0.4$), and in Figures 2(a) to 2(d) for Method 2 (Proposed adaptive SMC using the WLSE: $\gamma = 0, \quad c_e = 0.5$) where the total sampling number is assumed as 200. In Methods 1 and 2, the parameter values of γ and c_e are selected from the time evolutions of the variables shown in these figures by performing the simulation experiment. It is seen from Figures 1(a) to 1(c) and 2(a) to 2(c) that the proposed WEKF and WLSE much improve the performance of the estimation because $\hat{x}_1(k)$ and $\hat{x}_2(k)$ respectively converge to $x_1(k)$ and $x_2(k)$ at short sampling instant, and that $\hat{\theta}_1(k)$ and $\hat{\theta}_2(k)$ respectively do to θ_1 and θ_2 at almost $100th$ sampling instant.

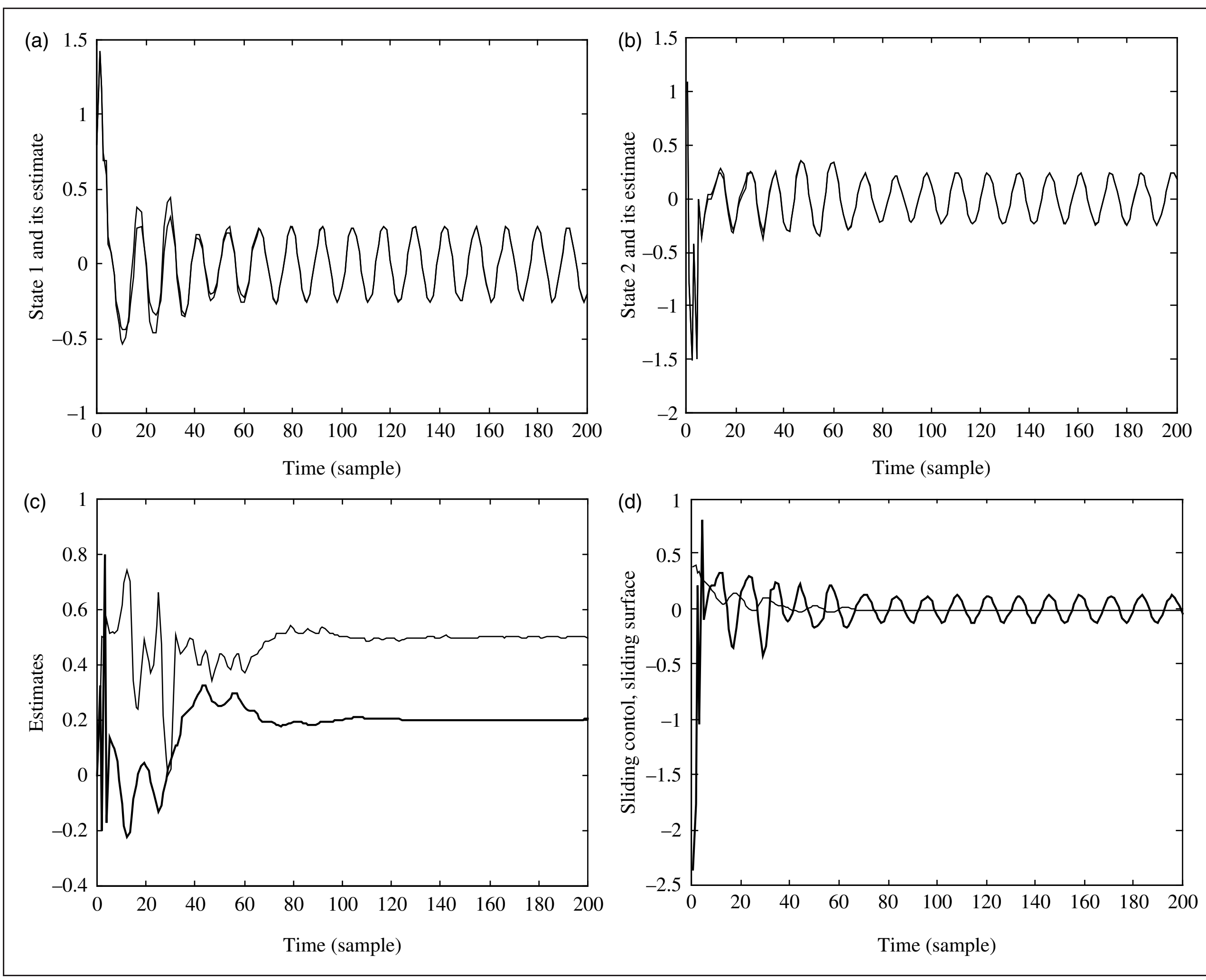

Figure 1. (a). The time evolutions of $x_1(k)$ (thick line) and $\hat{x}_1(k)$ (thin line) obtained from Method 1.; (b). The time evolutions of $x_2(k)$ (thick line) and $\hat{x}_2(k)$ (thin line) obtained from Method 1.; (c). The time evolutions of $\hat{\theta}_1(k)$ (thick line) and $\hat{\theta}_2(k)$ (thin line) obtained from Method 1.; (d). The time evolutions of $u_s(k)$ (thick line) and $\sigma(k)$ (thin line) obtained from Method 1.

These figures denote that the WEKF is better in the performance of the estimation than the WLSE. Figures 1(d) and 2(d) show that the time evolutions of $u_s(k)$ are respectively subject to the effect of the external disturbance, and the trajectory of the states is kept in the vicinity of $\sigma(k) = 0$ as k increases. The root mean squares of $x_1(k)$, $x_2(k)$ and $u_s(k)$ are, respectively, evaluated as 0.2566, 0.2536 and 0.2615 for Method 1, and 0.3942, 0.5067 and 0.5348 for Method 2. It indicates that Method 1 using the WEKF is much better in the performance of the estimation and the control than Method 2 using the WLSE because the former decreases the amplitudes of $x_1(k)$ and $x_2(k)$ with smaller amplitude of $u_s(k)$ more than the latter. However, the former increases the computational load more than the latter.

The proposed adaptive SMC is compared with the adaptive SMC already presented (Yoshimura, 2008) where $\sigma(k)$ is assumed as

$$\sigma(k) = \mathbf{S}\mathbf{x}_a(k) \tag{49}$$

Then, $u_s(k)$ is obtained by

$$u_s(k) = -(\mathbf{S}\mathbf{G}_a)^{-1}[\mathbf{S}(\mathbf{F}_a - \mathbf{I})\hat{\mathbf{x}}_a(k) + \mathbf{\Gamma}_a(\hat{\mathbf{x}}_a, k)\hat{\boldsymbol{\theta}}(k) + \gamma'\hat{\sigma}(k)] \tag{50}$$

where $\hat{\sigma}(k) = \mathbf{S}\hat{\mathbf{x}}_a(k)$ and $0 < \gamma' < 2$. The simulation experiment is performed using Method 3 (adaptive SMC already presented using the WEKF: $\gamma' = 1$, $c_e = 0.4$) and Method 4 (adaptive SMC already presented using the WLSE: $\gamma' = 1$, $c_e = 0.5$) where $\mathbf{S}$ in

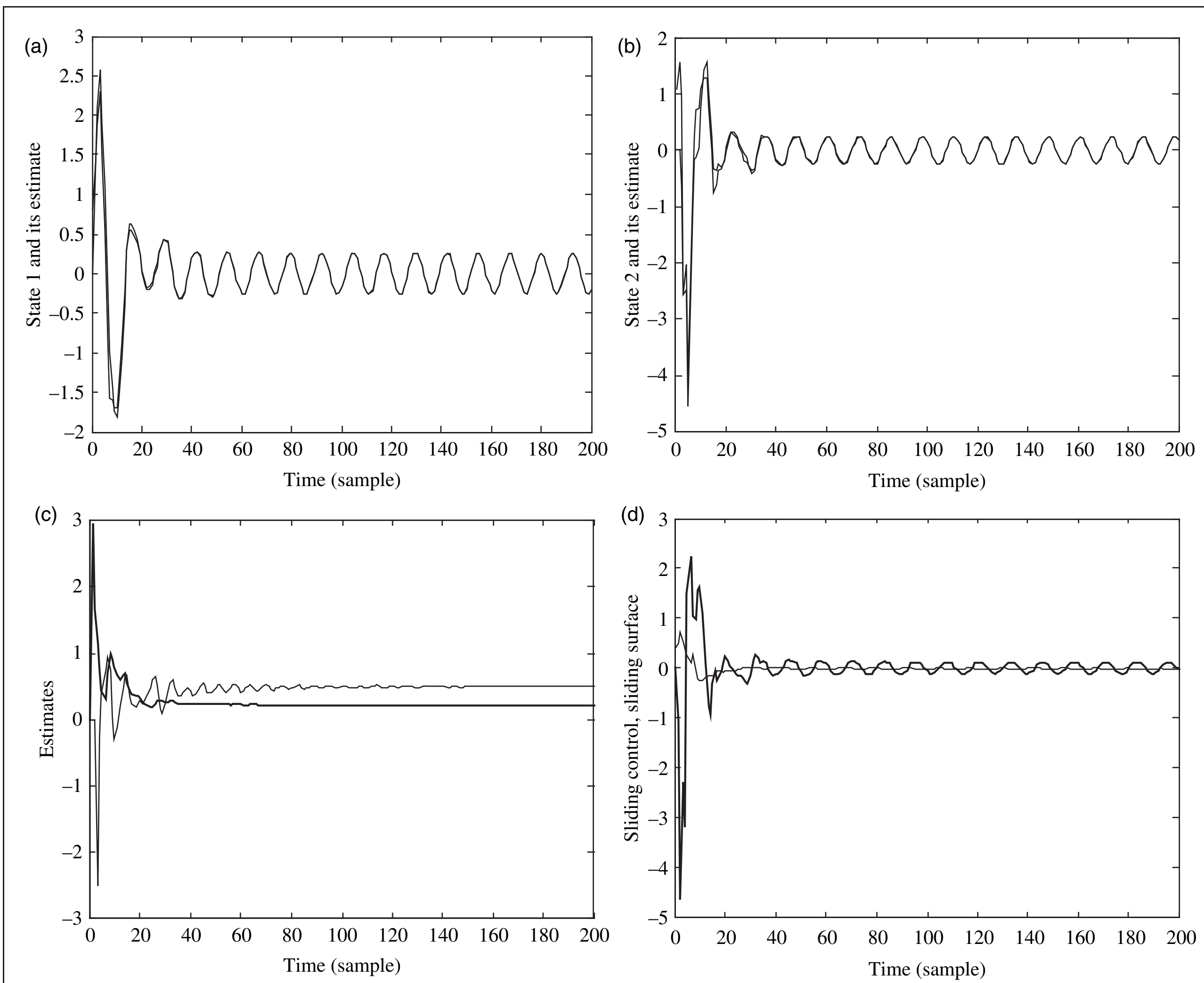

Figure 2. (a) The time evolutions of $x_1(k)$ (thick line) and $\hat{x}_1(k)$ (thin line) obtained from Method 2.; (b) The time evolutions of $x_2(k)$ (thick line) and $\hat{x}_2(k)$ (thin line) obtained from Method 2; (c) The time evolutions of $\hat{\theta}_1(k)$ (thick line) and $\hat{\theta}_2(k)$ (thin line) obtained from Method 2; (d) The time evolutions of $u_s(k)$ (thick line) and $\sigma(k)$ (thin line) obtained from Method 2.

Methods 3 and 4 is equal to $\mathbf{T}$ in Methods 1 and 2. Other conditions for the simulation experiment are the same as Methods 1 and 2 already presented, and the parameter values of γ' and c_e in Methods 3 and 4 are, respectively, selected by applying the same procedure as Methods 1 and 2. In the simulation experiment, the root mean squares of $x_1(k)$, $x_2(k)$ and $u_s(k)$ are, respectively, evaluated as 0.2913, 0.3063 and 0.3573 for Method 3, and 0.4008, 0.5214 and 0.5385 for Method 4. It indicates that Method 3 is better in the performance of the estimation and the control, but increases the computational load more than Method 4.

Comparing the result obtained by Method 1 with that obtained by Method 3, the former is better than the latter, and comparing the result obtained by Method 2 with that obtained by Method 4, the former is better than the latter. It is concluded from the simulation experiment that the proposed discrete-time adaptive SMC is very effective in the performance of the estimation and the control.

7. Conclusion

This paper proposes a discrete-time adaptive SMC for a class of uncertain time delay systems. It was assumed that the dynamic systems were described by a discrete-time time delay state equation with mismatched uncertainties, and that complete information of the states was untenable by the measurement restriction of available sensors and by the contamination with independent random noises. The augmented state equation was derived based on the time delay of the states, and the EEKF and the WLSE were designed to take the estimates for the augmented states and the uncertainties. The proposed adaptive SMC was designed using the integral-type sliding surface and the output information from the estimators. It was seen from the simulation experiment in a simple system that the WEKF improved the performance of the estimation and the control, but also increased the computational load, more than the WLSE. It was verified that the estimation errors obtained from the proposed WEKF converged to zero as time increased, and the states of the dynamic systems were ultimately bounded under the action of the proposed discrete-time adaptive SMC. The simulation experiment indicated that the proposed method was more excellent in the performance of the estimation and the control than the adaptive SMC already presented.

Appendix

Convergence of the estimation errors obtained from the WEKF

Defining the estimation error $\tilde{\mathbf{x}}_e(k)$ as $\tilde{\mathbf{x}}_e(k) = \mathbf{x}_e(k) - \hat{\mathbf{x}}_e(k)$ where $\mathbf{x}_e(k)$ and $\hat{\mathbf{x}}_e(k)$ are, respectively, assumed bounded, and using equations (8), (9), (24) and (25), the equation for $\tilde{\mathbf{x}}_e(k)$ provides

$$\begin{aligned}\tilde{\mathbf{x}}_e(k+1) = {} & [\mathbf{I} - \mathbf{K}_e(k+1)\mathbf{H}_e]\mathbf{F}_e(\hat{\mathbf{x}}_e, k)\tilde{\mathbf{x}}_e(k) \\ & - \mathbf{K}_e(k+1)\mathbf{v}(k+1)\end{aligned} \tag{A-1}$$

where the higher order terms of $\tilde{\mathbf{x}}_e(k)$ are assumed negligibly small. It is verified in equation (A-1) that $\tilde{\mathbf{x}}_e(k+1)$ converges to zero from $\tilde{\mathbf{x}}_e(0) \neq \mathbf{0}$ as k increases at the first stage where $\mathbf{v}(k+1)$ is ignored, and secondly the effect of $\mathbf{v}(k+1)$ is disappeared as k increases.

To verify that $\tilde{\mathbf{x}}_e(k+1)$ converges to zero from $\tilde{\mathbf{x}}_e(0) \neq \mathbf{0}$ as k increases in equation (A-1) at the first stage, the Lyapunov function candidate is defined as

$$V_e(k) = \tilde{\mathbf{x}}_e^T(k)\mathbf{P}_e^{-1}(k)\tilde{\mathbf{x}}_e(k) \tag{A-2}$$

It is seen that $\tilde{\mathbf{x}}_e(k)$ converges to zero as k increases if the time difference of $V_e(k)$ defined as $\Delta V_e(k+1) = V_e(k+1) - V_e(k)$ is always negative except at $\tilde{\mathbf{x}}_e(k) = \mathbf{0}$. The $\Delta V_e(k+1)$ is evaluated as (Yoshimura, 2008, 2010)

$$\begin{aligned}\Delta V_e(k+1) &= \tilde{\mathbf{x}}_e^T(k+1)\mathbf{P}_e^{-1}(k+1)\tilde{\mathbf{x}}_e(k+1) - \tilde{\mathbf{x}}_e^T(k)\mathbf{P}_e^{-1}(k)\tilde{\mathbf{x}}_e(k) \\ &= \tilde{\mathbf{x}}_e^T(k+1|k)[\mathbf{I} - \mathbf{K}_e(k+1)\mathbf{H}_e]^T[\mathbf{exp}(-c_e)\mathbf{P}_e^{-1}(k+1|k) + \mathbf{H}_e^T\mathbf{R}_v^{-1}\mathbf{H}_e] \\ &\quad \times [\mathbf{I} - \mathbf{K}_e(k+1)\mathbf{H}_e]\tilde{\mathbf{x}}_e(k+1|k) - \tilde{\mathbf{x}}_e^T(k)\mathbf{P}_e^{-1}(k)\tilde{\mathbf{x}}_e(k) \\ &= -\tilde{\mathbf{x}}_e^T(k)\mathbf{Q}_e(k)\tilde{\mathbf{x}}_e(k) - \tilde{\mathbf{x}}_e^T(k+1|k)\mathbf{R}_e(k+1)\tilde{\mathbf{x}}_e(k+1|k)\end{aligned} \tag{A-3}$$

where

$$\mathbf{Q}_e(k) = [1 - \mathbf{exp}(-c_e)]\mathbf{P}_e^{-1}(k) \tag{A-4}$$

$$\begin{aligned}\mathbf{R}_e(k+1) = {} & \mathbf{H}_e^T\mathbf{R}_v^{-1}\mathbf{H}_e - \mathbf{H}_e^T\mathbf{R}_v^{-1}\mathbf{H}_e \\ & \times [\mathbf{exp}(-c_e)\mathbf{P}_e^{-1}(k+1|k) + \mathbf{H}_e^T\mathbf{R}_v^{-1}\mathbf{H}_e]^{-1} \\ & \times \mathbf{H}_e^T\mathbf{R}_v^{-1}\mathbf{H}_e\end{aligned} \tag{A-5}$$

It is seen that $\mathbf{Q}_e(k)$ is symmetric positive definite matrix because $\mathbf{P}_e(k)$ is symmetric positive definite matrix and c_e is a positive constant. Using the fact that $\mathbf{R}_e(k+1)$ is positive semi-definite matrix, $\Delta V_e(k+1)$ is denoted as

$$\begin{aligned}\Delta V_e(k+1) &\leq -\lambda_{\min}[\mathbf{Q}_e(k)]\left\|\tilde{\mathbf{x}}_e(k)\right\|^2 - \tilde{\mathbf{x}}_e^T(k+1|k)\\ &\quad\times \mathbf{R}_e(k+1)\tilde{\mathbf{x}}_e(k+1|k)\\ &\leq -\lambda_{\min}[\mathbf{Q}_e(k)]\left\|\tilde{\mathbf{x}}_e(k)\right\|^2\end{aligned} \tag{A-6}$$

where $\lambda_{\min}[\mathbf{Q}_e(k)]$ is the minimum eigenvalue of $\mathbf{Q}_e(k)$. It is seen from (A-6) that $\Delta V_e(k+1)$ is always negative except at $\tilde{\mathbf{x}}_e(k)=\mathbf{0}$ so that $\tilde{\mathbf{x}}_e(k)$ converges to zero as k increases.

Secondly, setting $\tilde{\mathbf{x}}_e(0)=\mathbf{0}$ in equation (A-1) and computing repeatedly from $k=0$ till k gives

$$\tilde{\mathbf{x}}_e(k+1) = -\mathbf{L}_e(k+1)\sum_{i=1}^{k+1}\mathbf{L}_e^{-1}(i)\mathbf{K}_e(i)\mathbf{v}(i) \tag{A-7}$$

where

$$\begin{aligned}\mathbf{L}_e(i+1) &= [\mathbf{I}-\mathbf{K}_e(i+1)\mathbf{H}_e]\mathbf{F}_e(\hat{\mathbf{x}}_e,i)[\mathbf{I}-\mathbf{K}_e(i)\mathbf{H}_e]\\ &\quad\times \mathbf{F}_e(\hat{\mathbf{x}}_e,i-1)\times\cdots\times[\mathbf{I}-\mathbf{K}_e(2)\mathbf{H}_e]\mathbf{F}_e(\hat{\mathbf{x}}_e,1)\\ &= [\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\\ &\quad\times \mathbf{F}_e(\hat{\mathbf{x}}_e,i)[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i-1)\\ &\quad\times \mathbf{P}_e(i-1)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i-1)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\\ &\quad\times \mathbf{F}_e(\hat{\mathbf{x}}_e,i-1)\times\cdots\times[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,1)\\ &\quad\times \mathbf{P}_e(1)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,1)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\mathbf{F}_e(\hat{\mathbf{x}}_e,1)\end{aligned}$$

$$(i\geq 1)\quad \mathbf{L}_e(1)=\mathbf{I} \tag{A-8}$$

Using equations (26) to (28) provides

$$\mathbf{P}_e^{-1}(i+1) = [\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)]^{-1} + \mathbf{H}_e^T\mathbf{R}^{-1}\mathbf{H}_e \tag{A-9}$$

Since $\mathbf{P}_e(i)$ is positive definite and decreasing matrix with time, and $\mathbf{F}_e(\hat{\mathbf{x}}_e,i)$ is nonsingular and bounded matrix, it is assumed that

$$M_{P1}\frac{1}{i} \leq \left\|\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\right\| \leq M_{P2}\frac{1}{i} \tag{A-10}$$

$$\begin{aligned}M_{Q1}\frac{i}{i+1} &\leq \left\|[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\mathbf{H}_e^T\mathbf{R}_e\mathbf{H}_e]^{-1}\right\|\\ &\leq M_{Q2}\frac{i}{i+1}\end{aligned} \tag{A-11}$$

where M_{P1}, M_{P2}, M_{Q1} and M_{Q2} are respectively positive constants. Using (A-9), (26) and (28) provides

$$\begin{aligned}\mathbf{L}_e^{-1}(i+1)\mathbf{K}_e(i+1) &= \{[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\\ &\quad\times[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i-1)\mathbf{P}_e(i-1)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i-1)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\mathbf{F}_e(\hat{\mathbf{x}}_e,i-1)\times\cdots\\ &\quad\times[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,1)\mathbf{P}_e(1)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,1)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\mathbf{F}_e(\hat{\mathbf{x}}_e,1)\}^{-1}\\ &\quad\times[\mathbf{I}+\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\mathbf{H}_e^T\mathbf{R}_e^{-1}\mathbf{H}_e]^{-1}\mathbf{exp}(c_e)\mathbf{F}_e(\hat{\mathbf{x}}_e,i)\mathbf{P}_e(i)\mathbf{F}_e^T(\hat{\mathbf{x}}_e,i)\mathbf{H}_e^T\mathbf{R}_e^{-1}\end{aligned} \tag{A-12}$$

and applying (A-10) and (A-11) yields

$$M_{R1} \leq \left\|\mathbf{L}_e^{-1}(i+1)\mathbf{K}_e(i+1)\right\| \leq M_{R2} \tag{A-13}$$

where M_{R1} and M_{R2} are positive constants. Therefore, equation (A-7) becomes

$$\begin{aligned}\left\|\tilde{\mathbf{x}}_e(k+1)\right\| &= \left\|\mathbf{L}_e(k+1)\sum_{i=1}^{k+1}\mathbf{L}_e^{-1}(i)\mathbf{K}_e(i)\mathbf{v}(i)\right\|\\ &\leq \left\|\frac{M_S}{k+1}\sum_{i=1}^{k+1}\mathbf{L}_e^{-1}(i)\mathbf{K}_e(i)\mathbf{v}(i)\right\|\end{aligned} \tag{A-14}$$

where M_S is a positive constant. Since $\mathbf{v}(i)$ $i=1,2,\ldots,k+1$ are zero-mean independent random sequences, the right hand of (A-14) converges to zero as k increases by applying the ergodic theorem so that $\tilde{\mathbf{x}}_e(k+1)$ converges to zero as k increases. Hence, it is verified that $\tilde{\mathbf{x}}_e(k+1)$ described by equation (A-1) converges to zero as k increases.

References

Bartolini G, Pisano A, Punta E and Usai E (2003) A survey of applications of second-order sliding mode control to mechanical systems. *International Journal of Control* 76(9/10): 875–892.

Chen X (2006) Adaptive sliding mode control for discrete-time multi-input multi-output systems. *Automatica* 42(3): 427–435.

Cheng C-C, Lin M-H and Hsiao J-M (2000) Sliding mode controllers design for linear discrete-time systems with matching perturbations. *Automatica* 36: 1205–1211.

Chou C-H and Cheng C-C (2001) Design of adaptive variable structure controllers for perturbed time-varying state delay systems. *Journal of Franklin Institute* 338: 35–46.

Edwards C and Spurgeon SK (1998) *Sliding Mode Control: Theory and Application*. London: Taylor & Francis.

Edwards C, Spurgeon SK and Hebden RG (2003) On the design of sliding mode output feedback controllers. *International Journal of Control* 76(9/10): 893–905.

Efe MÖ, Unsal C, Kaynak O and Yu X (2004) Variable structure control of a class of uncertain systems. *Automatica* 40(1): 59–64.

Feng Y, Yu X and Man Z (2002) Non-singular terminal sliding mode control of rigid manipulators. *Automatica* 38: 2159–2167.

Furuta K and Pan Y (2000) Variable structure control with sliding sector. *Automatica* 36(2): 211–228.

Gouaisbaut F, Dambrine M and Richard JP (2002) Robust control of delay systems: a sliding mode control design via LMI. *Systems & Control Letters* 46: 219–230.

Hua C-C, Wang O-G and Guan X-P (2008) Memoryless state feedback controller design for time delay systems with matched uncertain nonlinearities. *IEEE Transactions on Automatic Control* 53(3): 801–807.

Huang AC and Chen YC (2004) Adaptive multiple-surface sliding control for non-autonomous systems with mismatched uncertainties. *Automatica* 40(11): 1939–1945.

Hwang CL (2004) Discrete sliding-mode tracking control of high-displacement piezoelectric actuator systems. *ASME Journal of Dynamic Systems, Measurement, and Control* 126: 721–731.

Janardhanan S and Bandyopadhyay B (2006) Output feedback sliding mode control for uncertain systems using fast output sampling technique. *IEEE Transactions on Industrial Electronics* 53(5): 1677–1682.

Jazwinski AH (1970) *Stochastic Processes and Filtering Theory*. New York: Academic Press.

Lai NO, Edwards C and Spurgeon SK (2007) On output tracking using dynamic output feedback discrete-time sliding mode controllers. *IEEE Transactions on Automatic Control* 52(10): 1975–1981.

Li X and Decarlo RA (2003) Robust sliding mode control of uncertain time delay systems. *International Journal of Control* 76(13): 1296–1305.

Meditch JS (1969) *Stochastic Optimal Linear Estimation and Control*. New York: McGraw-Hill.

Niu Y, Lam J, Wang X and Ho DWC (2004) Observer-based sliding mode control for nonlinear state-delayed systems. *International Journal of Systems Science* 35: 139–150.

Niu Y, Ho D and Lam J (2005) Robust integral sliding mode control for uncertain stochastic systems with time-varying delay. *Automatica* 41(5): 873–880.

Qucheriah S (2005) Adaptive variable structure control of linear delayed systems. *ASME Journal of Dynamic Systems, Measurement, and Control* 127(4): 710–714.

Sam YM, Osman JHS and Ghani MRA (2004) A class of proportional-integral sliding mode control with application to active suspension system. *Systems & Control Letters* 51: 217–223.

Utkin VI, Guldner J and Shi J (1999) *Sliding Mode Control in Electromechanical Systems*. London: Taylor & Francis.

Wang J, Brussel HV and Swevers J (2003) Robust perfect tracking control with discrete sliding mode controller. *ASME Journal of Dynamic Systems, Measurement, and Control* 125(1): 27–32.

Xia Y-Q and Jia Y-M (2003) Robust sliding-mode control for uncertain time-delay systems: an LMI approach. *IEEE Transactions on Automatic Control* 48: 1086–1092.

Xia Y, Liu G-P, Shi P, Chen J and Rees D (2008) Robust delay-dependent sliding mode control for uncertain time delayed systems. *International Journal of Robust and Nonlinear Control* 18(11): 1142–1161.

Yoshimura T (2008) Adaptive sliding mode control for nonlinear discrete-time systems with mismatched uncertainty. *International Journal of Modeling, Identification and Control* 3(4): 427–433.

Yoshimura T (2010) Adaptive discrete sliding mode control for mechanical systems with mismatched uncertainties. *Journal of Vibration and Control* [in press].