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Nonlinear Unknown Input Observer Design for Nonlinear Systems: A New Method

T. Sedighi * A. J. Koshkouei ** K. J. Burnham ***

* Control Theory and Applications Centre, Coventry University, Priory
Street, Coventry CV1 5FB, UK (e-mail: tabassoms@yahoo.com).
** (e-mail: a.koshkouei@coevntry.ac.uk)
*** (e-mail: k.burnham@coventry.ac.uk)

Abstract: This paper provides new developments in the design of observers for nonlinear systems with unknown inputs. The proposed methods guarantee the error system stability and yield many additional degrees of freedom available to the designer. The algorithms for designing a nonlinear observer for nonlinear systems with unknown inputs is derived in detail. The proposed observers may be used for fault detection and isolation. A nonlinear mass-spring-damper model is given in order to highlight the efficiency of the proposed method.

Keywords: Nonlinear control systems; Nonlinear observer; Unknown input observer; Error stability analysis; Mass-spring-damper system;.

1. INTRODUCTION

Observer design is an important problem that has various applications such as output feedback control, system monitoring, process identification and fault detection.

The observer was first proposed and developed by Luenberger (1971) in the early 1960s. After the pioneering work of Luenberger on estimation problems, various types of estimators have been developed for both linear and nonlinear systems. An observer for nonlinear systems whose nonlinearity is globally Lipschitz has been developed by many researchers including Abbaszadeh and Marquez (2010). The full order Luenberger-like observer for nonlinear systems whose nonlinearity satisfy the well known Lipschitz condition has been proposed in recent years, see for example Raghavan and Hedrick (1994).

It is of importance to design observers for multivariable linear or nonlinear systems partially driven by unknown inputs. In many control systems, the measurement of all the input signals in the system is impossible. In some cases, the uncertainties of certain parameters of the system can be modelled as unknown inputs for designing a robust observer. For that type of system, an observer, which has the capability of estimating the states in the presence of unknown inputs, is required in order to design an appropriate control for the system. Such a problem arises in systems subject to disturbance or with inaccessible inputs and it may appear in many applications such as fault detection and isolation or parameter identification, see Mondal et al. (2009).

Design of observers for linear systems subject to unknown inputs has attracted considerable attention in the last three decades. Different approaches have been considered to design unknown input observers (UIO) for linear systems following the conventional Luenberger design procedure or using sliding mode control theory. For both approaches the existence conditions are exactly the same. In Floquet and Barbot (2005) the proposed observation algorithm allows the user to recover the state and the unknown inputs in finite time.

Under less restrictive conditions several approaches for designing reduced order observers have been proposed by Guan and Saif (1991), Hou and Müller (1992), and Mehta et al. (2010) and for full order observers many methods have been presented including Hou et al. (1999) and Darouach (2009). Achieving less restrictive existence conditions and more direct design procedures has been a challenge in this area.

Since the 1990s, many attempts have been made to extend the existing UIO design from linear systems to nonlinear systems. UIOs for bilinear systems were designed by many researchers including Zasadzinski et al. (1998), and Hamidi et al. (2008). UIOs for more general nonlinear systems were also proposed in Seliger and Frank (1991), Yang and Saif (1996), Ha et al. (2003), Barbot et al. (2007) and Barbot et al. (2009).

More recently, observer architectures utilising the concept of sliding mode control/observer for uncertain systems, see for example, Zak (2003), Koshkouei and Zinober (2004), and Barbot and Floquet (2009). Also a direct extension of the linear results to the nonlinear case was referred to as nonlinear unknown input observer (NUIO) and considered systems with nonlinearities that are functions of inputs and outputs.

An UIO for nonlinear systems using H_∞ approach has been designed by Pertew et al. (2005) and a nonlinear observer for descriptive type of nonlinear systems with unknown inputs based on linear matrix inequality (LMI) approach has been presented by Koeing (2006), Chen and Saif (2006a), and Chen and Saif (2006b). This paper present an alternative method to design the NUIOs for nonlinear systems under certain conditions. Section 2 presents a mathematical description of the nonlinear system. Design of the NUIO for the nonlinear system along with theorems and error dynamics and stability analysis are addressed in Sections 3 and 4, respectively. A

numerical example is provided in Sections 5. Conclusions are presented in Section 6.

2. SYSTEM DESCRIPTION

Consider the following nonlinear system

$$\dot{x}(t) = Ax(t) + Bu(t) + D\mu(t) + Sg(x, u, t) + K_a f_a(t)$$
$$y(t) = Cx(t) + K_s f_s(t) \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ represent the state, input and output vectors respectively. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $S \in \mathbb{R}^{n \times s}$, $K_a \in \mathbb{R}^{n \times r}$ and $K_s \in \mathbb{R}^{p \times s}$ are known matrices, f_a and f_s are the actuator and sensor faults. $D \in \mathbb{R}^{n \times q}$, is referred to as the known input disturbances distribution matrix and $\mu(t) \in \mathbb{R}^q$ is an unknown bounded vector. The function $\mu(t)$ describes the disturbances and any kind of modelling uncertainties such as noise, time-varying terms, and parameter variations. In system (1), the function $g(x, u, t) \in \mathbb{R}^s$ represents the known nonlinearity function.

Prior to nonlinear observer design, the following assumption is made:

Assumption 1:

- There is no fault in the output, $f_s(t) = 0$.
- $CD \neq 0$ is of full rank.
- D, S, and K_a are full column rank matrices.
- The output $y(t)$ and its derivative $\dot{y}(t)$ are available.

3. NONLINEAR UNKNOWN INPUT OBSERVER DESIGN (NUIO):

In this section the design procedure of an observer is considered. The conditions which eliminate some terms from the error system are given. These conditions are sufficient conditions for designing the observer.

3.1 Observer design

An unknown input observer (UIO) for the system (1) is designed such that its state estimation error vector $e(t)$, approaches zero asymptotically, regardless of the presence of the unknown input term in the system.

3.2 NUIO Design

An full-order NUIO could be presented in the following form:

$$\dot{z}(t) = Nz(t) + Ly(t) + Gu(t) + H^* Sg(\hat{x}, u, t) + H^* D\hat{\mu}(t)$$
$$+ H^* K_a \hat{f}_a(t)$$
$$\hat{x}(t) = z(t) - Ey(t) \tag{2}$$

where $z \in \mathbb{R}^n$ is the state observer, $g(\hat{x}, u, t)$, $\hat{\mu}(t)$ and $\hat{f}_a(t)$ are the estimate of $g(x, u, t)$, $\mu(t)$ and $f_a(t)$ respectively. Matrices $N \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{n \times p}$, $G \in \mathbb{R}^{n \times m}$, $H^* \in \mathbb{R}^{n \times n}$ with $E \in \mathbb{R}^{n \times p}$, and $\hat{x}$ is an estimate of x. It is desired to design the observer such that $\hat{x}$ eventually tends to x.
The error equation for system (1) and observer (2) is

defined as follows:

$$e_x(t) = x(t) - \hat{x}(t) = x(t) - z(t) + Ey(t) \tag{3}$$

By substituting the system output defined in (1) into the error equation (3), the time-derivative of the error equation (3) will have the following form:

$$\dot{e_x}(t) = \dot{x}(t) - \dot{z}(t) + EC\dot{x}(t) \tag{4}$$

Then substituting (1) and (2) into (4), where $H = I_n + EC$ and $H = H^*$, yields

$$\dot{e_x}(t) = Ne_x(t) + (HA - NH - LC)x(t) + (HB - G)u(t)$$
$$+ HS(g(t, u, x) - g(\hat{x}, u, t)) + HD(\mu(t) - \hat{\mu}(t))$$
$$+ HK_a(f_a(t) - \hat{f}_a(t)) \tag{5}$$

If the following conditions hold,

$$HD = E_0 \tag{6}$$
$$HB - G = 0 \tag{7}$$
$$HA - NH - LC = 0 \tag{8}$$

where $E_0 \in \mathbb{R}^{n \times m}$ has a small norm, then the error equation (5) is presented as follows,

$$\dot{e_x}(t) = Ne_x(t) + A_g HSe_g + A_d HDe_\mu(t) + A_a HK_a e_a(t) \tag{9}$$

where $e_g(x, u, t) = g(x, u, t) - g(\hat{x}, u, t)$, $e_\mu(t) = \mu(t) - \hat{\mu}(t)$ and $e_a(t) = f_a(t) - \hat{f}_a(t)$ represent the nonlinearity, disturbance and actuator fault errors respectively. Matrices A_g, A_d, and A_a are the design matrices.
To obtain matrices N, E, H, G and the observer gain L the following steps should be followed:

- First substitute $H = I_n + EC$ into (6), the matrix E may be presented as follows:

$$E = (E_0 - D)(CD)^+ + \Gamma(I - (CD)(CD)^+) \tag{10}$$

where $(CD)^+$ is the pseudo-inverse of (CD), Γ is an arbitrary matrix.

- Then by substituting E into $H = I_n - EC$, matrix H will be obtained.

- After that substitute H into (7) to obtain matrix G.

- Assume that (HA, C) is an observable pair and P_0 is the symmetric positive definite (s.p.d.) solution of the following algebraic Riccati equation, then the gain K is selected such that the matrix $N = HA - KC$ is stable.

$$(HA)^T P_0 + P_0(HA) - P_0 C^T R^{-1} C P_0 = -Q_0 \tag{11}$$

where $Q_0 \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{p \times p}$ are arbitrary s.p.d. matrices. By selecting $K = P_0 C^T R^{-1}$ the matrix N will be an stable matrix.

- Finally by substituting N, K and (6) into (8) the observer gain L will be obtained:

$$L = HAE + K(I_p - CE). \tag{12}$$

4. STABILITY ANALYSIS OF THE ERROR SYSTEM

In this Section the behaviour of the error system (9) is considered. The desire is to obtain the stability of $e_\mu(t)$, $e_a(t)$ and $e_g(x, u, t)$, to show that each of these errors are stable, then the stability of error (9) is guaranteed.

4.1 The disturbance error $e_\mu(t)$ stability analysis

The unknown input (disturbance) $\mu(t)$, could be estimated from the available known signals as follows:

$$\hat{\mu}(t) = M_{11}y(t) + M_{12}\dot{y}(t) + M_{13}\hat{x}(t) + M_{14}\dot{\hat{x}}(t)$$
$$+ M_{15}Sg(x, u, t) + M_{16}u(t) \qquad (13)$$

where matrices $M_{11} \in \mathbb{R}^{q \times p}$, $M_{12} \in \mathbb{R}^{q \times p}$, $M_{13} \in \mathbb{R}^{q \times n}$, $M_{14} \in \mathbb{R}^{q \times n}$, $M_{15} \in \mathbb{R}^{q \times n}$ and $M_{16} \in \mathbb{R}^{q \times m}$ will be designed to obtain the observer $\hat{\mu}(t)$.
$\mu(t)$ is obtained from equation (1) as follows:

$$\mu(t) = D^+\dot{x}(t) - D^+Ax(t) - D^+Bu(t) - D^+Sg(x, u, t)$$
$$- D^+K_af_a(t) \qquad (14)$$

Using (13) and (14), the disturbance error $e_\mu(t)$ can be rewritten as:

$$e_\mu(t) = \mu(t) - \hat{\mu}(t)$$
$$= -(D^+A + M_{11}C)x(t) + (D^+ - M_{12}C)\dot{x}(t)$$
$$- (D^+B + M_{16})u(t) - (D^+ + M_{15})Sg(x, u, t)$$
$$- M_{13}\hat{x}(t) - M_{14}\dot{\hat{x}}(t) - D^+K_af_a(t)$$
$$(15)$$

Since $e_x(t) = x(t) - \hat{x}(t)$ then by substituting $\hat{x}(t)$ into (15), the following is obtained:

$$e_\mu(t) = -(D^+A + M_{11}C - M_{13})x(t) + (D^+ - M_{12}C -$$
$$M_{14})\dot{x}(t) - (D^+B + M_{16})u(t) - (D^+ + M_{15})$$
$$+ Sg(x, u, t)M_{13}e(t) + M_{14}\dot{e}(t) - D^+K_af_a(t)$$
$$(16)$$

Therefore, if the following conditions hold:

$$M_{14} = D^+ - M_{12}C$$
$$M_{13} = -D^+A - M_{11}C$$
$$M_{16} = -D^+B$$
$$M_{15} = -D^+$$
$$D^+K_a = 0$$
$$(17)$$

then (16) can be written as follows:

$$e_\mu(t) = (-D^+A - M_{11}C)e_x(t) + (D^+ - M_{12}C)\dot{e}_x(t) \quad (18)$$

Equation (18) shows that the disturbance error $e_\mu(t)$ is a function of $e_x(t)$, and if $e_x(t)$ is stable then $e_\mu(t)$ tends to zero.

4.2 The actuator fault error $e_a(t)$ stability analysis

The actuator fault $f_a(t)$, could also be estimated from the available known signals as follows:

$$\hat{f}_a(t) = F_{11}y(t) + F_{12}\dot{y}(t) + F_{13}\hat{x}(t) + F_{14}\dot{\hat{x}}(t)$$
$$+ F_{15}Sg(x, u, t) + F_{16}u(t) \qquad (19)$$

where matrices $F_{11} \in \mathbb{R}^{r \times p}$, $F_{12} \in \mathbb{R}^{r \times p}$, $F_{13} \in \mathbb{R}^{r \times n}$, $F_{14} \in \mathbb{R}^{r \times n}$, $F_{15} \in \mathbb{R}^{r \times n}$ and $F_{16} \in \mathbb{R}^{r \times m}$ need to be designed. From (1), $f_a(t)$ is giving by:

$$f_a(t) = K_a^+\dot{x}(t) - K_a^+Ax(t) - K_a^+Bu(t) - K_a^+Sg(x, u, t)$$
$$- K_a^+D\mu(t) \qquad (20)$$

Using (19) and (20), $e_a(t)$, the actuator fault error is obtained:

$$e_a(t) = f_a(t) - \hat{f}_a(t)$$
$$= -(K_a^+A + F_{11}C)x(t) + (K_a^+ - F_{12}C)\dot{x}(t)$$
$$- (K_a^+B + F_{16})u(t) - (K_a^+ + F_{15})Sg(x, u, t)$$
$$- F_{13}\hat{x}(t) - F_{14}\dot{\hat{x}}(t) - K_a^+D\mu(t) \qquad (21)$$

From $e_x(t) = x(t) - \hat{x}(t)$ substitute $\hat{x}(t)$ into (21), yields:

$$e_a(t) = -(K_a^+A + F_{11}C - F_{13})x(t) + (K_a^+ - F_{12}C - F_{14})$$
$$\dot{x}(t) - (K_a^+B + F_{16})u(t) - (K_a^+ + F_{15})Sg(x, u, t)$$
$$+ F_{13}e(t) + F_{14}\dot{e}(t) - K_a^+D\mu(t) \qquad (22)$$

Thus if the following conditions hold:

$$F_{14} = K_a^+ - F_{12}C$$
$$F_{13} = -K_a^+A - F_{11}C$$
$$F_{16} = -K_a^+B$$
$$F_{15} = -K_a^+$$
$$0 = K_a^+D$$
$$(23)$$

then the fault error (22) is presented as:

$$e_a(t) = (-K_a^+A - F_{11}C)e_x(t) + (K_a^+ - F_{12}C)\dot{e}_x(t) \quad (24)$$

Equation (24) shows that the actuator fault error $e_a(t)$ is a function of $e_x(t)$, and tends to zero if $e_x(t)$ goes to zero.

4.3 The nonlinearity error $e_g(t)$ stability analysis

The nonlinearity $g(x, u, t)$, could be estimated from the available known signals as follows:

$$g(\hat{x}, u, t) = G_{11}y(t) + G_{12}\dot{y}(t) + G_{13}\hat{x}(t) + G_{14}\dot{\hat{x}}(t)$$
$$+ G_{15}u(t) + G_{16}\hat{f}_a(t) \qquad (25)$$

where matrices $G_{11} \in \mathbb{R}^{s \times p}$, $G_{12} \in \mathbb{R}^{s \times p}$, $G_{13} \in \mathbb{R}^{s \times n}$, $G_{14} \in \mathbb{R}^{s \times n}$, $G_{15} \in \mathbb{R}^{s \times m}$ and $G_{16} \in \mathbb{R}^{s \times q}$ need to be designed. From equation (1), $g(x, u, t)$ is obtained as:

$$g(x, u, t) = S^+\dot{x}(t) - S^+Ax(t) - S^+Bu(t) - S^+K_af_a(t)$$
$$- S^+D\mu(t) \qquad (26)$$

Using (25) and (26), $e_g(x, u, t)$, the nonlinearity error is giving by:

$$e_g(x, u, t) = g(x, u, t) - g(\hat{x}, u, t)$$
$$= -(S^+ A + G_{11}C)x(t) + (S^+ - G_{12}C)\dot{x}(t)$$
$$- (S^+ B + G_{15})u(t) - (S^+ K_a + G_{16})f_a(t)$$
$$- G_{13}\hat{x}(t) - G_{14}\dot{\hat{x}}(t) - S^+ D\mu(t) \qquad (27)$$

Substituting $e_x(t) = x(t) - \hat{x}(t)$ into (27), $e_g(x, u, t)$ becomes:

$$e_g(x, u, t) = -(S^+ A + G_{11}C - G_{13})x(t) + (S^+ - G_{12}C$$
$$- G_{14})\dot{x}(t) - (S^+ B + G_{15})u(t) - (S^+ K_a$$
$$+ G_{16})f_a(t) + G_{13}e(t) + G_{14}\dot{e}(t) + G_{16}e_a(t)$$
$$- S^+ D\mu(t) \qquad (28)$$

Then if the following conditions hold:

$$G_{13} = -S^+ A - G_{11}C$$
$$G_{14} = S^+ - G_{12}C$$
$$G_{15} = -S^+ B$$
$$G_{16} = -S^+ K_a$$
$$S^+ D = 0$$
$$\qquad (29)$$

then the nonlinear error estimation $e_g(x, u, t)$ is giving by:

$$e_g(x, u, t) = (-S^+ A - G_{11}C)e_x(t) + (S^+ - G_{12}C)\dot{e}_x(t)$$
$$+ G_{16}e_a(t) \qquad (30)$$

Now by substituting e_a into (30), the error e_g is giving by:

$$e_g(x, u, t) = (-S^+ A - G_{11}C - G_{16}K_a^+ A - G_{16}F_{11}C)e_x(t)$$
$$+ (S^+ - G_{12}C + G_{16}K_a^+ - G_{16}F_{12}C)\dot{e}_x(t) \quad (31)$$

Equation (31) shows that the nonlinearity error $e_g(x, u, t)$ is a function of $e_x(t)$, so it tends to zero if $e_x(t)$ goes to zero.

Substituting (18), (24) and (31) into equation (9) yields:

$$\bar{M}\dot{e}_x(t) = \bar{G}e_x(t) \qquad (32)$$

with

$$\bar{M} = I_n - A_d HD(D^+ - M_{12}C) - A_a HK_a(K_a^+ - F_{12}$$
$$C) - A_g HS(-S^+ A - G_{11}C - G_{16}K_a^+ A - G_{16}$$
$$F_{11}C) \qquad (33)$$

where I_n is an identity matrix of size n, and,

$$\bar{G} = N + A_a HK_a(-K_a^+ A - F_{11}C) + A_d HD(-D^+$$
$$- M_{11}C) + A_g HS(S^+ - G_{12}C + G_{16}K_a^+ - G_{16}$$
$$F_{12}C). \qquad (34)$$

Since $\bar{M}$ is nonsingular, $A_g \in \mathbf{R}^{n \times n}$, $A_a \in \mathbf{R}^{n \times n}$ and $A_d \in \mathbf{R}^{n \times n}$ should be selected to make $\bar{M}^{-1}\bar{G}$ Hurwitz. If $\bar{M}$ was not singular and $\bar{M}^{-1}$ does not exist, then the singular value decomposition (SVD) technique needs to be used. Hence the state error equation (32) is represented as:

$$\dot{e}_x(t) = \bar{M}^{-1}\bar{G}e_x(t) \qquad (35)$$

which satisfies the asymptotic stability of the state error estimation (9). In this method sufficient design degrees of freedom E_0, M_{ij}, F_{ij} and G_{ij} and nonuniquness of A_g, A_d and A_a also provides extra design degrees of freedom. Since error (32) is a function of $e_x(t)$, the error is eventually stable and tends to zero.
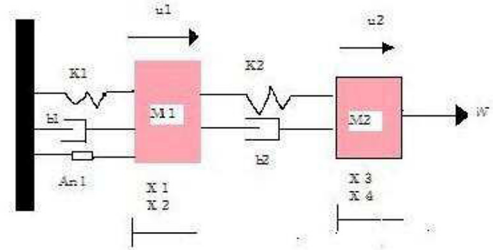
5. EXAMPLE SYSTEM

Fig. 1. The mass-spring-damper system.

Consider the mass-spring-damper (M-S-D) system in Figure 1 where two masses, springs and dampers are connected together serially (Roch-Cozalt *et al.* (2005)). x_1 and x_2 are the position and velocity of the first mass and x_3 and x_4 are the position and velocity of the second mass, respectively. A_{nl} is a nonsingular damping device whose damping force is $F_{A_{nl}} = C_{nl}\text{sign}(x_2)\ln(1 + |x_2|)$, where $C_{nl} \geq 0$. On the other hand, an arbitrary and unknown force $\mu(t)$ is applied on the second mass. u_1 and u_2 are the known input forces applied to masses 1 and 2, respectively. The state equations of the system are presented as

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_1 + k_2}{m_1} & -\dfrac{b_1 + b_2}{m_1} & \dfrac{k_2}{m_1} & \dfrac{b_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_2}{m_2} & \dfrac{b_2}{m_2} & -\dfrac{k_2}{m_2} & -\dfrac{b_2}{m_2} \end{pmatrix} x + \begin{pmatrix} 0 \\ \dfrac{1}{m_1} \\ 0 \\ \dfrac{1}{m_2} \end{pmatrix} u$$
$$+ \begin{pmatrix} 0 \\ -\dfrac{1}{m_1} \\ 0 \\ 0 \end{pmatrix} g(x_2) + \begin{pmatrix} 0 \\ 0.02 \\ 0 \\ 0 \end{pmatrix} f_a(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{m_2} \end{pmatrix} \mu(t). (36)$$

The actuator fault $f_a(t)$ is any unwanted change in the length of the springs. The outputs of the system are $y_1 = x_3$ and $y_2 = x_4$. Using the following values for parameters: $m_1 = 5kg$, $m_2 = 1kg$, $k_1 = 30N/m$, $k_2 = 10N/m$, $b_1 = 4Ns/m$, $b_2 = 2Ns/m$, $C_{nl} = 5N$, and $\mu(t) = 0.04\sin(t) + 2N$, will give:

$$A = \begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -8.0 & -1.2 & 2.0 & 0.4 \\ 0 & 0 & 0 & 1.0 \\ 10.0 & 2.0 & -10.0 & -2.0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0.20 & 0 \\ 0 & 0 \\ 0 & 1.0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 \\ -0.2 \\ 0 \\ 0 \end{pmatrix}$$

$$K_a = \begin{pmatrix} 0 \\ 0.2 \\ 0 \\ 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.9990 \end{pmatrix}, \quad E_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.0001 \end{pmatrix}$$
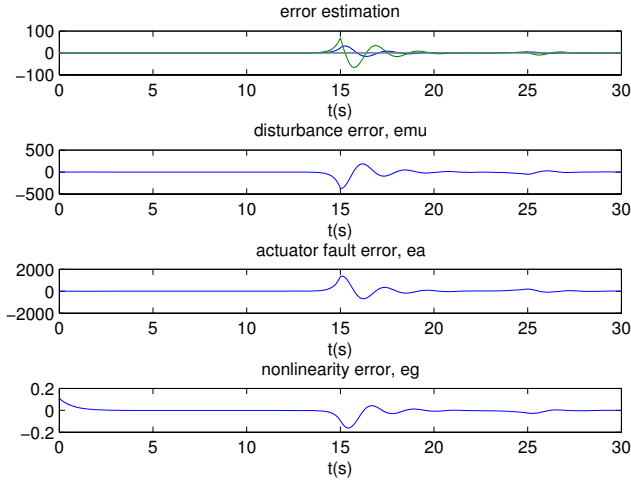
Fig. 2. The behaviour of state errors $e_x(t)$, disturbance error $e_\mu(t)$, actuator fault error $e_a(t)$ and nonlinearity error $e_g(x, u, t)$.

$$L = \begin{pmatrix} -12.574 & 0.0012 \\ 3.179 & 0.399 \\ 3.5250 & 1.000 \\ 0.9915 & -0.0006 \end{pmatrix}, \quad K = \begin{pmatrix} -12.57 & 1.20 \\ 3.17 & -0.41 \\ 3.52 & 0.99 \\ 0.99 & 1.40 \end{pmatrix}$$

$$H = \begin{pmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 0 \\ 0.2 & 0 \\ 0 & 0 \\ 0 & 0.001 \end{pmatrix}$$

$$N = \begin{pmatrix} 0 & 1.00 & 12.57 & -1.207 \\ -8.00 & -1.20 & -1.17 & 0.819 \\ 0 & 0 & -3.52 & 0.0085 \\ 0.010 & 0.002 & -1.001 & -1.404 \end{pmatrix}$$

The matrix $\bar{M}$, in equation (33) is

$$\bar{M} = \begin{pmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 2.0 & 0 & -0.040 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0008 \end{pmatrix}$$

which is invertible

$$\bar{M}^{-1} = \begin{pmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 0.50 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0.9992 \end{pmatrix}.$$

Also the matrix $\bar{G}$, in (34) is:

$$\bar{G} = \begin{pmatrix} 0 & 1.0 & 12.57 & -1.20 \\ -16.00 & -2.40 & 0.82 & 1.37 \\ 0 & 0 & -3.52 & 0.0085 \\ 0.020 & 0.0040 & -1.011 & -1.406 \end{pmatrix}.$$

The design matrices A_d, A_a and A_g are selected as follows to make $\bar{M}^{-1}\bar{G}$ hurwitz:

$$A_d = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad A_a = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
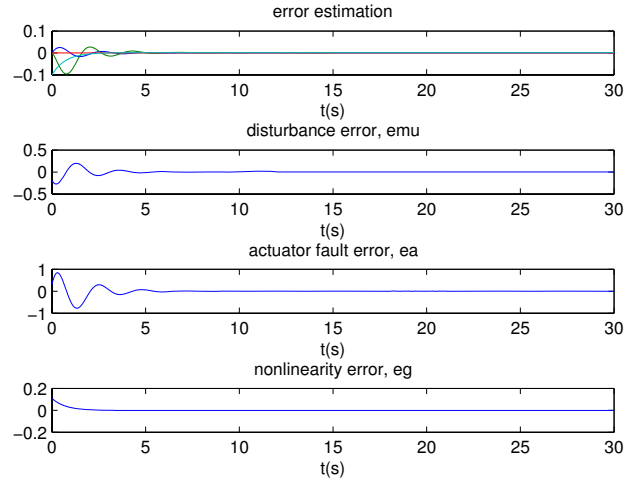

Fig. 3. The behaviour of state errors $e_x(t)$, disturbance error $e_\mu(t)$, actuator fault error $e_a(t)$ and nonlinearity error $e_g(x, u, t)$ where there is no fault on the system.

$$A_g = \begin{pmatrix} -0.20 & 0 & 0 & 0 \\ 0 & -0.20 & 0 & 0 \\ 0 & 0 & -0.20 & 0 \\ 0 & 0 & 0 & -0.20 \end{pmatrix}.$$

Since the matrix $\bar{M}^{-1}\bar{G}$ is hurwitz with following eigenvalues :

$$(-0.603 + 2.76i \quad -0.603 - 2.76i \quad -1.4033 \quad -3.5207).$$

then the error system (32) is asymptotically stable.

Figure (2) shows that in the time of no fault, the errors tend to zero asymptotically. When the fault occurs, the error responses are quite far from zero. If the fault was small or occurs in a small period of time then the system needs sufficient time to isolate the fault and reduce the effect of the fault on the system. Otherwise the system with fault is not stable unless the fault is bonded, then the error under some conditions is eventually ultimately bounded (see Sedighit *et al* (2008)).

Figure (3) shows the error behaviour of states, disturbance, fault and nonlinearity where there is no fault in the system. In this case, the errors are stable and converge to zero asymptotically. Figure (4) shows that the error estimation of the system in the absence of faults is stable regardless of any unknown disturbances.

6. CONCLUSIONS

The robust nonlinear UIO has been designed for nonlinear systems and the stability of the error systems have been demonstrated. Sufficient existence conditions were derived for the nonlinear UIO. In this method sufficient design degrees of freedom (E_0, M_{1i}, F_{1i} and G_{1i} for $i = 1, \cdots, n+2$) and nonuniqness of A_g, A_d and A_a also provides extra design degrees of freedom. This observer may be useful for state estimation and fault diagnosis for nonlinear systems. The effectiveness of the observer is shown with a numerical example. The simulation results show that the designed NUIO guarantees the asymptotically convergence of the state to zero in the presence of the unknown inputs.
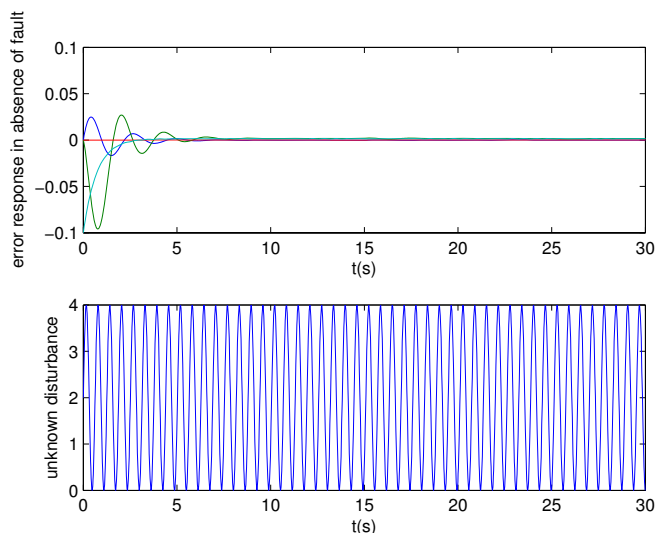
Fig. 4. The behaviour of error estimation and the disturbance.

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