Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-4 (2018) 942–947

A Pole Placing PID Type Controller

Anna Soffía Hauksdóttir * **Sven Þ. Sigurðsson** **

* *Faculty of Electrical and Computer Engineering, School of Engineering and Natural Sciences, University of Iceland, Reykjavík, Iceland (e-mail: ash@hi.is)*
** *Faculty of Industrial Engineering, Mechanical Engineering and Computer Science, School of Engineering and Natural Sciences, University of Iceland, Reykjavík, Iceland*

Abstract: The problem of designing a pole placing PID type (ppPIDt) controller is considered, motivated by educational considerations. Effectively, the number of controller zeros, integrators and filtering poles can be extended to aid in the response shaping and stabilizing of the closed loop, while simultaneously avoiding spikes in the control signal. In contrast to the state feedback observer–controller, we do not emphasize maintaining the original systems order, rather we utilize the increased order of the closed loop to shape its response by a suitable choice of poles in the closed loop. This is similar to the classical PID which typically increases the order of the original system by two in the closed loop. We also propose a prefilter to cancel stable system zeros and ppPIDt zeros and replace them by new zeros if desirable. This results in a new overall transfer function of the controlled system, with full pole placement including good input tracking and disturbance rejection properties and guaranteed closed loop stability. The material is presented in a tutorial way suitable for basic undergraduate control courses and has been used successfully by the first author in such a course.

Keywords: Control Education; Control Design; Linear Control Systems

1. INTRODUCTION

PID (Proportional, Integral, Differential) controllers are the most common controllers in industry today. They remain a very active research area, as evident in the numerous books, e.g., Åström et al. (2006), Åström et al. (1997), Skogestad et al. (2007) and Datta et al. (2013) on the topic. A recent survey paper on the evolution of control by Åström et al. (2014), provides an excellent historical perspective, wherein PID controllers are a key player in the highly multidisiplinary area of control. A much referenced survey paper on automatic tuning and adaptation for PID controllers, is Åström et al. (1993). A new paper provides an interesting experimental comparison of PID autotuners Berner (2018).

Polynomial based design frequently leads to the Diophantine equation and, subsequently, to the Sylvester matrix in control problems, see, e.g., Kucera (1993) and Åström et al. (1997). The classical observer state feedback controller also belongs to this group, when presented in transfer function form, see Kailath (1980).

Closed form expressions of linear system responses have been developed and used for optimal computation of PID zeros, see Herjólfsson et al. (2012), and other references therein. A polynomial based approach towards the design of PID controllers was taken in Hauksdóttir et al. (2011), however, closed loop stability was not guaranteed.

In this paper, we propose a pole placing PID type controller. Effectively, we extend the number of PID zeros, PID integrators and PID high frequency filtering poles to aid in reponse shaping and stabilizing of the closed loop as well as to avoid spikes in the control signal. As it turns out, if the number of poles of the plant model is n and the order of the integrator in the controller is N, then by adding $N + n - 1$ controller zeros and $n - 1$ filtering poles, thus maintaining a relative degree of zero for the controller, the unknown coefficients of the corresponding polynomials are determined uniquely by the closed loop poles. Thus, full placement, and thereby stability of all closed loop poles, is guaranteed. These are simply obtained by equating coefficients of like powers resulting in a linear system with a Sylvester matrix. In addition, a prefilter is used in the shaping of the reference input. The design is polynomial based. A regular PID with a low pass filter on the D part results in the case of $n = 2$.

A word of caution is appropriate here, as it is known that higher order controllers, and in particular an increased number of integrators in an outer control loop, may lead to systems with low stability margins. Such controllers may also be sensitive to variations in the nominal plant, in particular for challenging unstable nonminimum phase plants, see Keel (1997). We note here, though, that nominal stability is guaranteed as the problem is posed as a closed loop pole placement problem. Further, the controlled system was found to be quite insensitive to variations in a nominal underdamped benchmark problem, Åström et al. (2000).

⋆ This work was supported by the University of Iceland. The authors are grateful to the numerous helpful comments on the manuscript from anonymous reviewers.

10.1016/j.ifacol.2018.06.101

We present the ppPIDt type controller in Section 2. We discuss the choice of closed loop poles in Section 3 and the prefilter design in Section 4. Examples are given in Section 5 and conclusions are discussed in Section 6. The Diophantine equation and the Sylvester matrix in control systems are shortly covered in Appendix A.

2. A POLE PLACING PID TYPE CONTROLLER

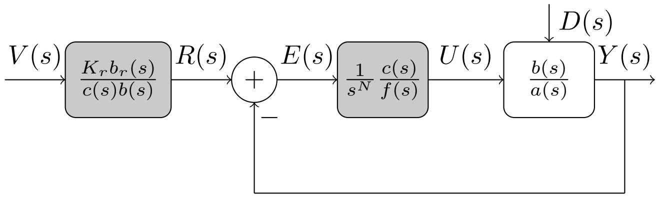

Fig. 1. A pole placing PID type controller with a prefilter.

We would like to build a pole placing PID type controller. The controller should be effective in input tracking and disturbance rejection, yet without a high cost in the control signal by keeping it a zero relative degree controller, thus guaranteeing its realizability. In addition, closed loop stability must be guaranteed and closed loop design requirements should be fulfilled. Towards this end, we extend the number of PID integrators, zeros and use high frequency filtering, typically implemented in a regular PID to avoid spikes in the control signal, to aid in the pole placement of the closed loop. This controller will move poles without cancellation and it will not cancel unstable system zeros. Thus, it can deal with unstable systems as well as systems with some right half plane zeros.

Consider a SISO system, see Fig. 1, where the transfer function for the plant $G(s)$ is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}. \quad (1)$$

The transfer function of the ppPIDt controller is given by

$$G_c(s) = \frac{c(s)}{s^N f(s)} = \frac{c_{m_c}s^{m_c} + c_{m_c-1}s^{m_c-1} + \cdots + c_0}{s^N\left(s^{n_f} + f_{n_f-1}s^{n_f-1} + \cdots + f_0\right)}. \quad (2)$$

The term ppPIDt controller needs some further explaining. A regular PID has a proportional term P, multiplying the present time value of the output error; an integral term I of the output error, effectively describing the history of the output error; and, attempting to predict the future, a differential term D of the output with a low pass filter to curb the noise in the D term. When written as a transfer function, the standard PID has two zeros ($m_c = 2$), one integrator ($N = 1$) and one pole ($n_f = 1$) in the form of (2). Extending the thought of a single P term, but adding further information of the history of the output error by including a regular I term, a double integral term etc., and similarly enhancing the prediction of the future by adding a double differential term with two lowpass filters etc., to the regular D term, leads to a controller of the transfer function (2). Including a sufficient number of coefficients in $c(s)$ and $f(s)$ further ensures a full pole placement capability of the closed loop by a set of fully determined linear equations, thus the term ppPIDt controller.

Thus, we propose more than two zeros, possibly more than one integrator and the number of poles is chosen such that $m_c = N + n_f$, so that the controller has a relative degree of zero. All the $n_f + m_c + 1$ free coefficients of the ppPIDt controller aid in the pole placement of the closed loop. The integrators ensure a PID like behavior in closed loop in terms of input signal tracking and disturbance rejection, $N = 1$ for steps, $N = 2$ for steps and ramps etc.

The closed loop transfer function becomes

$$\frac{Y(s)}{R(s)} = \frac{\frac{c(s)b(s)}{s^N f(s)a(s)}}{1 + \frac{c(s)b(s)}{s^N f(s)a(s)}} = \frac{c(s)b(s)}{s^N f(s)a(s) + c(s)b(s)}. \quad (3)$$

The disturbance hits the plant at the input, at the output or somewhere in between. We thus split the plant into two parts, i.e., the part before the disturbance $b_b(s)/a_b(s)$ and the part after the disturbance $b_a(s)/a_a(s)$. The transfer function from the disturbance to the error is given by

$$\frac{E(s)}{D(s)} = \frac{-\frac{b_a(s)}{a_a(s)}}{1 + \frac{c(s)b(s)}{s^N f(s)a(s)}} = -\frac{s^N f(s)a_b(s)b_a(s)}{s^N f(s)a(s) + c(s)b(s)}. \quad (4)$$

Thus, $\lim_{t\to\infty} e(t) = 0$ can be guaranteed by a proper selection of N for the different standard types of disturbance, step, ramp etc. We note here that $c(s) = 0$ does not have any zero roots and, further, that the choice of N is affected by possible zero roots in $b(s) = 0$.

The basic idea is to choose the coefficients of $c(s)$ and $f(s)$ in such a way that the closed loop (CL) poles given by

$$\begin{aligned} \delta(s) &= s^N f(s)a(s) + c(s)b(s) \\ &= s^{n_\delta} + \delta_{n_\delta-1}s^{n_\delta-1} + \cdots + \delta_1 s + \delta_0, \end{aligned} \quad (5)$$

are prescribed. In order for this choice to be unique or, equally, for the problem to be a fully determined one, the number of CL poles has to be equal to the number of unknown coefficients, i.e.,

$$n_\delta = n_f + m_c + 1. \quad (6)$$

In addition, we want the relative degree of the controller to be zero, such that it will have a PID like behavior without direct differentiating effects, yet maintaining an aggressive control signal with a direct transfer from the error to the control signal, i.e.,

$$m_c = N + n_f. \quad (7)$$

Given that $m \leq n$, it follows from (5) that

$$n_\delta = N + n_f + n \quad (8)$$

and hence from (6)

$$m_c = N + n - 1, \quad (9)$$

then from (7) that

$$n_f = n - 1 \quad (10)$$

and, finally from (8)

$$n_\delta = N + 2n - 1. \quad (11)$$

The classical PID controller typically increases the degree of the system and the closed loop normally has a degree of $n + 2$. Here we are not focusing on maintaining the closed loop of the same degree as the system to be controlled, as in the pole placement problem (see discussion in Appendix

A), rather we use the higher order closed loop to our benefit in shaping the closed loop dynamics. We note though that, if desired, one may begin by cancelling feasible stable parts of the system zeros and/or poles, before proceeding with the design of the remaining ppPIDt controller.

The design procedure, given $a(s)$ and $b(s)$, is composed of the following steps:

(1) Choose N, compute m_c and n_f from (9) and (10), respectively.
(2) Choose the desired n_δ roots of $\delta(s) = 0$, and hence the coefficients δ_i, $i = 0, 1, \ldots, n_\delta - 1$.
(3) Equating the coefficients of (5) on each side, solve for the $n_f + m_c + 1$ unknown coefficients of the controller, f_i, $i = 0, 1, \ldots, n_f - 1$ and c_i, $i = 0, 1, \ldots, m_c$ from the following linear system, by making, e.g., use of Matlab's backslash command.

$$
\begin{bmatrix}
1 & 0 & \cdots \rightarrow & & & \\
a_{n-1} & 1 & 0 \ \cdots \rightarrow & & & \\
a_{n-2} & a_{n-1} & \ddots & & & \\
\vdots & a_{n-2} & \searrow & & & \\
\downarrow & \vdots & & & & \\
 & & & & \vdots & \uparrow \\
 & & & \nwarrow & b_2 & \vdots \\
 & & & \ddots & b_1 & b_2 \\
 & & \longleftarrow \ \cdots & 0 & b_0 & b_1 \\
 & & \longleftarrow \ \cdots & & 0 & b_0
\end{bmatrix}_{n_\delta \times n_\delta}
\times
\begin{bmatrix}
f_{n_f-1} \\ f_{n_f-2} \\ \vdots \\ f_0 \\ c_{m_c} \\ c_{m_c-1} \\ \vdots \\ c_0
\end{bmatrix}_{(n_\delta = m_c + 1 + n_f) \times 1}
=
\begin{bmatrix}
\delta_{n_\delta-1} - a_{n-1} \\ \delta_{n_\delta-2} - a_{n-2} \\ \vdots \\ \delta_1 \\ \delta_0
\end{bmatrix}_{n_\delta \times 1}
. \tag{12}
$$

Note that the $n_\delta \times n_\delta$ Sylvester matrix is of full rank as long as $a(s) = 0$ and $b(s) = 0$ have no common factors, see e.g., Kailath (1980).

Remark 1. Equation (5) is a form of the Diophantine equation which here leads to a solution involving the Sylvester matrix in (12), see Appendix A for a further discussion of their appearances in control.

Remark 2. If we want to restrict ourselves to a regular PID without differentiating effects and thus set $m_c = 2$, $n_f = 1$, $N = 1$, (6) can only be satisfied if $n_\delta = 4$, and hence (5) can only hold true if $n = 2$. In the case $n > 2$ we have the option of only satisfying (5) in a least squares sense, i.e., by solving the overdetermined system

$$
\begin{bmatrix}
1 & & & \\
a_{n-1} & & & \\
a_{n-2} & & & \\
\vdots & \uparrow & & \\
\downarrow & \vdots & \uparrow & \\
 & b_2 & \vdots & \uparrow \\
 & b_1 & b_2 & \vdots \\
 & b_0 & b_1 & b_2 \\
 & 0 & b_0 & b_1 \\
 & 0 & 0 & b_0
\end{bmatrix}_{n_\delta \times 4}
\times
\begin{bmatrix}
f_0 \\ c_2 \\ c_1 \\ c_0
\end{bmatrix}_{4 \times 1}
\approx
\begin{bmatrix}
\delta_{n_\delta-1} - a_{n-1} \\ \delta_{n_\delta-2} - a_{n-2} \\ \vdots \\ \delta_1 \\ \delta_0
\end{bmatrix}_{n_\delta \times 1}
\tag{13}
$$

which can again, e.g., be done by the use of Matlab's backslash command. However, in this case, closed loop stability is not guaranteed. This is similar to the approach taken in Hauksdóttir et al. (2011). There, however, $f(s)$ was not utilized to place the closed loop poles, but simply implemented as a regular derivative divisor in the PID.

3. CHOICE OF CLOSED LOOP POLES

We now need a systematic way of selecting the desired closed loop poles, i.e., the roots of $\delta(s) = 0$. It is useful to

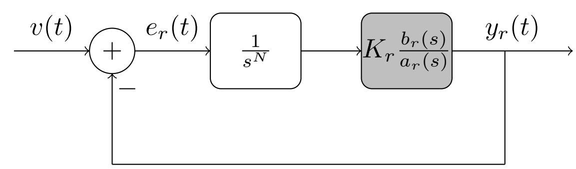

Fig. 2. A SISO reference system.

choose a simple reference system to represent the design requirements, see Fig. 2. The transfer function of the reference system is chosen of the general form

$$
\begin{aligned}
G_r(s) &= K_r \frac{b_r(s)}{s^N a_r(s)} \\
&= K_r \frac{s^{m_r} + b_{m_r-1,r} s^{m_r-1} + \cdots + b_{0,r}}{s^N \left(s^{n_r} + a_{n_r-1,r} s^{n_r-1} + \cdots + a_{0,r}\right)}.
\end{aligned}
\tag{14}
$$

The closed loop of the reference system, also representing the closed loop design requirements, is given by

$$
\begin{aligned}
\frac{Y_r(s)}{V(s)} &= \frac{K_r \frac{b_r(s)}{s^N a_r(s)}}{1 + K_r \frac{b_r(s)}{s^N a_r(s)}} \\
&= \frac{K_r b_r(s)}{s^N a_r(s) + K_r b_r(s)} = \frac{K_r b_r(s)}{\delta(s)},
\end{aligned}
\tag{15}
$$

where

$$n_r = n_\delta - N. \tag{16}$$

The roots of $\delta(s) = 0$ are selected as follows:

(1) We choose N to have the same value as in the system to be controlled in Fig. 1.
(2) We choose the desired dominant closed loop pole pair termed $s_{1,2} = \sigma \pm j\omega$, i.e., two of the $\delta(s) = 0$ roots, such that they meet some design criteria, typically maximum overshoot and maximum settling time. Alternatively, s_1 and s_2 may be chosen to be located separately on the negative real axis.

(3) We select parts of $\frac{b_r(s)}{a_r(s)}$ such that the root locus of the reference system goes through $s_{1,2}$.
(4) The remaining zeros and poles of $\frac{b_r(s)}{a_r(s)}$ are placed in pairs close to each other and to the left of the $s_{1,2}$ area, such as to have a minimal effect.
(5) K_r is fixed by the length condition such that the dominant closed loop roots of the reference system are located at $s_{1,2}$.

In summary, the closed loop design requirements are described by the time response corresponding to (15). We further note, that $b_r(s)$ and $a_r(s)$ are chosen such that the corresponding root loci is in general considerably simpler than the controlled system's root loci.

4. PREFILTER DESIGN

The closed loop controlled system has the transfer function

$$\frac{Y(s)}{R(s)} = \frac{c(s)b(s)}{\delta(s)}. \tag{17}$$

If $c(s)b(s) = 0$ has stable roots, we may prefilter the input $r(t)$ of the controlled system by a unity DC gain prefilter given by

$$\frac{R(s)}{V(s)} = \frac{K_r b_r(s)}{c(s)b(s).} \tag{18}$$

In that case perfect tracking of the reference system results and we have

$$\frac{Y(s)}{V(s)} = \frac{K_r b_r(s)}{\delta(s).} \tag{19}$$

We note that the prefilter has a unity DC gain, i.e.,

$$\lim_{s\to 0}\frac{R(s)}{V(s)} = \frac{\lim\limits_{s\to 0} K_r b_r(s)}{\lim\limits_{s\to 0} c(s)b(s)} = 1 \tag{20}$$

as we have from (5) and (15) that

$$\lim_{s\to 0}\delta(s) = \lim_{s\to 0} c(s)b(s) = \lim_{s\to 0} K_r b_r(s). \tag{21}$$

Naturally, the prefilter does not affect the disturbance rejection, as it is outside the feedback loop.

We may also leave parts of or all of the dynamics given by $c(s)b(s)$, if it improves the response or if parts of $c(s)b(s)$ contain unstable roots and should therefore not be cancelled. Further, if the relative degree of the prefilter is negative, we may use a set of padding poles each of the form $f_{pre}(s) = \tau s + 1$ in the prefilter, to avoid a negative relative degree. Those are simply placed to the left of the active overall pole/zero zone.

5. EXAMPLES

Example 1: We now consider a highly under-damped 3rd order benchmark plant from Åström et al. (2000), with $\zeta = 0.2$ and $\omega_n = 1$rad/s in the underdamped part and a third pole at $s = -2$, given by

$$\frac{b(s)}{a(s)} = \frac{2}{(s+2)}\frac{1}{(s^2+0.2s+1)}, \tag{22}$$

this plant has an overshoot of 65%, a 2% settling time of 38.5s and a DC gain of unity.

(1) Given $n = 3$, selecting $N = 1$, we have

$$m_c = 1 + 3 - 1 = 3 \tag{23}$$

and

$$n_f = 3 - 1 = 2. \tag{24}$$

(2) We choose a reference system

$$K_r\frac{b_r(s)}{sa_r(s)} = 2\frac{(s+2.9)(s+3.9)(s+4.9)(s+5.9)}{s(s+2)(s+3)(s+4)(s+5)(s+6)} \tag{25}$$

which has dominant closed loop poles at $s_{1,2} = -1 \pm j0.9$.

(3) We now solve

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_2 & 1 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & b_0 & 0 & 0 & 0 \\ a_0 & a_1 & 0 & b_0 & 0 & 0 \\ 0 & a_0 & 0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_0 \end{bmatrix}\begin{bmatrix} f_1 \\ f_0 \\ c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} \delta_5 - a_2 \\ \delta_4 - a_1 \\ \delta_3 - a_0 \\ \delta_2 \\ \delta_1 \\ \delta_0 \end{bmatrix} \tag{26}$$

This results in the ppPIDt controller given by

$$\begin{aligned} &\frac{166.06s^3 + 536.35s^2 + 562.3s + 362.97}{s(s^2 + 17.8s + 116.44)} \\ &= 166.06\frac{(s+2.05)(s+0.59 \pm j0.78)}{s(s+8.9 \pm j6.10)} \end{aligned} \tag{27}$$

and a prefilter of

$$\begin{aligned} &\frac{K_r b_r(s)}{c(s)b(s)} \\ &= \frac{2(s+2.9)(s+3.9)(s+4.9)(s+5.9)}{166.06(s+2.05)(s+0.59 \pm j0.78)2(s/20+1)}, \end{aligned} \tag{28}$$

where the pole at $s = -20$ is a padding pole.

The system is subject to a unit step input at time $t = 0$ and to a unit step disturbance at time $t = 10$. The disturbance hits the plant right into the highly underdamped part, after the real pole at $s = -2$. The resulting closed loop control signal and output and the corresponding root locus are shown in Fig. 3. The resulting gain margin is 14.4dB and the phase margin was found to be 43 degrees. Again, the prefiltering results in a perfect match with the reference system response during input tracking and excelling disturbance rejection for the highly underdamped plant. The control signal remains moderate during the input tracking as well as the disturbance rejection period. Naturally, here the closed loop poles of the ppPIDt controlled system and the reference system closed loop poles do fully coincide.

The controller turned out to be quite robust with respect to changes in the nominal plant. The nominal plant was perturbed like this

$$\frac{b(s)}{a(s)} = \frac{2\rho}{(s+2\rho)}\frac{\rho}{(s^2+0.2\rho s+\rho)}, \tag{29}$$

stability was maintained for $0.5 \leq \rho \leq 3.2$, results are shown for the case $\rho = 0.6$ in Fig. 4. Naturally, here the closed loop poles of the ppPIDt controlled system and the reference system closed loop poles do not coincide.

An IMC was also tested here and effectively resulted in a type 1 controller with zeros completely cancelling the system dynamics and two real poles, when examined as a controller directly preceeding the plant. The input tracking phase was excellent with a moderate control signal, thus a

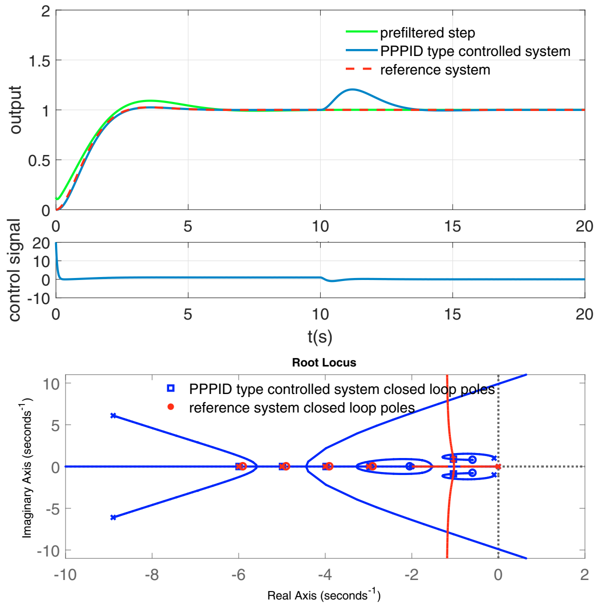

Fig. 3. Example 1. A ppPIDt controller for a highly under-damped plant.

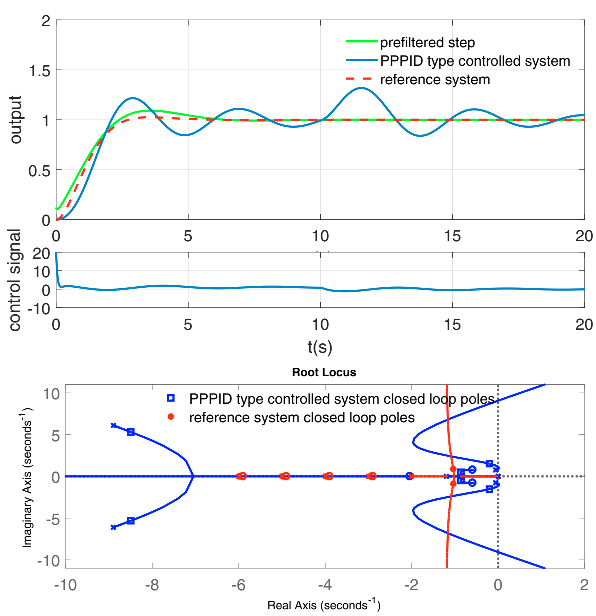

Fig. 4. Example 1. A ppPIDt controller for a highly under-damped plant, robustness study with respect to changes in nominal plant, $\rho = 0.6$.

prefilter was not used to curb the control signal. However, as typically happens when highly underdamped system poles are cancelled, the disturbance rejection was not acceptable. It should be noted that a feed forward (FF) was not implemented to aid in the disturbance rejection. In fact, both controllers might benefit from a FF if the disturbance can be measured. In the case when a disturbance can not be measured, a disturbance observer may be successfully used with both controllers Hauksdóttir et al. (2011).

Example 2: We now present a regular PID designed by the same approach, i.e., by solving (13) in a least squares sense.

(1) Given $n = 2$, selecting $N = 1$, we have
$$m_c = 1 + 2 - 1 = 2 \tag{30}$$
and
$$n_f = 2 - 1 = 1. \tag{31}$$
(2) We choose a reference system
$$K_r \frac{b_r(s)}{s a_r(s)} = 2\frac{(s+2.9)(s+3.9)(s+4.9)}{s(s+2)(s+3)(s+4)(s+5)} \tag{32}$$
which has dominant closed loop poles at $s_{1,2} = -1.04 \pm j0.902$.
(3) We now solve
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 0 \\ a_1 & b_0 & 0 & 0 \\ a_0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_0 \end{bmatrix} \begin{bmatrix} f_0 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix} \approx \begin{bmatrix} \delta_4 - a_2 \\ \delta_3 - a_1 \\ \delta_2 - a_0 \\ \delta_1 \\ \delta_0 \end{bmatrix} \tag{33}$$
This results in the ppPIDt controller given by
$$\begin{aligned} &\frac{67.41s^2 + 75.64s + 55.42}{s(s+28.99)} \\ &= 67.41\frac{(s+0.56 \pm j0.71)}{s(s+28.99)}. \end{aligned} \tag{34}$$

and a prefilter of
$$\frac{K_r b_r(s)}{c(s)b(s)} = \frac{2(s+2.9)(s+3.9)(s+4.9)}{6.74(s+0.56 \pm j0.71)2(s/20+1)}, \tag{35}$$
where the pole at $s = -20$ is a padding pole.

The resulting closed loop control signal and output and the corresponding root locus are shown in Fig. 5. The resulting gain margin is 16.3dB and the corresponding phase margin was found to be 13 degrees, a bit low. Naturally, the closed loop poles of the PID controlled system and the reference system do not coincide here and the input tracking response and the disturbance rejection are considerably more oscillating. The control signal remains moderate throughout the simulation.

6. CONCLUSIONS

The problem of designing a pole placing PID type controller was considered by extending the number of PID zeros, PID integrators and PID poles to aid in the response shaping and stabilizing of the closed loop. Several examples were tested with quite good results. The method can deal with highly oscillatory plants with good input tracking and disturbance rejection. This controller moves poles without cancellation and it does not cancel plant zeros. Thus, it can deal with unstable plants, as well as plants with right half plane zeros. Robustness has also been successfully tested.

It is also possible to use the same method for computing a regular PID, in that case the pole placement becomes a least squares solution to an overdetermined system of

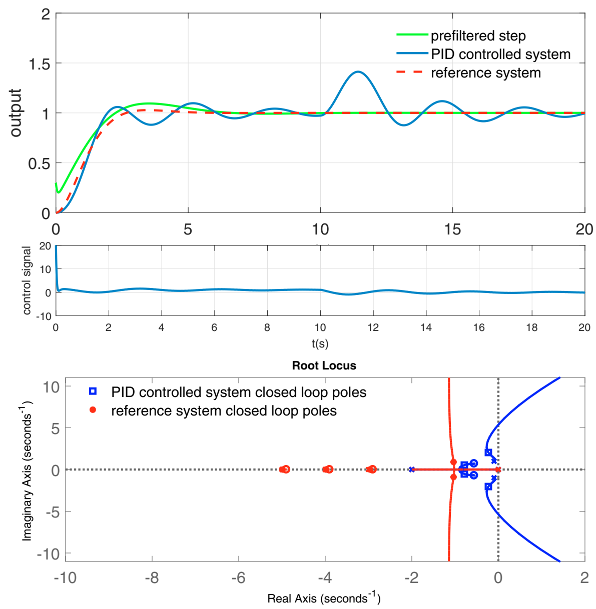

Fig. 5. Example 2. A regular PID for a highly underdamped plant.

equations. However, closed loop stability is not guaranteed in that case.

In summary, the ppPIDt controller is easily computed and implemented, effective in input tracking and disturbance rejection, without a high cost in the control signal, in addition to closed loop stability being guaranteed. Finally, the methodology is easily presented in basic control courses.

REFERENCES

K.J. Åström, T. Hägglund. Advanced PID control. ISA-The Instrumentation, Systems and Automation Society, 2006.

K.J. Åström, T. Hägglund. Benchmark systems for PID control. *Proceedings of the IFAC workshop, Digital Control: Past, Present and Future of PID Control.* Terassa, Spain:165–166, 2000.

K.J. Åström, B. Wittenmark. Computer–Controlled Systems, Theory and Design. Prentice Hall, Information and Systems Sciences Series, 1997, 186-195.

A. Datta, M.-T. Ho, and S. P. Bhattacharyya. Structure and synthesis of PID controllers. Springer Science & Business Media, 2013.

K.J. Åström, and P.R. Kumar. Survey paper-Control: a perspective. *Automatica,* 50:3-43, 2014.

K.J. Åström, T. Hägglund, C.C. Hang, and W.K. Ho. Automatic tuning and adaptation for PID controllers–a survey. *Control Engineering Practice,* 1:4:699-714, 1993.

J. Berner, K. Soltesz, T. Hägglund, and K.J. Åström. An experimental comparison of PID autotuners. *Control Engineering Practice,* 73:124-133, 2018.

A.S. Hauksdóttir, S.Þ. Sigurðsson. The matching coefficients PID controller. *The 2011 American Control Conference.* San Fransisco, USA, June 29 - July 1:710–715, 2011.

G. Herjólfsson, A.S. Hauksdóttir, S.Þ. Sigurðsson. L_2/H_2-*optimal zeros two stage optimization.* The *IFAC Conference on Advances in PID Control, PID'12.* March 28-30, 2012, Brescia, Italy.

T. Kailath. Linear Systems. Prentice Hall, Information and Systems Sciences Series, 1980, 297-310.

L.H. Keel, S.P. Bhattacharyya. Robust, Fragile, or Optimal? *IEEE Transactions on Automatic Control.* 42:8: 1098–1105, 1997.

V. Kucera. Diophantine Equations in Control–A Survey. *Automatica,* 29:6:1361-1375, 1993.

S. Skogestad, I. Postlethwaite. Multivariable feedback control: analysis and design. Wiley New York, 2007.

Appendix A. THE DIOPHANTINE EQUATION AND THE SYLVESTER MATRIX IN CONTROL PROBLEMS

There are many occurrences of the Diophantine equation and the subsequent Sylvester matrix in control, see, e.g., Kailath (1980), Kucera (1993) and Åström et al. (1997).

Even the simple pole placement problem applying state feedback without an observer, i.e., assuming all states are available for feedback, leads to a very simple form of the Diophantine equation, where the original characteristic equation plus a shifting characteristic equation leads to the new characteristic equation. This is easily seen by considering the regular pole placement problem where $c(s)/f(s)$ is implemented in the feedback. If $f(s) = b(s)$ for stable $b(s)$ there results

$$\frac{\frac{b(s)}{a(s)}}{1+\frac{c(s)b(s)}{f(s)a(s)}} = \frac{f(s)b(s)}{f(s)a(s)+c(s)b(s)} = \frac{b(s)}{a(s)+c(s)} = \frac{b(s)}{\delta(s)}. \tag{A.1}$$

Thus, the number of poles remains unchanged if the degree of $c(s)$ is no larger than that of $a(s)$. This corresponds to

$$a(s) = \det(sI - A), \tag{A.2}$$
$$b(s) = C\mathrm{Adj}(sI - A)B \tag{A.3}$$

and

$$c(s) = K\mathrm{Adj}(sI - A)B \tag{A.4}$$

in the state space form $\{A, B, C\}$, where K is the state feedback vector.

In the more general case of an observer controller designed using a transfer function formulation Kailath (1980), we end up with a more general Diophantine equation, leading to a polynomial of order $2n$ and a subsequent Sylvester matrix. The polynomial is then selected to be equal to the new characteristic polynomial times the observer poles. We then end up with an observer controller setup, wherein the observer dynamics are cancelled, effectively making the observer unobservable. This leads to a new transfer function of order n, having a new set of poles, but keeping the original zeros.

Various other setups in control lead to the Diophantine equation and the Sylvester matrix, see Kucera (1993) for a survey paper on the subject and Åström et al. (1997) for setups for discrete time systems.