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PID Tuning for Time-Varying Delay Systems Based on Modified Smith Predictor ⋆

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Abstract: This paper presents a new linear matrix inequality (LMI) based control strategy for second-order systems with time-varying delay based on a modified Smith predictor with a proportional-integral-derivative (PID) controller. The main idea consists in representing the predictor model as a closed-loop observer, which takes into account only the estimated average value of the time-delay, such that no real time measurement of the delay is required. A numerical example is introduced to illustrate the effectiveness of the proposed method.

Keywords: Process time-delay, Smith predictor, PID controller, linear matrix inequalities.

1. INTRODUCTION

Time-delays appear naturally in different kinds of real world applications, in which the transportation or propagation of material, energy or information is present, as well as they can also be induced by sensors and actuators in the controller loop. As a matter of fact, time-delays can introduce instability and performance degradation and the control design should take it into account.

Among well-known methods in the literature to deal with time-delays, a class of them is based on the compensation of the time-delay in which the free delayed output of the system is predicted and feedbacked to the controller. The Smith predictor is one example of this kind of method and it has been largely used in the industry (Normey-Rico and Camacho, 2008). However, the main issue in the standard Smith predictor is that it is not robust to parameter uncertainties in the model as well as uncertain delays. Notice also that the standard Smith predictor is just applied to stable SISO (Single Input - Single Output) systems. For further details about advantages and disadvantages see Palmor (1996), Normey-Rico and Camacho (2008), and references therein.

As the Smith predictor has a great appeal for the industrial context, a lot of effort has been done to improve its robustness as, for example, including explicitly uncertain parameters (Santacesaria and Scattolini, 1993; Palmor and Blau, 1994; Lee. et al., 1999), or changing its standard structure as in Normey-Rico et al. (1997) and Normey-Rico and Camacho (1999, 2009). More recently, different alternatives have also been introduced in the literature to deal explicitly with time-varying delay processes using, for example, linear matrix inequalities (LMIs) and H_∞ performance (Oliveira and Karimi, 2013; Normey-Rico et al., 2012; Bolea et al., 2014).

In this paper, we introduce a control strategy for second-order systems with time-varying delay based on a modified Smith predictor with a PID controller. The main contribution of the proposed approach is predicting the system output using a closed-loop observer, which takes into account only the constant estimated average value of the time-delay. Thus, real time measurement of the delay is not required. The main design results are derived as LMIs conditions. A numerical example is introduced to illustrate the effectiveness of the proposed method.

Notation: "$*$" denotes symmetric terms in matrices. The superscript "T" stands for transpose. For a real symmetric matrix M, $M > 0$ (< 0) means that M is positive (negative) definite. $\text{sm}\{M\}$ denotes $M+M^T$, I the identity matrix, and $\text{diag}\{\cdot\}$ stands for a diagonal matrix with the entries on the main diagonal.

2. PROBLEM FORMULATION

Consider second-order plants with time-delay in the form:

$$G(s) = \frac{b_0 e^{-d_\tau s}}{s^2 + a_1 s + a_0} \quad (1)$$

where the time-delay, d_τ, is assumed uncertain but belonging to a given interval, namely $d_\tau \in [\tau - \mu, \tau + \mu]$, with τ representing the estimated average delay and $0 \leq \mu \leq \tau$ is a scalar parameter used to define the time-delay range.

The aim is to design a PID controller to the system (1) modifying appropriately the standard Smith predictor, illustrated in Fig. 1, in order to design a control system more robust to uncertain delay.

In this paper, to compensate the time-delay uncertainties the predictor in the traditional Smith structure, which is

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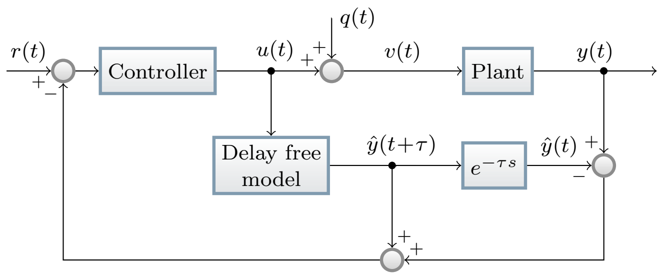

Fig. 1. Standard Smith predictor structure.

typically an open-loop observer, is replaced by a kind of closed-loop observer. The proposed structure is depicted in Fig. 2. Moreover the offered methodology is based on two steps: i) firstly the PID is tuned assuming the system free of delay, following the idea in the traditional Smith predictor, and in the sequel, ii) the observer design is performed considering the pre-tuned PID controller.

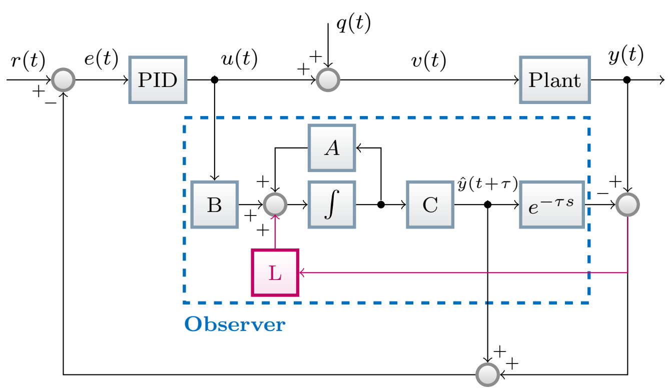

Fig. 2. The controller structure proposed.

In order to formulate the problem in mathematical form, note that one possible state-space realization for (1) with input $v(t) = u(t) + q(t)$ is given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) \\ y(t) &= Cx(t - d_\tau(t)) \end{aligned} \tag{2}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \text{ and } C = [b_0 \; 0].$$

In the sequel, whenever the uncertain delay d_τ is assumed time-varying, i.e. $d_\tau(t)$, one should refer to the time-varying state-space realization (2) instead of the time-invariant transfer-function (1).

The observer structure adopted is the standard one:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t - \tau) \end{cases} \tag{3}$$

where L has to be computed and the remaining matrices are given in (2). Further, for the observer only the estimated average time-delay, τ, is considered. One advantage of this approach is that no real time measurement of the time-delay is needed.

Moreover assume the PID controller

$$C(s) = k_p + \frac{k_i}{s} + k_d \frac{\alpha s}{s + \alpha} \tag{4}$$

where α is the derivative filter parameter.

One possible state-space realization for the PID controller (4) is given as:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c e(t) \\ u(t) = C_c x_c(t) + D_c e(t), \end{cases} \tag{5}$$

with

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C_c^T = \begin{bmatrix} k_i \alpha \\ k_i \\ k_d \alpha^2 \end{bmatrix}, \; D_c = k_p + k_d \alpha.$$

where the matrices A_c, B_c, C_c, and D_c should be designed for the system free of time-delay previously than the observer in (3).

According Fig. 2, $e(t)$ is defined as:

$$e(t) = r(t) - \hat{y}(t + \tau) - y(t) + \hat{y}(t).$$

Using the fact that:

$$x(t - d_\tau(t)) = x(t - \tau) - \int_{d_\tau(t)}^{\tau} \dot{x}(t - \xi) d\xi,$$

and defining $\tilde{x}(t) = x(t) - \hat{x}(t)$, $\bar{x}^T = [x^T(t) \; \tilde{x}^T(t) \; x_c^T(t)]^T$, and $z^T(t) = [q(t) \; r(t)]$, the closed-loop system, described from Fig. 2, is given by:

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_d \bar{x}(t-\tau) + \bar{A}_t \int_{d_\tau(t)}^{\tau} \dot{\bar{x}}(t-\xi) d\xi + \bar{D}z(t), \tag{6}$$

with

$$\bar{A} = \begin{bmatrix} A - BD_cC & BD_cC & BC_c \\ 0 & A & 0 \\ -B_cC & B_cC & A_c \end{bmatrix}, \quad \bar{A}_d = \begin{bmatrix} 0 & -BD_cC & 0 \\ 0 & -LC & 0 \\ 0 & -B_cC & 0 \end{bmatrix},$$

$$\bar{A}_t = \begin{bmatrix} BD_cC & 0 & 0 \\ LC & 0 & 0 \\ B_cC & 0 & 0 \end{bmatrix}, \quad \text{and} \quad \bar{D} = \begin{bmatrix} B & BD_c \\ B & 0 \\ 0 & -B_c \end{bmatrix}. \tag{7}$$

The main results of this paper are stated in the next section.

3. MAIN RESULTS

Firstly an LMI delay dependent stability condition for the closed-loop system in (6) is presented. Afterwards such stability condition is used as the starting point to design the observer (3) used as the predictor model.

***Theorem** 1.* Let $\tau > 0$, the estimated average time-delay, $0 \leq \mu \leq \tau$, the upper bound to its uncertainty range, be given. Then the closed-loop system in (6) is asymptotically stable if there exist matrices $P = P^T$, Q, $R_1 = R_1^T$, R_2, $R_3 = R_3^T$, $S = S^T$, $Z = Z^T$, $U = U^T$, F, and $G \in \mathbb{R}^{6\times 6}$, such that the following LMIs hold:

$$\begin{bmatrix} P & Q \\ * & \frac{1}{\tau}S \end{bmatrix} > 0, \quad \bar{R} = \begin{bmatrix} R_1 & R_2^T \\ * & R_3 \end{bmatrix} > 0, \tag{8}$$

$$\Sigma = \left[\begin{array}{c|c} \Pi & \Phi^T \\ \hline * & -\mu U \end{array}\right] < 0 \tag{9}$$

with

$$\Phi = \left[(\mu F \bar{A}_t)^T \; (\mu G \bar{A}_t)^T \; 0 \; 0 \right], \tag{10}$$

$$\Pi = \begin{bmatrix} \Pi_{(1,1)} & \Pi_{(1,2)} & -Q + \frac{1}{\tau}R_3 + F\bar{A}_d & -\frac{1}{\tau}R_2 + \frac{2}{\tau}Z \\ * & \Pi_{(2,2)} & G\bar{A}_d & Q \\ * & * & -S - \frac{1}{\tau}R_3 & \frac{1}{\tau}R_2 \\ * & * & * & -\frac{1}{\tau}R_1 - \frac{2}{\tau^2}Z \end{bmatrix}, \tag{11}$$

$$\Pi_{(1,1)} = S + \mathrm{sm}\{Q\} + \tau R_1 - \frac{1}{\tau}R_3 + \mathrm{sm}\{F\bar{A}\} - 2Z,$$
$$\Pi_{(1,2)} = \tau R_2^T + P + \bar{A}^T G^T - F,$$
$$\Pi_{(2,2)} = 2\mu U + \tau R_3 - \mathrm{sm}\{G\} + \frac{\tau^2}{2}Z.$$

Proof. Consider the Lyapunov-Krasovskii functional candidate:

$$V(\bar{x}_t) = \bar{x}^T(t)P\bar{x}(t) + 2\bar{x}^T(t)\int_{-\tau}^{0} Q\bar{x}(t+\xi)d\xi \tag{12}$$
$$+ \int_{-\tau}^{0}\int_{t+s}^{t} \chi^T(\xi)\bar{R}\chi(\xi)d\xi ds$$
$$+ \int_{-\tau}^{0} \bar{x}^T(t+\xi)S\bar{x}(t+\xi)d\xi$$
$$+ \int_{-\mu}^{\mu}\int_{t+s-\tau}^{t} \dot{\bar{x}}^T(\xi)U\dot{\bar{x}}(\xi)d\xi ds$$
$$+ \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+s}^{t} \dot{\bar{x}}^T(\xi)Z\dot{\bar{x}}(\xi)d\xi ds d\theta$$

with $\chi^T(\xi) \triangleq [\bar{x}^T(\xi)\ \ \dot{\bar{x}}^T(\xi)]$, $\bar{R}$ given as in (8), and $\bar{x}_t$ is the value of $\bar{x}(\phi)$ with $\phi \in [t-\tau-\mu,\ t]$. The main advantage of this functional is the introduction of triple integral term, which contributes to reduction of conservatism (Sun et al., 2010).

Notice that the closed-loop system (6) is asymptotically stable if (12) satisfies the following both conditions:

$$V(\bar{x}_t) \geq \epsilon||\bar{x}(t)||^2 \tag{13}$$

and

$$\dot{V}(\bar{x}_t) \leq -\epsilon||\bar{x}(t)||^2 \tag{14}$$

for $\epsilon > 0$ sufficiently small.

Firstly we show that the functional (12) satisfies the positiveness condition in (13) if LMIs (8) and (9) hold.

Notice that if (8) is true then $S > 0$. Therefore, by Jensen inequality (Gu et al., 2003), it follows:

$$\int_{-\tau}^{0}\bar{x}^T(t+\xi)S\bar{x}(t+\xi)d\xi \geq \frac{1}{\tau}\int_{-\tau}^{0}\bar{x}^T(t+\xi)d\xi S\int_{-\tau}^{0}\bar{x}(t+\xi)d\xi.$$

Then using the above inequality and (12) we have that:

$$V(\bar{x}_t) \geq \zeta^T \begin{bmatrix} P & Q \\ * & \frac{1}{\tau}S \end{bmatrix} \zeta$$
$$+ \int_{-\tau}^{0}\int_{t+s}^{t} \chi^T(\xi)\bar{R}\chi(\xi)d\xi ds$$
$$+ \int_{-\mu}^{\mu}\int_{t+s-\tau}^{t} \dot{\bar{x}}^T(\xi)U\dot{\bar{x}}(\xi)d\xi ds$$
$$+ \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+s}^{t} \dot{\bar{x}}^T(\xi)Z\dot{\bar{x}}(\xi)d\xi ds d\theta \tag{15}$$

with $\zeta^T = \left[\bar{x}^T(t)\ \ \int_{-\tau}^{0}\bar{x}^T(t+\xi)d\xi\right]$.

Therefore, if LMIs (8) hold then the first two terms in the inequality above are positive, and if (9) is true then U and Z in (15) are positive definite, implying that (13) is satisfied.

On the other hand, it is also necessary to show that if the LMIs conditions hold then the functional (12) satisfies condition (14).

Taking the time derivative of the functional it follows that:

$$\dot{V}(\bar{x}_t) = 2\bar{x}^T(t)P\dot{\bar{x}}(t) + 2\dot{\bar{x}}^T(t)\int_{-\tau}^{0} Q\bar{x}(t+\xi)d\xi \tag{16}$$
$$+2\bar{x}^T(t)Q\bar{x}(t) - 2\bar{x}^T(t)Q\bar{x}(t-\tau)$$
$$+\tau\chi^T(t)\bar{R}\chi(t) - \boxed{\int_{-\tau}^{0}\chi^T(t+\xi)\bar{R}\chi(t+\xi)d\xi}$$
$$+\bar{x}^T(t)S\bar{x}(t) - \bar{x}^T(t-\tau)S\bar{x}(t-\tau)$$
$$+2\mu\dot{\bar{x}}^T(t)U\dot{\bar{x}}(t) - \int_{t-\tau-\mu}^{t-\tau+\mu}\dot{\bar{x}}^T(s)U\dot{\bar{x}}(s)ds$$
$$+\frac{\tau^2}{2}\dot{\bar{x}}^T(t)Z\dot{\bar{x}}(t) - \boxed{\int_{-\tau}^{0}\int_{t+s}^{t}\dot{\bar{x}}^T(\xi)Z\dot{\bar{x}}(\xi)d\xi ds}.$$

Applying the Jensen inequality to each quadratic form in the boxes, we have:

$$\dot{V}(\bar{x}_t) \leq 2\bar{x}^T(t)P\dot{\bar{x}}(t) + 2\bar{x}^T(t)Q\bar{x}(t) \tag{17}$$
$$+2\dot{\bar{x}}^T(t)\int_{-\tau}^{0}Q\bar{x}(t+\xi)d\xi - 2\bar{x}^T(t)Q\bar{x}(t-\tau)$$
$$-\left[\int_{-\tau}^{0}\chi^T(t+\xi)d\xi\right]\bar{R}\left[\int_{-\tau}^{0}\chi(t+\xi)d\xi\right]$$
$$+\tau\chi^T(t)\bar{R}\chi(t) + \bar{x}^T(t)S\bar{x}(t) - \bar{x}^T(t-\tau)S\bar{x}(t-\tau)$$
$$+2\mu\dot{\bar{x}}^T(t)U\dot{\bar{x}}(t) - \int_{t-\tau-\mu}^{t-\tau+\mu}\dot{\bar{x}}^T(s)U\dot{\bar{x}}(s)ds$$
$$+\frac{\tau^2}{2}\dot{\bar{x}}^T(t)Z\dot{\bar{x}}(t) - 2\bar{x}^T(t)Z\bar{x}(t)$$
$$+\frac{4}{\tau}\bar{x}^T(t)Z\int_{-\tau}^{0}\bar{x}(t+\xi)d\xi$$
$$-\frac{2}{\tau^2}\left[\int_{-\tau}^{0}\bar{x}^T(t+\xi)d\xi\right]Z\left[\int_{-\tau}^{0}\bar{x}(t+\xi)d\xi\right].$$

Now considering (6) with $z(t) = 0$ the following null term is obtained with two free weighting matrices F and G:

$$0 = 2[\bar{x}^T(t)F + \dot{\bar{x}}^T(t)G]\left[-\dot{\bar{x}}(t) + \bar{A}\bar{x}(t) + \bar{A}_d\bar{x}(t-\tau)\right.$$
$$\left.+\bar{A}_t\int_{d_\tau(t)}^{\tau}\dot{\bar{x}}(t-\xi)d\xi\right] \tag{18}$$
$$= 2[\bar{x}^T(t)F + \dot{\bar{x}}^T(t)G][-\dot{\bar{x}}(t)+\bar{A}\bar{x}(t)+\bar{A}_d\bar{x}(t-\tau)] + f(t)$$

where

$$f(t) = 2\Lambda\bar{A}_t\int_{d_\tau(t)}^{\tau}\dot{\bar{x}}(t-\xi)d\xi,$$

and $\Lambda = [\bar{x}^T(t)F + \dot{\bar{x}}^T(t)G]$.

Moreover, invoking the inequality $2a^Tb \leq a^TUa + b^TU^{-1}b$ where a and b are vectors and U is a defined positive matrix, $f(t)$ can be bounded as

$$f(t) \leq \int_{d_\tau(t)}^{\tau} (\Lambda\bar{A}_t)U^{-1}(\Lambda\bar{A}_t)^T d\xi + \int_{d_\tau(t)}^{\tau} \dot{\bar{x}}^T(t-\xi)U\dot{\bar{x}}(t-\xi)d\xi$$
$$\leq \mu(\Lambda\bar{A}_t)U^{-1}(\Lambda\bar{A}_t)^T + \int_{t-\tau-\mu}^{t-\tau+\mu} \dot{\bar{x}}^T(s)U\dot{\bar{x}}(s)ds.$$

Adding (18) to (17), and considering the upper bound to $f(t)$ and the definitions of $\bar{R}$ and χ, it follows that:

$$\dot{V}(\bar{x}_t) \leq \varphi^T(t)\left[\Pi + \Phi^T(\mu^{-1}U^{-1})\Phi\right]\varphi(t) \qquad (19)$$

with $\varphi^T(t) = \left[\bar{x}^T(t)\ \dot{\bar{x}}^T(t)\ \bar{x}^T(t-\tau)\ \int_{-\tau}^{0}\bar{x}^T(t+\xi)d\xi\right]$, and Φ is given in (10) and Π is defined as in (11).

Thus, from (19) notice that (14) is satisfied if

$$\Pi + \Phi^T(\mu^{-1}U^{-1})\Phi < 0$$

or, after applying Schur's complement, if $\Sigma < 0$, which is LMI (9). Concluding, if LMIs (8) and (9) hold then (14) is true. □

The complete design is carried out also considering an H_∞ performance index in the sense of the following definition.

***Definition** 1.* Consider (2) and (5). The observer (3) is said to be γ-admissible if:

(1) $\lim_{t\to\infty} [x(t) - \hat{x}(t)] = 0$ for $r(t) = q(t) = 0$.

(2) For zero initial conditions, there exists a positive scalar γ such that (Mansouri et al., 2009):

$$\int_0^\infty \tilde{x}^T(t)W\tilde{x}(t)dt \leq \gamma^2 \int_0^\infty [r^T(t)r(t) + q^T(t)q(t)]dt, \qquad (20)$$

in which W is a weighting positive definite matrix and γ is the attenuation level prespecified.

Basically the first condition in the previous definition requires the estimation error to be stable and the second one ensures robustness to exogenous inputs.

The proposed observer design method is presented in the sequel.

***Theorem** 2.* Let $\tau > 0$, the estimated average time-delay, $0 \leq \mu \leq \tau$, the upper bound to its uncertainty range, and nonzero scalars η_1, η_2, and η_3 be given. Then the closed-loop system (6) is asymptotically stable and the observer (3) is γ-admissible if there exist matrices $G_2 \in \mathbb{R}^{2\times2}$, $\hat{L} \in \mathbb{R}^{2\times1}$, F_1, F_2, G_1, $G_3 \in \mathbb{R}^{6\times2}$, and $P = P^T$, Q, $R_1 = R_1^T$, R_2, $R_3 = R_3^T$, $S = S^T$, $Z = Z^T$, $U = U^T \in \mathbb{R}^{6\times6}$, such that the LMIs in (8) and the following LMI hold:

$$\hat{\Sigma} = \left[\begin{array}{c|c} \hat{\Pi} & \hat{\Phi}^T \\ \hline * & -\mu U \end{array}\right] < 0 \qquad (21)$$

with

$$\hat{\Phi}^T = \begin{bmatrix} \mu(F_1BD_c + I_\eta\hat{L} + F_2B_c)CI_1 \\ \mu(G_1BD_c + I_I\hat{L} + G_3B_c)CI_1 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{(1,1)} & \hat{\Pi}_{(1,2)} & \hat{\Pi}_{(1,3)} & 2\delta Q - \frac{1}{\tau}R_2 + \frac{2}{\tau}Z & \hat{\Pi}_{(1,5)} \\ * & \hat{\Pi}_{(2,2)} & \hat{\Pi}_{(2,3)} & Q & \hat{\Pi}_{(2,5)} \\ * & * & -\frac{1}{\tau}S + R_3 & \frac{1}{\tau}R_2 & 0 \\ * & * & * & -\frac{1}{\tau}R_1 - \frac{2}{\tau^2}Z & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

in which

$$\hat{\Pi}_{(1,1)} = S + \mathrm{sm}\{Q\} + \tau R_1 - \frac{1}{\tau}R_3 - 2Z$$
$$+\mathrm{sm}\{F_1[(A - BD_cC)I_1 + BD_cCI_2 + BC_cI_3]$$
$$+F_2[-B_cC(I_1 - I_2) + A_cI_3] + I_\eta G_2AI_2\} + \bar{W}$$
$$\hat{\Pi}_{(1,2)} = \tau R_2^T + P + [I_3^TC_c^TB^T - I_1^T(C^TD_c^TB^T - A^T)]G_1^T$$
$$+[I_3^TA_c^T + (I_2^T - I_1^T)C^TB_c^T]G_3^T + I_2^TA^TG_2^TI^T$$
$$-F_1I_1 - I_\eta G_2I_2 - F_2I_3$$
$$\hat{\Pi}_{(1,3)} = -Q + \frac{1}{\tau}R_3 - (F_1BD_c - I_\eta\hat{L} - F_2B_c)CI_2$$
$$\hat{\Pi}_{(1,5)} = F_1(BI_i + BD_cI_{ii}) + I_\eta G_2BI_i + F_2B_cI_{ii}$$
$$\hat{\Pi}_{(2,2)} = 2\mu U + \tau R_3 + \frac{\tau^2}{2}Z - \ \mathrm{sm}\{G_1I_1 + I_IG_2I_2 + G_3I_3\}$$
$$\hat{\Pi}_{(2,3)} = -(G_1BD_c + I_I\hat{L} + G_3B_c)CI_2$$
$$\hat{\Pi}_{(2,5)} = G_1(BI_i + BD_cI_{ii}) + I_IG_2BI_i + G_3B_cI_{ii}$$

and

$$I_I = \begin{bmatrix} I \\ I \\ I \end{bmatrix},\ I_\eta = \begin{bmatrix} \eta_1 I \\ \eta_2 I \\ \eta_3 I \end{bmatrix},\ I_1^T = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix},\ I_2^T = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix},\ I_3^T = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix},$$
$$I_i = [\,I_d\ 0\ 0\,],\ I_{ii} = [\,0\ I_d\ 0\,],\ \text{and } I_d = [\,I\ 0\,]. \qquad (22)$$

In the affirmative case, the observer gain in (3) is given by: $L = G_2^{-1}\hat{L}$.

Proof. It will be shown that if all conditions in this theorem are satisfied, then the closed-loop system in (6) with the observer matrix gain $L = G_2^{-1}\hat{L}$ satisfies the conditions of Theorem 1. Therefore it suffices to show that if LMI (21) is satisfied then LMI (9) is also satisfied.

LMI (21) is derived following similar steps as before in Theorem 1 to obtain LMI (9), but with a particular null term as the one in (18) obtained considering the closed-loop system in (6) with the input $z(t) \neq 0$. Besides, matrices F and G are partitioned as:

$$F = [F_1\ I_\eta G_2\ F_2] \text{ and } G = [G_1\ I_IG_2\ G_3]$$

where F_1, F_2, G_1, and $G_3 \in \mathbb{R}^{6\times2}$, $G_2 \in \mathbb{R}^{2\times2}$, and I_I and I_η are defined as in (22).

The H_∞ index[1] in (20) is considered by imposing that

$$\int_0^\infty [\dot{V}(\bar{x}_t) + \bar{x}^T(t)\bar{W}\bar{x}(t) - \gamma^2z^T(t)z(t)]dt < 0.$$

Finally, the change of variable $\hat{L} = G_2L$ is used. Notice that if the proposed LMI in the Theorem holds, then G_2 is nonsingular since the block $\hat{\Pi}_{(2,2)}$ in LMI (21) imposes:

[1] Note that the H_∞ criterion in (20) can be rewritten as:

$$\int_0^\infty \bar{x}^T(t)\bar{W}\bar{x}(t)dt \leq \gamma^2\int_0^\infty z^T(t)z(t)dt, \text{ with } \bar{W} = \mathrm{diag}\{0, W, 0\}.$$

$$\mathrm{sm}\left\{\left[\begin{array}{c|c|c} & G_2 & \\ G_1 & G_2 & G_3 \\ & G_2 & \end{array}\right]\right\} > 0.$$

Then the observer gain follows in a straightforward way.

Therefore, if LMIs (8) and (21) are satisfied, it ensures that the observer (3) is γ-admissible as stated in Definition 1 and the observer gain is given by $L = G_2^{-1}\hat{L}$. □

4. PID TUNING VIA LMIs

The PID controller considered in the observer design is previously tuned for the system (1) free of delay. Clearly this follows the same idea for the control design using Smith predictor.

A possible strategy for the PID design is to use the one proposed in Parada et al. (2011, 2017) that translates the problem of designing the PID controller

$$C(s) = k_p + \frac{k_i}{s} + k_d s \tag{23}$$

for the system in (1) in a state feedback control problem, where the closed-loop system is given by:

$$\begin{aligned} \dot{x}(t) &= (A_a + B_a K E)x_a(t) \\ y(t) &= C_a x_a(t) \end{aligned} \tag{24}$$

in which $x_a^T(t) = [x^T(t)\ x_3(t)]^T$, $x_3 = \int y(t)dt$,

$$A_a = \left[\begin{array}{c|c} A & 0 \\ \hline C & 0 \end{array}\right],\ B_a = \begin{bmatrix} B \\ 0 \end{bmatrix},\ C_a = \begin{bmatrix} C \\ 0 \end{bmatrix}^T,$$

with A, B, and C given as in (2), and

$$K = [-k_p\ \ -k_d\ \ -k_i],\quad E = \mathrm{diag}\{b_0,\ b_0,\ 1\} \tag{25}$$

where b_0 is given in (1).

Therefore, the PID tuning problem is related to finding an appropriate matrix K in (25).

In the state-space context we consider the standard regional pole placement via LMIs in the $\mathcal{D}$ region given by the intersection of regions: *i*) half-plane $\mathbb{R}(s) < -\beta$, *ii*) conic sector with apex at the origin and inner angle 2θ, and *iii*) semicircle of radius r and center at the origin. The following theorem details how the pole placement in a given $\mathcal{D}$ region can be done via PID design.

Theorem 3. System (24) is asymptotically $\mathcal{D}$-stable if and only if there exist matrices $X > 0 \in \mathbb{R}^{3\times 3}$ and $Y \in \mathbb{R}^{1\times 3}$ satisfying:

$$2\beta X + \Upsilon + \Upsilon^T < 0,\quad \begin{bmatrix} -rX & \Upsilon \\ * & -rX \end{bmatrix} < 0,$$

$$\text{and } \begin{bmatrix} \sin\theta(\Upsilon + \Upsilon^T) & \cos\theta(\Upsilon - \Upsilon^T) \\ * & \sin\theta(\Upsilon + \Upsilon^T) \end{bmatrix} < 0$$

with $\Upsilon = A_a X + B_a Y$.

In the affirmative case, the static state feedback gain is given by: $K = YX^{-1}E^{-1}$.

Proof. It follows directly from Theorem 2.2 in Chilali and Gahinet (1996) replacing A by $A_a + B_a K E$ and using the change of variables $Y = KEX$.

5. CASE STUDY

Consider a water heater with a long pipe system described in Normey-Rico and Camacho (2007) and depicted in Fig. 3. In this system, the water is heated in the tank using an electric resistor and driven by a pump along a thermally insulated pipe to the output of the system. The control input is the power $\wp(t)$ at the resistor and the plant output is the temperature T at the end of the pipe. $\varpi(t)$ is the water flow. Since the temperature depends on the water flow, the time-delay is time-varying.

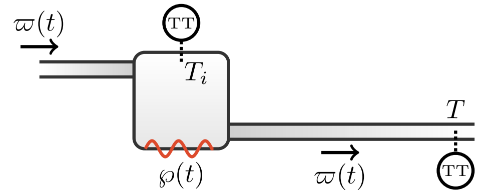

Fig. 3. Water heater adapted from Normey-Rico and Camacho (2007).

A second order model for this process is given by (Normey-Rico and Camacho, 2007, p. 46):

$$G(s) = \frac{1}{(1.5s+1)(0.4s+1)} e^{-d_\tau s} \tag{26}$$

with d_τ denoting an uncertain delay or time-varying delay.

Assume that the time-delay is varying inside the interval $[3.15,\ 3.85]$ and the design constraint for the maximum overshoot is 5%.

To deal with this problem the design is carried out in two steps: *i*) firstly the PID controller in (23) is tuned for the system (26) free of delay via Theorem 3 and, in the sequel, *ii*) the observer is designed via Theorem 2 considering the pre-tuned PID parameters and a given derivative filter parameter, α, as in (4).

The PID tuning is performed based on the regional pole placement for the closed-loop system free of delay choosing $\beta = 0.75$, $\theta = 45°$, and $r = 4$. The PID gains obtained via Theorem 3 are listed in Table 1.

Using the PID controller found by Theorem 3 and considering the derivative filter parameter $\alpha = 20$, we are in position to design the observer as proposed in Theorem 2. Then choosing $\eta_1 = \eta_3 = 1$, $\eta_2 = 5$, $\gamma = 0.2537$, and $W = \mathrm{diag}\{0.1,\ 0.1\}$, and using Theorem 2 the observer gain is obtained:

$$L = \begin{bmatrix} 0.0901 \\ -0.2009 \end{bmatrix}.$$

In order to check the effectiveness of the proposed method, we compare it with other approaches: the standard Smith predictor and PID control design methodology in Palmor and Blau (1994) and the PID control design method for time-varying delay systems in Mozelli and Souza (2016) [2]. The PID control gains obtained using Palmor and Blau (1994) and Mozelli and Souza (2016) are listed in Table 1.

Fig. 4 depicts the closed-loop time response of each method using the PID gains in Table 1. The time-delay in the

[2] The PID controller tuned using the method by Mozelli and Souza (2016) was computed using the parameters: $\delta = 0.1250$ and $\alpha = 1.15$.

Table 1. The PID control gains.

Method	k_p	k_i	k_d
Proposed method	**5.0822**	**3.5501**	**1.4335**
Palmor and Blau (1994)	1.6286	0.5143	0.5143
Mozelli and Souza (2016)	0.2184	0.1156	0.0681

simulation is considered as a time-varying signal. Table 2 presents the performance criteria, t_r rising time, t_s settling time, and M_p overshoot attained by each controller designed. The simulations reveal that the controller designed by the methodology proposed presents better performance than the standard Smith predictor/PID designed following Palmor and Blau (1994) and the PID controller designed as proposed by Mozelli and Souza (2016).

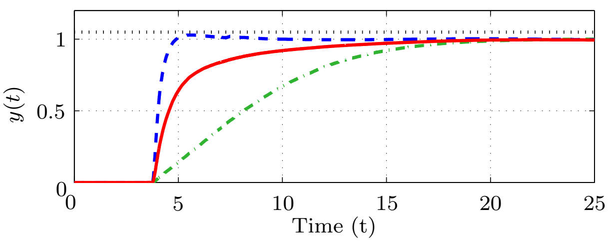

Fig. 4. Step response of the system (26) in closed-loop using the controller designed as proposed: in this paper (dashed line), the method in Palmor and Blau (1994) (solid line), and the one in Mozelli and Souza (2016) (dash-dot line).

Table 2. Performance criteria.

Method	t_r	t_s	$M_p(\%)$
Proposed Method	**0.6323**	**7.7473**	**3.2691**
Palmor and Blau (1994)	4.9614	16.1681	0
Mozelli and Souza (2016)	9.6100	19.0654	0

6. CONCLUSION

A control strategy for second-order systems with uncertain delay based on a modified Smith predictor combined with a PID controller has been formulated as a constrained LMI set. Basically the proposed control structure replaces the traditional Smith predictor model by a closed-loop observer in order to compensate the uncertain delay. The main advantage of the proposed method is its robustness to uncertain or even time-varying delay.

A numerical example has been shown significant improvements over some existing results. It is worth to mention that the proposed method performance is conditioned to the performance of the observer designed.

A possibly extension is to consider uncertainties in all model parameters, which is under further study.

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