Abstract—The dynamic response in the dynamic launching process of electromagnetic rail launcher (EMRL) is very important for the design of launchers and armatures. With the Bernoulli–Euler beam located on an elastic support as an object of the study, the analytical model of EMRL dynamic characteristics under the influence of EM repulsive force and contact force between the armature and the rail is obtained, and the corresponding 3-D finite-element simulation model [moving load–rail model (MR model)] is established. On this basis, the 3-D finite-element simulation model [projectile–rail model (PR model)] considering the projection vibration is established to obtain the dynamic response of EMRL due to projectile vibration. Studies show that the calculation results of the analytical model agree well with that of the MR model. When the moving load works at a critical speed and reaches the point of action, the rail will have an increased deformation and then a harmonic vibration, whose amplitude is five or six times than that under the noncritical operating condition. The analysis of the impact of projectile vibration on the rail deformation makes it known that the vibration amplitude of the PR model is four or five times than that of the MR model because of projectile vibration. The greater the gap between the rail and the armature is, the more violent is the projectile vibration and the greater is the vertical deformation of the rail, and the greater the density of the payload with the same volume is, the more intense is the projectile vibration and the greater is the vertical deformation of the rail. When launching reaches the critical speed, the rail deformation of the PR model is less than that of the MR model, which is mainly caused by the vibration of the projectile, but the vibration periods of those two models are the same.

Index Terms—Critical velocity, dynamic launching, dynamic response, electromagnetic rail launcher (EMRL), electromagnetic (EM) repulsive force, rail deformation.

I. INTRODUCTION

W ith the advent of the armature 5900 m/s in 1978, the electromagnetic (EM) rail launch technology has drawn wide attention [1]. The EM rail launcher (EMRL) utilizes the Lorentz force acting on the armature to quickly push the armature and payload for dynamic launching. It has advantages over traditional cannons in its projectile muzzle velocity being high and controllable and having a long firing range [2]. At first, the design of EMRL focuses on power characteristics but did not refer to dynamic characteristics on the device so that the launcher is difficult to apply to practical engineering. To solve the problem, it is necessary to make the EMRL light in weight and simple in its structure while superhigh-speed launch capability. In addition, the structured response of the launcher will directly affect the life of the rail and the launch accuracy [3]. For this reason, research on the dynamic response of EMRL in its dynamic launching process is extremely important.

Timoshenko [4] realized the problem of rail dynamic response and derived an analytic solution to the critical velocity of the beam under the condition of the constant moving load by the use of the Bernoulli–Euler beam on the elastic support as an object of the study. In order to find an analytical solution to the EMRL dynamic response, there is a need to simplify its model. Recently, a lot of scholars have made simulations of the dynamic response, with the EMRL simplified as a beam with the elastic support and the armature as a moving load. In the EMRL model, the beam represents the rail, the elastic support involves the insulating support and containment structure, the elastic coefficient of the support refers to the interaction between the rail and containment structure, and the Heaviside step function is used to simulate the EM force identical to the velocity of the projectile along the rail. Thus, the analytical results of dynamic response can be obtained with the numerical integration method [3], [5]–[6]. On this basis, Bin Cao et al. [7] established a thermal expansion model of the armature, with changes in armature temperature in the launching excluded from consideration, thereby achieving the analytical results of dynamic response, which are quiet consistent with the experimental results. Using the EM railgun rapid fire railgun in the French-German Research Institute of Saint-Louis as an object of the study, Gildutis et al. [8] adopted the finite-element method (FEM) to analyze the deformation of EMRL quantitatively and qualitatively and found that the bolt elongation accounts for the largest proportion of the vertical rail deformation and the bolt extension in the dynamic simulation doubles that under the static working condition.

With the development of computer technology, the 3-D finite-element software was first used by Nechitailo and Lewis [9] for the 3-D visualization simulation of the critical velocity of the superhigh-speed launcher. The simulation result is in good agreement with that calculated by the theoretical model [9]. Based on this, Nechitailo and Lewis [9] further studied the influence of the rail cross-sectional size on the critical velocity of the launcher and thought that the critical velocity would decrease as the radius-to-wall thickness of the
rail cross section [10]. Meanwhile, they calculated the critical velocity of the launcher according to Young’s modulus as regards the rail, support, and barrel under different operating conditions. This has provided a good reference for the design of superhigh-speed launchers [11].

During the dynamic launch of the EMRL, the repulsive of the armature to the rail acts on the rail in the form of physical force. However, the analytical model usually uses the equivalent distributed load instead of the armature to directly act on the rail, ignoring the influence of the projectile (including armature and payload) vibration on the rail deformation. Therefore, a 3-D moving load–rail model (MR model) and a 3-D projectile–rail model [projectile–rail model (PR model)] have been established to study the influence of the projectile motion on the rail dynamic response. Considering the rail repulsive force and the contact force between the rail and the armature, an analytical model is first derived, and then, the FEM is used to carry out the dynamic simulation of MR model and PR model.

II. PHYSICAL MODEL AND MATHEMATICAL MODEL

A. Physical Model

In order to study the effect of the projectile motion on the rail dynamic response, an analytical model, an MR model, and a PR model are established. With the Bernoulli–Euler beam located on an elastic support as an object of the study, the analytical model of EMRL dynamic characteristics under the influence of EM repulsive force and the contact force between the armature and the rail is established, as shown in Fig. 2—A. According to the model and boundary conditions in [3], a finite-element MR model is constructed, as shown in Fig. 2—B.

The latest experiment of our research group found that the projectile vibrates violently during the dynamic launching process and the vertical vibration overloading reaches 2000 g \((g = 9.8 \text{ m/s}^2)\). Therefore, a PR model is established to study the projectile vibration on the rail dynamic response. In this model, the two beams, respectively, represent the upper and lower rails, the elastic support involves the insulating support and containment structure, the elastic coefficient of the support refers to the interaction between the rail and containment structure, and the contact force between the rail and the armature through the contact surface in the manner of body load. Compared with the MR model and analytical model, the PR model not only takes into account the effect of the spatial distribution of EM force but also considers the effect of the projectile vibration. Theoretically, the PR model is more close to the actual EMRL.

The rail size is 2.0 m \((L) \times 0.02 \text{ m} \times 0.02 \text{ m} \times 0.05 \text{ m} \times (h1)\). The armature outer dimension is 0.021 m \((d) \times 0.02 \text{ m} \times 0.02 \text{ m} \times 0.01 \text{ m} \times (h2)\). The moving load is equal in length to the armature \((0.021 \text{ m})\). The rails are made of copper with an elastic modulus of 100 GPa and a density of 8100 kg/m³. The armature is made of aluminum with an elastic modulus of 71 GPa and a density of 3000 kg/m³.

B. Mathematical Model

For the analytical model, the governing equation of the dynamic launch process of EMRL is [3]

$$EI \frac{\partial^4 \omega(x, t)}{\partial x^4} + m \frac{\partial^2 \omega(x, t)}{\partial t^2} + k \omega(x, t) = f(x, t)$$

(1)

where \(\omega(x, t)\) is the rail deformation depending on time \(t\) and axial position \(x\), \(m\) is the mass per unit length \((m = \rho bh)\), \(\rho\) is the density of rail material, \(b\) is the rail width, \(h\) is the rail thickness, \(d\) is the armature length, \(E\) is the elastic modulus of the rail, \(I\) is the moment of inertia of the rail cross section, and \(k\) is the elastic constant of the elastic support. To solve \(k\), loads of 50, 100, and 150 Mpa are, respectively, applied to the interior rail surface by the use of the statics FEM. Thus, the amounts of deformation obtained are, respectively, 9.86 \(10^{-5}\) mm, 1.96 \(10^{-4}\) mm, and 2.98 \(10^{-4}\) mm, and the elastic rigid coefficients are, respectively, 2.028 \(10^{10}\) N/m, 2.017 \(10^{10}\) N/m, and 2.013 \(10^{10}\) N/m, from which an average value is taken, that is, \(k\) is 2.0231\(10^{10}\) N/m.

The load equation \(f(x, t)\) is the sum of the EM repelling force of the rail and the contact force between the rail and the armature, so it can be expressed as

$$f(x, t) = \begin{cases} q_1, & \text{if } x < vt \\ q_2, & \text{if } vt \leq x < vt + d \\ 0, & \text{if } x \geq vt + d. \end{cases}$$

(2)

The homogeneous vertical deformation \(\omega(x, t)\) is expressed as

$$\omega(x, t) = \phi(t)\theta(x).$$

(3)
The substitution of (3) into (1) makes it known that the influence of moving velocity of the load on the rail deformation is related to the pressure on the rail, according to Timoshenko [4]. When the compressive force on the rail surface reaches the critical value, the critical velocity of the rail will come into existence, so the theoretical expression is as follows:

$$V_{cr} = \sqrt{\frac{2\sqrt{E}k}{\rho A}}.$$  \hspace{1cm} (4)

Nechitailo and Lewis [9] used the characteristic equation based on the existing solution of a binary equation and obtained the same analytical solution of critical velocity. The further derivation leads to the following equation of the rectangular cross-sectional track:

$$V_{cr} = \sqrt{\frac{Ekh}{3\rho k^2}}.$$  \hspace{1cm} (5)

In recent years, Tzeng [3] has obtained the same expression of critical velocity according to the fourth-order partial differential control equation of EMRL dynamic launching process; using the Laplace–Carson integral transform and inverse transform, Daneshjoo et al. [12] have created an analytical model of rail deformation under the dynamic conditions in which the EM repulsive force is exerted on the rail; on the basis of this, Jin et al. [13] derived an analytical model of dynamic response of the EMRL under the influence of EM repulsive force and armature–rail contact force. Both of them have used the analytical result and the result obtained from (5) for comparison, which shows the results are in good agreement.

Based on the above-mentioned work and with the analytical model in Fig. 1—A as an object of the study, this paper has deduced the analytical model of dynamic response by the use of the Laplace–Carson integral transformation and the inverse transformation. The results are

$$\omega(x, t) = \frac{2}{L} \sum_{n=1}^{\infty} \sum_{j=1}^{3} W_j(n, t) \sin \left(\frac{n\pi x}{L}\right)$$  \hspace{1cm} (6)

where

$$W_1(n, t) = \frac{q_1 n \pi v^2}{\rho A L^2 \mu^2} + \frac{q_1 n \pi v^2}{\rho A L (\lambda_n^2 - \mu_n^2)} \times \left(\frac{\cos(\lambda_n t)}{\lambda_n^2} - \frac{\cos(\mu_n t)}{\mu_n^2}\right)$$  \hspace{1cm} (7)

$$W_2(n, t) = \frac{q_2 L}{\rho A n \pi (\lambda_n^2 - \mu_n^2)} \left[1 - \cos\left(n\pi \frac{d}{L}\right)\right] \times \left[1 + \frac{1}{\lambda_n^2 - \mu_n^2} \left(\cos \mu_n t - \cos \lambda_n t\right)\right]$$  \hspace{1cm} (8)

$$W_3(n, t) = \frac{q_3 n \pi v^2 (\mu_n t - \mu_n \sin(\mu_n t))}{\rho A (\lambda_n^2 - \mu_n^2)} \times \sin\left(\frac{n\pi d}{L}\right)$$  \hspace{1cm} (9)

$$\lambda_n = \sqrt{\frac{E n^4 \pi^4}{\rho A L^4} + \frac{k}{\rho A}}$$  \hspace{1cm} (10)

$$\mu_n = \frac{n \pi v}{L}$$  \hspace{1cm} (11)

where \(t\) is the movement time of the moving load and \(x\) is the axial distance of the rail.

III. RESULT AND ANALYSIS

A. Comparison Between the Analytical Model and MR Model

In order to analyze the dynamic response of the EMRL operating at a critical speed or noncritical speed, the MR model is calculated by the analytical method and FEM for calculation. The critical velocity of the MR model obtained from the calculation is 1703 m/s when the model parameters are substituted into (5).

First, the dynamic launching process of EMRL is simulated when that launching velocity is 1700 m/s. The calculation results of the midpoint vertical deformation due to the change of launching range are obtained with the FEM and analytical method, as shown in Fig. 3. Fig. 4 shows the results of the vertical deformation of the midline of the rail inner surface when the launch range is 1 m. The calculation results show that the midpoint vertical displacement of the rail inner surface assumes a harmonic vibration during the operation of the moving load. The vibration period calculated by the analytical method is 1.13× than that by the FEM, with amplitudes basically the same. The result of the theoretical analysis agrees with that of the FEM. When the launching range is 1 m, a periodic phase difference appears in the rail deformation in the range of 0.6 m from the breech, and both the vibration amplitudes and periods in using these two methods conform to each other.
As shown in Fig. 5, the midpoint vertical displacement of the rail inner surface changes with the launching range when the launch velocity is 1000 and 1700 m/s. Fig. 6 indicates the vertical deformation of the midline of the rail inner surface when the launch speed is 1000 and 1700 m/s and the launching range is 1 m. The results show that the vertical deformation at the acting point can be ignored before the moving load reaches the action point, that after the moving load arrives at the acting point, the deformation increases rapidly and then the point comes into vibration at a certain equilibrium position, and that when the moving load reaches the critical velocity, the vertical deformation amplitude of the rail at the same acting point is five or six times than that under noncritical conditions.

### B. Comparison Between the MR Model and the PR Model

Fig. 7 shows that the midpoint vertical deformation of the inner surface of the MR model and PR model varies with the launch time when the launch speed is 1000 m/s. Fig. 8 shows the vertical deformation of the midline of the rail inner surface when the launch range is 1 m and the launch speed is 1000 m/s. The results show that the vertical deformation of the PR model at the same position is about four or five times that of the MR model and that the vibration periods of the two models are almost the same.

In a real EMRL, the projectile mainly includes the armature and the payload, with nonmetallic insulating material between the payload and the projectile and between the projectile and the rail, and the entire payload is in a cantilevered state. The experimental results show that the projectile vibrates violently during the launching process, with an overload above 3000 g ($g = 9.86 \text{ m/s}^2$) in the horizontal and normal direction. To prevent the payload from colliding with the rails and the insulator, it is necessary to add nonmetallic protective material around the payload.

In the MR model, the contact force between the armature and rail directly acts on the rail surface in the manner of surface load; on the other hand, in the PR model, the contact force between them acts on the rail through the contact surface in the manner of body load, which is just the practical role of EM force. Therefore, according to the primary analysis, different results of dynamic response in the MR model and PR model are due to the projectile vibration.

To verify the above-mentioned conclusion, the vertical vibrations of the projectile and rail during the dynamic launching process are analyzed.

As shown in Fig. 9, the vertical acceleration of the projectile movement varies with time when the launch velocity of the PR model is 1000 m/s. Fig. 10 shows the vertical acceleration of the rail inner surface when the launch velocity is 1000 m/s and the launching range is 1 m. The calculation results show that the vertical vibration acceleration of the projectile is less than 1000 g during its initial movement (0–0.3 m), and then, the vertical acceleration rapidly increases and changes in a simple harmonic mode, with its peak value up to 6000 g. Under the same conditions of launching, the vertical acceleration of the PR model is obviously greater than that
of the MR model. Therefore, it can be determined that the projectile vibration will have a great influence on the dynamic response of EMRL.

The EMRL projectile vibration is mainly influenced by the two factors: 1) the gap between the projectile and the rail and 2) the mass of the payload. In order to further verify the above-mentioned conclusion, a finite-element simulation of dynamic launching process needs to be carried out, respectively, according to the above-mentioned two factors.

As shown in Fig. 11, the vertical deformation of the midpoint of the rail inner surface varies with the launching time when the launch velocity of the PR model is 1000 m/s and the unilateral clearance between the armature and rail is 0.05, 0.15, and 0.25 mm, respectively. The calculation results show that the larger the gap between the rail and the armature is, the more violent is the projectile vibration and the greater is the vertical deformation of the rail inner surface.

As shown in Fig. 12, the vertical deformation of the midpoint of the rail inner surface varies with the launching time when the launch velocity of the PR model is 1000 m/s and the density of the payload is 8900, 9900, and 10 900 kg/m^3, respectively. The calculation results show that the greater the density of the payload with the same volume is, the intense is the projectile vibration and the greater is the vertical deformation of the rail inner surface.

The above-mentioned two conclusions further confirm that the projectile vibration is the main cause of different amplitudes of the dynamic response of the MR model and PR model.

As shown in Fig. 13, the vertical deformation of the midpoint of the inner surface of the MR model and PR model changes with the launching range when the launch velocity is 1700 m/s (critical velocity). Fig. 14 shows the vertical deformation of the midline of the rail inner surface when the launch speed is 1000 m/s and the launching range is 1 m. It is surprising to find that when launching reaches the critical velocity, the rail deformation of the PR model is smaller than that of the MR model, and the vibration of the projectile makes the maximum amplitude of the rail deformation smaller but the vibration period is the same. This phenomenon is caused by a reduction in the rail vibration resulting from the collision between crests and troughs during the high-speed vibration of the projectile and the rail.
Fig. 14. Rail deflection when the moving load reaches \( x = 1 \) m.

IV. CONCLUSION

To deal with the dynamic response of the EMRL, this paper takes the Bernoulli–Euler beam located on an elastic support as an object of the study and uses the Laplace–Carson integral transformation and inverse transformation to derive an analytical model of EMRL dynamic characteristics under the influence of the EM repulsive force and the contact force between the armature and the rail. On this basis, the FEM is adopted to establish the MR model and PR model for the research on the influence of projectile motion on the dynamic response of EMRL. The conclusion is as follows.

1) The results of the theoretical analysis are in good agreement with those of the MR model using FEM. The vertical deformation at the acting point can be ignored before the moving load reaches the acting point. After the moving load arrives at the acting point, the deformation will increase rapidly, and then the point will harmonically vibrate at a certain equilibrium position. When the moving load operates at a critical speed and gets to the acting point, the vertical deformation of the rail increases quickly, with the amplitude being five or six times than that under the noncritical working conditions.

2) Under the same conditions, the vertical deformation of the PR model at the same position is about four or five times that of the MR model, which is mainly caused by the projectile vibration. It is found from the further study that the longer the gap between the rail and the armature, the more violent the projectile vibration and the greater the vertical deformation of the rail and that the larger the density of the payload with the same volume, the intense the projectile vibration and the greater the vertical deformation of the rail.

3) When launching reaches the critical velocity, the rail deformation of the PR model is less than that of the MR model and the projectile vibration makes the maximum amplitude of the rail deformation smaller but the vibration period is the same. This phenomenon is caused by a reduction in the rail vibration resulting from the collision between crests and troughs during the high-speed vibration of the projectile and the rail.

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