Fault Detection of Nonlinear Systems by Parity Relations

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***Abstract*—Recently a lot of works have been done to detect faults in nonlinear systems. In this paper a new method, based on parity relations for linear systems, is proposed to detect faults in nonlinear systems that can be modeled by Takagi-Sugeno (T.S) fuzzy system. This method is an intuitive generalization of parity relations, because T.S fuzzy system uses local linear models. Results of simulation and implementation on a rotary inverted pendulum show that faults can be detected very well.**

I. Introduction

OVER the last three decades, the growing demand for safety and reliability in control systems resulted in significant research in Fault Detection and Isolation (FDI); see survey papers [1]-[4]. Such efforts have led to the development of many FDI techniques. The Parity relation method is one of the most commonly used techniques. Dynamic parity relation first introduced in [5]. Reference [6] proposed a procedure to generate parity equations from the state-space representation of a dynamic system. This technique is further developed in [7]. Most of studies concentrate on linear systems and some also consider bilinear [8] and nonlinear systems [9]-[10].

Moreover, in the past two decades, many researchers have studied a class of nonlinear systems described by a Takagi-Sugeno (T.S) fuzzy model, see [11]-[13]. The model proposed by Takagi and Sugeno [14] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a T.S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models. This fuzzy model has been shown to be able to approximate a large class of nonlinear systems [15].

Motivated by the capability of the T.S fuzzy model, this paper generalizes the parity space approach for linear systems to T.S fuzzy systems with faults. The effectiveness of the proposed design procedure is demonstrated through simulation and implementation on a rotary inverted pendulum.

This paper is organized as follows. In section II, parity space approach for linear systems and generalization to nonlinear systems is represented. In section III, simulation and experimental results of applying this method on inverted pendulum are shown to validate this approach. Finally in section IV, the conclusion is drawn.

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II. Parity Based Fault Detection for Nonlinear Systems

The main objective of this section is to extend parity space method to T.S fuzzy systems. In order to make the paper self-contained, we would begin with a brief introduction to parity space for linear systems.

A. Elementary Background on Parity Space

Consider the following linear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) + Gf(t) \\ y(t) &= Cx(t) + Du(t) + Jd(t) + Hf(t) \end{aligned} \quad (1)$$

Where $x \in \Re^n$ denotes state vector, $u \in \Re^m$ is input vector, $d \in \Re^p$ is unknown disturbance/uncertainty, $y \in \Re^l$ is output (measurement), $f \in \Re^q$ is fault vector to be detected. A, B, E, G, C, D, J and H are known constant matrices of appropriate dimensions.

By getting differentials of y(t) we have [16]:

$$Y(t) = Tx(t) + H_{u,s}U(t) + H_{d,s}D(t) + H_{f,s}F(t) \quad (2)$$

$$Y(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(s)}(t) \end{bmatrix}, U(t) = \begin{bmatrix} u(t) \\ \dot{u}(t) \\ \vdots \\ u^{(s)}(t) \end{bmatrix}, D(t) = \begin{bmatrix} d(t) \\ \dot{d}(t) \\ \vdots \\ d^{(s)}(t) \end{bmatrix}$$

$$F(t) = \begin{bmatrix} f(t) \\ \dot{f}(t) \\ \vdots \\ f^{(s)}(t) \end{bmatrix}, T = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}, H_{u,s} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s-1}B & \cdots & CB & D \end{bmatrix}$$

$$H_{d,s} = \begin{bmatrix} J & 0 & \cdots & 0 \\ CE & J & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s-1}E & \cdots & CE & J \end{bmatrix}, H_{f,s} = \begin{bmatrix} H & 0 & \cdots & 0 \\ CG & H & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s-1}G & \cdots & CG & H \end{bmatrix}$$

Selecting $v_s \in \Re^{(s+1)l}$ from the parity space P_s defined by $P_s = \{v_s \mid v_s T = 0\}$ gives:

$$\begin{aligned} r_s(t) &= v_s(Y(t) - H_{u,s}U(t)) \\ &= v_s(H_{d,s}D(t) + H_{f,s}F(t)) \in \Re \end{aligned} \quad (3)$$

Where $s > 0$ is called the order of the parity relations. The residual generated by (3) must be robust against disturbances and simultaneously sensitive to the faults. This can be achieved by choosing the parity vector v_s in some optimum senses; see e.g. [17]-[19].

B. Parity Based Fault Detection for Nonlinear Systems

Consider the following nonlinear system:

$$\begin{aligned}\dot{x}(t) &= f_1(x(t)) + f_2(x(t))u(t) + f_3(x(t))d(t) + f_4(x(t))f(t)\\ y(t) &= g_1(x(t)) + g_2(x(t))u(t) + g_3(x(t))d(t) + g_4(x(t))f(t)\end{aligned} \quad (4)$$

Where x(t), u(t), d(t) and f(t) are defined as in (1). $f_i(x(t))$ and $g_i(x(t))$ are functions of $x(t)$. Suppose this nonlinear system can be modeled by a fuzzy T.S model as follows:

Plant Rule i :

IF $z_1(t)$ *is* M_{i1} *and ... and* $z_k(t)$ *is* M_{ik}

THEN (1)

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + B_i u(t) + E_i d(t) + G_i f(t)\\ y(t) &= C_i x(t) + D_i u(t) + J_i d(t) + H_i f(t)\end{aligned}$$

Where i = 1 … r, r is the number of rules, M_{ij} (j = 1 … k) are fuzzy sets and $z(t) = [z_1(t) \cdots z_k(t)]$ are the premise variables. The defuzzified output of the T.S fuzzy system (5) is represented as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t) + E_i d(t) + G_i f(t)\}\\ y(t) &= \sum_{i=1}^{r} h_i(z(t))\{C_i x(t) + D_i u(t) + J_i d(t) + H_i f(t)\}\end{aligned} \quad (2)$$

Where

$$w_i(z(t)) = \prod_{j=1}^{k} M_{ij}(z_j(t))$$

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$$

and $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}.

The idea for generalizing parity relations to nonlinear systems is to write parity relations for every linear system and then blend through fuzzy IF-THEN rules that are used for modeling the original system. In fact we can rewrite equation (2) as:

$$Y(t) = Tx(t) + H_{u,s}U(t) + H_{d,s}D(t) + H_{f,s}F(t) \quad (3)$$

$$Y(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(s)}(t) \end{bmatrix}, U(t) = \begin{bmatrix} u(t) \\ \dot{u}(t) \\ \vdots \\ u^{(s)}(t) \end{bmatrix}, D(t) = \begin{bmatrix} d(t) \\ \dot{d}(t) \\ \vdots \\ d^{(s)}(t) \end{bmatrix}$$

$$F(t) = \begin{bmatrix} f(t) \\ \dot{f}(t) \\ \vdots \\ f^{(s)}(t) \end{bmatrix}, T = \sum_{i=1}^{r} h_i(z) \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^s \end{bmatrix}$$

$$H_{u,s} = \sum_{i=1}^{r} h_i(z) \begin{bmatrix} D_i & 0 & \cdots & \\ C_i B_i & D_i & \cdots & \\ \vdots & \ddots & \ddots & \vdots \\ C_i A_i^{s-1} B_i & \cdots & & D_i \end{bmatrix}$$

$$H_{d,s} = \sum_{i=1}^{r} h_i(z) \begin{bmatrix} J_i & 0 & \cdots & \\ C_i E_i & J_i & \cdots & \\ \vdots & \ddots & \ddots & \vdots \\ C_i A_i^{s-1} E_i & \cdots & & J_i \end{bmatrix}$$

$$H_{f,s} = \sum_{i=1}^{r} h_i(z) \begin{bmatrix} H_i & 0 & \cdots & \\ C_i G_i & H_i & \cdots & \\ \vdots & \ddots & \ddots & \vdots \\ C_i A_i^{s-1} G_i & \cdots & & H_i \end{bmatrix}$$

So as linear systems, we can generate residual as follows:

$$r_s(t) = v_s(Y(t) - H_{u,s}U(t)) \quad \in \Re \quad (8)$$

It is like extracting $v_{s,i}$ and $H_{u,s,i}$ from every linear model and then blend using fuzzy rules. In the next section we show that this method works well for a nonlinear system.

III. Illustrative Example

In this section, we simulate our method with an inverted pendulum system and also implement in a set called Quanser. The system, as shown in Fig. 1, consists of a vertical pendulum, a horizontal arm, a gear chain, and a servomotor which drives the pendulum through the gear transmission system. An encoder is attached to the arm shaft to measure the rotating angle of the arm. At the end of rotating arm there is a hinge instrumented with an encoder. The pendulum is attached to the hinge.

Fig. 1. Rotary inverted pendulum module of Quanser.

A. System Description

Consider a rotary inverted pendulum as shown in Fig. 2. The simplified dynamic equations of a rotary inverted pendulum are given as follows:

$$a \ddot{} \quad \cos(\alpha) \ddot{} \quad {}_m$$

$$-b\cos(\alpha) \ddot{} \quad (\alpha) = 0$$

Where α is pendulum arm deflection (radians), θ is load gear angle (radians) and V_m is input voltage. For complete description of variables, refer to [20].

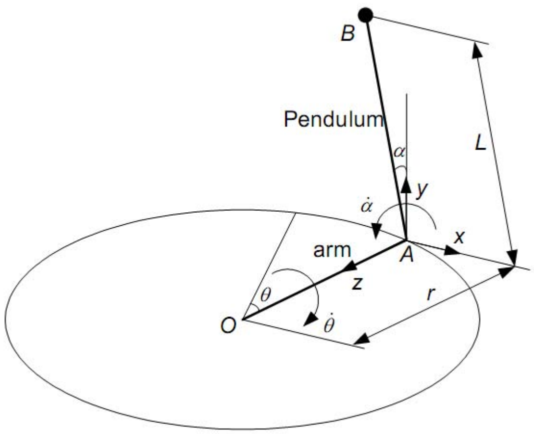

Fig. 2. Schematic diagram of a rotary inverted pendulum.

Define state variables as:

$$x_1 = \theta, \; x_2 = \alpha$$

$$x_3 = \dot{\theta}, \; x_4 \quad \dot{}$$

We have following state equations:

$$\begin{aligned} &\dot{} \\ &\dot{} \\ &\dot{} \quad \frac{1}{ac - b^2\cos^2(x_2)} \\ &\qquad\qquad bd\sin(x_2)\cos(x_2) - cex_3 + cfV_m] \\ &\dot{} \quad \frac{1}{ac - b^2\cos^2(x_2)} \\ &\qquad\qquad -be\cos(x_2)x_3 + bf\cos(x_2)V_m] \end{aligned} \tag{10}$$

The T.S fuzzy model for the system (10) is given by the following six-rule fuzzy model:

$Rule 1: IF\ x_2\ is\ about\ 0\ and\ x_4\ is\ about\ 0$

$THEN$

$$\begin{cases} \dot{} \quad B_1 u \\ y = C_1 x \end{cases}$$

$Rule 2: IF\ x_2\ is\ about \pm\frac{\pi}{2}\ and\ x_4\ is\ about\ 0$

$THEN$

$$\begin{cases} \dot{} \quad B_2 u \\ y = C_2 x \end{cases}$$

$Rule 3: IF\ x_2\ is\ about \pm\frac{\pi}{4}\ and\ x_4\ is\ about\ 0$

$THEN$

$$\begin{cases} \dot{} \quad B_3 u \\ y = C_3 x \end{cases}$$

$Rule 4: IF\ x_2\ is\ about\ 0\ and\ x_4\ is\ about\ 1$

$THEN$

$$\begin{cases} \dot{} \quad B_4 u \\ y = C_4 x \end{cases}$$

$Rule 5: IF\ x_2\ is\ about \pm\frac{\pi}{2}\ and\ x_4\ is\ about\ 1$

$THEN$

$$\begin{cases} \dot{} \quad B_5 u \\ y = C_5 x \end{cases}$$

$Rule 6: IF\ x_2\ is\ about \pm\frac{\pi}{4}\ and\ x_4\ is\ about\ 1$

$THEN$

$$\begin{cases} \dot{} \quad B_6 u \\ y = C_6 x \end{cases}$$

Where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 28.9568 & -10.7340 & 0 \\ 0 & 71.4267 & -10.2773 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 18.8766 \\ 18.0733 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -17.7154 & -6.5679 & 0 \\ 0 & 0.1056 & -0.0679 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 11.5501 \\ 0.1194 \end{bmatrix} \text{III}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -5.0830 & -8.1704 & 0 \\ 0 & 20.1884 & -5.5613 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \\ 14.3682 \\ 9.7799 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 28.2942 & -10.7340 & 0 \\ 0 & 70.7923 & -10.2773 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ 0 \\ 18.8766 \\ 18.0733 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -17.7163 & -6.5679 & -0.8108 \\ 0 & 0.4937 & -0.0679 & -0.0084 \end{bmatrix}, B_5 = \begin{bmatrix} 0 \\ 0 \\ 11.5501 \\ 0.1194 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -5.2703 & -8.1704 & -0.7094 \\ 0 & 20.2998 & -5.5613 & -0.4829 \end{bmatrix}, B_6 = \begin{bmatrix} 0 \\ 0 \\ 14.3682 \\ 9.7799 \end{bmatrix}$$

$$C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad i = 1, \cdots$$

A_i and B_i are extracted by linearization of nonlinear system about given state points. States that are not mentioned in the antecedent part are considered as zero. By choosing parity order s=2, parity vectors are chosen as follows:

$$v_{s,1} = [0 \quad 0.4664 \quad -0.8811 \quad 0 \quad -0.0734 \quad 0.0293]$$

$$v_{s,2} = [0 \quad 0.7165 \quad 0.4436 \quad 0 \quad 0.0366 \quad 0.5371]$$

$$v_{s,3} = [0 \quad -0.2227 \quad -0.9723 \quad 0 \quad -0.0701 \quad -0.0067]$$

$$v_{s,4} = [0 \quad 0.461 \quad -0.8839 \quad 0 \quad -0.0731 \quad 0.0286]$$

$$v_{s,5} = [0 \quad -0.9001 \quad -0.4231 \quad -0.0383 \quad -0.0395 \quad 0.0887]$$

$$v_{s,6} = [0 \quad -0.234 \quad 0.9681 \quad 0.0617 \quad 0.0603 \quad 0.0257]$$

We stabilize system by a state feedback controller $u(t) = -kx(t)$ with:

$$k = [-2.2361 \quad 21.0858 \quad -2.0052 \quad 2.8616]$$

B. *Simulation Results*

In our simulation, there are three kinds of sensor faults:

(1) Abrupt fault which represents bias in the monitored signal (α).

(2) Intermittent fault which is a combination of impulses with different amplitudes.

(3) Incipient fault which represent drift of the monitored signal (α).

We use a filter in applying faults to smooth changes so the results would be clearer. This is more important when we want to apply fault in practice. The results, as shown in Fig. 3, offer that generated residual is zero when fault is zero and it will be nonzero, when fault is nonzero.

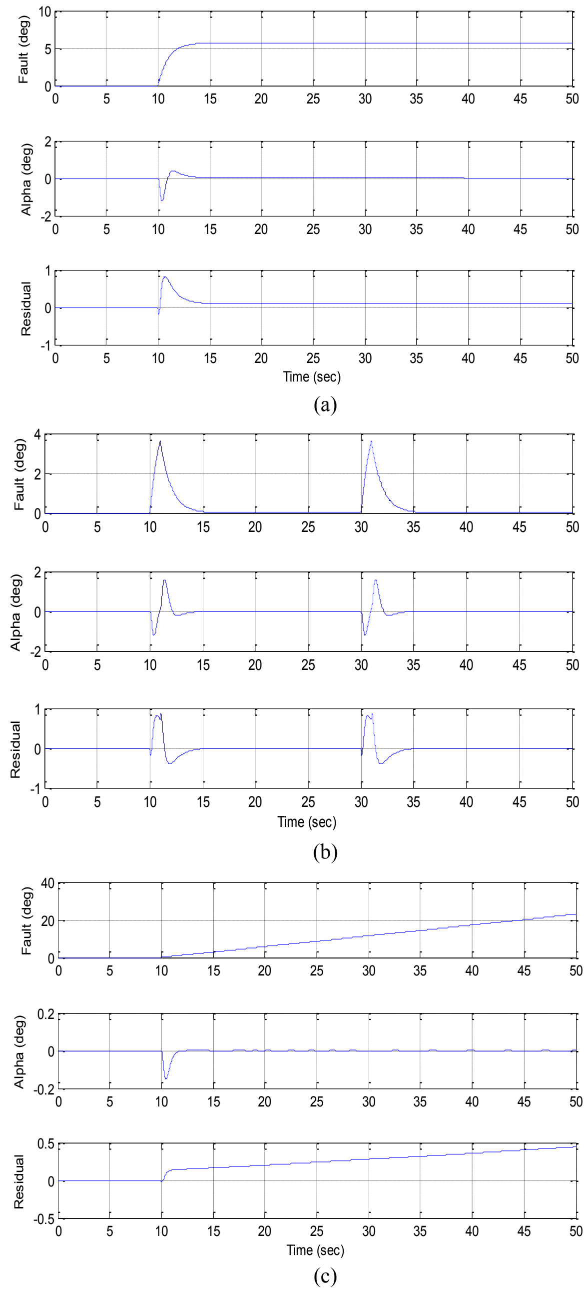

Fig. 3. Results of simulation in three kinds of faults. Each result shown in three diagrams; first diagram shows fault in degrees, second is pendulum arm deflection in degrees, and third is generated residual with the proposed method. (a) An abrupt fault occurs in 10th second. (b) Intermittent fault occur in 10th and 30th seconds and last 5 seconds. (c) An incipient fault starts in 10th second.

C. *Implementation Results*

We apply the proposed method in inverted pendulum module of Quanser. Because of some limitations in load gear angle (θ) and control effort, we only use intermittent fault. As shown in Fig. 4, generated residual is nonzero when fault occurs. This proves that the method works well in practice.

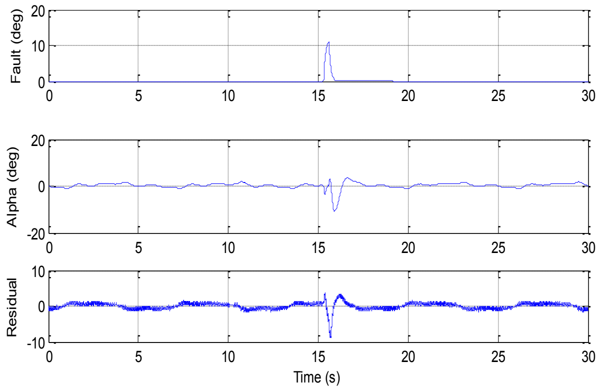

Fig. 4. Result of implementing a fault in 15th second. First diagram shows fault in degrees, second is pendulum arm deflection in degrees, and third is generated residual with the proposed method

IV. CONCLUSION

In this paper, based on T.S fuzzy model, a method is proposed to generalize parity space approach of linear systems to nonlinear systems. To show effectiveness, this method simulated and implemented on a rotary inverted pendulum. Early detection of faults in this system is very important because a minor fault can lead the system to instability. This method proposed for continuous-time systems, however, it can be used in discrete-time systems.

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